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The phase diagram of strong interactions from lattice QCD simulations

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Summary. — We summarize the present status of our knowledge of the phase diagram of strong interactions, as it emerges from lattice QCD simulations.

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1. – Introduction

Many of the open questions regarding the Standard Model of particle physics concern strong interactions, which are described by Quantum Chromodynamics (QCD). While we have a clear and consistent view of the theory in the high energy limit, where it is perturbative, thanks to property of asymptotic freedom, we still cannot solve the opposite, low energy limit, where the coupling grows and the theory is non-perturbative. A result of this limitation is that we still do not understand why colored degrees of freedom, quarks and gluons, are confined into hadrons. One would also like to known if color confinement is a permanent state of matter, or if particular, extreme conditions may exist, characterized by high temperature, baryon density or background fields, where strongly interacting matter is found in different phases, *e.g.*, a deconfined Quark-Gluon plasma phase.

Such possibility, which was first proposed by Cabibbo and Parisi in 1975 [1], is of particular interest in order to describe the early stages of evolution of the Universe, the inner core of some compact astrophysical objects, and is experimentally probed by heavy ion collision experiments.

Presently, the only known first principle approach to QCD in the non-perturbative regime, with improvable control over systematic errors, is to discretize the theory on a Euclidean space-time lattice, and compute it by numerical Monte Carlo simulations. In

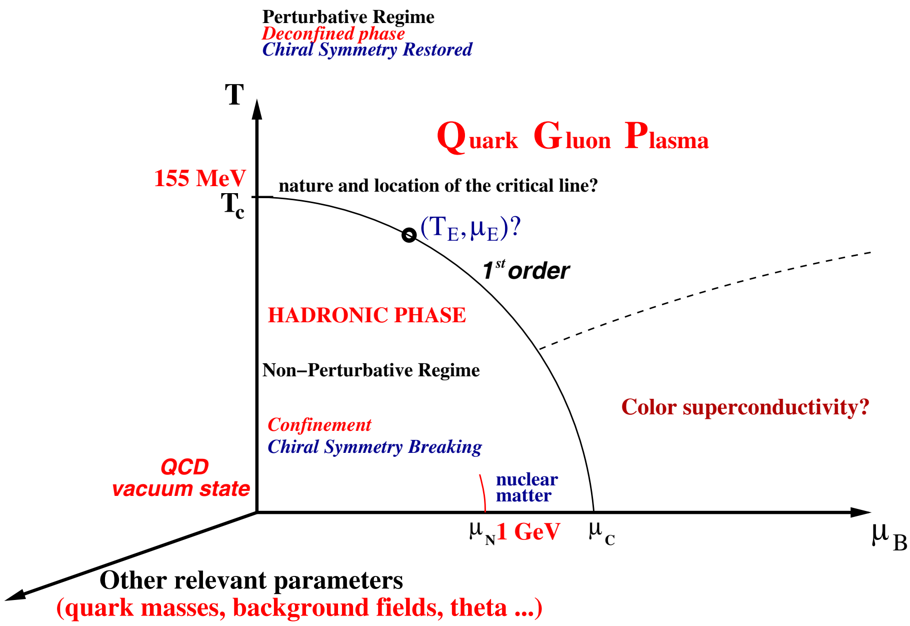

Fig. 1. – Schematic view of the QCD phase diagram. The T-μ_B plane is shown in more detail, together with some questions that lattice simulations still leave open.

practice, one rewrites the QCD thermal partition function, formulated in the Feynman path integral formalism

$$Z(V,T) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-(S_G[U]+\bar{\psi}M[U]\psi)} = \int \mathcal{D}Ue^{-S_G[U]} \det M[U],$$

where U are the gauge link variables (elementary parallel transports) and $\bar{\psi}M\psi$ is a discretization of the quark action. The temperature is related to the extension of the compactified time dimension τ, $T = 1/\tau = 1/(N_t a)$ where a is the lattice spacing and N_t is the number of lattice sites in the time direction. $S_G[U]$ is the pure gauge action while $\det M$ encodes the contribution of dynamical fermions and its correct inclusion into the probability distribution of gauge configurations is the most demanding task from a computational point of view. Lattice simulations are ideally suited to compute equilibrium quantities, like

$$\langle O\rangle_T = \frac{\int \mathcal{D}Ue^{-S_G[U]} \det M[U]O[U]}{\int \mathcal{D}Ue^{-S_G[U]} \det M[U]} = \int \mathcal{D}U\mathcal{P}[U]O[U],$$

where O is a generic physical observable, via Monte Carlo sampling.

In this way we can obtain information, with a systematically increasable precision, about basic equilibrium thermodynamics (*e.g.*, pressure and energy density), equilibrium particle and quantum number distributions (*e.g.*, quadratic and higher order susceptibilities of baryon number and electric charge) and the location and order of the transitions to the different phases of strongly interacting matter, *i.e.* about the QCD phase diagram, a schematic view of which is reported in fig. 1. Unfortunately, the important case regarding the inclusion of a baryonic chemical potential μ_B, which is necessary to consider QCD at finite density, is plagued by the so-called sign problem: $\det M[\mu_B \neq 0]$ is complex, so that the path integral measure is not positive and direct Monte Carlo methods are not usable. Approximate methods works well for $\mu_B/T \ll 1$, which is the case for the strongly interacting medium produced in high energy heavy ion collisions ($\mu_B/T \sim 10^{-2}$ at LHC).

2. – Present knowledge about the QCD phase diagram

The liberation of color degrees of freedom, at a transition temperature T_c, is clearly visible from the sudden increase of various thermodynamical quantities, like the energy density, the pressure or the quark number susceptibilities, and roughly coincides with the restoration of chiral symmetry, which is instead spontaneously broken at low T. There is now fair agreement between different collaborations, adopting different discretizations of QCD, regarding the location of the transition: the value of T_c, extracted by looking at chiral symmetry restoration, is $T_c \sim 155\,\mathrm{MeV}$ [2, 3].

The behaviour of thermodynamical quantities is consistent with the absence of a true transition, *i.e.* no discontinuity seems to develop as the thermodynamical limit is approached [4], meaning that either the transition is extremely weak (hence not phenomenologically relevant), or just a rapid change of physical properties takes place, instead of a real transition. However, while this is true for physical quark masses, in numerical simulations the quark mass spectrum can be changed at will, so it makes sense to study the nature of the transition as a function of u/d and s quark masses. A true transition is present in the limit of very light or very heavy quark masses, where exact order parameters can be defined, and unsettled issues still exist regarding the chiral limit of the two flavor theory [5], where the transition could first order or second order in the $O(4)$ universality class.

As one introduces more external parameters into the game, like a baryon chemical potential μ_B, one would like to know how T_c changes and if a true transition appears at some stage, *e.g.*, at a critical endpoint in the T-μ_B plane. Direct numerical simulations at $\mu_B \neq 0$ are not feasible, because of the sign problem, however reliable numerical results can be obtained in a restricted region of high temperatures and small chemical potentials, where approximate solutions to the problem can be found, among which reweighting techniques [6, 7], analytic continuation from imaginary chemical potentials [8-10] and Taylor expansion techniques [11, 12]. In fig. 2 we show a comparison (see ref. [13]) of the critical line $T_c(\mu_B)$ determined in the case of four degenerate flavors by different techniques (the pseudocritical coupling β_c is reported in place of T_c, which is a monotonically increasing function of T_c): consistency among different determinations is obtained only as long $\mu/T \leq 1$ ($\mu \equiv \mu_B/3$), meaning that at least the curvature of the pseudocritical line can be determined with good control over systematic uncertainties.

In the more physical case of 2 or 2+1 flavors, one typically obtains values for the curvature of the critical line, $T_c(\mu)/T_c(0) = 1 - A(\mu/T)^2$, in the range $A \sim 0.05$–0.07 [9, 14-16]. Such values are substantially smaller than those obtained for chemical freeze-out curves in heavy ion collisions, which are a factor ~ 3 larger. There is no apriori reason for deconfinement and freeze-out to coincide, since hadrons produced from the thermal medium may undergo inelastic interactions even after the transition, however it is true that the gap between them leaves space for speculations about new possible phases, like the so-called quarkyonic phase [17]. It is therefore interesting to notice

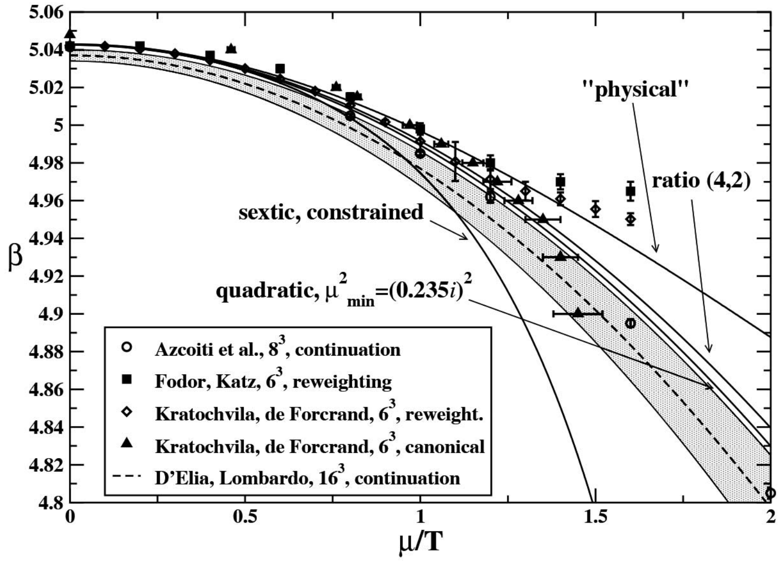

Fig. 2. – Determination of the line of pseudo-critical couplings for deconfinement as a function of the quark chemical potential μ of the theory with four degenerate flavors, as obtained by various methods, including different extrapolations from simulations at imaginary chemical potentials.

that a recent re-analysis of heavy-ion data, which takes better into account baryon-antibaryon annihilations, seems to bring the freeze-out curves in fair agreement with lattice predictions for the pseudocritical line [18, 19].

Unfortunately the same techniques, working well for small baryon chemical potentials, have failed, up to now, to provide clear and consistent evidence for the presence and location of a critical endpoint, at which the pseudo-transition present at $\mu_B = 0$ would turn into a first order transition.

Contrary to the case of a finite μ_B, the introduction of magnetic background fields does not encounter any technical problem, so that interesting questions, regarding the fate of the transition and of its order in the presence of magnetic [20-22] or chromo-magnetic [23, 24] background fields can be approached systematically. Such issues are of great interest for the physics of the early Universe and of non-central heavy ion collisions; for example, magnetic fields as a large as 10^{15} Tesla may be produced at LHC. Lattice results show that the pseudocritical temperature decreases as a function of the external field, while its strength slightly increases, even if no evidence has been found till now for a critical endpoint in the T-B plane; deconfinement and chiral symmetry restoration continue to stay entangled also in the presence of the external field.

Finally, let us mention that different extensions of the phase diagram, even if not relevant from a phenomenological point of view, may unveil interesting aspects of strong interactions in the non-perturbative regime. An example is given by the recent studies regarding the relation between deconfinement and the dependence on the topological, CP-breaking parameter θ [25, 26].

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