IL NUOVO CIMENTO Vol. 37 C, N. 1 Gennaio-Febbraio 2014
DOI 10.1393/ncc/i2014-11670-1

Colloquia: IFAE 2013

Perturbative QCD at the LHC

G. Ferrera([1]) and L. Trentadue([2])(*)

([1]) *Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano - Milan, Italy*

([2]) *Dipartimento di Fisica e Scienze della Terra, Università di Parma and INFN, Sezione di Milano Bicocca - Milan, Italy*

ricevuto l'1 Ottobre 2013

Summary. — In this brief overview, we discuss selected topics of perturbative QCD at the Large Hadron Collider (LHC). In particular we presents some recent results on parton densities, higher-order calculations, Sudakov resummation and parton showers.

PACS `12.38.-t` – Quantum chromodynamics.
PACS `12.38.Bx` – Perturbative calculations.
PACS `12.38.Cy` – Summation of perturbation theory.
PACS `13.85.-t` – Hadron-induced high- and super-high-energy interactions (energy > 10 GeV).

1. – Introduction

The CERN Large Hadron Collider (LHC) is a hadronic machine: all the interesting high-p_T reactions initiate by a hard scattering of partons governed by QCD. A good control of the QCD cross sections and the related kinematical distributions is thus necessary in order to perform detailed studies on the Standard Model (SM) processes and, in case, to claim for new-physics signals. This requires, in particular, the computation of perturbative QCD radiative corrections.

The standard framework to perform perturbative QCD calculations at colliders is provided by the factorization theorem of mass singularities (for a review see ref. [1]).

(*) Speaker.

According to it, a generic hard cross section $\sigma(Q^2)$ can be written as([1])

$$(1)\quad \sigma(Q^2) = \sum_{a,b}\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 f_{a/h_1}(x_1,\mu_F^2) f_{b/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ab}(x_1x_2s,Q^2,\mu_F^2) + \mathcal{O}((\Lambda/Q)^p),$$

where $f_{a/h}(x,\mu_F)$ are the non-perturbative (universal) parton densities giving the distribution of the parton a in the hadron h at the factorization scale μ_F as a function of the longitudinal momentum fraction x carried by the parton, $\hat{\sigma}_{ab}$ are the (process-dependent) partonic cross sections and s ($x_1x_2s = \hat{s}$) is the hadronic (partonic) centre-of-mass energy squared. The hard scale of the process, Q, is typically set by the invariant mass or the transverse momentum of the triggered hard probe, and have to be much larger of the QCD scale, $Q \gg \Lambda$, to be in a perturbative domain. The term $\mathcal{O}((\Lambda/Q)^p)$ on the right-hand side of eq. (1) represents power suppressed ($p \geq 1$) non-perturbative contributions to the cross section (the so-called higher-twist component). The first term on the right-hand side of eq. (1) (the leading-twist component of the cross section), can be calculated in perturbation theory. A generic([2]) partonic cross section $\hat{\sigma}_{ab}$ can be calculated through a power expansion in the strong coupling α_S

$$(2)\qquad \hat{\sigma}_{ab}(\hat{s},Q^2,\mu_F^2) = \sum_{n=0}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{\pi}\right)^n \hat{\sigma}_{ab}^{(n)}(\hat{s},Q^2,\mu_F^2,\mu_R^2),$$

where μ_R is the renormalization scale and the QCD coupling at lowest order is

$$(3)\qquad \frac{\alpha_S(\mu_R^2)}{\pi} = \frac{1}{\beta_0 \log(\mu_R^2/\Lambda^2)} \ll 1\,,$$

for $\mu_R \sim Q \gg \Lambda$([3]).

2. – Parton densities

Perturbation theory allows us to evaluate the Q-evolution of the parton densities or parton distribution functions (PDFs) $f_{a/h}(x,Q^2)$ in eq. (1) via the DGLAP equation [2]

$$(4)\qquad \frac{d\, f_{a/h}(x,Q^2)}{d\ln Q^2} = \sum_b \int_x^1 \frac{\mathrm{d}z}{z} P_{ab}(\alpha_\mathrm{S}(Q^2),z)\, f_{a/h}(x/z,Q^2),$$

where the kernels $P_{ab}(\alpha_\mathrm{S},z)$ are the Altarelli–Parisi (AP) splitting functions which can be computed as a power series expansion in α_S

$$(5)\qquad P_{ab}(\alpha_\mathrm{S},z) = \alpha_\mathrm{S}P_{ab}^{(0)}(z) + \alpha_\mathrm{S}^2P_{ab}^{(1)}(z) + \alpha_\mathrm{S}^3P_{ab}^{(2)}(z) + \mathcal{O}(\alpha_\mathrm{S}^4).$$

([1]) When the process involves observed final-state hadrons or photons, the factorization formula contains additional convolutions with the corresponding fragmentation functions.
([2]) However the corresponding hadronic observable has to be infrared and collinear safe: it has to be independent of the number of soft and collinear particles in the final state.
([3]) β_0 is the (positive) first order coefficient of the QCD beta function defined by the equation $\mathrm{d}\ln\alpha_S(\mu^2)/\mathrm{d}\ln\mu^2 = \beta(\alpha_S) = -\sum_{n=0}^{\infty}\beta_n(\alpha_S/\pi)^{n+1}$.

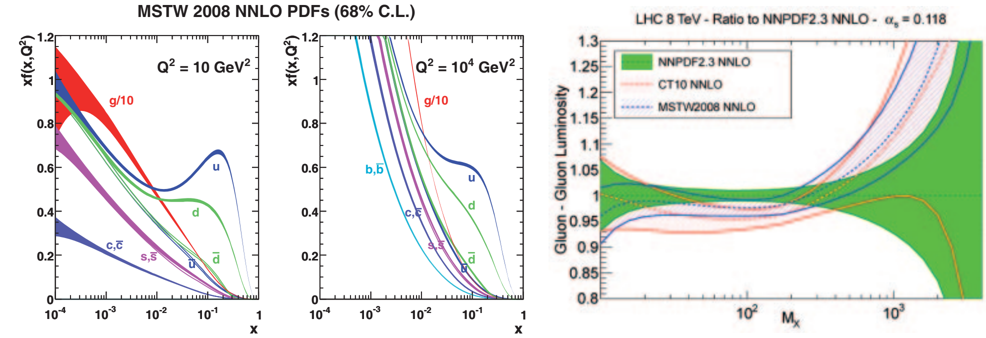

Fig. 1. – MSTW 2008 NNLO parton densities at $Q^2 = 10\,\mathrm{GeV}^2$ and $Q^2 = 10^4\,\mathrm{GeV}^2$ [7] (left panel). Uncertainty on gg luminosity function ([4]) (which is relevant for the Higgs boson production) at the LHC $\sqrt{s} = 8\,\mathrm{TeV}$ from MSTW 2008, CT10 and NNPDF2.3 NNLO PDFs sets taken as the ratio to NNPDF2.3 [10] (right panel).

The absolute normalization of the parton densities at an arbitrary scale is not computable in perturbation theory and has to be determined otherwise (basically by a comparison with experimental data). Having determined $f_{a/h}(x, Q_0^2)$ at a given input scale Q_0, the evolution equation (4) can be used to compute the parton densities at different perturbative scales and larger values of the momentum fraction x. Moreover parton densities are universal (process-independent) quantities: once determined from cross section measurements, they can be used to predict cross sections for other hard-scattering processes.

The parton densities are an essential ingredient to study hard-scattering collisions since their knowledge enter in the computation of any cross section. At present parton densities are determined by fitting data from deep-inelastic scattering (DIS), Drell–Yan (DY) and jet production from a wide variety of experiments. Various groups have produced different sets of PDFs which are publicly available: ABKM/ABM [3], CTEQ/CT [4], GJR/JR [6], HERAPDF [5], MRST/MSTW [7] and NNPDF [8].

The method used to determine PDFs sets consists in parametrizing the parton densities at some input scale Q_0 with a given functional form and then adjusting the parameters to fit the data. However existing PDFs sets differ in many aspects: input data, perturbative order in the evolution and in the partonic cross sections, parton densities parametrization, treatment of α_S, treatments of heavy quarks and criteria for determining uncertainties.

The state of the art in the determination of the PDFs is reviewed in ref. [9]. In fig. 1 (left side) the typical x-shape of the parton densities of the proton is shown.

An important aspect of the PDFs determination regards the quantification of the corresponding uncertainty. The uncertainty is difficult to estimate since it depends on the correlations between experimental errors and on the assumptions and the systematics which affect the fit procedures. A recent comparison between various PDFs sets and the implication of their uncertainties on the LHC physics is performed in ref. [10] (see right side of fig. 1).

([4]) Parton luminosities are defined as $\mathcal{L}_{ab}(x, Q^2) \equiv \int_x^1 \frac{\mathrm{d}z}{z}\, f_{a/h_1}(z, Q^2)\, f_{b/h_2}(\frac{x}{z}, Q^2)$.

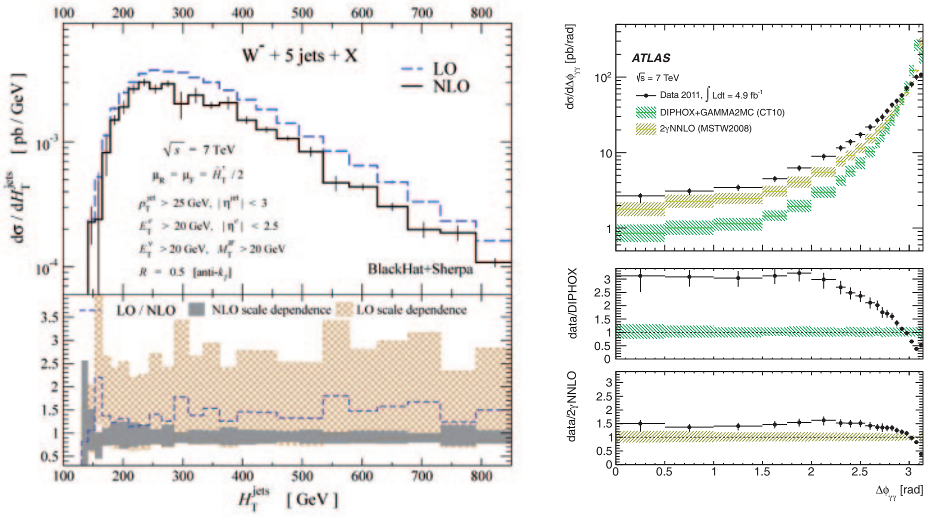

Fig. 2. – $W^- + 5$jets production at the LHC: total hadronic energy $H_T = \sum_i p_J^{(i)}$ spectrum in LO and NLO QCD [15] (left panel). Diphoton production at the LHC: azimuthal angle $\Delta\phi_{\gamma\gamma}$ spectrum measured by ATLAS compared with NLO and NNLO QCD corrections [12] (right panel).

3. – Higher-order calculations

Perturbative QCD calculations at leading order (LO) give only the order of magnitude of the corresponding cross sections and distributions. A reliable theoretical estimate can be obtained by the knowledge of next-to-leading order (NLO) and by the higher-order QCD corrections. The large amount of precise experimental predictions accumulated by the LHC requires NLO calculations for a wide variety of process.

The evaluation of perturbative QCD corrections to hard-scattering processes requires the computation of higher-order real and virtual contributions which is not an easy task. While the standard renormalization procedure remove the ultraviolet singularities affecting the virtual contributions, difficulties arise from the presence of infrared singularities at intermediate stages of the calculation which prevents a straightforward implementation of numerical techniques. The amplitudes which contribute at NLO are the *real* corrections and the *one-loop virtual* corrections to the LO subprocess. Both contributions are separately infrared divergent while their sum is finite. However exploiting the universality of the soft and collinear radiation, general algorithms [11] based on the subtraction method were developed. They allow relatively straightforward calculations for *any* infrared-safe observable once the corresponding QCD amplitudes are available.

Nowadays efficient automated tools for the computation of multi-legs tree level matrix elements which contribute to the Born and to the real correction process exist [13]. On the contrary the computation of the virtual corrections with more than two particle in the final states is still complicated. Nonetheless recently important advances in this respect were possible thanks both to new semi-numerical methods based on on-shell recursion relations and unitarity and to improved reduction technique for loop tensor integrals (for a review see ref. [14]). Following these lines, NLO results for process of increasing complexity (see left side of fig. 2) and with a high degree of automation [16] were recently published by different groups.

Next-to-next-to leading (NNLO) corrections are important to have a good control of theoretical uncertainties and are especially useful in two cases: i) when NLO corrections are particularly large (*e.g.* for Higgs production in gluon fusion), ii) for benchmark processes measured with high precision whose knowledge have a wide impact for the LHC physics program (*e.g.* for the Drell–Yan process). At this order three kind of corrections contribute: *double real*, *real-virtual* (*i.e.* real at one-loop order) and *two-loop virtual* corrections to the LO subprocess. In this case the complicated structure of the infrared divergence has still prevented an automated and fully general extension of the subtraction schemes. For this reason, fully exclusive NNLO calculations have been computed for the following few processes: Higgs boson production in gluon fusion [17, 18] and in association with a W boson [19], Drell–Yan process [20, 21] and diphoton production [22] (see right side of fig. 2).

4. – Sudakov resummation

In particular kinematical region where the real emissions are strongly inhibited, the cancellation of infrared singularities which are present in the real and virtual QCD matrix elements is highly unbalanced. As a result, the convergence of the fixed-order perturbative expansion is spoiled by the presence of large double-logarithmic terms due to soft and/or collinear gluon emissions. To obtain reliable predictions, these terms have to be resummed to all orders in perturbation theory.

The Sudakov resummation, which is the all-order summation of soft-gluon contributions, can be performed when the dynamics and the kinematics of the process factorize. While dynamical factorization is a general property of the QCD matrix elements in the soft limit, the factorization of kinematics strongly depends on the observable under consideration. If the phase-space in the soft limit can be factorized, resummation allows to write multi-gluon emissions in form of generalized exponentiation of the single-gluon emission probability and to perform an improved perturbative expansion that systematically resums to all orders the exponentiated leading (LL), next-to-leading (NLL), next-to-next-to-leading (NNLL) (and so on) logarithmic contributions. Moreover the logarithmic resummed calculation can be consistently matched (*i.e.* avoiding double counting) with the fixed order ones in order to obtain QCD prediction with uniform accuracy for the entire kinematical region.

Well-known example of Sudakov resummation which are particularly relevant for the LHC are the transverse-momentum (q_T) resummation at low q_T [23] and threshold resummation [24].

Transverse-momentum logarithms, occur in the distribution of transverse momentum of systems with high mass M ($M \gg q_T$) that are produced with a vanishing q_T in the LO subprocess. Some examples of such systems are DY lepton pairs [25] (see left side of fig. 3), Higgs bosons [26] and diphoton production [27].

In such cases when $q_T \ll M$ the emission of real radiation at higher orders is strongly suppressed. As a result large double-logarithms of the type $L = \ln M^2/q_T^2$ (which diverge in the $q_T \to 0$ limit) appear order by order in the perturbative expansion.

Threshold logarithms occur when the observed high mass system is forced to carry a very large fraction x ($x \to 1$) of the available centre-of-mass energy $\sqrt{s}$. Also in this case, the suppression of real emissions at higher orders give rise to large double-logarithms of the type $L = \log(1-x)/(1-x)$. Some examples of such systems are DIS at large value of the Bjorken variable x, production of DY lepton pairs [24] or Higgs bosons with large invariant mass Q ($x = Q/\sqrt{s}$) (see right panel of fig. 3), production of heavy quark–

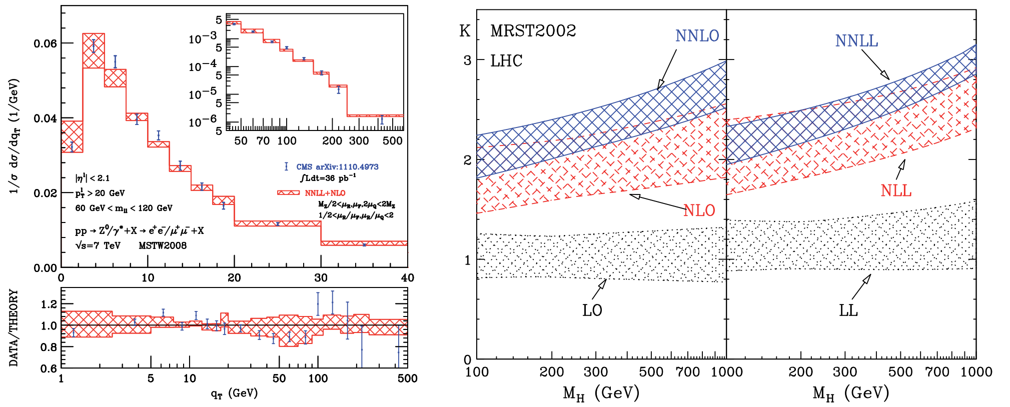

Fig. 3. – Transverse-momentum resummed prediction for Drell-Yan lepton pair ($e^+e^-/\mu^+\mu^-$) production at the LHC [25] compared to CMS data (left panel). Fixed-order and resummed K-factors for Higgs production at the LHC from ref. [28] (right panel).

antiquark pairs ($x = 2m_Q/\sqrt{s}$) [29], production of single particle at large transverse momentum p_T ($x = 2p_T/\sqrt{s}$) [30].

5. – Monte Carlo parton showers

As we have seen in the previous sections, in the soft and collinear regions the convergence of the standard perturbative expansion is spoiled by the presence of Sudakov logarithms. Reliable perturbative results can be obtained resumming these logarithmic corrections to all order with the *analytic* soft-gluon resummation techniques described in the previous section.

A different approach to perform the resummation of the Sudakov logarithms is based on Monte Carlo parton showers [31] which give an all-order approximation of the partonic cross section in the soft and collinear regions.

The parton shower algorithms start from a LO cross section and generates multiple parton emissions as a probabilistic Markov process. The probability distribution approximates the squared QCD matrix elements. This approximation is based on the universal (process-independent) factorization properties of the multi-parton QCD matrix elements in the soft and collinear limits. However in the soft limit, factorization applies at the amplitude level and its square contains quantum interferences which prevent a probabilistic formulation. This problem can be overcomed to leading infrared accuracy by exploiting QCD coherence [32]: the interference is entirely destructive for soft gluons radiated outside angular-ordered regions of phase space. Therefore it is possible to correctly take into account soft-gluon interference by implementing in the parton shower algorithm an angular-ordering constraint on the phase space available for parton emissions.

However while resummed analytic calculations can in principle be performed to any logarithmic accuracy, the logarithmic accuracy achievable by parton showers is instead limited by quantum interference effects which cannot be systematically (at higher orders) taken into account with a probabilistic approach. Moreover the QCD running coupling α_S which is used in the parton showers corresponds only to the lowest order coupling, while in resummed analytical calculation the QCD coupling can be consistently defined at higher orders.

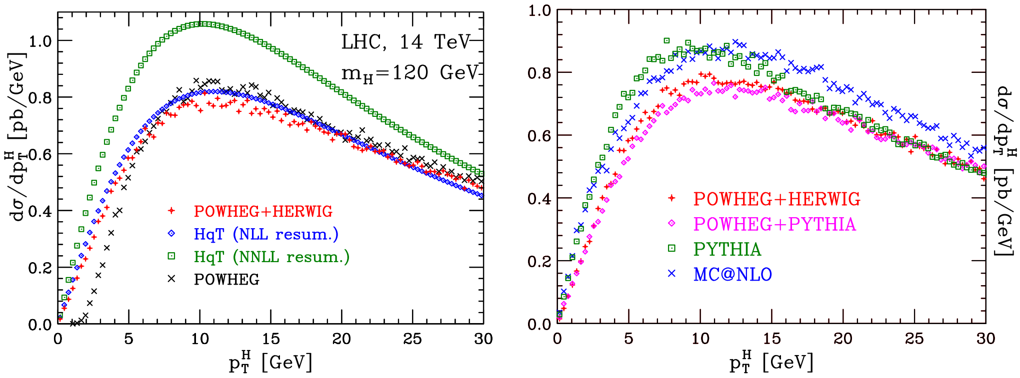

Fig. 4. – Higgs q_T spectrum at the LHC: comparison of various NLO+PS, Monte Carlo and analytic resummation predictions (from ref. [36]).

Nonetheless, apart from these limitations regarding the higher-order accuracy, parton-shower calculations have some advantages with respect to analytic resummation. Most of all, parton showers can treat multi-parton kinematics exactly while analytic resummation is always inclusive over the soft/collinear parton emissions. This aspect makes parton showers suitable to be supplemented with a non-perturbative model of hadronization effects. In this way, it is possible to obtain a *QCD Monte Carlo event generator* which provides a complete description (at hadron level) of the hard-scattering process [33].

Although parton showers give a good approximation of the QCD matrix elements in the dominant soft and collinear regions they cannot provide a good description of the hard non-collinear emissions.

There exist two main alternative methods for combining fixed-order and parton-shower QCD calculation avoiding double counting. The approach for *merging* tree-level high-multiplicity QCD matrix elements and parton showers (ME+PS) [34] considers multi-parton configurations generated by the parton shower and corrects them in the hard non-collinear region by using the exact expressions of the matrix elements. This can be obtained by implementing a truncated vetoed parton shower evolution. The overall accuracy obtained for the inclusive cross section is at LO but hard multi-jet configurations are correctly described and the logarithmic accuracy of the parton shower is preserved. Another approach consists in exactly *matching* NLO calculations and parton showers (NLO+PS). In this case total rates and hard emissions have a NLO accuracy while soft/collinear emissions are handled by the parton shower [35] (see fig. 4).

6. – Conclusions

In this contribution we reviewed some selected topics on perturbative QCD and their phenomenological implication to the physics program of the LHC. We presented recent results on parton densities, higher-order calculations, Sudakov resummation and parton showers.

There are however many other important aspects we could not cover in this brief review. Some of them are: jets algorithms, flavour physics, top and Higgs physics, effective theories and the determination of α_S.

Nowadays perturbative QCD techniques allow us to perform accurate prediction which are essential to fully exploit the physics potential of the LHC.

REFERENCES

[1] Collins J. C., Soper D. E. and Sterman G., in *Perturbative Quantum Chromodynamics*, edited by Mueller A. H. (World Scientific, Singapore) 1989, p. 1.
[2] Gribov V. N. and Lipatov L. N., *Sov. J. Nucl. Phys.*, **15** (1972) 438 (*Yad. Fiz.*, **15** (1972) 781); Altarelli G. and Parisi G., *Nucl. Phys. B*, **126** (1977) 298; Dokshitzer Y. L., *Sov. Phys. JETP*, **46** (1977) 641 (*Zh. Eksp. Teor. Fiz.*, **73** (1977) 1216).
[3] Alekhin S., Blumlein J. and Moch S., *Phys. Rev. D*, **86** (2012) 054009; Alekhin S., Blumlein J., Klein S. and Moch S., *Phys. Rev. D*, **81** (2010) 014032.
[4] Gao J. *et al.*, arXiv:1302.6246 [hep-ph]; Guzzi M. *et al.*, arXiv:1101.0561 [hep-ph].
[5] Aaron F. D. *et al.* (H1 and ZEUS Collaboration), *JHEP*, **01** (2010) 109.
[6] Gluck M., Jimenez-Delgado P. and Reya E., *Eur. Phys. J. C*, **53** (2008) 355; Jimenez-Delgado P. and Reya E., *Phys. Rev. D*, **79** (2009) 074023.
[7] Martin A. D. *et al.*, *Eur. Phys. J. C*, **63** (2009) 189.
[8] Ball R. D. *et al.* (NNPDF Collaboration), *Nucl. Phys. B*, **855** (2012) 153; Ball R. D. *et al.* (NNPDF Collaboration), *Nucl. Phys. B*, **867** (2013) 244.
[9] Forte S. and Watt G., arXiv:1301.6754 [hep-ph].
[10] Ball R. D. *et al.*, *JHEP*, **04** (2013) 125.
[11] Frixione S., Kunszt Z. and Signer A., *Nucl. Phys. B*, **467** (1996) 399; Catani S. and Seymour M. H., *Nucl. Phys. B*, **485** (1997) 291 **510** (1998) 503(E).
[12] Aad G. *et al.* (ATLAS Collaboration), *JHEP*, **01** (2013) 086
[13] Maltoni F. and Stelzer T., *JHEP*, **02** (2003) 027; Alwall J. *et al.*, *JHEP*, **06** (2011) 128; Gleisberg T. *et al.*, *JHEP*, **02** (2009) 007.
[14] Ellis R. K., Kunszt Z., Melnikov K. and Zanderighi G., *Phys. Rep.*, **518** (2012) 141.
[15] Bern Z. *et al.*, *Phys. Rev. D*, **88** (2013) 014025.
[16] Berger C. F. *et al.*, *Phys. Rev. D*, **78** (2008) 036003; Hirschi V. *et al.*, *JHEP*, **05** (2011) 044; Badger S. *et al.*, *Comput. Phys. Commun.*, **182** (2011) 1674; Cascioli F. *et al.*, *Phys. Rev. Lett.*, **108** (2012) 111601; Cullen G. *et al.*, *Eur. Phys. J. C*, **72** (2012) 1889; Bevilacqua G. *et al.*, *Comput. Phys. Commun.*, **184** (2013) 986.
[17] Anastasiou C., Melnikov K. and Petriello F., *Nucl. Phys. B*, **724** (2005) 197.
[18] Catani S. and Grazzini M., *Phys. Rev. Lett.*, **98** (2007) 222002.
[19] Ferrera G., Grazzini M. and Tramontano F., *Phys. Rev. Lett.*, **107** (2011) 152003.
[20] Melnikov K. and Petriello F., *Phys. Rev. Lett.*, **96** (2006) 231803; *Phys. Rev. D*, **74** (2006) 114017.
[21] Catani S. *et al.*, *Phys. Rev. Lett.*, **103** (2009) 082001.
[22] Catani S. *et al.*, *Phys. Rev. Lett.*, **108** (2012) 072001.
[23] Parisi G. and Petronzio R., *Nucl. Phys. B*, **154** (1979) 427; Curci G., Greco M. and Srivastava Y., *Phys. Rev. Lett.*, **43** (1979) 834; Kodaira J. and Trentadue L., *Phys. Lett. B*, **112** (1982) 66; **123** (1983) 335; preprint SLAC-PUB-2934 (1982); Collins J. C., Soper D. E. and Sterman G., *Nucl. Phys. B*, **250** (1985) 199.
[24] Sterman G., *Nucl. Phys. B*, **281** (1987) 310; Catani S. and Trentadue L., *Nucl. Phys. B*, **327** (1989) 323.
[25] Bozzi G., Catani S., Ferrera G., de Florian D. and Grazzini M., *Phys. Lett. B*, **696** (2011) 207 and references therein.
[26] Bozzi G., Catani S., de Florian D. and Grazzini M., *Nucl. Phys. B*, **737** (2006) 73; Catani S. and Grazzini M., *Nucl. Phys. B*, **845** (2011) 297 and references therein; de Florian D., Ferrera G., Grazzini M. and Tommasini D., *JHEP*, **11** (2011) 064.
[27] Balazs C., Berger E. L., Nadolsky P. M. and Yuan C.-P., *Phys. Rev. D*, **76** (2007) 013009.
[28] Catani S., de Florian D., Grazzini M. and Nason P., *JHEP*, **07** (2003) 028; de Florian D. and Grazzini M., *Phys. Lett. B*, **718** (2012) 117 and references therein.
[29] Cacciari M., Czakon M., Mangano M., Mitov A. and Nason P., *Phys. Lett. B*, **710** (2012) 612 and references therein.
[30] Catani S., Grazzini M. and Torre A., arXiv:1305.3870 [hep-ph] and references therein.

[31] Ellis R. K., Stirling W. J. and Webber B. R., *QCD and Collider Physics* (Cambridge University Press, Cambridge) 1996 and references therein.
[32] Dokshitzer Y. L., Khoze V. A., Mueller A. H. and Troian S. I., *Basics of Perturbative QCD* (Editions Frontières, Gif-sur-Yvette, Singapore) 1991 and references therein.
[33] Corcella G. *et al.*, *JHEP*, **01** (2001) 010; Sjöstrand T. *et al.*, *Comput. Phys. Commun.*, **135** (2001) 238; Gleisberg T.*et al.*, *JHEP*, **02** (2004) 056.
[34] Catani S., Krauss F., Kuhn R. and Webber B. R., *JHEP*, **11** (2001) 063; Mangano M. L., Moretti M. and Pittau R., *Nucl. Phys. B*, **632** (2002) 343.
[35] Frixione S. and Webber B. R., *JHEP*, **06** (2002) 029; Nason P., *JHEP*, **11** (2004) 040; Frixione S., Nason P. and Oleari C., *JHEP*, **11** (2007) 070.
[36] Dittmaier S. *et al.* (LHC Higgs Cross Section Working Group Collaboration), arXiv:1101.0593 [hep-ph].