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Reggeization of the Phillips-Barger model of high-energy hadron scattering

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Summary. — The Phillips-Barger model, successful in describing elastic hadron scattering, is generalized to include explicitly energy dependence *à la* Regge.

PACS 13.75.Cs – Nucleon-nucleon interactions (including antinucleons, deuterons, etc.).
PACS 13.85.-t – Hadron-induced high- and super-high-energy interactions (energy > 10 GeV).

1. – Introduction

Using a generic expression for the differential cross section suggested by Phillips and Barger (PB) [1] in analyzing the ISR data, including the dip phenomenon first observed in 1972, we have performed an analysis of the evolution with energy of this phenomenon. The PB approach is the simplest universal way of modelling the dip-bump structere: the dip (diffraction minimum) arises there from the interference of two exponentials in t with a relative phase ϕ.

The PB ansatz reads (it will be convenient to use the scattering amplitude rather than cross sections):

$$A(s,t) = i[\sqrt{A}\exp(Bt/2) + \exp(i\phi(s))\sqrt{C}\exp(Dt/2)], \quad (1)$$

where A, B, C, D and ϕ were fitted to each ISR energy independently, *i.e.* energy dependence in the PB ansatz enters parametrically.

The PB formula was tested [2-5] against the TOTEM data [6], and, contrary to many alternative models, it works well. However, one should remember the limitations of the PB ansatz, namely that it does not contain energy dependence. In this paper we try to remedy this limitation, by combining the appealingly simple and efficient form of its t dependence with energy dependence according to the Regge-pole model. Attempts in this direction were undertaken also in papers [2-5].

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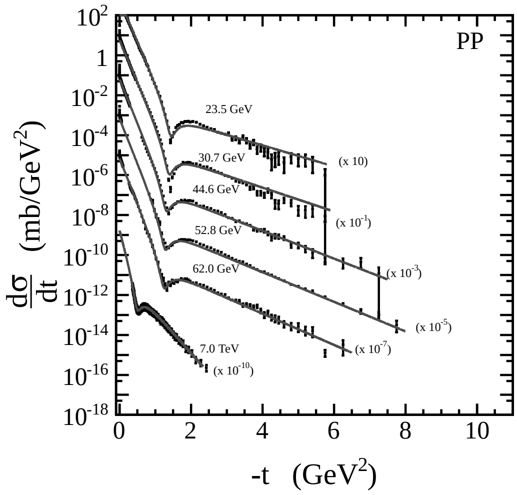

Fig. 1. – The PB model fitted to the pp data.

2. – Energy dependence

To understand better the existence of any connection between ansatz (1) and the Regge-pole model, we plot the values of the parameters A, B, C, D and ϕ against s and fit their "experimental" values to Regge-pole formulas.

In the present paper we use this formula to fit the pp elastic scattering data from the ISR energy region up to LHC's 7 TeV. For the sake of completeness, we do the same for $p\bar{p}$ scattering using the SPS and Tevatron data. The PB formula fits the data, as shown in figs. 1 and 2.

These unbiased "data points" can be used as a reference frame in subsequent model building.

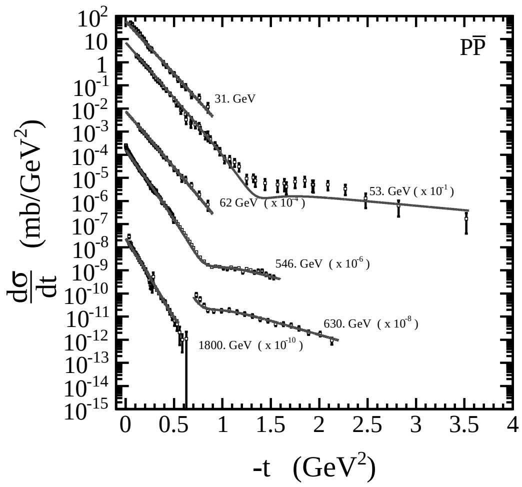

Fig. 2. – The PB model fitted to the $\bar{p}p$ data.

Having fixed A, B, C, D and ϕ to each energy set, we next assume their simplest energy dependence *à la* Regge, namely: the Donnachie-Landshoff [7] type parametrization for the rise with energy of the cross sections, $A \to A(s) = A_1 s^{\epsilon_1}$, $C \to C(s) = C_1 s^{\epsilon_2}$, and with shrinking cones, $B \to B(s) = B_0 + B_1 \ln s$, $D \to D(s) = D_0 + D_1 \ln s$.

An alternative way of fitting these parameters is by inserting the above energy dependence of $A, B, \ldots$ in the PB formula eq. (1) and consequently fitting $A_0, \ldots, D_1$ directly to the data. The results of this alternative approach with relevant fits, with more figures and the values of the fitted parameters can be found in papers [5].

3. – Further developments

The success of the simple PB parametrization motivates its further improvement, extension and utilization. In papers [3, 4] the low-$|t|$ behaviour of the PB ansatz was improved by modifying its simple exponential behaviour by: a) inclusion of a two-pion threshold required by analyticity, and b) by means of a multiplicative factor reflecting the proton form factor. Achieving perfect fits to the LHC data at 7 TeV, the modified ansatz was used to predict the behaviour of the observables at future energies as well as the expected asymptotic behaviour of the cross sections.

3˙1. *Regge poles, trajectories; the Odderon.* – The Odderon is the odd C counterpart of the Pomeron in the sense that, like the Pomeron, it has vacuum quantum numbers and is assumed to survive at asymptotic energies, which implies that its intercept is equal or greater than one. The existence of the Odderon (Odd) does not contradict neither the analytic S-matrix theory, nor quantum chromodynamics, where it corresponds to the exchange of three gluons. In spite of such legitimacy, so far it has not been confirmed experimentally. At low energies, it is masked by odd-C secondary Reggeons, namely by ω exchange. To see clearly the Odderon signal at high energies, one needs simultaneous (= at the same energy) measurements of pp and $\bar{p}p$ cross sections, whereby, since $A_{pp}^{\bar{p}p} = P \pm Odd$, the Odderon contribution may be extracted unambiguously from the difference of the pp and $\bar{p}p$ cross sections. Such data are not available; the existing high-energy accelerators study either pp or $\bar{p}p$ and they never meet. (This might become possible if the LHC energy will be lowered to 1.8–2 TeV). The unique coincidence occurred at the ISR, where, at the dip, pp and $\bar{p}p$ appeared different, although this may be attributed to the contribution of ω. The PB ansatz [1] or its modifications [2-4], providing perfect fits to both pp and $\bar{p}p$ scattering data, may help in the identification of the Odderon. Since, as noted, pp and $\bar{p}p$ are being measured at different energies, an interpolation in energy is mandatory. This is possible by using the parametrizations quoted in sect. **2**, where all parameters, but the phase ϕ, show regular and monotonic behaviour.

As show in [8], the contribution from secondary Reggeons is negligible at the LHC, but the inclusion of the Odderon is mandatory, even for the description of pp scattering alone. Thus, any realistic model should include:

– the dip-bump structure typical of high-energy diffractive processes;

– non-linear Regge trajectories;

– possible Odderon (odd-C asymptotic Regge exchange), and be

– compatible with s- and t-channel unitarity.

Generally speaking, the scattering amplitude is a sum of four terms, two asymptotic (Pomeron (P) and Odderon (O)) and two non-asymptotic ones (secondary Regge pole contributions).

The P and f terms have positive C-parity, thus entering in the scattering amplitude with the same sign in pp and $\bar{p}p$ scattering, while the Odderon and ω have negative C-parity, present in pp and $\bar{p}p$ scattering with opposite signs:

$$A\left(s,t\right)_{pp}^{\bar{p}p} = A_P\left(s,t\right) + A_f\left(s,t\right) \pm \left[A_\omega\left(s,t\right) + A_O\left(s,t\right)\right], \tag{2}$$

where the symbols P, f, O, ω stand for the relevant Regge-pole amplitudes and the super(sub)script, evidently, indicate $\bar{p}p(pp)$ scattering with the relevant choice of the signs in the sum (2). This sum can be extended by adding more Reggeons, whose role may become increasingly important towards lower energies; their contribution can be effectively absorbed by f and ω.

We treat the Odderon, the C-odd counterpart of the Pomeron, on equal footing, differing by its C-parity and the values of its parameters (to be fitted to the data).

Regge trajectories are non-linear complex functions. In a limited range and with limited precision, they can be approximated by linear ones (a common practice, reasonable when non-linear effects can be neglected). This non-linearity is manifest, *e.g.* as the "break" *i.e.* change of the slope $\Delta B \approx 2\,\mathrm{GeV}^{-2}$ around $t \approx -0.1\,\mathrm{GeV}^2$, and at large $|t|$, beyond the second maximum, $|t| > 2\,\mathrm{GeV}^2$, where the cross section flattens and the trajectories are expected to slow down logarithmically.

Representative examples of the Pomeron trajectories are [8]: 1) a linear one, eq. (TR.1); 2) that with a square-root threshold, eq. (TR.2) required by t-channel unitarity and accounting for the small-t "break", see [9, 10] and earlier references therein, as well as the possible "Orear", $e^{\sqrt{-t}}$, behavior in the second cone; and 3) a logarithmic one, eq. (TR.3), anticipating possible "hard effects" at large $|t|$,

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t, \tag{TR.1}$$

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t - \alpha_{2P}\left(\sqrt{4\alpha_{3P}^2 - t} - 2\alpha_{3P}\right), \tag{TR.2}$$

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P - \alpha_{1P}\ln\left(1 - \alpha_{2P} t\right). \tag{TR.3}$$

Alternatives choices for the nonlinear trajectories and fits can be found, *e.g.* in [9, 10].

Further studies of the small-t curvature (the "break" or fine structure of the Pomeron), with the Coulombic term added will reproduce (and predict) the behaviour of elastic cross sections in the Coulomb interference region, while the intermediate- and large-t behavior can be accounted for by using a Pomeron trajectory with logarithmic asymptotics.

Given the perfect description of the pp and $\bar{p}p$ data in a wide energy span by the PB model [1] and its modifications [3, 4], we propose to use it for the identification of a possible odd-C exchange at high energies, *i.e.* of the Odderon. An open question remains here, namely extrapolation in energy of the phase ϕ, showing irregular behaviour. Given this irregularity, the simplest things to do is to fix the phase at its "average" value, say 2.8 for pp. A more advanced way, by reggeizing the PB model, is discussed in the next subsection.

3˙2. *Reggeization of the PB ansatz.* – Before going into details, below we try to find a link between the successful PB model and Regge poles. Both terms in eq. (1) show

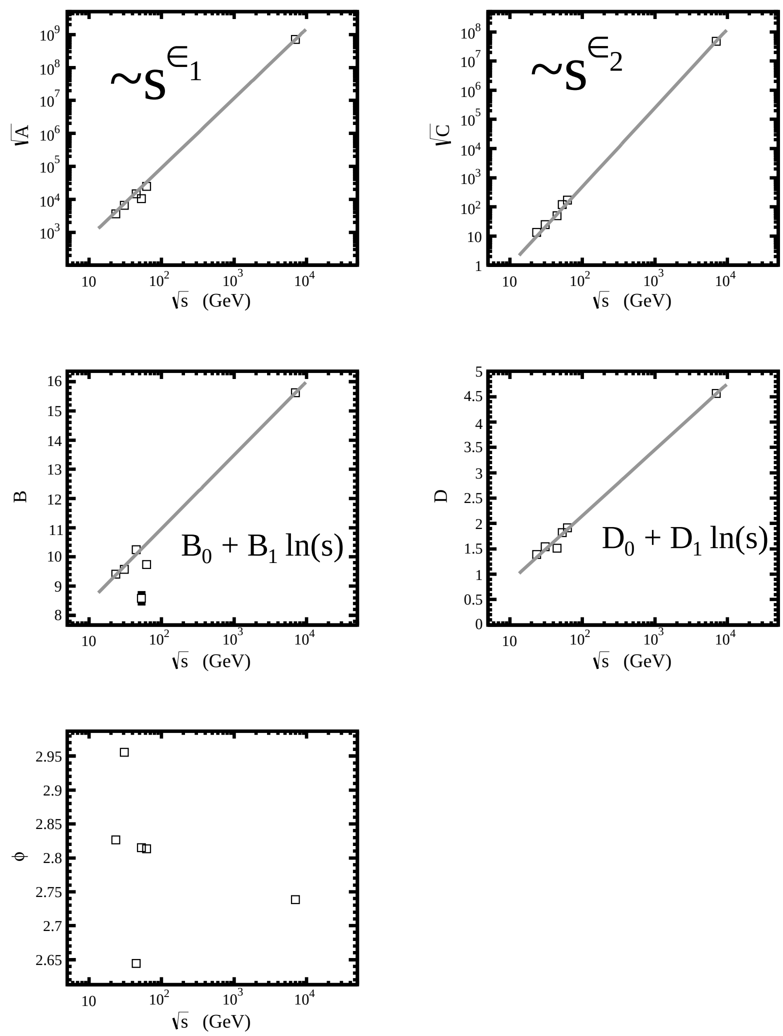

Fig. 3. – Values of the parameters of the PB ansatz extracted form a fit to *pp* data.

energy dependence typical of Regge pole models: cross sections slowly rising with energy (a supercritical Pomeron) and shrinkage of the cone.. The parameters A, B, C, D show monotonic behaviour, but the phase ϕ does not. In any case, in the Regge pole model, the phase should depend on t, rather than on s.

We identify the second term in the PB ansatz by the "soft" Pomeron and replace the "random" (see figs. 3 and 4) s-dependent phase there by a t-dependent one, the t-dependence entering through the Regge (here: Pomeron) trajectory (see, *e.g.*, [9]), $\exp(\phi(s)) \to \exp(-i\pi\alpha(t)/2)$.

For simplicity, we start with linear Regge (here: Pomeron or Odderon) Regge trajectories, $\alpha(t) = \alpha_0 + \alpha' t$. Nonlinear trajectories may provide better fits in the very small and large $|t|$ regions, see [9, 10].

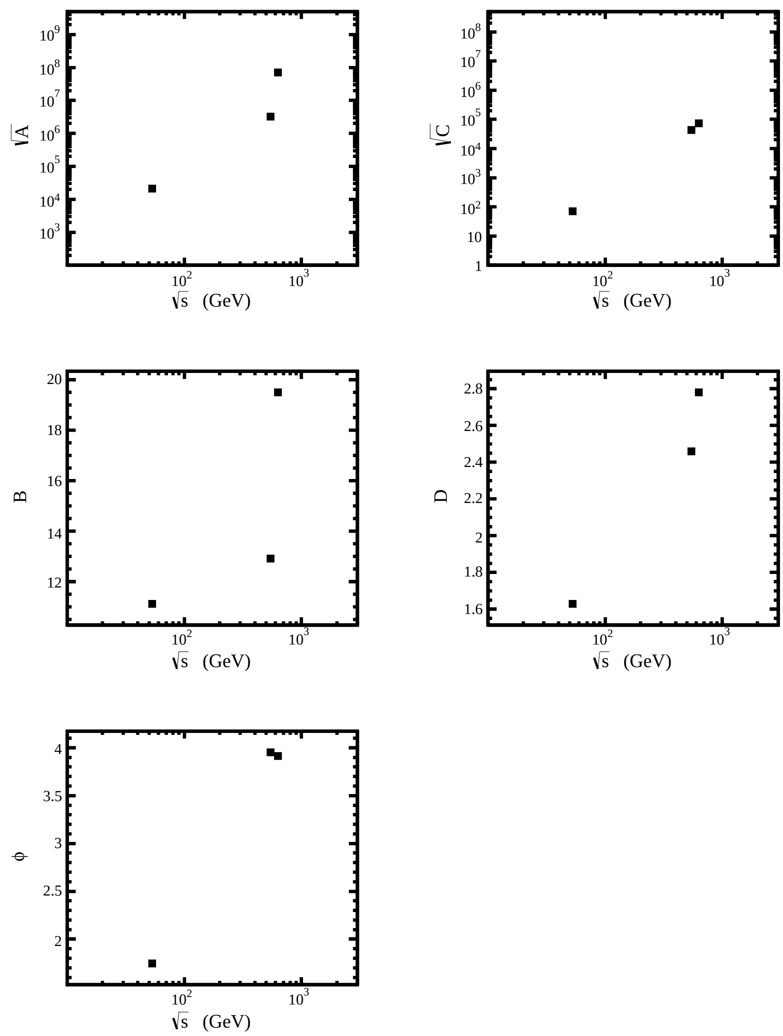

Fig. 4. – Same for $\bar{p}p$. Contrary to the previous case of pp, the behavior of the parameteres here is less regular.

The first (phase-independent) term of the PB ansatz may be identified with a "hard", ("flat" *i.e.* with a small slope) Pomeron. Its smallness, $\alpha' \approx 0.01$ provides a nearly constant real phase in the first term of the PB ansatz, P_1.

Let us rewrite the second term in the PB ansatz (1) $A = P_1 + P_2$ as

$$P_2(s,t) \sim -\exp[-(i\pi(\alpha(t)-1))/2]e^{b[\alpha(t)-1]}(s/s_0)^{[\alpha(t)-1]/2}, \tag{3}$$

where, for simplicity, we set $s_0 = 1\,\mathrm{GeV}^2$, and b is a free parameter. With a linear trajectory, eq. (3) reproduces, apart from the phase, the second term in the PB ansatz (1). The minus sign in (3) provides positivity of the imaginary part (total cross section, $\sigma_{tot} = \Im A(s,t=0)$).

For the first term P_1 in (1) we have two options:

1) P_1 corresponds to a "hard" Pomeron: it has the same form as the "soft" one (as above), but its slope is very low, resulting in an almost constant phase, as in the first term of the ansatz (1), while its intercept is high, typically $\alpha(0) \sim 1.3$;

2) it corresponds to the Odderon, with the same functional form as (3), just multiplied by i (opposite C-parity!) and with different values of the parameters.

The relative weight of two terms in both cases is a free parameter, to be fitted to the data, that will also help in choosing the right option.

* * *

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