IL NUOVO CIMENTO Vol. 37 C, N. 2 Marzo-Aprile 2014
DOI 10.1393/ncc/i2014-11751-1

Colloquia: LC13

Hadronic contribution to the anomalous magnetic moment of the muon

G. Venanzoni(*)
INFN - Laboratori Nazionali di Frascati - Frascati, Italy

ricevuto il 17 Febbraio 2014

Summary. — I will present the status and the prospects of the leading-order hadronic contribution to the muon $g-2$, with particular emphasis on the recent achievements on the hadronic cross section measurements at low energy.

PACS 13.40.Gp – Electromagnetic form factors.
PACS 13.60.Hb – Total and inclusive cross sections (including deep-inelastic processes).
PACS 13.66.De – Lepton production in e^-e^+ interactions.
PACS 13.66.Jn – Precision measurements in e^-e^+ interactions.

1. – The muon anomaly as a precision test of the Standard Model

The muon anomaly $a_\mu = (g-2)/2$ is a low-energy observable, which can be both measured and computed to high precision [1]. Therefore it provides an important test of the Standard Model (SM) and allows a sensitive search for new physics [2]. Since the first precision measurement of a_μ from the E821 experiment at BNL in 2001 [3], there has been a discrepancy between its experimental value and the SM prediction. This discrepancy has been slowly growing due to recent impressive theory and experiment achievements. Figure 1 (from ref. [4]) shows an up-to-date comparison of the SM predictions by different groups and the BNL measurement for a_μ. Evaluations of different groups are in very good agreement, showing a persisting 3σ discrepancy (as, for example, $26.1 \pm 8.0 \times 10^{-10}$ [4]). It should be noted that both theoretical and experimental uncertainties have been reduced by more than a factor of two in the last ten years(1).

The accuracy of the theoretical prediction ($\delta a_\mu^{\mathrm{SM}}$, between 5 and 6×10^{-10}) is limited by the strong interaction effects which cannot be computed perturbatively at low energies. Table I shows their contribution to the error for three recent estimates [6, 7, 4](2).

(*) E-mail: graziano.venanzoni@lnf.infn.it
(1) In 2001 this discrepancy was $(23.1 \pm 16.9) \times 10^{-10}$ [5].
(2) Reference [6] uses a more conservative error analysis.

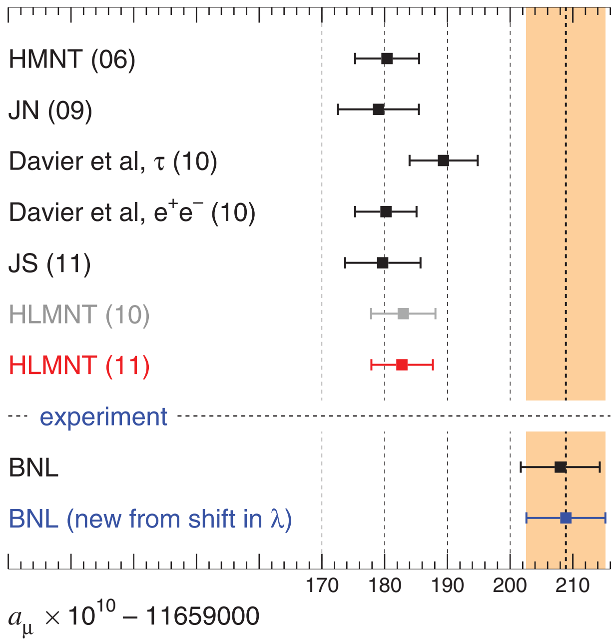

Fig. 1. – Standard Model predictions of a_μ by several groups compared to the measurement from BNL (from ref. [4]).

The leading-order hadronic vacuum polarization contribution, $a_\mu^{\rm HLO}$, gives the main uncertainty (between 4 and 5×10^{-10}). It can be related by dispersion integral to the measured hadronic cross sections, and it is known with a fractional accuracy of 0.7%, *i.e.* to about 0.4 ppm. The $O(\alpha^3)$ hadronic Light-by-Light contribution, $a_\mu^{\rm HLbL}$, is the second dominant error in the theoretical evaluation. It cannot at present be determined from data, and relies on specific models. Although its value is almost one order of magnitude smaller than $a_\mu^{\rm HLO}$, it is much worse known (with a fractional error of the order of 30%) and therefore it still gives a significant contribution to $\delta a_\mu^{\rm SM}$ (between

Table I. – *Estimated uncertainties* δa_μ *in units of* 10^{-10} *according to refs.* [6, 7, 4] *and (last column) prospects in case of improved precision in the* e^+e^- *hadronic cross section measurement (the prospect on* $\delta a_\mu^{\rm HLbL}$ *is an educated guess). Last row: Uncertainty on* Δa_μ *assuming the present experimental error of* 6.3 *from BNL-E821* [8] *(first two columns) and of* 1.6 *(last column) as planned by the future* $(g-2)$ *experiments* [9, 10].

Error	[6]	[7]	[4]	Prospect
$\delta a_\mu^{\rm SM}$	6.5	4.9	4.9	3.5
$\delta a_\mu^{\rm HLO}$	5.3	4.2	4.3	2.6
$\delta a_\mu^{\rm HLbL}$	3.9	2.6	2.6	2.5
$\delta(a_\mu^{\rm SM} - a_\mu^{\rm EXP})$	8.8	8.0	8.0	4.0

2.5 and 4×10^{-10}). From the experimental side, the error achieved by the BNL E821 experiment is $\delta a_\mu^{\rm EXP} = 6.3 \times 10^{-10}$ (0.54 ppm) [8]. This impressive result is still limited by the statistical error, and experiments to measure the muon $g - 2$ with a fourfold improvement in accuracy have been approved at Fermilab [9] and J-PARC [10].

2. – Recent progress on the hadronic contribution to a_μ

Differently from the QED and Electroweak contributions to a_μ, which can be calculated using perturbation theory, and therefore are well under control, the hadronic ones (LO VP and HLbL) cannot be computed reliably using perturbative QCD. The lowest order hadronic contribution $a_\mu^{\rm HLO}$ can be computed from hadronic e^+e^- annihilation data via a dispersion relation, and therefore its uncertainty strongly depends on the accuracy of the experimental data. For the hadronic Light-by-Light contribution $a_\mu^{\rm HLbL}$ there is no direct connection with data and therefore only model-dependent estimates exist. As the hadronic sector dominates the uncertainty on the theoretical prediction $a_\mu^{\rm SM}$, considerable effort has been put on it by experimental and theoretical groups, reaching the following main results:

- A precise determination of the hadronic cross sections at the e^+e^- colliders (VEPP-2M, DAΦNE, BEPC, PEP-II and KEKB) which allowed a determination of $a_\mu^{\rm HLO}$ with a fractional error below 1%. These efforts led to the development of dedicated high precision theoretical tools, like the inclusion of high-order Radiative Corrections (RC) and the non-perturbative hadronic contribution to the running of α (*i.e.* the vacuum polarisation, VP) in Monte Carlo (MC) programs used for the analysis of the data [11];
- Use of *Initial State Radiation* (ISR) [12-14] which opened a new way to precisely obtain the electron-positron annihilation cross sections into hadrons at particle factories operating at fixed beam-energies [15,16];
- A dedicate effort on the evaluation of the Hadronic Light-by-Light contribution, where two different groups [17,6] found agreement on the size of the contribution (with slightly different errors), and therefore strengthening our confidence in the reliability of these estimates;
- An impressive progress on QCD calculation on the lattice, where an accuracy better than 3% was reached on the two-flavor QCD correction to $a_\mu^{\rm HLO}$ [18];
- Better agreement between the e^+e^- and the τ based evaluation of $a_\mu^{\rm HLO}$, thanks to improved isospin corrections [7]. These two sets of data are eventually in agreement (with τ data moving towards e^+e^- data) after including vector meson and $\rho - \gamma$ mixing [19,20].

3. – σ_{had} measurements at low energy

In the last few years, big efforts on e^+e^- data in the energy range below a few GeV led to a substantial reduction in the hadronic uncertainty on $a_\mu^{\rm HLO}$. Figure 2 shows an up-to-date compilation of these data. The main improvements have been achieved in the region below 5 GeV: between 2 and 5 GeV (where the data are now closer to the prediction of pQCD), the BESII collaboration reduced the error to $\sim 7\%$ [21] (before it was $\sim 15\%$); between 1 and 4.5 GeV BaBar measured various final states with more

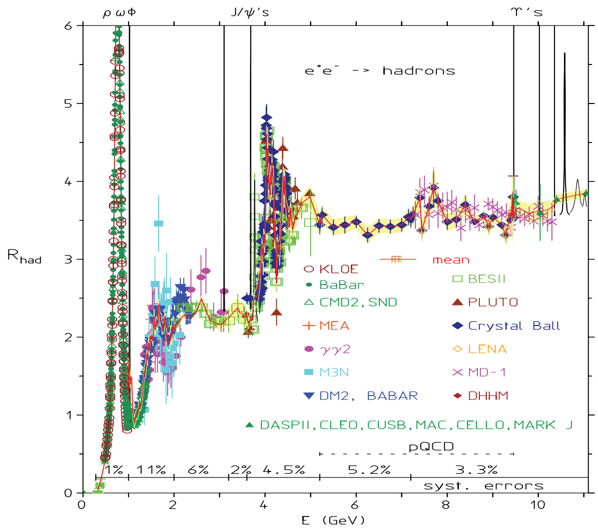

Fig. 2. – An updated compilation of R measurements. In the bottom line the overall uncertainties of the different regions are reported (*courtesy of Fred Jegerlehner*).

than two hadrons with a systematic accuracy between 3% and 15%, and the K^+K^- cross section with a systematic uncertainty below 1% at the ϕ peak [22]; below 1 GeV, the CMD-2 [23-25] and SND [26] collaborations at Novosibirsk, KLOE [27-29] at Frascati and BaBar [30] at Stanford measured the pion form factor in the energy range around the ρ peak with a systematic error of 0.8%, 1.3%, 0.9%, and 0.5%, respectively.

The CMD-2 and SND collaborations at Novosibirsk and BESII in Beijing were performing the hadronic cross section measurements in a traditional way, *i.e.*, by varying the e^+e^- beam energies. KLOE, BaBar, and more recently Belle used ISR (also called *radiative return*) as reviewed in refs. [11, 15, 16]. Figure 2 shows that, despite the recent progress, the region between 1 and 2 GeV is still poorly known, with a fractional accuracy of $\sim 6\%$. Since about 50% of the error squared, $\delta^2 a_\mu^{\mathrm{HLO}}$ comes from this region, it is evident how desirable an improvement on the hadronic cross section of this region is.

4. – Prospects on a_μ^{HLO}

Much of the prospects on the hadronic contribution to a_μ is based on the improvement of the hadronic cross section measurements.

4˙1. *Novosibirsk*. – The VEPP2M machine has been upgraded to VEPP-2000. The maximum energy has been increased from $\sqrt{s} = 1.4\,\mathrm{GeV}$ to 2.0 GeV. Additionally, the SND detector has been upgraded and the CMD2 detector was replaced by the much-improved CMD3 detector. The cross section will be measured from threshold to 2.0 GeV using an energy scan, filling in the energy region between 1.4 GeV, where the previous scan ended, up to 2.0 GeV, the lowest energy point reached by the BES collaboration in their measurements. Engineering runs began in 2009, and data collection started in 2011. So far two independent energy scans between 1.0 and 2.0 GeV were performed in 2011 and 2012. The peak luminosity of $3 \times 10^{31}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ was achieved, which was limited by the positron production rate. The new injection facility, scheduled to be commissioned during the 2013–2014 upgrade, should permit the luminosity to reach $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Data collection resumed in late 2012 with a new energy scan over energies below 1.0 GeV. The goal of experiments at VEPP-2000 is to achieve a systematic error 0.3–0.5% in the $\pi^+\pi^-$ channel, with negligible statistical error in the integral. The high statistics, expected at VEPP-2000, should allow a detailed comparison of the measured cross-sections with ISR results at BaBar and DAΦNE. After the upgrade, experiments at VEPP-2000 plan to take a large amount of data at 1.8–2 GeV, around the $N\bar{N}$ threshold. This will permit ISR data with the beam energy of 2 GeV, which is between the PEP2 energy at the $\Upsilon(4S)$ and the 1 GeV ϕ energy at the DAΦNE facility in Frascati. The dual ISR and scan approach will provide an important cross check on the two central methods used to determine the HVP.

4·2. *The BESIII Experiment.* – The BESIII experiment at the Beijing tau-charm factory BEPC-II has already collected several femtobarns of integrated luminosity at various centre-of-mass energies in the range 3–4.5 GeV. The ISR program includes cross section measurements of: $e^+e^- \to \pi^+\pi^-$, $e^+e^- \to \pi^+\pi^-\pi^0$, $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ — the final states most relevant to $(g-2)_\mu$. Presently, a data sample of $2.9\,\mathrm{fb}^{-1}$ at $\sqrt{s} = 3.77\,\mathrm{GeV}$ is being analyzed, but new data at $\sqrt{s} > 4\,\mathrm{GeV}$ can be used for ISR physics as well and will double the statistics. Using these data, hadronic invariant masses from threshold up to approximately 3.5 GeV can be accessed at BESIII. Although the integrated luminosities are orders of magnitude lower compared to the B-factory experiments BaBar and BELLE, the ISR method at BESIII still provides competitive statistics. BESIII is aiming for a precision measurement of the ISR R-ratio $R_{\mathrm{ISR}} = N(\pi\pi\gamma)/N(\mu\mu\gamma)$ with a precision of about 1%. This requires an excellent pion-muon separation, which is achieved by training a multi-variate neural network. As a preliminary result, an absolute cross section measurement of the reaction $e^+e^- \to \mu^+\mu^-\gamma$ has been achieved, which agrees with the QED prediction within 1% precision [31].

Moreover, at BESIII a new energy scan campaign is planned to measure the inclusive R ratio in the energy range between 2.0 and 4.6 GeV. Thanks to the good performance of the BEPC-II accelerator and the BESIII detector a significant improvement upon the existing BESII measurement can be expected. The goal is to arrive at an inclusive R ratio measurement with about 1% statistical and 3% systematic precision per scan point.

4·3. *Energy upgrade of DAΦNE.* – With a specific luminosity of $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, DAΦNE upgraded in energy, could perform a scan in the region from 1 to 2.5 GeV, collecting an integrated luminosity of $20\,\mathrm{pb}^{-1}$ per point corresponding to few days of data taking for each energy bin [32]. By assuming an energy step of 25 MeV, the whole region would be scanned in one year of data taking. The statistical yield would be one order of magnitude higher than what would have been achieved with $1\,\mathrm{ab}^{-1}$ at BaBar, and better than what is to be expected at BESIII with $10\,\mathrm{fb}^{-1}$ at 3 GeV.

5. – Prospects on a_μ

With the new experiments planned at Fermilab and J-PARC the uncertainty of the difference Δa_μ between the experimental and the theoretical value of a_μ will be dominated by the uncertainty of the hadronic cross sections at low energies, unless new experimental efforts at low energy are undertaken. The last column of table I shows a future scenario based on realistic improvements in the $e^+e^- \to$ *hadrons* cross sections measurements. Such improvements could be obtained by reducing the uncertainties of

TABLE II. – *Overall uncertainty of the cross-section measurement required to get the reduction of uncertainty on* $a_\mu^{\rm HLO}$ *in units* 10^{-10} *for three regions of* $\sqrt{s}$ *(from ref.* [33]*).*

	$\delta(\sigma)/\sigma$ present	$\delta a_\mu^{\rm HLO}$ present	$\delta(\sigma)/\sigma$ prospect	$\delta a_\mu^{\rm HLO}$ prospect
$\sqrt{s} < 1\,{\rm GeV}$	0.7%	3.3	0.4%	1.9
$1 < \sqrt{s} < 2\,{\rm GeV}$	6%	3.9	2%	1.3
$\sqrt{s} > 2\,{\rm GeV}$		1.2		1.2
total		5.3		2.6

the hadronic cross sections from 0.7% to 0.4% in the region below 1 GeV and from 6% to 2% in the region between 1 and 2 GeV as shown in table II.

In this scenario the overall uncertainty on Δa_μ could be reduced by a factor 2. In case the central value would remain the same, the statistical significance would become 7–8 standard deviations, as it can be seen in fig. 3.

The effort needed to reduce the uncertainties of the $e^+e^- \to$ *hadrons* cross-sections according to table II is challenging but possible, and certainly well motivated by the excellent opportunity the muon g-2 is providing us to unveil (or constrain) "new-physics" effects.

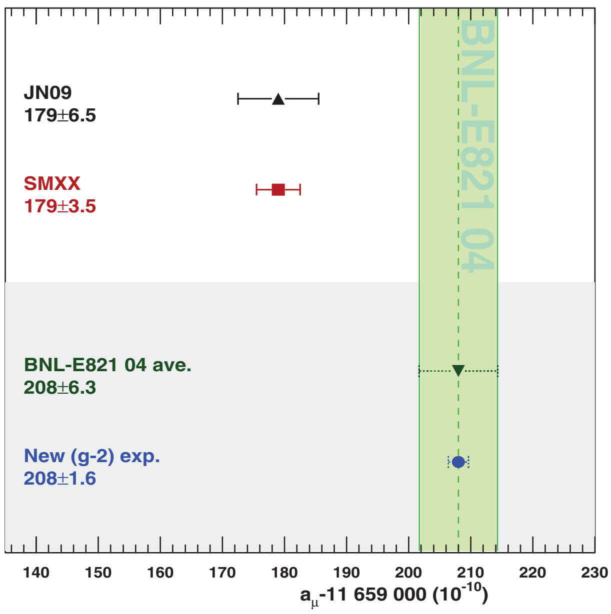

Fig. 3. – Comparison between $a_\mu^{\rm SM}$ and $a_\mu^{\rm EXP}$. "JN09" is the current evaluation of $a_\mu^{\rm SM}$ using ref. [6]; "SMXX" is the same central value with a reduced error as expected by the improvement on the hadronic cross section measurement (see text); "BNL-E821 04 ave." is the current experimental value of a_μ; "New (g-2) exp." is the same central value with a fourfold improved accuracy as planned by the future (g-2) experiments [9, 10].

* * *

It is a pleasure to thank Lia Pancheri, Gennaro Corcella, Massimo Passera, and the other members of the LC13 local organizing committee, for running a smooth and productive meeting in a very friendly atmosphere. Support from ECT* is warmly acknowledged.

REFERENCES

[1] Jegerlehner F., *The anomalous magnetic moment of the muon Springer Tracts Mod. phys.* **226** (Berlin, Springer) 2008.
[2] Stöckinger D., "Muon (g-2) and physics beyond the standard model", in *Lepton dipole moments 393-438*, edited by Roberts, Lee B., Marciano and William J. Advanced Series on Directions in High Energy Physics **20**.
[3] Brown H. N. *et al.* (Muon g-2 Collaboration), *Phys. Rev. Lett.*, **86** (2001) 2227.
[4] Hagiwara K., Liao R., Martin A. D., Nomura D. and Teubner T., *J. Phys. G*, **38** (2011) 085003.
[5] Prades J., hep-ph/0108192.
[6] Jegerlehner F. and Nyffeler A., *Phys. Rep.*, **477** (2009) 1.
[7] Davier M., Hoecker A., Malaescu B. and Zhang Z., *Eur. Phys. J. C*, **71** (2011) 1515.
[8] Bennett G. W. *et al.* (Muon G-2 Collaboration), *Phys. Rev. D*, **73** (2006) 072003.
[9] Carey R. M. *et al.* (The New Muon (g-2) Collaboration), see `http://lss.fnal.gov/archive/testproposal/0000/fermilab-proposal-0989.shtml`
[10] Imazato J., *Nucl. Phys. Proc. Suppl.*, **129** (2004) 81.
[11] Actis S. *et al.* (Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies Collaboration), *Eur. Phys. J. C*, **66** (2010) 585.
[12] Chen M. S. and Zerwas P. M., *Phys. Rev. D*, **11** (1975) 58.
[13] Binner S., Kuhn J. H. and Melnikov K., *Phys. Lett. B*, **459** (1999) 279.
[14] Benayoun M., Eidelman S. I., Ivanchenko V. N. and Silagadze Z. K., *Mod. Phys. Lett. A*, **14** (1999) 2605.
[15] Kluge W., *Nucl. Phys. Proc. Suppl.*, **181-182** (2008) 280.
[16] Druzhinin V. P., Eidelman S. I., Serednyakov S. I. and Solodov E. P., *Rev. Mod. Phys.*, **83** (2011) 1545.
[17] Prades J., de Rafael E. and Vainshtein A., Advanced Series on Directions in High Energy Physics 20.
[18] Feng X., Jansen K., Petschlies M. and Renner D. B., arXiv:1103.4818 [hep-lat].
[19] Jegerlehner F. and Szafron R., *Eur. Phys. J. C*, **71** (2011) 1632.
[20] Benayoun M., David P., Del Buono L. and Jegerlehner F., *Eur. Phys. J. C*, **72** (2012) 1848.
[21] Bai J. Z. *et al.* (BES Collaboration), *Phys. Rev. Lett.*, **88** (2002) 101802.
[22] Lees J. P. *et al.* (BaBar Collaboration), *Phys. Rev. D*, **88** (2013) 032013 [arXiv:1306.3600 [hep-ex]].
[23] Akhmetshin R. R. *et al.* (CMD-2 Collaboration), *Phys. Lett. B*, **648** (2007) 28.
[24] Akhmetshin R. R. *et al.* (CMD-2 Collaboration), *JETP Lett.*, **84** (2006) 413 (*Pisma Zh. Eksp. Teor. Fiz.*, **84** (2006) 491).
[25] Akhmetshin R. R. *et al.* (CMD-2 Collaboration), *Phys. Lett. B*, **578** (2004) 285.
[26] Achasov M. N. *et al.*, *J. Exp. Theor. Phys.*, **103** (2006) 380.
[27] Ambrosino F. *et al.* (KLOE Collaboration), *Phys. Lett. B*, **700** (2011) 102.
[28] Ambrosino F. *et al.* (KLOE Collaboration), *Phys. Lett. B*, **670** (2009) 285.
[29] Aloisio A. *et al.* (KLOE Collaboration), *Phys. Lett. B*, **606** (2005) 12.
[30] Aubert B. *et al.* (BABAR Collaboration), *Phys. Rev. Lett.*, **103** (2009) 231801.
[31] Scumann E., presentation at the International Workshop on e+e- collisions from Phi to Psi (PHIPSI13), September 9-12 2013, Rome.
[32] Babusci D. *et al.*, arXiv:1007.5219 [hep-ex].
[33] Jegerlehner F., *Nucl. Phys. Proc. Suppl.*, **181-182** (2008) 26.