## Sterile neutrinos

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Summary. - We review the results of global analyses of short-baseline neutrino oscillation data in $3+1$ and $3+2$ neutrino mixing schemes.

Neutrino oscillations have been measured with high accuracy in solar, atmospheric and long-baseline neutrino oscillation experiments (see [1-3]). Hence, we know that neutrinos are massive and mixed particles (see $[4,5]$ ) and that there are two independent squaredmass differences: the solar $\Delta m_{\text {SOL }}^{2} \simeq 7.5 \times 10^{-5} \mathrm{eV}^{2}$ and the atmospheric $\Delta m_{\text {ATM }}^{2} \simeq$ $2.3 \times 10^{-3} \mathrm{eV}^{2}$. This is in agreement with the standard three-neutrino mixing paradigm, in which the three active neutrinos $\nu_{e}, \nu_{\mu}, \nu_{\tau}$ are superpositions of three massive neutrinos $\nu_{1}, \nu_{2}, \nu_{3}$ with respective masses $m_{1}, m_{2}, m_{3}$. The two measured squared-mass differences can be interpreted as $\Delta m_{\mathrm{SOL}}^{2}=\Delta m_{21}^{2}$ and $\Delta m_{\mathrm{ATM}}^{2}=\left|\Delta m_{31}^{2}\right| \simeq\left|\Delta m_{32}^{2}\right|$, with $\Delta m_{k j}^{2}=$ $m_{k}^{2}-m_{j}^{2}$.

The completeness of the three-neutrino mixing paradigm has been challenged by the following indications in favor of short-baseline neutrino oscillations, which require the existence of at least one additional squared-mass difference, $\Delta m_{\mathrm{SBL}}^{2}$, which is much larger than $\Delta m_{\text {SOL }}^{2}$ and $\Delta m_{\text {ATM }}^{2}$ :

1. The LSND experiment, in which a signal of short-baseline $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ oscillations has been observed with a statistical significance of about $3.8 \sigma[6,7]$.
2. The reactor antineutrino anomaly [8], which is a deficit of the rate of $\bar{\nu}_{e}$ observed in several short-baseline reactor neutrino experiments in comparison with that expected from a new calculation of the reactor neutrino fluxes [9,10]. The statistical significance is about $2.8 \sigma$.
3. The Gallium neutrino anomaly [11-15], consisting in a short-baseline disappearance of $\nu_{e}$ measured in the Gallium radioactive source experiments GALLEX [16] and SAGE [17] with a statistical significance of about $2.9 \sigma$.

In this review, we consider $3+1$ [18-21] and $3+2$ [22-25] neutrino mixing schemes in which there are one or two additional massive neutrinos at the eV scale and the masses of the three standard massive neutrinos are much smaller. Since from the LEP measurement of the invisible width of the $Z$ boson we know that there are only three
active neutrinos (see [4]), in the flavor basis the additional massive neutrinos correspond to sterile neutrinos [26], which do not have standard weak interactions.

The possible existence of sterile neutrinos is very interesting, because they are new particles which could give us precious information on the physics beyond the Standard Model (see [27,28]). The existence of light sterile neutrinos is also very important for astrophysics (see [29]) and cosmology (see [30,31]).

In the $3+1$ scheme, the effective probability of $\stackrel{(-)}{\nu}_{\alpha} \rightarrow \stackrel{(-)}{\nu}$ 位 transitions in short-baseline experiments has the two-neutrino-like form

$$
\begin{equation*}
P_{\substack{(-)(-) \\ \nu_{\alpha} \rightarrow \nu_{\beta}}}^{(-)}=\delta_{\alpha \beta}-4\left|U_{\alpha 4}\right|^{2}\left(\delta_{\alpha \beta}-\left|U_{\beta 4}\right|^{2}\right) \sin ^{2}\left(\frac{\Delta m_{41}^{2} L}{4 E}\right), \tag{1}
\end{equation*}
$$

where $U$ is the mixing matrix, $L$ is the source-detector distance, $E$ is the neutrino energy and $\Delta m_{41}^{2}=m_{4}^{2}-m_{1}^{2}=\Delta m_{\mathrm{SBL}}^{2} \sim 1 \mathrm{eV}^{2}$. The electron and muon neutrino and antineutrino appearance and disappearance in short-baseline experiments depend on $\left|U_{e 4}\right|^{2}$ and $\left|U_{\mu 4}\right|^{2}$, which determine the amplitude $\sin ^{2} 2 \vartheta_{e \mu}=4\left|U_{e 4}\right|^{2}\left|U_{\mu 4}\right|^{2}$ of $\stackrel{(-)}{\nu}{ }_{\mu} \rightarrow \stackrel{(-)}{\nu}$ transitions, the amplitude $\sin ^{2} 2 \vartheta_{e e}=4\left|U_{e 4}\right|^{2}\left(1-\left|U_{e 4}\right|^{2}\right)$ of $\stackrel{(-)}{\nu} e$ disappearance, and the amplitude $\sin ^{2} 2 \vartheta_{\mu \mu}=4\left|U_{\mu 4}\right|^{2}\left(1-\left|U_{\mu 4}\right|^{2}\right)$ of $\stackrel{(-)}{\nu} \mu$ disappearance.

Since the oscillation probabilities of neutrinos and antineutrinos are related by a complex conjugation of the elements of the mixing matrix (see [4]), the effective probabilities of short-baseline $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ transitions are equal. Hence, the $3+1$ scheme cannot explain a possible CP-violating difference of $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ transitions in short-baseline experiments. In order to allow this possibility, one must consider a $3+2$ scheme, in which, there are four additional effective mixing parameters in shortbaseline experiments: $\Delta m_{51}^{2}$, which is conventionally assumed $\geq \Delta m_{41}^{2},\left|U_{e 5}\right|^{2},\left|U_{\mu 5}\right|^{2}$ and $\eta=\arg \left[U_{e 4}^{*} U_{\mu 4} U_{e 5} U_{\mu 5}^{*}\right]$. Since this complex phase appears with different signs in the effective $3+2$ probabilities of short-baseline $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ transitions, it can generate measurable CP violations.

Global fits of short-baseline neutrino oscillation data have been presented recently in Refs. [32,33]. These analyses take into account the final results of the MiniBooNE experiment, which was made in order to check the LSND signal with about one order of magnitude larger distance $(L)$ and energy $(E)$, but the same order of magnitude for the ratio $L / E$ from which neutrino oscillations depend. Unfortunately, the results of the MiniBooNE experiment are ambiguous, because the LSND signal was not seen in neutrino mode [34] and the signal observed in 2010 [35] with the first half of the antineutrino data was not observed in the second half of the data [36]. Moreover, the MiniBooNE data in both neutrino and antineutrino modes show an excess in the low-energy bins which is widely considered to be anomalous because it is at odds with neutrino oscillations [37,38].

In the following we summarize the results of the analysis of short-baseline data presented in Ref. [33] of the following three groups of experiments:
(A) The ${ }_{(-)}^{\nu}{ }_{\mu} \rightarrow \stackrel{(-)}{\nu}$ e appearance data of the LSND [7], MiniBooNE [36], BNL-E776 [39], KARMEN [40], NOMAD [41], ICARUS [42] and OPERA [43] experiments.
(B) The ${ }_{\left({ }_{\nu}\right.}^{(-)}$disappearance data described in Ref. [15], which take into account the reactor $[9,8,10]$ and Gallium [11-13, 44, 14] anomalies.
(C) The constraints on $\stackrel{(-)}{\nu}$ disappearance obtained from the data of the CDHSW experiment [45], from the analysis [24] of the data of atmospheric neutrino oscillation experiments, from the analysis [37] of the MINOS neutral-current data [46] and from the analysis of the SciBooNE-MiniBooNE neutrino [47] and antineutrino [48] data.

TABLE I. - Results of the fit of short-baseline data [33] taking into account all MiniBooNE data (LOW), only the MiniBooNE data above 475 MeV (HIG), without MiniBooNE data (noMB) and without LSND data (noLSND) in the 3+1 and 3+2 schemes. The first three lines give the minimum $\chi^{2}\left(\chi_{\min }^{2}\right)$, the number of degrees of freedom (NDF) and the goodness-of-fit (GoF). The following five lines give the quantities relevant for the appearance-disappearance (APP$D I S)$ parameter goodness-of-fit $(P G)$ [49]. The last three lines give the difference between the $\chi^{2}$ without short-baseline oscillations and $\chi_{\min }^{2}\left(\Delta \chi_{N O}^{2}\right)$, the corresponding difference of number of degrees of freedom $\left(N D F_{N O}\right)$ and the resulting number of $\sigma^{\prime} s\left(n \sigma_{N O}\right)$ for which the absence of oscillations is disfavored.

|  | $\begin{gathered} 3+1 \\ \text { LOW } \end{gathered}$ | $\begin{aligned} & 3+1 \\ & \text { HIG } \end{aligned}$ | $\begin{gathered} 3+1 \\ \text { noMB } \end{gathered}$ | $3+1$ <br> noLSND | $\begin{gathered} 3+2 \\ \text { LOW } \end{gathered}$ | $\begin{aligned} & 3+2 \\ & \text { HIG } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\chi_{\text {min }}^{2}$ | 291.7 | 261.8 | 236.1 | 278.4 | 284.4 | 256.4 |
| NDF | 256 | 250 | 218 | 252 | 252 | 246 |
| GoF | 6\% | $29 \%$ | 19\% | 12\% | 8\% | $31 \%$ |
| $\left(\chi_{\text {min }}^{2}\right)_{\text {APP }}$ | 99.3 | 77.0 | 50.9 | 91.8 | 87.7 | 69.8 |
| $\left(\chi_{\text {min }}^{2}\right)_{\text {DIS }}$ | 180.1 | 180.1 | 180.1 | 180.1 | 179.1 | 179.1 |
| $\Delta \chi_{\text {PG }}^{2}$ | 12.7 | 4.8 | 5.1 | 6.4 | 17.7 | 7.5 |
| $\mathrm{NDF}_{\text {PG }}$ | 2 | 2 | 2 | 2 | 4 | 4 |
| $\mathrm{GoF}_{\mathrm{PG}}$ | 0.2\% | 9\% | 8\% | 4\% | 0.1\% | 11\% |
| $\Delta \chi_{\text {NO }}^{2}$ | 47.5 | 46.2 | 47.1 | 8.3 | 54.8 | 51.6 |
| $\mathrm{NDF}_{\text {NO }}$ | 3 | 3 | 3 | 3 | 7 | 7 |
| $n \sigma_{\text {NO }}$ | $6.3 \sigma$ | $6.2 \sigma$ | $6.3 \sigma$ | $2.1 \sigma$ | $6.0 \sigma$ | $5.8 \sigma$ |

Table I summarizes the statistical results obtained in Ref. [33] from global fits of the data above in the $3+1$ and $3+2$ schemes. In the LOW fits all the MiniBooNE data are considered, including the anomalous low-energy bins, which are omitted in the HIG fits. There is also a $3+1$-noMB fit without MiniBooNE data and a $3+1$-noLSND fit without LSND data.

From Tab. I, one can see that in all fits which include the LSND data the absence of short-baseline oscillations is disfavored by about $6 \sigma$, because the improvement of the $\chi^{2}$ with short-baseline oscillations is much larger than the number of oscillation parameters.

In both the $3+1$ and $3+2$ schemes the goodness-of-fit in the LOW analysis is significantly worse than that in the HIG analysis and the appearance-disappearance parameter goodness-of-fit is much worse. This result confirms the fact that the MiniBooNE lowenergy anomaly is incompatible with neutrino oscillations, because it would require a small value of $\Delta m_{41}^{2}$ and a large value of $\sin ^{2} 2 \vartheta_{e \mu}[37,38]$, which are excluded by the data of other experiments (see Ref. [33] for further details). Note that the appearancedisappearance tension in the $3+2$-LOW fit is even worse than that in the $3+1$-LOW fit, since the $\Delta \chi_{\mathrm{PG}}^{2}$ is so much larger that it cannot be compensated by the additional degrees of freedom (this behavior has been explained in Ref. [50]). Therefore, we think


Fig. 1. - Allowed regions in the $\sin ^{2} 2 \vartheta_{e \mu}-\Delta m_{41}^{2}, \sin ^{2} 2 \vartheta_{e e}-\Delta m_{41}^{2}$ and $\sin ^{2} 2 \vartheta_{\mu \mu}-\Delta m_{41}^{2}$ planes obtained in the global (GLO) 3+1-HIG fit [33] of short-baseline neutrino oscillation data compared with the $3 \sigma$ allowed regions obtained from ${ }_{(-)}^{\nu} \rightarrow{ }_{\mu}^{(-)}{ }_{\nu}$ short-baseline appearance data (APP) and the $3 \sigma$ constraints obtained from $\left(-{ }_{\nu}^{(-)}\right.$short-baseline disappearance data ( $\nu_{e}$ DIS) ${ }_{( }^{(-)}{ }_{\mu}$ shortbaseline disappearance data ( $\nu_{\mu}$ DIS) and the combined short-baseline disappearance data (DIS). The best-fit points of the GLO and APP fits are indicated by crosses.
that it is very likely that the MiniBooNE low-energy anomaly has an explanation which is different from neutrino oscillations and the HIG fits are more reliable than the LOW fits.

The $3+2$ mixing scheme, was considered to be interesting in 2010 when the MiniBooNE neutrino [34] and antineutrino [35] data showed a CP-violating tension. Unfortunately, this tension reduced considerably in the final MiniBooNE data [36] and from Tab. I one can see that there is little improvement of the $3+2$-HIG fit with respect to the $3+1$-HIG fit, in spite of the four additional parameters and the additional possibility of CP violation. Moreover, since the p-value obtained by restricting the $3+2$ scheme to $3+1$ disfavors the $3+1$ scheme only at $1.2 \sigma$ [33], we think that considering the larger complexity of the $3+2$ scheme is not justified by the data $\left({ }^{1}\right)$.

Figure 1 shows the allowed regions in the $\sin ^{2} 2 \vartheta_{e \mu}-\Delta m_{41}^{2}, \sin ^{2} 2 \vartheta_{e e}-\Delta m_{41}^{2}$ and $\sin ^{2} 2 \vartheta_{\mu \mu}-\Delta m_{41}^{2}$ planes obtained in the 3+1-HIG fit of Ref. [33]. These regions are relevant, respectively, for $\stackrel{(-)}{\nu}_{\mu} \rightarrow \stackrel{(-)}{\nu}$ appearance,,$\stackrel{(-)}{\nu}_{e}$ disappearance and $\stackrel{(-)}{\nu}_{\mu}$ disappearance searches. Figure 1 shows also the region allowed by $\stackrel{(-)}{\nu} \rightarrow \stackrel{(-)}{\nu}{ }_{\mu}$ appearance data and the
 combined disappearance constraint in the $\sin ^{2} 2 \vartheta_{e \mu}-\Delta m_{41}^{2}$ plane excludes a large part of the region allowed by $\stackrel{(-)}{\nu} \rightarrow \stackrel{(-)}{\nu}$ appearance data, leading to the well-known appearancedisappearance tension $[51,52,37,38,53,54,50,32]$ quantified by the parameter goodness-of-fit in Tab. I.

[^0]It is interesting to investigate what is the impact of the MiniBooNE experiment on the global analysis of short-baseline neutrino oscillation data. With this aim, the authors of Ref. [33] performed two additional 3+1 fits: a $3+1$-noMB fit without MiniBooNE data and a $3+1-$ noLSND fit without LSND data. From Tab. I one can see that the results of the $3+1$-noMB fit are similar to those of the $3+1-$ HIG fit and the exclusion of the case of no-oscillations remains at the level of $6 \sigma$. On the other hand, in the $3+1$-noLSND fit, without LSND data, the exclusion of the case of no-oscillations drops dramatically to $2.1 \sigma$. In fact, in this case the main indication in favor of short-baseline oscillations is given by the reactor and Gallium anomalies which have a similar statistical significance. Therefore, it is clear that the LSND experiment is still crucial for the indication in favor of short-baseline $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ transitions and the MiniBooNE experiment has been rather inconclusive.

In conclusion, the results of the global fit of short-baseline neutrino oscillation data presented in Ref. [33] show that the data can be explained by $3+1$ neutrino mixing and this simplest scheme beyond three-neutrino mixing cannot be rejected in favor of the more complex $3+2$ scheme. The low-energy MiniBooNE anomaly cannot be explained by neutrino oscillations in any of these schemes. Moreover, the crucial indication in favor of short-baseline $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ appearance is still given by the old LSND data and the MiniBooNE experiment has been inconclusive. Hence new better experiments are needed in order to check this signal.

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[^0]:    ( ${ }^{1}$ ) See however the somewhat different conclusions reached in Ref. [32].

