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Lorentz violation as the origin of dynamical flavour oscillations

J. Alexandre(1), J. Leite(1), N. E. Mavromatos(1)(2)

(1) *King's College London, Department of Physics - London WC2R 2LS, UK*

(2) *CERN, Physics Department - Geneva 23 CH-1211 Switzerland*

Summary. — We present an alternative mechanism to generate masses and oscillations dynamically for two fermion flavours coupled to a Lorentz-Invariance-Violating (LIV) gauge field. The LIV field plays the role of a regulator for the gap equations and the non-analytic form of the dynamical masses allows to consider the limit where the LIV field decouples from the (Dirac or Majorana) fermions and, then, Lorentz invariance is recovered.

1. – Introduction

The generation of quark, lepton and vector boson masses, as described in the standard model due to their coupling with the Higgs field, seems to have been confirmed at the LHC [1]. However, the origin of neutrino masses is still not well established, although the seesaw mechanism seems the most elegant and simple for such a purpose [2].

The possibility, therefore, of generating neutrino masses without the involvement of right-handed states is still at play. In [3], preliminary steps in this direction have been taken, in which flavour oscillations can arise dynamically, from the flavour-mixing interaction of two massless bare fermions with a LIV gauge field. Lorentz violation is achieved by higher order space derivatives suppressed by a mass scale M, which allows the dynamical generation of fermion masses, as shown in [4], and also regulates the model.

An important point is the structure of the dynamical mass [4]: $m_{dyn} \simeq M\exp(-a/e^2)$, where a is a positive constant and e is the gauge coupling. As one can see, m_{dyn} has a non-analytical form which can be derived from the Schwinger-Dyson approach, for example. In addition, we note that the simultaneous limits $M \to \infty$ and $e \to 0$ can be taken, in such a way that the dynamical mass remains finite, corresponding to a *physical* fermion mass.

2. – Model and solutions

The LIV model considered in [3] is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(1-\frac{\Delta}{M^2})F^{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - \tau A\!\!\!/)\Psi \ , \tag{1}$$

where M is a large mass scale that suppresses $\Delta = -\partial_i \partial^i$ and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \quad \Psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \ , \quad \tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} \ . \tag{2}$$

To begin with, the flavour states in Ψ are considered to be Dirac, but the structure of the gap equations remains the same in the case of Majorana fermions.

Then, we assume the possibility of generating the following mass matrix

$$\mathbf{M} = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix} \ . \tag{3}$$

Thus, from (1) and (3), $D_{\mu\nu}$, S and G, the bare gauge propagator and, the bare and dressed fermion propagators can be written, respectively, as

$$D_{\mu\nu} = -\frac{i}{1+\vec{p}^2/M^2}\left(\frac{\eta_{\mu\nu}}{\omega^2-\vec{p}^2} + \zeta\frac{p_\mu p_\nu}{(\omega^2-\vec{p}^2)^2}\right) \ , \quad S = i\frac{p\!\!\!/}{p^2} \ , \tag{4}$$
$$G = i\frac{p^2 + p\!\!\!/(m_1+m_2) + m_1 m_2 - \mu^2}{(p^2-m_1^2)(p^2-m_2^2) - 2\mu^2(p^2+m_1 m_2) + \mu^4}\begin{pmatrix} p\!\!\!/ - m_2 & \mu \\ \mu & p\!\!\!/ - m_1 \end{pmatrix} \ .$$

Now, neglecting corrections to the wave functions, vertices and gauge propagator, the Schwinger-Dyson equation for the fermion propagator is

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu}\ \tau\gamma^\mu\ G\ \tau\gamma^\nu \ . \tag{5}$$

Such an integral is finite as a consequence of the LIV term $\vec{p}^2/M^2$ in $D_{\mu\nu}$.

Consequently, eq.(5) leads to four gap equations which must be satisfied by m_1, m_2, μ. We solve these gap equations analytically respecting all the constraints and present, in table I, two interesting cases.

TABLE I. – *Two possible cases, where $X = \frac{8\pi^2}{4+\zeta}$ and $\lambda_\pm$ are mass eigenvalues.*

Case	$\lambda_\pm$	θ, mixing angle	Solution
I: $m_1 = m_2 \neq 0 : \epsilon = 0, e_1 = e_2, \mu^2 = m^2$	$2m;\ 0$	$\mp\pi/4(\mu = \pm m)$	$m = \frac{M}{2}\exp(-X/e^2)$
II i): $\mu = 0, \epsilon = 0$ and $m_i \neq 0, i = 1, 2$	m_i	0	$m_i = \frac{M}{2}\exp(-X/e_i^2)$
II ii): $\mu = 0, \epsilon = 0$ and $m_1 = 0, m_2 \neq 0$	$0, m_2$	0	$m_2 = \frac{M}{2}\exp(-X/e_2^2)$

3. – Discussion

Looking at table I, we note that both cases allow for finite dynamical mass generation, even when setting $M \to \infty$ and $e \to 0$. However, only in "I", oscillations take place, since for this, we need that $\lambda_+^2 \neq \lambda_-^2$ and $\theta \neq 0$. Nevertheless, in this case, one fermion is left massless and θ is maximal.

Then, calculating the one-loop fermion self-energy and setting $M \to \infty$ and $e \to 0$, we find: $\Sigma(\omega, \vec{p}) \to -[1/(4+\zeta)](\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})\mathbf{1} - \mathbf{M_R}$, where $\mathbf{M_R}$ is the "renormalized" mass matrix. Therefore, the dispersion relations are relativistic, since space and time are dressed in the same way.

Furthermore, two extensions (related to II i) and ii)) to Majorana neutrinos have been considered in [3]. One of them has shown that masses and oscillations can be dynamically generated when Ψ in (1) represents two left-handed Majorana fields. While for the other, where the fields in (1) are in the background of a Higgs field and Ψ represents a left-handed and a right-handed field, it was shown that the mechanism described here and the Higgs mechanism can be, respectively, responsible for the generation of the Majorana mass term and the Dirac mass term present in seesaw mechanisms.

Finally, we plan to extend our model to the physical case of three generations, including CP violation, to look for realistic phenomenology.

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