IL NUOVO CIMENTO Vol. 37 C, N. 3 Maggio-Giugno 2014
DOI 10.1393/ncc/i2014-11791-5

Communications: SIF Congress 2013

Boson sampling with integrated optical circuits

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ricevuto il 17 Febbraio 2014; approvato il 18 Marzo 2014

Summary. — Simulating the evolution of non-interacting bosons through a linear transformation acting on the system's Fock state is strongly believed to be hard for a classical computer. This is commonly known as the Boson Sampling problem, and has recently got attention as the first possble way to demonstrate the superior computational power of quantum devices over classical ones. In this paper we describe the quantum optics approach to this problem, highlighting the role of integrated optical circuits.

PACS 03.67.Ac – Quantum algorithms and protocols, quantum information.
PACS 03.67.Lx – Quantum computation architectures and implementations.
PACS 42.79.Gn – Optical waveguides and couplers.

1. – Introduction

The large efforts towards development of quantum simulators and, in the long term, of quantum computers are motivated by the "Feynman's Conjecture", which states that quantum mechanics is exponentially hard to simulate by classical computers [1, 2]. In this sense, quantum devices would be able to break a key statement of computer science, the Extended Church-Turing Thesis (ECT), which states that any *effective* calculation (*i.e.* any calculation performed by any physical process) can be simulated efficiently by a classical computer [3].

The first hint for this hard-computability can be searched in the very probabilistic nature of quantum mechanics. Trying to simulate a system composed of N particles, one would need to know the probabilities to find each of them at some specific positions $x_1, x_2, \ldots x_N$. Working with a discretized space of M points, this would lead to a number of possible configurations of M^N, which means an overhead of the required resources which is exponential in the number of particles to simulate [4].

More interestingly, the dynamic behaviour of quantum systems is itself believed to be hard to simulate, and at the same time it holds the promise to solve otherwise intractable

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computational problems. The most known example is the time needed to factorize a large number, which goes sub-exponentially with the size of the number on a classical computer but only polynomially on a quantum computer performing Shor's algorithm [5].

In synthesis, ECT is strongly called into question by quantum computation. Still, it has not been disproved, and the number of controllable qubits which can be realized in practice is expected to remain orders of magnitude too low in the near future. This is why scientists have begun to look for non-universal quantum computers able to outperform classical devices in specific tasks, which would be enough to dismiss ECT. To this extent, the solution could be a computational problem in a well-defined hard computational class, which have a natural efficient implementation on a quantum device.

2. – The Boson Sampling problem

Recently, Aaronson and Arkhipov introduced the so-called Boson Sampling problem [6], which we are going to describe. Consider an $m \times n$ matrix A ($m \geq n$) with complex elements, whose columns are orthonormal. We define $\Phi_{m,n}$ as the set of possible strings of m non-negative integers $T = t_1, t_2, \dots t_m$ which sum up to n. Each of these strings T define a $n \times n$ unitary matrix A_T, composed repeating t_k times the k^{th} row of A, where k goes from 1 to m. Typically, $\Phi_{m,n}$ is required to contain strings which are composed only of 0 and 1 (*collision-free* scheme). The Boson Sampling problem consists in fairly sampling from the probability distribution over $\Phi_{m,n}$ given by the following formula:

$$P(S) = \frac{|\mathrm{Per}(A_T)|^2}{t_1!t_2!\dots t_m!}\,. \tag{1}$$

The permanent of an $n \times n$ matrix A is given by

$$\mathrm{Per}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}, \tag{2}$$

where S_n contains all permutations of the integers $1, 2, \dots n$. Simply speaking, the permanent is defined similarly to the determinant, with plus signs on all terms of the summation instead of alternating signs.

The motivations for studying this problem are the following.

2$\cdot$1. *Hard computability.* – It is well known that the calculation of the permanent is a $\#P$-complete computational problem [7,8]. $\#P$ identifies the complexity class of finding the number of solutions to an NP problem, which in turn are the problems for which a solution can be guessed and verified in polynomial time by a non-deterministic algorithm. Every problem in $\#P$ can be reduced to a $\#P$-complete problem in an efficient way, which means that the calculation of the permanent is "at least as complex" as any other $\#P$ problem.

All decisions problems solvable by a probabilistic algorithm in polynomial time with a bounded error probability are in BPP. A complexity class of the form A^B, where A and B are two given complexity class, includes all problems that can be solved by algorithms of class A having access to an *oracle* able to solve problems of class B. An *oracle* is a sort of "black box" which can solve certain decision problems, belonging to a specific class, with a single operation. For example, the class $P^{\#P}$ consists of all the problems which can be solved in polynomial time, having access to instantaneous answers to any counting problem in $\#P$. This defines a more general set of classes, which can be ordered in what is called the "polynomial hierarchy" in terms of complexity.

It has been demonstrated [6] that if one could find a classical algorithm able to solve Boson Sampling with polynomial resources, this would imply $P^{\#P} = \mathrm{BPP}^{NP}$ and the consequent "polynomial hierarchy" collapse, a striking result in the theory of computational complexity which is considered extremely unlikely.

2˙2. *Physical analogous.* – It has been proven by Knill, Laflamme and Milburn (KLM) that universal quantum computation could in principle be performed using only linear optics, single-photon sources and detectors, and effective nonlinearities induced by measurements [9]. To perform Boson Sampling, moreover, not only linear optics approach is natural, but the adaptive measurements to produce nonlinearities are not necessary. Actually, Boson Sampling can be seen as a sort of *ad hoc* hard computational problem, because of its natural implementation through a linear optical quantum circuit. In fact, its abstract mathematical formulation resemble the evolution of n bosons distributed on m paths of a linear interferometer. To see this, consider a unitary transformation U, described by an $m \times m$ matrix, acting on the Fock state of n identical bosons distributed on m spatial modes. If we limit ourselves to the *collision-free* case, so that we have no more than one boson in each mode, both initial state (S) and final state (T) can be described by a string of zero and ones of lenght m, which sum up to n. The transition amplitude between S and T is given by the formula

$$\langle T|U|S\rangle = \frac{\mathrm{Per}(U_{S,T})}{\sqrt{s_1!s_2!\dots s_n!}\sqrt{t_1!t_2!\dots t_n!}}\,. \tag{3}$$

This means that sending n photons through a m modes linear optical interferometer and collecting the output with single photon detectors is equivalent to sample from the distribution 1. The $m \times n$ complex matrix A defined at the beginning to describe the Boson Sampling problem is a submatrix of the $m \times m$ unitary matrix describing the whole interferometer, given by the n columns corresponding to the input modes. The protocol is naturally efficient in the sense that the required resources scale polynomially with m and n. At the same time, a quantum Boson Sampler possesses the hard-to-compute characteristics related to the Feynman conjecture, since the dimension of Hilbert space describing the photonic Fock state is exponential in n.

Even considering only the complexity related to the calculation of the permanent, it turns out that in the regime $n \simeq$ 20–30, $m \simeq 400$ the complexity of the system is so high that a quantum device would start to outperform a classical computer in the task of solving the Boson Sampling problem [6].

3. – Experimental implementation

Even if not as hard as building a universal quantum computer, the experimental challenges in reaching the required numbers of photons and modes are indeed huge. Novel techniques for the realization of integrated optical circuits hold the promise to allow the realization of large optical interferometers in the forthcoming years. Nevertheless, the realization of a photon source able to generate 20 indistinguishable photons is indeed far from the present technology. At the same time, the Boson Sampling research is still in its embryonic stage, and it is very likely that in the near future new experimental implementations, conceptual schemes or even different computational problems will enter in the game. Some examples of the latest developments will be shown in sect. **4**.

Since the problem formulation, there has been a rush of experimental activitiy [10-13] to demonstrate the in-principle feasibility of experimental Boson Sampling. The general approach is the same in all the experiments: three or four indistinguishable photons are generated through the second-order nonlinear process of Spontaneous Parametric DownConversion (SPDC), injected into a 5- or 6-mode integrated interferometer, and detected at the output with single photon avalanche photodiodes.

3˙1. *SPDC multiphoton sources.* – The largest part of quantum optics experiments currently rely on the SPDC to generate single photons [14, 15]. The process, which is the result of second-order interaction of light with a nonlinear crystal, consists in the conversion of a photon from the pump beam in two photons of lower energy. The efficiency of the process depends from the second-order electric susceptibility $\chi^{(2)}$ of the material, ranging around 10^{-8}–10^{-10}. These very low values demands for very strong pump pulses, which are typically given by mode-locked lasers.

The SPDC process can be used as an *heralded* single photon source: it cannot generate a single photon on demand (the process is nondeterministic) but the detection of one of the two generated photons (which is called idler) informs us that a second photon (signal) has been generated. Such nondeterministic sources, while not good for quantum communication tasks, are useful for quantum computation problems like the Boson Sampling. The main problem with SPDC is the scalability: the generation rate of more than one pair of photons decrease exponentially with the number of photon pairs, so that, in practice, it is not possible to reach a signal of more than 4–6 photons.

The first proof-of-principle implementations of Boson Sampling [10-13] have made use of SPDC sources because of their reliability and accessibility. However, to reach the required number of 20–25 photons, other approaches will be necessary.

3˙2. *Integrated interferometers.* – It is important to note that the demonstration of the hardness of Boson Sampling reported in [6] is given in a probabilistic way: not *all* unitaries give rise to a hard Boson Sampling problem. The most trivial example can be the identity matrix. In general, any regularity in the unitary A_T in eq. (1) could simplify the problem, spoiling the computational complexity. Because of this, the $m \times m$ interferometer matrix has to be chosen randomly, with respect to the Haar measure, in the space of the unitaries, which requires a very good control over the engineering protocol of the interferometric device.

It has been demonstrated [16] that any unitary transformation acting on the Fock state of a photon ensemble can be implemented using only beamsplitters and phase shifters. However, the realization of a large interferometer with bulk optics is not feasible in terms of subwavelength phase stability. On the other hand, techniques for realization of integrated optical circuits, the analogous of integrated electronics for quantum optics, are becoming more and more powerful and reliable [17-20]. The monolithic nature of this devices makes them scalable and stable solutions for complex quantum circuits.

An integrated beam splitter, called directional coupler, can be realized writing two wave guides which are put close enough, with two S-bends, along a specific region in which the light is redistributed from one guide to the other via evanescent coupling. A different choice [13] consists in crossing the two waveguides with a specific angle, which will give a specific transmissivity. These structures, called cross-couplers, require shorter interaction regions and less bends, thus reducing the size of the chip and the photon losses [21].

The realizing process depends from the choice of materials. Silica-on-silicon waveguides [22] are realized through the deposition of a layer of a doped silica (core material), over a substrate of undoped silica (buffer material). The core is then shaped via UV lytography and finally overgrown with another layer of buffer material.

A simpler process, which does not require multiple layers deposition and is thus less similar to the electronic chip fabrication technique has recently been developed. The direct focusing of near-infrared femtosecond laser pulses in the bulk of a borosilicate [10] or pure silica glass [11] induces localized permanent change of the refraction index through nonlinear absorption [23]. A waveguide of arbitrary geometry can be produced by translating the chip during the laser focusing, with constant velocity, following the desired path. In particular, the possibility to write tridimensional structures open a whole new chapter for realizating complex quantum circuits [24, 25]. In femtosecond laser written interferometers, the trasmittivity of a directional coupler can be tuned by changing the distance between the two waveguides in the interaction region. This, in a 3D structure, can be done by rotating one arm of the waveguides respect to the plane of the circuit. In this way the control over the transmissivities can be made without modifying the length of the waveguides, so that the phase of the photons is left unchanged. At the same time, the shape of the S-bends can be enginereed to give a longer or shorter path, and thus arbitrary phase shifts, without modifying the transmissivities. In synthesis, the 3D capabilities of laser writing allows to control independently the transmissivities of the directional couplers and the phase shifts, which is mandatory if one has to implement an arbitrary unitary transformation.

At the same time, it is important to be able to efficiently characterize a given device, either to check the reliability of the fabrication technique or to know information about the unitary for validation purposes (see next section). The scheme reported in [26] gives a receipt to reconstruct transmissivities and phase shifts of the real chip from one- and two-photon measurement, respectively [10].

4. – Current status, open questions and perspectives

The main open point concerning the possibility of disproving ECT with an optical quantum device is clearly the scalability. The realization and operation of ~ 400 modes integrated interferometers, even if posing hard engineering challenges especially in terms of losses control, is expected to become feasible in the forthcoming years. On the other hand, scalable multiphoton sources are a critical point for all optical realizations of quantum information high-level tasks. Compared to other goals (one for all, the realization of a universal quantum computer) Boson Sampling has the advantage of being not (yet) scalable only in the number of photons, requiring an increase only of a factor of ten. Still, this gap has to be filled in some way.

A possible solution can rely on multiplexed SPDC parallel sources [27-29]. A recent scheme [30] use m synchronized SPDC sources, using idlers as triggers and connecting each signal to one of the m mode of the interferometer. This allows an enhancement of the photon generation rate, but the input mode string T is not fixed anymore. It is believed [30] that this "generalized" Boson Sampling scheme is still hard to compute. In any case, the same implementation can be modified adding a fast optical multiplexer routing the modes corresponding to detected idlers to the desired input state T [31]. With this scheme, a number of SPDC parallel sources of about 200 is expected to reach the threshold of 20 photons events with high probability for each pump pulse. In this sense, it has been argued that the bottleneck for scalalability of Boson Sampling experiment is given by the efficiency of photon detectors, rather than photon sources.

Finally, Boson Sampling raise a relevant problem of verifiability. As highlighted in [32], since the whole process is hard to simulate on a classical computer, when reaching the hard regime it could become hard also to verify that the device is working properly. First steps in solving this problem are reported in [33], where an efficient procedure to distinguish Boson Sampling data from a random, uniformly distributed output is depicted. The crucial point in this validation scheme, and most likely in the upcoming ones, is the efficient use of information about the unitary matrix. In fact, it has been demonstrated in [32] that in the so called "black-box setting", *i.e.* when no information about the unitary is known, even validating against uniform distribution cannot be done in an efficient way. As a consequence, the ability to control the implemented unitary, or at least to reconstruct it in an efficient way is again of fundamental importance. Following this approach, structures implementing specific not Haar-random unitaries can be used to check the genuine quantum behaviour of the Boson Sampling apparatus [34].

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