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From light polarization to the basic ideas of Quantum Mechanics. Experience carried out in a Technical Institute in Scampia (Naples)(*)

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Summary. — The article focuses on teaching/learning Quantum Mechanics (QM) in secondary schools and involves two fundamental aspects: teacher training and experimentation in the classroom, carefully reflecting on the content and highlighting innovation in the teaching methodology. The experience of training took place within a 2nd level post-graduate Master course for Physics teachers (The II level post-graduate Master course in Didactic Innovation in Physics and Guidance (Innovazione Didattica in Fisica e Orientamento IDIFO) `http://www.fisica.uniud.it/URDF/laurea/index.htm`) is directed by M. Michelini. The University of Udine organizes the course with the collaboration of the Physiscs Education Research Groups of eighteen Italian Universities, and is a part of a training project addressed to physics teachers focused on modern physics. IDIFO is part of a national plan of the Ministry of Education, Research and University for the Scientific Degrees project (Piano Lauree Scientifiche)), based on a module dedicated to teaching/learning QM according to the fundamental concepts of Dirac theory. The teacher achieved formation attending three e-learning courses about teaching/learning quantum mechanics in secondary schools, researching and discussing research materials of physics education, planning an intervention module and experimenting in a real classroom the planned innovative path for a situated formation. The experimentation, carried out in a fifth class of a Technical Institute at Scampia (Naples), made the conclusive phase and gave data concerning the thinking ways of the students about some basic concepts of QM collected and evaluated. In particular, average students focused more the functional/applicative aspects of formal and conceptual constructs rather than on their physical meanings.

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1. – Introduction

The present work concerns the introduction of Quantum Mechanics (QM) in secondary schools [1-7], according to the European and national indication on the need to deal with Modern Physics in upper secondary school curricula [8-10]. It also reports about the students conceptions [11] on QM basic ideas and its revolutionary way of looking at reality [12].

This report deals with educational experimentation on QM carried out in a school of Scampia (Naples), and is part of a long-term research effort on the best way to introduce Modern Physics in secondary schools that is still under discussion in school praxis [1, 2, 4-6, 13-16]. The didactic path experimented introduces QM through the conceptual approach based on Dirac theory [16-19, 24, 25]. According to the working hypothesis of this experimentation, the conceptual approach to QM could help overcome the well-documented difficulty of learning concepts due to mathematical formalism [18, 19]. Professional skills of Italian teachers do not yet include such topics, and in fact, they do not usually deal with them at school. When they approach quantum concepts, they only tackle them with a descriptive, qualitative approach, more focused on the Physics of quanta rather than on Quantum Theory [20, 21]. It is common opinion that to learn Modern Physics, one needs to deal preliminarly with classical physics. Research evidence and many experiences in this field [22] show that it is possible to pursue high-level goals, often considered too complex and ambitious, if the teacher plans the work considering the context, the type of school and the socio-cultural environment of the students.

This educational experimentation is a challenge within a challenge for two reasons: first because QM is an unusual topic in secondary schools, and, secondly because Scampia is a very deprived area of Naples and students live in close contact with organized crime.

It may seem quite difficult to conceive an innovative teaching proposal in such a context because of the surrounding conditions.

Nevertheless, formative practice is very important as a driving force to promote the emancipation of young people who live in difficult social environment [21]. In this context, school plays a very important role because it offers alternative social and cultural models to students. An innovating and inspiring educational offer is crucial for students in such contexts. Proposing contents and operating methods distant from students usual learning experiences was another challenge. In vocational education, the purpose of students training is letting them acquire professional skills so that they may be able to find a job as soon as possible. A conceptual approach to facts represents an additional chance for these students, even if they have little knowledge of QM theory, mostly related to technological applications (as for instance, tunneling). In spite of the intrinsic difficulties of QM formal contents, the project produced positive results both in terms of students engagement and of educational research outcome, according to the evidence discussed in this paper.

2. – The formative experience

The experimentation described below is the result of a training course held as part of the master IDIFO3, to support teachers' training and diffusion of innovative educational practices. The University of Udine promoted and coordinated the master; it has duration of two years and involves 18 universities on various issues of General Physics. The courses are based on e-learning address the various topics on the epistemological level and methodological one, with particular attention to the development of didactic experimental activities to be led in classrooms, both to increase the effectiveness of educational interventions and to investigate thinking ways of the students.

Three modules of IDIFO3 Master are the basis of the experience described herein.

I. Didactic proposals on QM: comparative analysis of research based didactic projects for teaching/learning QM documented in the literature [16-26]. This module was provided in order to discuss how to select QM's topics to be proposed at different high school and undergraduate levels focusing on the main difficulties of students in learning QM.

II. The new way of thinking of QM and the Dirac formalism that is a critical analysis of a research-based didactic proposal aimed at the fundamental concepts of QM, which has been considered as reference for didactic planning [17-24]. This module was essential to implement the didactic experimentation with students, provided the theoretical basis for developing the trail, and prepared teaching materials [27].

III. The conceptual knots in QM that is the analysis and reflection of the conceptual knots in QM aimed at the didactic planning. The main knots in QM, at the basis of this module, are four. The first one is looking for elements that characterize the quantum behaviour. The second one is recognizing that the knowledge of their properties does not imply the existence of quantum systems. The third one is how to change the function and meaning of the measurement in QM with respect to classical physics. The fourth one is the significance of the result of a measurement based on its predictability and on the objective nature of the measured properties [17].

In addition, an accurate analysis of the didactic materials helped to investigate on the students thinking ways about the meaning of quantum states and their properties, and of the superposition principle.

Finally, the participation, as an observer, in the Summer School of Modern Physics for high-school students (Udine, 2011), was very useful for the purposes of training, because it was possible to see how to implement the proposed teaching model and experience its effect on students [24, 25].

3. – The class context

As mentioned before, the experiment was carried out in a fifth year class in a technical institute of Scampia composed of 12 students, eight boys and four girls, attending the Electronics and Telecommunications course([1]). They had acquired concepts of basic Physics, Mechanics and Electromagnetism at the level of the first two years of secondary school and had attended the Electronics courses during the last three years. Then, the purpose of work's design consist of fitting it to the course in progress.

The overview topics referred to those already treated in the course of Telecommunications([2]) to motivate and encourage students to take the test. In particular, the tunnel effect, the wave-particle dualism, the de Broglie equation, the light quanta and Schrödinger's cat paradox.

([1]) The formative course was oriented to design, construct, operate, test and maintain electronic systems based on electrical parameters susceptible to long-distance transmission, as well as the realization of circuit components with logic functions. The goal of this course is the creation of professionals able to design and manage systems relating to signals transmission, *via etere* and cable telecommunications.

([2]) Mr. G. Ciardo, Telecommunication teacher.

4. – Objectives and working hypothesis

The double objective to face is, as a teacher to introduce a subject matter seldom treated in high schools and not at all in the secondary ones and, as a scholar, to review the fundamental methodological implication in tackling a very difficult issue at the formal level, by preferring the conceptual approach based on Dirac's formalism [15-17]. The methodological choice, started from the phenomenology of polarized light, encouraged the students involvement to achieve the third goal: to inspire students to compete in a field of Physics, —Quantum Mechanics— fascinating for them, but far from their educational and cultural experience. The proposed work, at least in the initial phase, captivated the students because it made QM easier to understand and understand phenomenology and facilitated students approach to the construction of more theoretical concepts of QM.

In particular, students were stimulated to build interpretative models:

a) to recognize and debate the experimental results of real and ideal experiments;

b) to elaborate different hypotheses on the phenomena to be analysed;

c) to select proofs and procedures in order to understand the topics;

d) to compare their hypotheses with the experimental results [20-22, 24-26].

The practical way of working provided students with tasks of increasing difficulty to deal with, until they reached the following specific objectives:

a) developing their cultural knowledge to promote interdisciplinary links and improvements about Quantum Physics, or in general about the cultural revolutions of the 20th century;

b) stimulating some reflections on the transition from Classical to Quantum Physics;

c) describing a photon as a quantum system by constructing its dynamic property to define its quantum state from light polarization state;

d) progressively achieving formalism by using the iconographic representation of quantum photonic properties.

5. – The didactic experimentation with the students

The educational path is inspired to the basic ideas of the Dirac approach, constructed in the phenomenological context of light polarization [16-19]. It originates by the concept of polarization states of light to introduce dynamic properties of the polarized photon, considered as a quantum particle, to which associate two fundamental orthogonal states, with their own properties, and any other state as a superposition of fundamental states. The analysis of the light polarization phenomenology led the construction of the photonic quantum concept.

Introducing formalism by a suitable iconographic representation of quantum state and its property [15-20], has helped to overcome any difficulties with the formalism.

Fig. 1. – The maximum light intensity transmitted by two aligned Polaroid filters with parallel transmission axes and the minimum intensity in case of two overlying Polaroid filters whose axes form a 90° angle.

The experimentation with the students started from the exploration of light Polaroid([3]) filters interaction, using experimental kits, purposedly designed and produced in multiple copies for active students learning([4]).

At the beginning, students observed the phenomenon of polarization by looking at what happened when the light, emitted from the projector, crosses the Polaroid filter laid on an OHP (fig. 1). During that initial activity, a guided discussion has highlighted the results from which to begin the phenomenological analysis.

All students were encouraged to look through the Polaroid, in order to investigate how the filter "influenced" the light in the room. By discussions in small groups, they found that, "apart from a light attenuation", nothing else happened, even by rotating the filter in every direction, so they tried again on the OHP changing the position of the Polaroid. They went on discussing about light attenuation, until they noticed that the filtered light ranged from a maximum to a minimum when two superimposed filters formed an angle that ranged from 0° to 90°.

By these first experimental observations, students were able to distinguish the properties of polarized light. They learned to identify a transversal direction (permitted or transmission direction) on each Polaroid by using a Polaroid as analyser, rotating it around the light propagation direction, as shown in fig. 2([5]).

In this way, the students learned to attribute an active role to the Polaroid filter by recognizing that it changes the polarization properties of the light crossing it: In a plane parallel to the permitted direction of the analyzer, when the light is transmitted; in a plane orthogonal to the permitted direction of the analyzer, when the light is absorbed, as shown in fig. 3.

As a result of the active role of the Polaroid, students concluded that: if two Polaroid filters are superimposed, light is totally transmitted if and only if the two transmission

([3]) A Polaroid filter is a plate of plastic material capable of transmitting only the component of the electric field parallel to a particular direction, called filter's polarization direction. The component along the direction perpendicular to the direction of polarization is completely absorbed.

([4]) The Polarization Kit is a tool for teaching/learning QM and is available in several copies in the context of the PLS-IDIFO, by the URDF of the University of Udine. This material is available for the schools.

([5]) The research group of the University of Udine (Italy) has granted figs. 2 to 5.

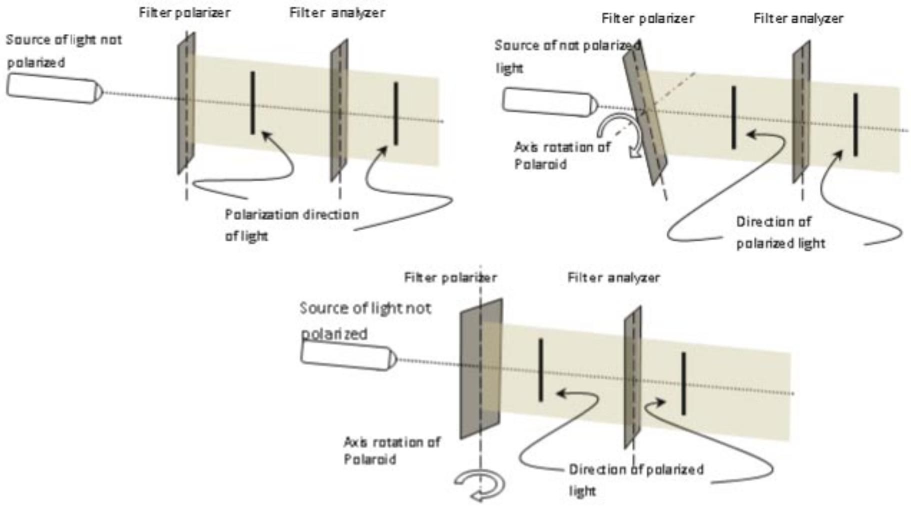


Fig. 2. – The light filtered from a Polaroid shows a polarization whose direction is orthogonal to that of the propagation of light and perpendicular to the plane defined by the direction of propagation and the polarization direction of the polaroid.

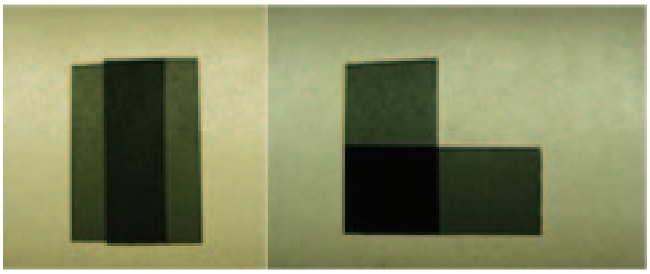


Fig. 3. – Two superimposed Polaroid filters, parallel on the left, perpendicular on the right.

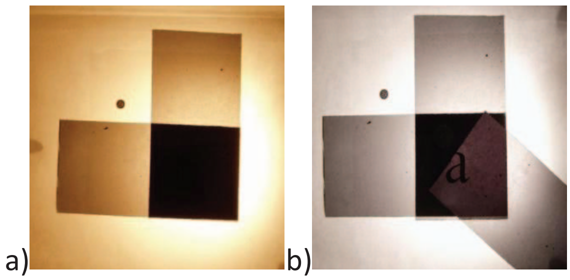


Fig. 4. – a) Two superimposed and crossed Polaroid filters. b) A third Polaroid is placed between the two first ones, its permission axis forms a 45° angle with respect the other axes.

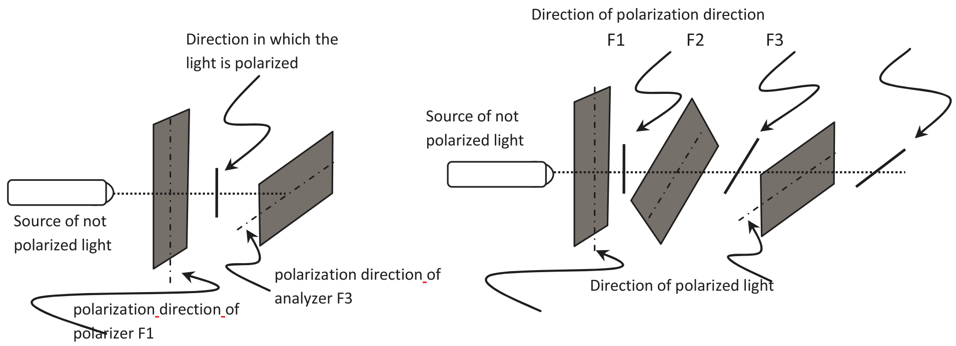


Fig. 5. – Left: light polarized by two Polaroid filters F1 (vertical transmission) and F3 (horizontal transmission). Right: light polarized by three Polaroid filters F1 (vertical transmission) F2 (45° transmission) and F3 (horizontal transmission).

axes are parallel; light is totally absorbed if the two filters are orthogonal (fig. 4a) and light is partially transmitted if they form a 45° angle (fig. 4b).

In fig. 5, the schematic representation shows a synthesis. In the left panel, the analyzer and polarizer F1 and F3 are crossed, filter F1 polarizes the non-polarized light according to its polarization direction. If the polarization direction of analyzer F3 is orthogonal to the direction in which the light is polarized, then it absorbs all the light. In the right panel, if the light transmitted from F1 polarizes orthogonally to the F2 polarization direction, then F2 partially transmits it with a bias determined by the F2 polarization direction. F3 partly transmits the light incident on the analyzer F3 with a polarization direction determined by the permission axis of the Polaroid. The polarization directions of the light transmitted by F1 and that transmitted by F2 are mutually orthogonal:

$$I(\theta) = I_{\max} \cos^2(\theta),$$

where $I(\theta)$ is the intensity of the polarized light and $I_{\max}$ is the maximum intensity detected. Measuring the intensity of the transmitted light by more parallel Polaroid filters allows to study the passive nature of Polaroid filters. The experimental data show that a Polaroid attenuates the polarized light, which impinges upon it by a constant factor T, also called transmission coefficient, given by

$$T = I_{\text{transmitted}}/I_{\text{incident}}, \qquad I_{\max} = I_{\text{incident}}T, \qquad I(\theta) = I_{\text{incident}}T\cos^2\theta,$$

where $I_{\text{transmitted}}$ is the transmitted light's fraction, I_{incident} is the incident light and θ is the angle between the axes of the superimposed Polaroid filters.

The $\cos^2\theta$ factor, which is an experimental result of Malus law, characterizes the property of polarized transmitted light according to a well-defined direction. The T factor takes into account all the processes that inevitably occur in the interaction of light by whatsoever means such as reflection, diffusion and absorption.

At the end of the initial phase, the students had learnt that polarized light had a well-defined property explored by using a third superimposed Polaroid (analyser). The initial explorations were led with specific student work-sheets, designed according to the

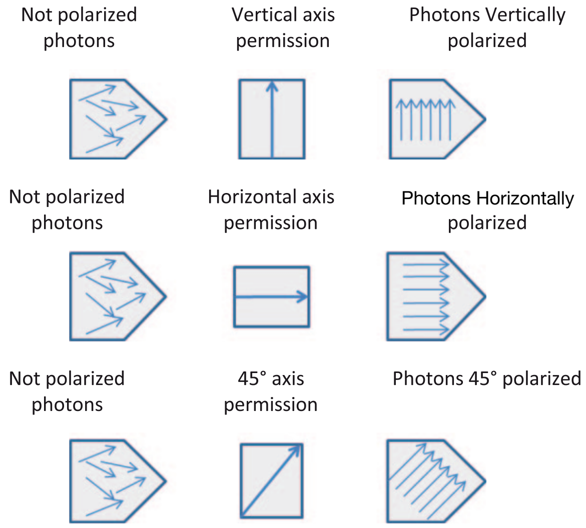


Fig. 6. – Representation of photons polarization states by arrows according to the transmission direction of filters.

PEC [30] strategy([6]) in order to stimulate students to try out to construct their own interpretations of the phenomena, Malus law, that might be analysed by using online light sensors [22], was introduced to carry out a quantitative analysis of the interaction between light and Polaroid. In addition, in this case the students were encouraged to think on their observations by answering to targeted questions. They had to examine several cases to describe the light-Polaroid([7]) interaction and predict effects, and finally they had to analyse cases with two or three Polaroid filters layered and rotated relative to one another at different angles, by applying Malus law.

Questions to small groups of students bring out their ideas and entice some of them to repeat experiments by looking for answers not only theoretical. During this activity, it was clear that the assignment was functional, not only to apply an abstract formula, but also to check its validity in real cases.

Then Malus law was applied to the interaction between single photon and perfect Polaroid. Now the problem was to make the introduction of the subject of photons plausible, that would become the "physical objects" to be treated from the point of view of QM([8]). In proceeding from light to photons, Malus law could give an interpretative model, in terms of probabilities, to predict whether polarized photons have a certain probability to pass through the Polaroid. To do that, it was necessary to describe a polarized photon as a particle with appropriate qualities. In other words, the photon has a dynamic property that uniquely describes its state of polarization after crossing the

([6]) PEC: Prediction, Experiment and Comparison.
([7]) Firstly, thoughts experiments with real Polaroid and then with perfect one.
([8]) The shift from the photon beam case to the single-photon case was very delicate both conceptually and operationally, and it required special attention in proposing it to the students. It refers to real experiments with light detectors to show that by decreasing the intensity of the beam more and more, it can be considered as made by a single photon.

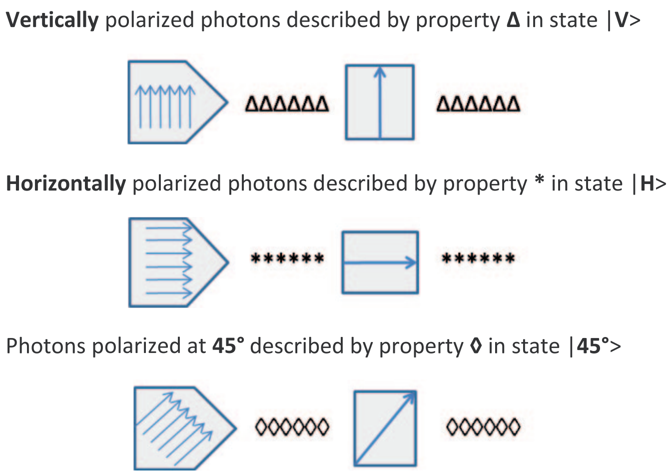


Fig. 7. – Representation of photons properties according to the transmission direction of the filters.

Polaroid. The Malus law, applied to ideal experiments with photons, is

$$N_t = N_i \cos^2 \theta,$$

where N_i and N_t represent the numbers of incident and transmitted photons. In this way, the factor $\cos^2 \theta$ is associated to the probability of a given Polaroid to transmit polarized photons.

The next step was to represent the polarization states of the polarized photons with arrows as shown in fig. 6.

States representation by vectors was used to assess the probabilistic forecasts of measurements using the square of a scalar product, according to Malus law. As described later, mutually orthogonal vectors are associated with mutually exclusive polarization properties.

To identify the polarization state as a quantum state of a polarized photon it is necessary to assign it a dynamic property univocally determined([9]). To complete the construction of the quantum state concept, an iconographic representation of the properties of the single photon polarized in different states has been introduced [15, 18, 19], as shown in fig. 7.

This representation provided students with a powerful tool to build their own interpretative hypotheses when they reviewed the photons-Polaroid interactions with thought experiments and compared their results with the experimental ones (application of Malus law). At the end of the activity, students realized that:

([9]) According to the aim of the educational path, degenerate states are not considered.

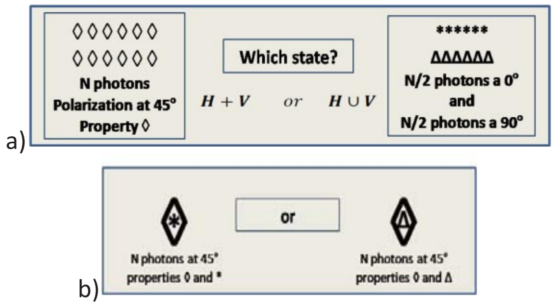


Fig. 8. – a) Statistic mixture of photons with two different properties. b) Photons that have two simultaneous properties.

a) When photons, prepared in the $|V\rangle$ state([10]) and with property $\triangle$, interact with a Polaroid-V([11]), they are always transmitted, they are totally absorbed by a horizontal Polaroid, and finally they are transmitted in half the cases by a Polaroid-45°.

b) When photons, prepared in the $|H\rangle$ state and with property $*$, interact with a Polaroid-H, they are totally transmitted, they are totally absorbed by a Polaroid-V, and finally they are transmitted in half the cases by a Polaroid-45°.

c) When photons, prepared in the $|H\rangle + |V\rangle$ state with property $\diamondsuit$, interact with Polaroid-45°, they are completely transmitted, they are transmitted in half the cases by a Polaroid-V or Polaroid-H.

At this point, thought experiments were proposed and the predictions of the experimental results discussed in small groups. Malus law, used probabilistically, has allowed to verify the results. At the end, a very important result was achieved: properties $\triangle$ and $*$ were mutually exclusive. The used formalism helped specifying that vectors that would constitute the base of the abstract two-dimensional space of the photons states could represent the two polarization states, horizontal and vertical. In this regard, the students scripts did not give significant results about their ways of thinking, suggesting that, being a crucial point, the vector space construction would have required a further in-depth analysis in class.

The next activity is focused on the description of photons in the $|45°\rangle$ state, to understand how to build the quantum state in that specific case, as a superposition of two orthogonal states, Vertical and Horizontal. The starting point was the ideal experiment: it has a certain number of photons in the $|45°\rangle$ state and it is required to describe the quantum properties owned by photons when they are prepared in that specific state.

The starting point was the discussion about two hypotheses on the nature of such photon state. The first one stated that the photons set was composed of a statistical mixture of photons having $\triangle$ and $*$ properties, the second one stated that the set was done with photons that had simultaneously $\triangle$ and $*$ properties in equal weight as represented in fig. 8.

([10]) Photon states are represented by bra-ket notation *i.e.* vertical polarization $|V\rangle$, horizontal polarization $|H\rangle$ and 45° polarization $|45°\rangle$.
([11]) The transmission axis of Polaroid is, respectively, Vertical, Horizontal, and 45°.

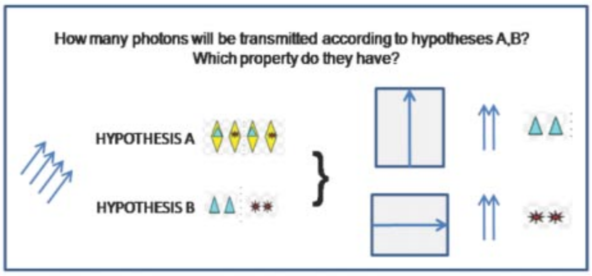


Fig. 9. – The experimental results: only photons with property △ are transmitted by the vertical Polaroid; only photons with property ∗ are transmitted by the horizontal Polaroid are reproduced by each of the two hypotheses.

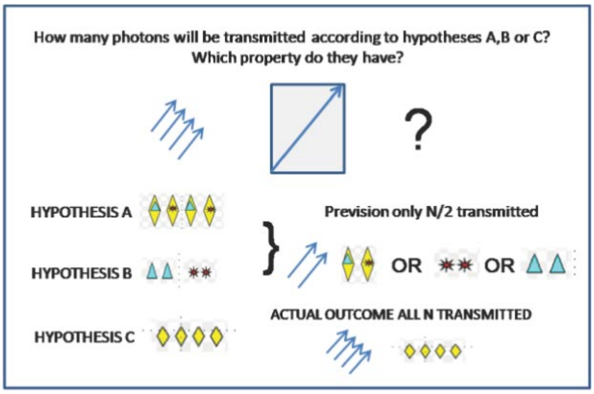


Fig. 10. – No agreement between predictions according to the hypotheses and experimental outcomes.

In this case, students discussed the predictions on the photon-Polaroid interaction, by using Malus law and compared them to the actual results.

According to Malus law, if N photons in $|45°\rangle$ state interact with Polaroid-V, then only $N/2$ are transmitted and, according to each hypothesis, they will have △ property, or ∗ property as shown in fig. 9.

On the other hand, if same photons interacted with Polaroid-45°, only $N/2$ photons would cross the filter, but this does not correspond to the experimental result, in fact, all the photons ♢ would cross the Polaroid-45°. In addition, the iconographic representation has been the model to depict the situation, as shown in fig. 10.

Students analyzed the thought experiments by completing specific forms and discussed them in small groups. At the end they realized that photons with properties ♢ could not ever own property-type ∗ or △, meaning that the properties ♢ and △ are incompatible([12]), and that a photon in $|45°\rangle$ state is in its own state, given by the superposition of two independent and orthogonal states, $|V\rangle$ and $|H\rangle$. This result has been achieved by constructive discussions among students and with the teacher, because debates directed their ways of thinking pointing out that, properties ∗ and △ were mutually exclusive and

([12]) Similarly to the two properties ♢ and ∗.

that each of them is incompatible with the $\diamondsuit$ property.

The interaction process between the Polaroid and the photon, proposed as a paradigmatic example of a quantum measurement process, has found its properly done interpretation as a transition between states (from the preparation state to the one in which the system has been found after the measurement).

6. – Strategies and methods classroom work stages

As explained above, the starting point was the phenomenological investigation of simple experiments explored first operationally with real Polaroid and then by thought experiments. This strategy was useful to support the interpretative hypotheses and subsepuently the construction of the formal entities required for modelling those interpretative hypotheses. Therefore, the class work dynamically developed alternating different moments based generally on interactive didactics, which were structured into: cooperative learning, brain storming, teaching workshop, debriefing, documentation. After introducing a topic, a guided discussion started in order to search for the most significant aspects of an experiment, both thought and real, to elaborate individual hypotheses by discussing, in little groups, spontaneous ideas and conveying each contribution towards common aims and shared results.

The worksheet was the main tool to start a constructive process, based on the activation of the PEC cycle, through the integration of stimulating questions and mini-paths based on the analysis of standard situations to draw general rules and conclusions to describe different results.

Seven worksheets were used [29] concerning several topics. The first four were about the polarization phenomenology on the operative introduction of polarization, the active/passive role of the Polaroid filters in the light interaction (worksheet W1), the probabilistic interpretation of Malus law (worksheet W2) and the situation analysis based on such interpretation (worksheet W3). The other ones regard the polarization dynamic property and the mutual exclusive properties (worksheet W4), the formulation of interpretative hypotheses on the polarization $|45°\rangle$ state (worksheet W5), the incompatible properties and the superposition of states (worksheet W6) and the superposition principle and quantum indeterminism (worksheets W7-W8).

The worksheets had a double role: encouraging the students to look for the cause of the discussed phenomena, directly/indirectly observed, and providing the students with the didactic researching material according to their ideas and their ways of reasoning. They went on speculating and trying, reasoning about the stimulus-questions, debating peer to peer or with the teacher, in order to elaborate predictions or plan experimental tests. With all these constructive activities, the students were at the centre of the learning/teaching process and had an active role to construct their own knowledge and solve the proposed problems.

The expert's and teacher's roles were generally different, in fact the former presented the examined situations and guided general discussions and discussions in small groups, while the latter helped students to feel self-confident with the general learning experience by supporting them to overcome their own difficulties. Now a more detailed description of the activities follows.

The first two meetings have been decisive to incite students towards the proposal and adapt it to the context class, and introduce immediately the pivotal concept of quantum state and the linear superposition principle of states.

The didactic proposal was divided into four segments: The first segment dealt with some QM key points, energy quanta, wave equation, the uncertainty principle, determinism or indeterminism, Schrödinger's cat paradox, from the light polarization to QM. The topics were chosen in agreement with the class teacher (Mr. G. Ciardo) in order to refer to the topics already studied and reinforce students motivation.

The second segment dealt with the qualitative exploration of the light-Polaroid interaction, by placing some filters on an OHP (fig. 1) and later only in small groups with a discussion, both free and guided, to elicit questions and reflections on the properties of polarized light, by discovering them through the passive/active role of the Polaroid.

As mentioned previously, besides direct observations, students have filled worksheets by following the PEC cycle in order to construct a first interpretative qualitative model of the light-Polaroid interaction.

The third segment focused on the thought experiments with photons, and students investigated the interactions with Polaroid in probabilistic terms by applying Malus law at a discrete number of photons. Finally, students used a different worksheet, based on an IBL filling schedule, to explore several ideal experiments by themselves. The schedule has been the source of data on which educational research is based.

The next segment aimed to deal with a very important conceptual problem: how to construct the quantum state concept of a polarized photon and how to recognize its dynamic properties. Moreover, the analysis of focused ideal experiments was performed to recognize the incompatible and mutually exclusive properties of suitably prepared photons. The elicitation of problem in a collective discussion set students out to reach their own elaborations to complete the student forms. The analysis data on students answers and their ways of thinking is the subject of the next section.

7. – Tools and methods of data analysis

Examination of students scripts in the worksheets, their individual interviews and videos recorded during the experiments conveyed the evaluation of the trial. The tutorial or worksheets [28], based on the PEC cycle to provide both the working tool to the students and the survey investigation instrument, dealt with the following topics and learning objectives:

Worksheet W1 Investigating the phenomenological light polarization, after having observed the light-Polaroid interaction on OHP to recognize how to produce polarized light, to identify the active/passive role of Polaroid, to describe polarized light, to associate the polarized-light property to the direction of the transmission axis of Polaroid, to formalize the previous results.

Worksheet W2 Constructing probabilistic interpretation of the quantum measurement process (single-photon processes) and reinterpretation of Malus law, and to construct quantum interpretation and associate probability $P(\theta)$ to $\cos^2\theta$.

Worksheet W3 Summary of Polaroid-photons interaction and the probabilistic interpretation in order to apply Malus law with two and three Polaroid filters and calculate the transmission probability.

Worksheet W4 Ideal experiments and iconographic representation in order to identify photon polarization by the Polaroid transmission direction, to recognize sure outcomes and photons properties, to describe polarized photon by its polarization state, to construct the property corresponding to the polarization state and represent a given state and its own properties([13]).

Worksheet W5 Thought experiments to recognize mutual exclusive properties in order to discuss thought experiments to identify exclusive properties owing to orthogonal states and construct Hilbert's space of *orthonormal* vectors.

Worksheet W6 Two interpretive hypotheses (statistical mixture of pure states) to examine incompatible photons properties in the $|45°\rangle$ state, to find the inconsistency between prediction and outcome, and the incompatibility of quantum properties and construct the $|45°\rangle$ state as a superposition of *orthonormal* states.

Worksheet W7 Quantum states, vector (unit vector) states and formal representation of the superposition principle in order to recognize how to represent states in the orthogonal vector space.

Worksheet W8 Uncertainty principle and quantum uncertainty in order to examine a polarized photon as a quantum particle and perceive the impossibility to simultaneously associate specified properties corresponding to incompatible variables.

In the next section, we are going to consider, the data analysis gathered from the worksheets by analysing some questions and key situations concerning:

Summary of Polaroid-photons interaction and the probabilistic interpretation by applying Malus law to calculate the transmission probability when photons interact with two or three Polaroid filters (see worksheets W2 and W3).

Recognizing incompatible photons properties by examining interpretive hypotheses (statistical mixture of pure states) to verify coexistence of predictions and outcomes of ideal experiments dealing with interaction between photons polarized in the $|45°\rangle$ state and different Polaroid filters (see worksheet W6).

Students answers were organized into a database and classified according to their types in comparison with those expected. The most significant answers have highlighted the students way of reasoning and in some cases have allowed to find a possible modelling.

8. – Data analysis

In the following, the analysis of data considers the answers given by students to the pivotal questions([14]) used to organise students worksheets W2, W3, W4. For each examined situation, the typologies of students answers (SA) and their frequencies were presented in the tables and then discussed.

Worksheet 2 (W2). The solution of problem is based on Malus law: a beam of polarized light of intensity I_0 hits a Polaroid filter, whose coefficient of transmission is T. According to the experiment, we find that the intensity of the transmitted light is

$$I_T(\theta) = I_0 T \cos^2 \theta.$$

([13]) Actually, polarization is a dynamic property of the photon, and the photon is seen as a quantum system.
([14]) "Q" stands for question and "SA" stands for students answers.

TABLE I. – *Typologies of students answers (SA) and their frequencies to Q A1.1, Q A1.2 and Q A2 questions (W2).*

Q A1.1	Which aspects of the phenomenon does the coefficient T describe?	
SA A1.1	type 1: *Intensity of transmission*	8%
SA A1.1	type 2: *transmission index*	8%
SA A1.1	type 3: *Decreasing of* I_T*: it is a constant that varies with* θ	84%
Q A1.2	Which is the experimental value of T? always > 1, always < 1, always $= 1$ Explain your answer	
SA A1.2	type 1: $T = 1$ *because* ... *"the calculation is simplified"* ... *to* ...	100%
SA A1.2	explanation 1: "*reasoning on ideal experiment*"	52%
SA A1.2	explanation 2: "*thinking about the formula*"	48%
Q A2	Which aspect of phenomenology does the factor $\cos^2\theta$ describe?	
SA A2	type 1: *The factor* $\cos^2\theta$ *describes a "probability of passage" of the individual microscopic entities involved in the process*	75%
SA A2	type 2: no answer	25%

As is evident from the prevailing types of answers given to Q A1.1, all the students consider the $\cos^2\theta$ factor as a systemic operator that transforms the intensity quantity from the input to the output (table I). One student recognises T as a transmission index (a-dimensional). The students answer the following question about the experimental value of T reasoning in the same way. Examining their answers, the students seem not to analyse the physical quantity in favour of the assumption of its value in ideal conditions, which represents a computing simplification because it is their only point of view and that many of them explicitly prefer the ideal context. They answer the Q A2 about the $\cos^2\theta$ *factor* maintaining that it has a functional role (of probability) without any association to the specific phenomenology of polarization, confirming the systemic-operational perspective with which they are used to reason.

Worksheet 3 (W3). In the following questions, the photons interaction with real Polaroid filters is analysed. Table II sums up the various types of answers and their frequency about the possibility to have a probability of transmission, respectively, equal to 1 (QC2.1) or to 0 (QC2.2) in the case of real Polaroid filters (for example, $T = 0.7$).

TABLE II. – *Typologies of answers and their frequency to Q B1, Q B2, and Q B3 questions of worksheet W3.*

Q B1	How many photons will F2 transmit?	
SA B1	type 1: $N_0/2$	100%
Q B2	Which incident beam's polarization direction should have for the ratio N_T/N_0 to be equal to 0? - 1? - 1/2?	
SA B2	type 1: $N_T/N_0 = 0$ *when* $*$; $N_T/N_0 = 1$ *when* $\triangle$; $N_T/N_0 = 1/2$ *when* $\diamondsuit$	100%
Q B3	What is the probability that a photon polarized at 45° be sent by an ideal Polaroid ($T = 1$) with the permitted direction: Vertical? Horizontal? at 45°?	
SA B3	type 1: *Vertical 50%* *Horizontal 50%* *at* 45° 100%	100%

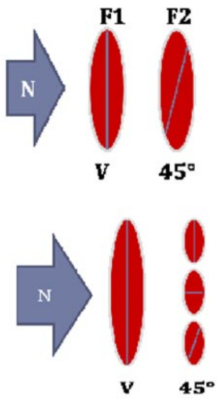
TABLE III. – *Typologies of answers and their frequency to QC1.1, QC2, and QC3, QC4 questions of worksheet W3.*

Q C1.1	What is the value of the probability that a photon be transmitted by the Polaroid with permission direction Vertical V? Horizontal H? 45° rotated with respect to V?	
SA C1.1	type 1: $P = 1$ *"only in the ideal case"* $P = 0$ *"in the case when the photons have property △"*	25%
SA C1.1	type 2: $P = 1$ *"only in the ideal case"* $P = 0$ *"in the case in which the photons have property △"* *(with $T = 0.7$ for example) and the Polaroid with property ∗*	25%
SA C1.1	type 3: $P = 1$ *"Only in the ideal case"* $P = 0$ *"in the case when the photons have property △"*	25%
SA C1.1	type 4: $P = 1$ *and* $P = 0$ *"only in the ideal case when $T = 1$"*	25%
Q C2	In case of real Polaroid, can you get transmission probability equal to 1? Equal to 0?	
SA C2	type 1: *equal to 1 only in case of perfect Polaroid $T = 1$* *(total transmission) equal to 0 in case photons have property △*	100%
Q C3	What is the maximum transmission probability of a single photon? When does it happen?	
SA C3	type 1: 70% *when photon and Polaroid have the same property*	50%
SA C3	type 2: 70% *when they have equal properties*	50%
Q C4	What is the ratio $\frac{I_T}{I_0} = \frac{N_T}{N_0}$ relative to the single photon?	
SA C4	type 1: *it represents the transmission index of Polaroid*	100%

Students' answers to the questions relative to *worksheet W3* and their frequencies are summarized in table III. The questions concerned phenomenology (N photons transmitted) and the probability of the photons transmitted. In fig. 11 the two questions and the ratio of the given answers are summarized. It can be noticed that almost all students can well master the phenomenology, and they manage to distinguish between ideal and real cases, by properly using the probabilistic meaning of Malus law.

About the question on which information Malus law provides, and on the result of the interaction between each photon and an ideal Polaroid filter ($T = 1$), the minority of the students (5/12) who have answered says that Malus law provides the "*probability with which (the photons) are transmitted through the Polaroid filter*". The understanding of the probabilistic meaning of Malus law is explicitly clear in the quantitative evaluations, while it is implicit for the other students who also show that they are able to handle Malus law with full knowledge either to evaluate its intensity or its probability.

Worksheet 6 (W6). Students had to get sense of photons incompatible properties. They had to describe a beam of 45° polarized photons, which have ◇ property, under the following hypothesis: the beam is equivalent to a quantum mixture of photons, formed 50% by photons with properties △ and 50% by photons with properties ∗, which means: $\diamondsuit\diamondsuit\diamondsuit\diamondsuit = \triangle\triangle + **$. The consistency of the hypothesis is evaluated by comparing the outcome of the interaction between photons and the Polaroid-45°.



B1 *N* photons, filtered by Polaroid-V, hit a second ideal F2 Polaroid-45° ($T = 1$).

B1.1 Which is the number N_t of transmitted photons by F2?
($N_t = \frac{N}{2}$) Students' answer: $N_t = \frac{N}{2}$

C1.1 Which is the probability that a beam's photon will be transmitted by a second real F2 Polaroid ($T = 0{,}7$) when its axis transmission is:

Questions	Student's answers	
	11/12	1/12
Vertical V-axis? ($N_t = 70\%$)	$N_t = 70\%$	$N_t = 70\%$
Horizontal H-axis? ($N_t = 0\%$)	$N_t = 0\%$	$N_t = 0\%$
45°-axis? ($N_t = 35\%$)	$N_t = 35\%$	$N_t = 70\%$

Fig. 11. – Questions on the phenomenology and on the probability of the photons transmission and students answers.

Hypothesis A. A beam of "◊ photons" corresponds to a set 50% made by "△ photons" and 50% by "∗ photons", formally: ◊◊◊◊ = △△ + ∗∗.

Figures 12, 13 reproduce the two key questions.

As shown in table IV, a common important aspect emerges from the four typologies of answers: many students consider that Polaroid and photons have the same properties, even though all students apply correctly the "formula" to calculate the number of transmitted photons. This important result has been widely discussed revealing students objective difficulties in understanding that it is not possible to attribute simultaneously two incompatible properties to photons in the $|45°\rangle$ state as quantum systems.

The last question of W6 concerns the students interpretation of the responses:

QB: What do we conclude from the above analysis about the hypothesis of quantum mixture?

RB: *The hypothesis is not consistent with the experimental results.*

Students answers (see table V) refer to the experimental result in the conclusions and to students switch to the interpretative field implying the conclusion of the experimental comparison, as the synthesis of the two fields will be achieved in a later stage. In the first

A1.1 IDEAL EXPERIMENT - A weak beam of *N* photons polarized at 45° (owing property ◊, eg transmitted through a Polaroid-45°) interacts with a second Polaroid-45°.

Questions.

A1.1 a) The number of transmitted photons is: ____________ (*N*)

A1.1 b) The polarization state is: ____________ (*45°*)

A1.1 c) Complete the figure on the right using appropriate symbols according to experimental data ____________ (◊◊◊◊)

Fig. 12. – Questions and answers of first part of W6.

A1.1 IDEAL EXPERIMENT - A weak beam of *N* photons polarized at 45°, on average made by 50% photons owing property Δ and 50% photons owing property *, interacts with a second Polaroid-45°.

Questions.

A1.1 a) The number of transmitted photons is: ___________(*N/2*)

A1.1 b) The polarization state is: _______________________(*45°*)

A1.1 c) Complete the figure on the right using appropriate symbols according to experimental data _________(◊◊)

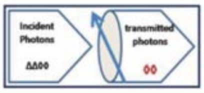


Fig. 13. – Questions and answers of second part of W6.

motivation it comes out that the property $\diamondsuit$ is incompatible either with the property $\triangle$ or the property $*$, even if the state is identified with the symbols which represent the system properties, a fact that highlights the well-known learning problem for which *eigenvalues* and eigenvectors are conceptually identified by the students [24]. In the second type of answers, some students refer to the concept of statistical mixture but they do not explain why.

9. – General results emerging from the experimentation and conclusions

The global answers highlight either the achievement of the learning objectives for the acquired self-assurance about previsions, and the correct management of the phenomenology in terms of functional operators. Some students show difficulties in constructing the physical meanings of the formal elements to represent the concepts; actually, they have reinterpreted physical quantities as transformation operators, compensating for the lack of complete knowledge of their physical meaning.

This data confirm a well-known crucial point of physics teaching [28]: how to make effective teaching in the switch from phenomenology to modelling and from this to the formalization of the concepts by introducing suitable physical quantities. In other words, it is necessary to investigate further, how students construct physical meanings and formalize the description of phenomena considering physical quantities as conceptual reference points.

For example, when some students deal with the $T\cos^2\theta$ factor as the only expression of the intensity modulation([15]), they reveal their looking at the interaction as an input/output process in which the formal black box ($T\cos^2\theta$) determines the intensity change. As already described in the previous section, their professional training significantly comes out in terms of skill and perspective with which they usually approach the physical phenomena.

Finally, the attribution of the properties to the Polaroid filters and photons as well, observed also in other experimentations [29,30], shows that there is an incomplete transition from the phenomenological exploration to the modelling: for example, in recognizing the photon-Polaroid interaction as a transition between states, that is, as a change from the preparation to the detection properties. It emerges how relevant it is to favour the link between the descriptive level and the interpretative one.

([15]) With no report on its physical meaning in describing the interaction photon-Polaroid, but according to the systemic analysis of the processes.

TABLE IV. – *Conformity of the outcome with experimental hypothesis statistical mixture.*

Q A1.1a	The number of transmitted photons is	
SA A1.1a	Type 1: *maximum ... number of photons*	25%
SA A1.1a	Type 2: *all ... photons*	75%
Q A1.1b	The polarization state is	
SA A1.1b	Type 1: 45°	25%
SA A1.1b	Type 2: ♢	75%
Q A1.1c	Complete the figure on the right by using appropriate symbols according to the experimental data	
SA A1.1c	Type 1: ♢♢♢♢	92%
SA A1.1c	Type 2: *no response*	8%
Q A1.2a	The number of transmitted photons is	
SA A1.2a	Type 1: *maximum ... number of photons*	25%
SA A1.2a	Type 2: *all ... photons*	75%
Q A1.2b	The state of polarization is	
SA A1.2b	Type 1: 45°	25%
SA A1.2b	Type 2: ♢	75%
Q A1.2c	Complete the figure on the right by using appropriate symbols according to the experimental data	
SA A1.2c	Type 1: ♢♢♢♢	92%
SA A1.2c	Type 2: no response	8%

TABLE V. – *Incompatible properties.*

Q	What can you conclude by the experimental results on properties ♢ and ∗ (♢ and △)?	
SA D	type 1: *we can consider it as a different state from the superposition of the △ and ∗ states and so the state ♢ does not have the states △ and ∗*	50%
SA D	type 2: *incompatibility: because the state at 45° is not a quantum mixture*	33%
SA D	type 3: *incompatibility: we can consider it as the union of photons in H e V states, a photon with a property at 45° cannot have also the horizontal or vertical properties and for that reason they are incompatible*	
SA D	type 4: *incompatibility: there is a contradiction in hypothesising that the property ♢ is "the union" of properties △ and ∗*	8%

Regarding the educational repercussion in such a context, it is interesting to notice that a couple of students introduced Quantum Mechanics in their final essays exam, emphasizing the tractability of topics, as quantum state and principle of superposition, which are usually out of their training experience.

As regards the feasibility of the experimentation, it is easy to get from a practical point of view with the kit of the University of Udine and it can be used in every type of school because, as shown, the students can approach this trail without the need for specific prerequisites. The experimentation allows dealing with topics that can involve the students, creating a challenge for interpreting and presenting modern physics. Furthermore, it allows exploring the microworld through light polarization, which is an easy to reproduce and interpret phenomenology by linking it to photons as quantum particles. This kind of experience is especially useful for constructively preparing the QM formal teaching without recurring to the historical approach that is less interesting for students and may be less effective for learning the key concepts.

Concerning the formative experience as a scholar, the IDIFO3 Master offers a real opportunity to plan a personalized and specialized trial of learning and training, such as formative requests enhanced in their contents and methods, in order to develop different skills that can also be spendable especially in the daily didactic practice.

An additional value of the e-learning formation compared to classroom courses is the easy access to the retraining and research material at any time. Moreover, this approach makes it easy to participate to discussions on different levels and in different moments and finally to initiate open debates on the relevant aspects of the conceptual knots of the topic from the disciplinary and methodological point of view. This is very important for planning the work and experimentation in schools.

* * *

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