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Teaching the basic concepts of the Special Relativity in the secondary school in the framework of the Theory of Conceptual Fields of Vergnaud

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Summary. — In this work, we investigate the conceptualization of the basic aspects of Special Relativity (SR) at secondary school level. We have conducted our research along the lines of the Theory of Conceptual Fields (TCF) proposed by Vergnaud (Vergnaud G., *Infancia y Aprendizaje*, **36** (2013) 131). The investigation consisted in the design, implementation and evaluation of a didactic sequence specially elaborated to conceptualize the basic aspects of SR. The proposal is composed by eight situations, complemented with a set of exercises. It was carried out in two classrooms with students of the last year of secondary level (17 years old, $N = 43$). The conceptualization was analyzed in a classroom context, where the selected situations are essential to promote the emergence of the relevant concepts.

1. – Introduction

The curriculum of the high school in Argentina proposes the study of the basic concepts of Relativistic Physics. In particular, in Buenos Aires province, the topic Special Relativity (SR) is part of the discipline Classical and Modern Physics, of 6th year high school, with natural-sciences orientation.

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Contrary to the stipulations of the high school curriculum, the contents of modern and contemporary physics are (in general) not studied at this level. However, commonly students show interest in modern physics topics, and have more information and knowledge than the supposed one, due to the variety of media at their disposal. It is clear that the study of SR is relevant due to the deep revolution it caused on common sense aspects about space and time. But from a wider perspective it also brings sense to the study of Galilean-Newtonian relativity, as a previous step to its conceptualization within the SR framework.

The researches on the teaching of the topic relativity, focusing on the conceptualization of the basic aspects, are scarce. Here we briefly review illustratively some of them. The works (Saltiel 1980; Hewson 1982; Posner 1982; Villani and Arruda 1998, Villani and Pacca 1987; Pietrocola 1999; de Hodson, Kermen and Parizot 2010; de Hodson and Kermen 2013) analyze the conceptualization of relative motion in the Galilean context, especially at University level. Regarding proposals to teach SR, some results indicate that students do not use SR concepts, but keep their pre-SR ideas to interpret SR results (Villani and Arruda 1998). Finally other works (Perez and Solbes 2003, 2006) explore epistemological, historical and conceptual aspects of the SR with teachers and students of physics education. They conclude that in general teachers introduce the concepts uncritically and with a weak knowledge of the basics of the theory, in detriment of an appropriate conceptualization by part of the students.

The aim of this work is to contribute to the development of a Didactics of the Special Theory of Relativity and the study of the conceptualization process of their fundamental notions, in students of the last years of high school. The didactic component of our research requires the specification of the Reference Conceptual Structure (RCS) for the SR (Otero 2006). This entails an epistemological and didactical analysis of fundamentals of SR, in order to propose a potentially viable sequence, adequate to high school. The didactic performance and viability of the sequence are experienced in 6th year high school physics courses, together with the analysis of the conceptualization by part of the students. The investigation assumes the complementarily of the didactic and cognitive dimensions.

2. – The theory of conceptual fields

The Theory of Conceptual Fields (TCF) is a cognitive theory that brings a coherent and operative framework, organized around a set of basic principles, to study the learning process and the development of complex concepts and competences. By providing a scenario for addressing learning aspects, the TCF is also relevant for Didactics (Vergnaud 1990). From the point of view of the TCF, the conceptualization takes place in all areas of human experience: family, compulsory school, professional training, employment, etc. However there are particularly suitable contexts, for instance, the learning of physics and mathematics topics requires a high level of conceptualization, which may emerge in situations that high school can recreate more likely than any other social institution (Otero M. R., Fanaro M. A., Sureda P., Llanos V. C. and Arlego M. 2014).

The TCF proposes that in every field of knowledge, certain processes of conceptualization are needed. These processes emerge in some kind of situations and events, evoking the development of certain types of activity. Therefore, it is necessary to explicit the knowledge of reference from which the teaching will be conceived, the knowledge to be taught and their transformations, as well as the one it is actually taught, taking into account the transpositive processes (Chevallard 1985).

The specificity of the acquisition processes in each conceptual field leads to Vergnaud linking cognitive development in a certain domain, with teaching, that is to say with Didactics (Vergnaud 2013).

3. – Operational form and predicative form of knowledge

The operational form of knowledge is what allows the subject to act in a given situation, whereas the predicative form consists in stating the relations between objects. There is a huge complexity in doing and speaking about what is done (Vergnaud 2007). But while teaching is irreplaceable in the process of conceptualization, it cannot be reduced to put into words the conceptual content of knowledge. The enunciation is essential in the process of conceptualization.

In particular, the difficulties students have in learning physics and mathematics show the complexity of the situations involved, and the thinking operations necessary to treat them.

3·1. *Concept.* – Vergnaud proposes a pragmatic —useful and functional— definition of concept. A concept can be defined by the conjunction of three different sets, which are not independent of each other (Vergnaud 2013).

$$\text{Concept} = \text{def } (S, I, L),$$

where,

S: is the set of situations that give sense to the concept.
I: is the set of operational invariants that integrate the schemes evoked in the situations.
L: is the set of linguistic and symbolic representations (algebraic, graphical, etc.) that allow representing the concepts and their relationships.

The operational invariants are of two types: concepts-in-action, defined as categories pertinent to the subject in the situation, and theorems-in-action, that are affirmations validated by the subject. In this way, the concept involves, on one hand, a component which is property of the subject but related to the situation, such as the operational invariants present in the schemes. On the other hand, a concept involves a link to "the real" as the types of situations that interact dialectically with the schemes. Finally, the concept comprises a semiotic component, which refers to the systems of signs or representations, used to enunciate the concepts, their relationships, and to refer to the objects (Vergnaud 2007, 2013).

3·2. *Investigation methodology.* – The design of a didactic sequence involves three main phases. The first one, known as *a priori* analysis, is the construction of a reference conceptual structure (RCS), which is the basis for the design of a number of situations, whose resolution requires the emergency of certain concepts. The second phase comprises the design and development of the didactic sequence itself, based in the priori analysis. Finally, the sequence is tested in one or more pilot projects to generate *a posteriori* analysis, which in turn will allow an eventual sequence reformulation. This process generates a cycle that leads to a relative stabilization of the main parts of the sequence, with the modification or addition of more tasks to enforce conceptualization of relevant topics if necessary, or conversely, reduce them.

The research has exploratory, qualitative and ethnographic character. In each class a situation is proposed to students, who work in small groups. Class by class student protocols are collected and scanned, to be returned the next class. In addition, all classes are audio-recorded, and the teacher, who is also the researcher, carries out participant observation. Other researchers of the team perform non-participant observation. The protocols are analyzed considering the situations, the theorems-in-action and representation systems used by students: verbal (oral and written) graphic, numeric and algebraic. Here we present the results of the testing of the original sequence in two courses of sixth year of secondary school ($N = 43$). This gives rise to a modification of the sequence, which is also described as part of the work.

3˙3. *Reference Conceptual Structure for Special Relativity theory*. – The SR describes the kinematic and dynamic behavior of objects without taking into account gravitational effects. It is possible to develop the SR on the basis of the two Einstein postulates:

P1: The Principle of Relativity: The laws of physics are the same for all inertial observers.
P2: Invariance of the speed of light: The speed of light c is constant for all inertial observers in the vacuum and is the upper bound for any speed.

In the first postulate, Einstein generalized the principle of relativity to all physical systems. The second postulate raises the speed of light in vacuum to the range of universal constant. Therefore, the laws of physics are invariant (principle of relativity) under the Lorentz transformation, which for low speeds compared with c reduces mathematically to the Galilean transformation.

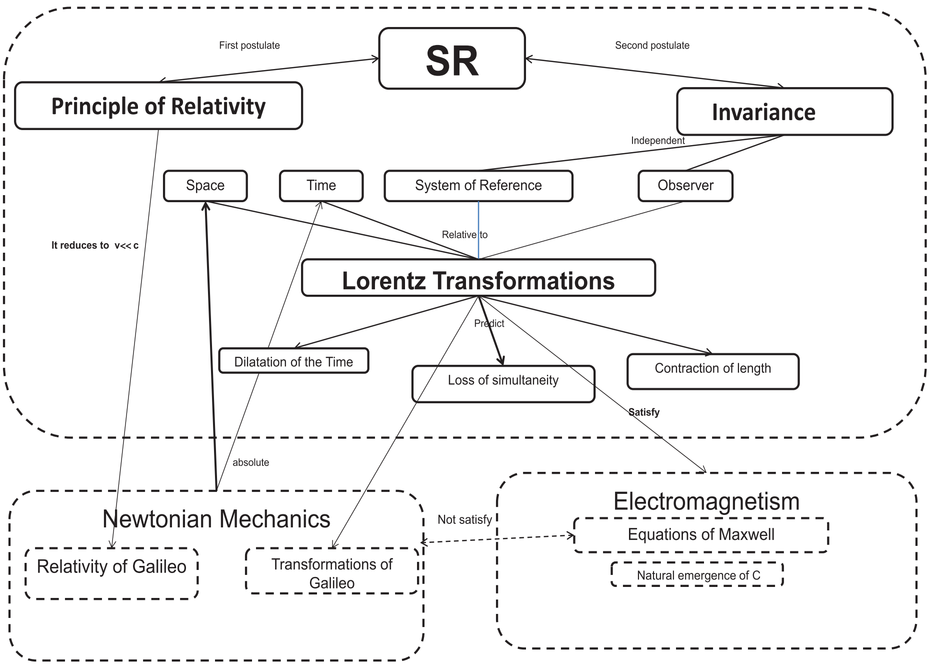

Fig. 1. – Scheme of the Reference Conceptual Structure (RCS) for the Special Relativity theory.

These two, seemingly harmless postulates, working together lead to a series of surprising predictions that challenge the ideas of space, time, mass and energy, deeply rooted in our everyday experience of low speeds (compared with c). Figure 1 shows schematically the interrelation between the different concepts involved in the RCS for SR.

In this proposal the effects of relativistic dynamics are not studied. Therefore some relevant topics, such as the relationship between mass and energy are beyond the scope of this research.

3·4. *Didactic sequence.*

Part 1: Classical kinematics and Galileo's Principle of Relativity

In this part the concepts of reference system, observer, measuring of length and time are discussed. After that, the concept of relative motion, *i.e.*, with respect to different reference systems, and the law of addition of velocities (much smaller than c) are discussed.

Finally, situations to evidence the impossibility of distinguishing between rest and uniform motion are proposed, and thus reconstructing the Galilean relativity principle in the RCS proposed.

Part 2: Postulates of Special Relativity

The Postulates are proposed as follows:

P1) it is impossible to distinguish the rest of the uniform translation of a reference system.

P2) the speed of light in vacuum c, is a universal constant, independent of the source movement.

Part 3: Main kinematic Special Relativity results

In this part, we propose situations to be analyzed from the SR point of view. In appearance, the SR postulates introduced, would seem "acceptable" from the point of view of intuition, rooted in a world of low speeds compared to c. That is, students, at the begging, do not suspect of the counter-intuitive consequences of these postulates.

We start by proposing a situation where the direct application of the principles reveals the relativity of simultaneity, *i.e.* that while some observers state that two events have happened at the same time, others measure different times.

After presenting this first counter-intuitive challenge to students, we propose situations where the phenomena of time dilation and length contraction are manifest. This gives rise to the analysis of relativistic law of velocities addition.

It is important to emphasize the range in which relativistic aspects are relevant. The large value of c compared to the ordinary speeds which we are used, deprive us of experiencing phenomena such as time dilation and length contraction.

To promote the conceptualization of these experimental results, we propose the didactical strategy of considering the hypothetical case of c being of same order of magnitude than ordinary speeds. In this case, relativistic effects would be observable and would not represent a contradiction to intuition.

3˙5. *Situations proposed in the original sequence*.

First part

S1 *How can anyone say that someone or something else is moving or not? Give examples, write your answer and if you want draw pictures.*

S2 *I am traveling by car on a straight and long road. I see another car coming from the front. My travel partner says that this car is approaching us at 150 km/h. The speed limit on this road is 80 km/h. My partner says that this car is violating the maximum allowed speed. I say no, because we are traveling at the speed limit. Who is right? Could you mathematically represent this situation?*

S3 *I am moving with a speed v with respect to the street to benefit from the "green wave" on an avenue. A car traveling in the right lane is going twice as fast as I with respect to the road. What is its speed relative to me? Does the other car benefits from the "green wave"?*

S4 *Build a pendulum by tying a rubber ball at the end of a string and analyze what happens when you perform the following actions:*

a) *You are walking in a straight line with your pendulum in one hand and you stop suddenly.*

b) *You are walking in a straight line with your pendulum in one hand without accelerating nor braking.*

c) *You are walking in a straight line with your pendulum in one hand and you start to run.*

d) *You are standing with your pendulum in one hand.*

e) *You go by car or bike and take a curve.*

Draw pictures or diagrams of the different cases and explain them

S5: *Suppose we were locked in a train wagon or in a car and can't see out, or take any external reference, but we have a pendulum. Is there anything we can do to find out if we are moving?*

Second part

S6: *A person stands right in the middle of an empty truck trailer, which moves on a straight road with a constant speed v (respect to the road). The observer has a device that can shoot rubber bullets or light beams (laser) forwards and backwards at the same time. If the person fires the rubber bullets*

Which one came first to the trailer edges?

What if the person does the same with the light?

Third part

S7: *An observer is sitting right in the middle of a closed truck trailer moving at a constant speed v, respect to the road. On the roof of the trailer there is a plane mirror. The observer has a device that can emit a beam of light perpendicular to the ceiling. The ray hits the mirror and is reflected back toward the viewer. How long does the ray of light take to go to the mirror and back to the observer?*

a) *For the observer in the truck*

b) *For another observer who is standing on the road.*

S8: *An observer is sitting right in the middle of a trailer that moves at a constant speed v, with respect to the road. The observer on the trailer says that the length of the trailer is L. What trailer length would measure another observer standing on the road? Consider different and coherent values for L and v.*

Answer previous question by supposing (hypothetically) that $c = 300\,\mathrm{km/h}$.

3˙6. *Data analysis*. – In situations 1–3 the students managed the use of Galilean speeds addition in one dimension, and we could said that the situations functioned properly. In the situation 4 the students conducted experiments in the schoolyard with enthusiasm. Some brought a bike to experience what was happening with the pendulum while they were turning. The students expressed their ideas in more than one system of representation, which indicates an appropriate level of conceptualization. It started implicit in action to become explicit in different representational formats. Most of the responses were correct and the drawings complete and coherent. This would indicate that students correctly understand the Galilean principle of relativity and the indistinguishability between uniform motion and rest in an inertial frame, as suggest the analysis of protocols (table I).

TABLE I. – *Relative frequency of the theorems-in-action identified in the situation 4.*

Theorems-in-action related to the Principle of Relativity identified in S4		
The pendulum accompanies the motion.	The pendulum moving with constant speed is the same as that at rest	If the pendulum moves with constant speed, it does not move respect to me.
38/43	41/43	23/43

As an example, in fig. 2, (left panel), the protocol of A23 student allows us to appreciate that this student understands what happens if he suddenly stops when walking in a straight line with constant speed, if he quickly starts moving from the rest, or moves in a straight line at constant speed (a)–(c) respectively, and when he is turning (e). Similar conclusions can be obtained from protocol of B7 student, shown in fig. 2 (right panel).

In the situation 5, the students had to use the ideas made explicit in the situation 4, assuming they were in an isolated system and had only a pendulum. Here they can predict what happens in the case of a speed variation but fail in the impossibility of distinguishing between uniform translation and rest. The difficulties are manifested by a drastic reduction of pictorial representations and the frequency of the theorems in action (table II).

TABLE II. – *Frequency of the theorems-in-action identified in the situation 5.*

Theorems-in-action related to the Principle of Relativity identified in S5	
I only realize if the train stops or turns.	I do not distinguish if the train is at rest or moves with constant speed.
33	14

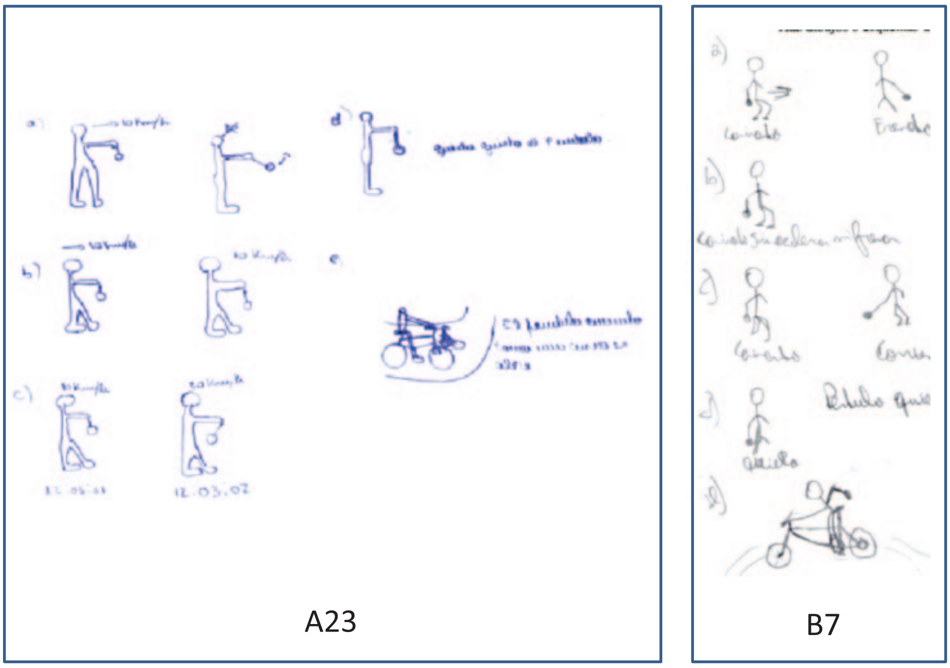

Fig. 2. – Protocols of A23 and B7 related to the Situation 4. The student A 23 has represented the speed increasing with the time. The protocols show how the students have drawn the pendulum motion when the bike turns.

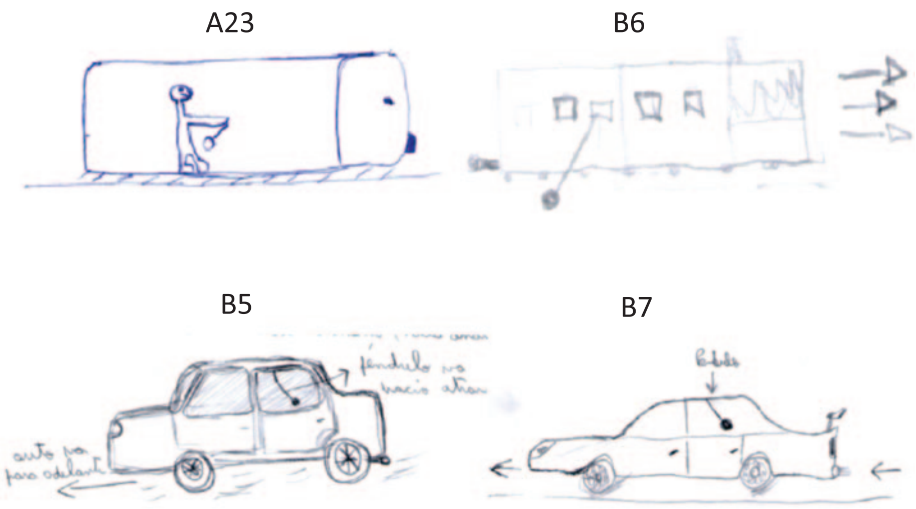

Fig. 3. – Pictures of the students A23, B6, B5 and B7 show arrows indicating velocity when the observer is inside, and therefore the motion is undetectable.

Regarding the few pictorial representations obtained in this situation, protocols of A23, B5, B6 and B7 students (fig. 3) show that students conceive the isolated system only seen from outside. The representation of the pendulum and arrows assume that the system is moving, which is undetectable from inside. This would indicate that they conceive the motion as absolute, rather than relative.

Table III. – *Frequency of the theorems-in-action identified in the situation 6 when the observer is inside the trailer.*

Observer inside the trailer			
Rubber bullets		Beams of Light	
They arrive together	It comes first the one that is going to behind	It comes first the beam of light that is going to behind	Both beams come together
15	27	2	37

Table IV. – *Frequency of the theorems-in-action identified in the situation 6 when the observer is outside the trailer.*

Observer outside the trailer		
Rubber bullets		Beams of Light
They arrive together	It comes first the one that is going to behind	Both beams come together
16	21	36

In the situation 6, students have to analyze the motion of two small balls and then two light beams fired from the center of a truck trailer, assuming two observer positions: inside outside the truck trailer.

The situation requires that students jointly apply the two postulates of relativity, and in doing so with light, put in evidence the non-simultaneity for the observer on the route. However, they conclude, mostly without surprise, that the balls will not come simultaneously to the walls of the trailer, neither when viewed from inside nor from outside the truck.

Even more unexpected is that they predict that the light rays come at the same time at the opposite sides of the truck, for both, the observer who is inside as well as the one is outside the truck trailer. In other words, their predictions are exactly contrary to the expected ones. A possible reason for these unexpected predictions could be that they seem to analyze the situation from outside the truck, *i.e.* they always consider the truck in motion, which is undetectable from inside. Therefore, they do not apply the principle of relativity, regarding the motion as absolute.

On the other hand, for them, the light propagates at such a high speed, that in practical terms it propagates "instantaneously", so, the arrival time of light to the walls is always the same, for all observers. The tables III and IV show the frequencies of the theorems-in-action for bullets and light for both observers, respectively.

According to the results obtained, we conclude that the sequence as it was originally designed must be modified. It has to be re-designed in order to allow the students to correctly apply the principle of relativity, in particular disregarding the speed of the truck when they are inside it, and taking it into account when they are outside. Therefore, we have modified the situation 6 as follows:

New S_6

An observer is sitting right in the middle of an empty truck trailer. Another observer standing at the side of the road determines that the truck moves with constant speed. The observer inside the truck has a device that can launch rubber bullets forward and backward at the same instant. Complete the following table V for each observer, proposing different speeds for the truck and the projectiles.

TABLE V. – *Complete the speed of bullets and the trailer considering the observer position.*

Observer inside the trailer			Observer outside the trailer		
V_t (m/s)	v_{br} (m/s)	v_{bl} (m/s)	v_{br} (m/s)	v_{bl} (m/s)	V_t (m/s)
	V_b	$-V_b$	$V_t + V_b$	$V_t - V_b$	V_t

a) Analyze for each observer, without doing calculations, if the bullets arrive simultaneously or not at each edge of the trailer.

b) Calculate the meeting point (position and time) between the bullets and trailer walls, for each observer, considering different values of speeds.

In the case of rubber bullets, students could complete a table, parametrizing with different speeds and formulate the equations of motion with the established parameters. After that, by using software calculate the meeting point and the corresponding time, verifying that it is the same, for both within and outside the truck. The aim here is that students would be able to write the equations of motion (at least numerically) as we can see in table VI.

TABLE VI. – *Equations of motion to find the meeting point in the case of the observer on the trailer or on the road.*

Observer on the trailer		Observer on the road	
Left	Right	Left	Right
$\chi_{wl} = -L$	$\chi_{wr} = L$	$\chi_{wl} = -L + v_c t_l$	$\chi_{wr} = L + v_t\, t_r$
$\chi_{bl} = -v_b\ t_l$	$\chi_{br} = v_b\ t_r$	$\chi_{bl} = (v_t - v_b) t_l$	$\chi_{br} = (v_t + v_b)\, t_r$
$\chi_{wl} = \chi_{bl}$	$\chi_{wr} = \chi_{br}$	$\chi_{wl} = \chi_{bl}$	$\chi_{wr} = \chi_{br}$
$t_l = \dfrac{-L}{-v_b} = \dfrac{L}{v_b}$	$t_r = \dfrac{L}{v_b}$	$-L + v_t t_l = (v_t - v_b)\, t_l$	$L + v_t t_r = (v_t + v_b)\, t_r$
		$-L + v_t\, t_l$	$L + v_t\, t_r$
		$= v_t\, t_l - v_b\ t_l$	$= v_t\, t_r + v_b\ t_r$
		$-L = -v_b\, t_l$	$L = v_b\, t_r$
		$t_l = \dfrac{-L}{-v_b} = \dfrac{L}{v_b}$	$t_r = \dfrac{L}{v_b}$

Having analyzed what happens with the rubber bullets, we propose considering the case of the rays of light in the new situation 7.

New S_7

An observer is sitting right in the middle of an empty truck trailer. Another observer standing at the side of the road determines that the truck moves with constant speed. The observer inside the truck has a device that can shoot laser light beams forward and backward at the same instant. Complete the following table VII for each observer, proposing different speeds for the truck.

TABLE VII. – *Complete the speed of the beams of light and the trailer considering the observer position.*

Observer inside the trailer		Observer outside the trailer		
V_{Lr}	V_{Lr}	V_t	V_{Lr}	V_{Lr}

a) Analyze for each observer, without doing calculations, if laser light arrives simultaneously or not at each edge of the trailer.

b) Calculate the meeting point (position and time) between the light beams and the trailer walls, considering different values of truck speed.

Here students should apply both principles of SR together. Although in this case the numerical solutions would not be appropriate to assess the difference in time, due to the high value of c, it is expected that the students would be able to write the analytical equations of motion. In particular, from outside the trailer, where the lack of simultaneity is explicit (table VIII).

TABLE VIII. – *Equations of motion to find the meeting point in the case of the observer on the trailer or on the road.*

Observer inside the trailer		Observer outside the trailer	
Left	Right	Left	Right
$\chi_{wl} = -L$	$\chi_{wr} = L$	$\chi_{wl} = -L + v_t\, t_l$	$\chi_{wr} = L + v_t\, t_r$
$\chi_{Ll} = -c\; t_l$	$\chi_{Lr} = c\; t_r$	$\chi_{Ll} = -c\; t_l$	$\chi_{Lr} = c\, t_r$
$\chi_{wl} = \chi_{Ll}$	$\chi_{wr} = \chi_{Lr}$	$\chi_{wl} = \chi_{Ll}$	$\chi_{wr} = \chi_{Lr}$
$t_l = \dfrac{-L}{-c} = \dfrac{L}{c}$	$t_r = \dfrac{L}{c}$	$-L + v_t\, t_l = -c\; t_l$	$L = v_t\, t_r = c\; t_r$
		$-L$	$L = (c - v_t)\, t_r$
		$= -(c + v_t)\, t_l$	$t_d = \dfrac{L}{(c - v_t)}$
		$t_l = \dfrac{-L}{-(c + v_t)} = \dfrac{L}{(c + v_t)}$	

After these situations, the sequence continues with situations 7 and 8 that are now renumbered into 8 and 9, respectively.

4. – Conclusions

We have designed, implemented and analyzed a didactic sequence for the teaching of basic aspects of Special Relativity Theory in high school level. The investigation focuses on the conceptualization process during the sequence, from the point of view of the Theory of Conceptual Fields of Vergnaud.

A careful analysis of the results based on 43 student protocols from a first cycle-implementation, let us conclude that the most complex aspects of the SR for students are related on the one hand with the Principle of Relativity itself. During the first five situations they tried to conceptualize this principle without success. In this case, we identify the main obstacle in the use of the underlying theorem-in-action: "motion is absolute" which is not correct. Note that this conceptual problem is not specifically related with SR, moreover it is a pre-Galilean misconception. On the other hand, regarding the second postulate, students accept the invariance of the light speed. However due to its very large value, compared with low speed everyday experience, we detect the use of the theorem-in-action: "the light is instantaneous". For this reason, they erroneously predict simultaneity for the case of light for all observers.

To face these obstacles, new situations were designed, aiming the emergence of appropriate operational invariants. To reach this higher level of conceptualization, the teaching of classical pre-relativistic kinematics is fundamental. Hence, it is necessary to conceptualize first, Galilean relativity as a previous step to its generalization in the framework of SR. This can only be accomplished by designing study programs revaluing classical physics, with a view towards relativistic physics.

Regarding the idea that light propagates instantaneously and therefore arrives simultaneously everywhere, the use of equations of motion to solve meeting point problems brings the possibility of direct application of the invariance of light and thus the prediction of lack of simultaneity. Although this only provides a mathematical root to the correct results, it may be considered as a first step to the conceptualization process of the relativity of the simultaneity.

Once the concept of absolute time is refused or at least accepted, the student is better prepared to deal with the concepts of time dilation and length contraction, but there is long way up to this point. Unfortunately, traditional textbook approaches usually comprise a brief analysis of the postulates to quickly go to the "spectacular" parts of the theory. It does not have any sense moving towards the main topics without *a prior* process of conceptualization of the basic postulates. In this sense, our contribution promotes a firm conceptual basis of postulates, paving the way to address significantly the core issues of the special relativity.

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