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Gravitino problem in $f(R)$ cosmology

G. Lambiase

Dipartimento di Fisica "E.R. Caianiello" Università di Salerno - I-84084 Fisciano (SA) and INFN - Gruppo Collegato di Salerno - Salerno, Italy

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Summary. — The gravitino problem is investigated in the framework of $f(R)$ cosmology. Since in $f(R)$ cosmology the expansion laws of the Universe are modified, as compared to the standard cosmology, it follows that also the thermal history of particles gets modified. We show that $f(R)$ models allow to avoid the late abundance of gravitinos. In particular, we found that for an appropriate choice of the parameters characterizing the $f(R)$ model, the gravitino abundance turns out to be independent of the reheating temperature.

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1. – Introduction

Supersymmetry (SUSY) [1] is certainly one of the most attractive extension of the Standard Model (SM). This follows for different (theoretical) reasons: 1) the stability of EW scale against radiative corrections finds its natural explanation in SUSY models; 2) the three gauge couplings of the SM meet at GUT scales 10^{16} GeV. However, apart from these strong motivations, at the moment there are no direct evidence of SUSY (superpartners). One therefore expects that if SUSY does exist, it must be broken at some scale. Besides SUSY, it is natural to consider its immediate extension, the so called Supergravity (SUGRA), which is a local SUSY [2]. According to SUGRA models, in the broken phase the super-Higgs effects take place so that the gravitino (the superpartner of the graviton) may acquire mass absorbing the Nambu-Goldstone fermion associated to SUSY-breaking symmetry, in analogy to the case occurring in the SM. Gravitino, in some SUSY models [3], plays a peculiar role since its mass is not directly related to the SUSY-breaking scales of ordinary particles of SM and their superpartners. Since its interaction is very weak, there is no chance to find it in collider experiments. Gravitino properties can be instead studied at cosmological scales, referring in particualr to early Universe. In this work we shall confine indeed to this case. One of the open issue in cosmology is the so-called gravitino problem (see for example [4]). Due to the fact that

SUSY particles couple to ordinary matter only through the gravitational interaction, their couplings are Planck suppressed. This circumstance implies a quite long lifetime of these particles

$$\tau \sim \frac{M_{Pl}^2}{m_{3/2}} \simeq 10^5 \left(\frac{1\,\mathrm{TeV}}{m_{3/2}}\right)^3 \mathrm{s},$$

where $m_{3/2}$ is the mass of the particle. The scale characterizing $m_{3/2}$ is of the order of 100 GeV, *i.e.* the electroweak energies, and is obtained by means of the SUSY breaking. Particles with so long lifetime give rise to non trivial problem in cosmology since if they decay after the big bang nucleosynthesis (BBN), their decay products (gauge bosons and their gaugino partners or high energy photons) would destroy light elements, affecting in such a way the successful predictions of BBN. This problem can be avoided by setting an upper bound on the reheating temperature. More specifically, from the Boltzamnn equation one finds that (in the framework of GR) the gravitino abundance $Y_{3/2}$ is proportional to the reheating temperature T_R [5]

$$Y_{3/2} \simeq 10^{-11} \frac{T_R}{10^{10}\,\mathrm{GeV}}. \tag{1}$$

Requiring that this abundance remains small for a successful prediction of BBN one gets the constraint [6]

$$T_R \lesssim (10^6\text{–}10^7)\,\mathrm{GeV} \quad \text{for} \quad m_{3/2} \sim \mathcal{O}(10^2\,\mathrm{GeV}). \tag{2}$$

This result opens a serious question for the inflationary scenarios, since the latter provide a reheating temperature larger than the upper bound (2) [7]. Studies aimed to bypass these problems have been faced in [8].

In this paper we present some preliminary results of the gravitino problem in the framework $f(R)$ cosmology, which represents the simplest extension of theories, the so-called Extended Theories of Gravity (ETG), that generalize/modify General Relativity (GR). These theories has been invoked for explaining the recent discovery of the accelerated expansion of the Universe [9], without introducing unknown forms of energy, the Dark Energy, and, in the astrophsyical context, unknown forms of matter, the Dark Matter. The appearance of these unknown form of dark components in the Universe is a clear signal of the breakdown of GR on large scales.

The gravitational Lagrangian for $f(R)$ models is [10-12]

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g}\, f(R) + S_m[g_{\mu\nu}, \psi_m], \tag{3}$$

where S_m is matter action and $\kappa^2 = 8\pi G = 8\pi/M_{Pl}^2$ ($M_{Pl} \simeq 10^{19}$ GeV is the Planck mass). The action (3) must be considered as an effective theory that allows to describe at phenomenological level the gravitational interactions. At the moment there are no indications, both on theoretical and experimental bases, on what could be the explicit form of the function $f(R)$. For simplicity we shall the model

$$f(R) = \alpha R^n, \tag{4}$$

(for other models and applications, see [10] and [13]). We assume that after inflation, from GUT scales to reheating scales, the Universe evolves according to (4), then its evolution enters in the regime governed by the cosmological standard model. As we shall see, $f(R)$ cosmology provides a scenario in which the thermal history of particles gets modified, and as a consequence, the Boltzmann equation too. This reflects on the gravitino abundance, which turns out to depend on parameters characterizing the $f(R)$ model and the transition temperature, *i.e.* the temperature at which the Universe passes from $f(R)$ evolution to the standard one. In this phase of the Universe evolution, the parameters $\{\alpha, n\}$ are taken arbitrary.

The paper is organized as follows. In sect. **2** we derive the $f(R)$ gravity field equations. The analysis of the gravitino problem in the framework of $f(R)$ cosmology is studied in sect. **3**. Conclusions are shortly drawn in sect. **4**.

2. – Field equations in $f(R)$ gravity

The field equations for $f(R)$ gravity are

$$G^c_{\mu\nu} = \kappa^2 T^m_{\mu\nu}, \quad G^c_{\mu\nu} \equiv f' R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} - \nabla_\mu \nabla_\nu f' + g_{\mu\nu} \Box f', \tag{5}$$

from which one infers the trace equation the trace is

$$3\Box f' + f'R - 2f = \kappa^2 T^m, \qquad T^m = \rho - 3p. \tag{6}$$

Here $f' \equiv \frac{\partial f}{\partial R}$, $T^m_{\mu\nu}$ and T^m are the energy-momentum tensor of matter and its trace, respectively. The tensor $G^c_{\mu\nu}$ is divergenceless $\nabla^\mu G_{\mu\nu} = 0$, as well as, for consistency, the energy-momentum tensor $T^{m\,\mu\nu}$. For a (spatially flat) Friedman-Robertson-Walker (FRW) metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2], \tag{7}$$

the nonvanishing components of $G^{c\,\mu\nu}$ are

$$G^{c\,0}_{\ \ 0} = f' R^0_0 - \frac{1}{2} f - 3H\dot{f}', \tag{8}$$

$$G^{c\,j}_{\ \ i} = f' R^j_i - \frac{f}{2}\delta^j_i + \left(\ddot{f}' + 2H\dot{f}'\right)\delta^j_i, \tag{9}$$

where we have used $\Box f' = \ddot{f}' + 3H\dot{f}'$, $H = \dot{a}/a$, and the dot stands for $\mathrm{d}/\mathrm{d}t$.

In this paper we consider power law solution of the scale factor, $a(t) = a_0 t^\beta$. The $0-0$ field equation and the trace equation read

$$\frac{\alpha}{2}\left[\frac{n(\beta + 2n - 3)}{2\beta - 1} - 1\right] R^n = \kappa^2 \rho, \tag{10}$$

$$\alpha\left[n - 2 - \frac{n(n-1)(2n-1)}{\beta(1-2\beta)} + \frac{3n(n-1)}{1-2\beta}\right] R^n = \kappa^2 (1 - 3w)\rho, \tag{11}$$

where ρ is the energy density, that in the radiation dominated era reads $\rho^m = \frac{\pi^2 g_*}{30} T^4$ (g_* counts the number of relativistic degrees of freedom). In a Universe radiation-dominated era $T^m = 0$, so that eqs. (12) and (13) give

$$\alpha \Omega_n R^n = \kappa^2 \rho, \tag{12}$$

$$\beta = \frac{n}{2}, \tag{13}$$

where

$$\Omega_n \equiv \frac{5n^2 - 8n + 2}{4(n-1)}. \tag{14}$$

From eq. (12) one obtains the relation between the cosmic time t and the temperature T

$$t = \Pi_n \left(\frac{T}{M_{Pl}}\right)^{-\frac{2}{n}} M_{Pl}^{-1}, \tag{15}$$

where

$$\Pi_n \equiv [3n|n-1|]^{1/2} \left(\frac{15\tilde{\alpha}\Omega_n}{4\pi^3 g_*}\right)^{\frac{1}{2n}}, \qquad \tilde{\alpha} = \frac{\alpha}{M_{Pl}^{2(1-n)}}. \tag{16}$$

Let us introduce the transition time (temperature) t_* (T_*) which characterizes the transition from the $f(R)$ cosmology to the standard cosmology, described by GR. This means to equate the equation of the evolution at the instant $t = t_*$, *i.e.* $\alpha \Omega_{\beta,n} R^n(t_*) = H^2_{GR}(t_*)$. One gets

$$t_* = [4\tilde{\alpha}\Omega_n [3n|n-1|]^n]^{\frac{1}{2(n-1)}} M_{Pl}^{-1}. \tag{17}$$

The expression of the transition temperature T_* is given by

$$T_* \equiv M_{Pl} [3n|n-1|]^{-\frac{n}{4(n-1)}} \left[\frac{15}{16\pi^3 g_*}\right]^{\frac{1}{4}} [4\tilde{\alpha}\Omega_n]^{-\frac{1}{4(n-1)}}, \tag{18}$$

so that the relation (17) can be cast in the form

$$t = t_* \left(\frac{T}{T_*}\right)^{-\frac{2}{n}}. \tag{19}$$

Moreover, notice that

$$\frac{t_* T_*^2}{M_{Pl}} = \sqrt{\frac{15}{16\pi^3 g_*}}. \tag{20}$$

Notice, finally, that the expansion rate of the Universe in $f(R)$ cosmology can be written as

$$H(T) = A(T) H^{(GR)}(T), \qquad A(T) \equiv 2\sqrt{3}\beta \left(\frac{T}{T_*}\right)^p, \quad p \equiv \frac{2}{n} - 2, \tag{21}$$

where the factor $A(T)$ is the so-called enhancement factor. Expressions similar to (21) are obtained in different frameworks: $p = 2$ in Randall-Sundrum type-II brane cosmology [14], $p = 1$ in kination models [15], $p = -1$ in scalar-tensor cosmology [16], $-1 \lesssim p \lesssim 0$ in $f(R)$ cosmology [17].

3. – Gravitino problem in $f(R)$ cosmology

As pointed out in the introduction, gravitino is generated by means of thermal scattering in the primordial plasma. This occurs during the reheating era after inflation. To describe the gravitino production one makes use of the Boltzmann equation for the number density of species in thermal bath. The relevant equation for the gravitino production is

$$\frac{\mathrm{d}n_{3/2}}{\mathrm{d}t} + 3Hn_{3/2} = \langle\sigma v\rangle n_{\rm rad}^2. \tag{22}$$

Here $n_{3/2,\rm rad}$ refers to gravitino and relativistic species, while $\langle\ldots\rangle$ stands for the thermal average of the gravitino cross section σv times the relative velocity of scattering radiation ($v \sim 1$). In (22) we have neglected the term $\frac{m_{3/2}}{\langle E_{3/2}\rangle}\frac{n_{3/2}}{\tau_{3/2}}$, where $\frac{m_{3/2}}{\langle E_{3/2}\rangle}$ is the average Lorentz factor. Introducing the gravitino and relativistic particles abundances $Y_{3/2} = n_{3/2}/s$ and $Y_{\rm rad} = n_{\rm rad}/s$, respectively, where $s = \frac{2\pi^2}{45}\, g_* T^3$ and $g_* \sim 300$, the Boltzmann equation (22) assumes the form

$$\frac{\mathrm{d}Y_{3/2}}{\mathrm{d}T} = \frac{s\langle\sigma v\rangle}{\dot{T}} Y_{\rm rad}^2, \tag{23}$$

where we have used

$$\frac{\dot{T}}{T} = -\frac{n}{2t} = -\frac{n}{2t_*}\left(\frac{T}{T_*}\right)^{\frac{2}{n}}.$$

Integrating from T_R ($\gg T_*$, see below) to a low temperature T_l ($\lesssim T_*$) in the era described by GR, the solution to (23) is

$$Y_{3/2} \simeq -\mathcal{B}\frac{M_{Pl}}{T_*}\left[\left(\frac{T_R}{T_*}\right)^{\Delta} - \left(\frac{T_l}{T_*}\right)^{\Delta}\right], \tag{24}$$

where

$$\Delta \equiv 3 - \frac{2}{n}, \tag{25}$$

$$\mathcal{B} \equiv \left[M_{Pl}\frac{\langle\sigma v\rangle s}{H_{GR}T} Y_{\rm rad}^2\right]_R \frac{1}{\sqrt{3}(3n-2)}. \tag{26}$$

Notice that (24) is independent of the parameter α.

The gravitino abundance derived in eq. (24) allows to solve the gravitino problem. In fact for $\Delta \approx 0$, *i.e.* $n \approx 2/3$, it follows that the gravitino abundance turns out to be $Y_{3/2} \ll 1$. The most stringent constrain on unstable massive relic particles with lifetime $\gtrsim 10^2$ s, obtained from ^{6}Li abundance, is $Y_{3/2} \lesssim 10^{-14}\frac{10^2\,\mathrm{GeV}}{m_{3/2}}$ [6]. Using this value

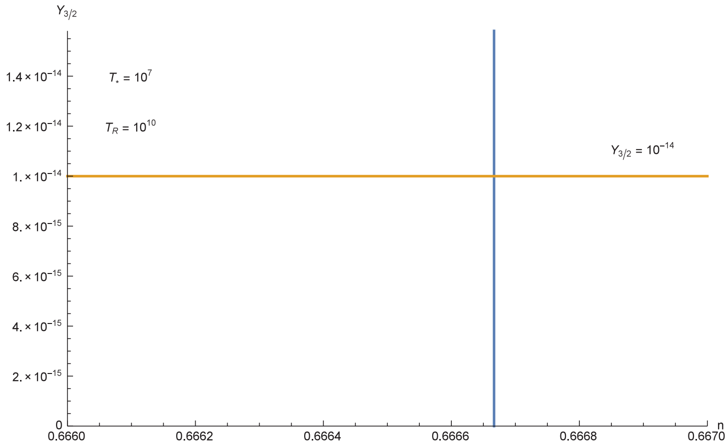

Fig. 1. – $Y_{3/2}$ *vs.* n.

it follows that the overproduction of gravitino is avoided if the transition temperature is $T_* \lesssim (10^6–10^7)\,\text{GeV}$, while the reheating temperature can be larger in order to be compatible with inflationary prediction values (see fig. 1).

4. – Conclusions

In this paper, we have presented some preliminary results related to the gravitino problem assuming that the background is described by $f(R)$ cosmology. Using the fact that the expansion rate of the Universe gets modified as $H = A(T)H_{GR}$, where the factor $A(T)$ (given in eq. (21)) accounts for the non canonical evolution of the cosmic background, we solve the Boltzmann equation that describes the time evolution of the gravitino abundance. We have shown that, under specific condition, $f(R)$ cosmology might provide cosmological scenarios able to avoid the late overproduction of gravitino. The analysis carried out in this paper relies on models in which $f(R)$ is a power-law expansion of the scalar curvature R. However, besides the possibility to find a more generic solution for the scalar factor solution of the field equations, other curvature invariants, like Riemann and Ricci ones and their derivatives, might play a relevant role for the gravitino problem here studied. In particular, in view of recent results obtained in the framework of black hole physics [18], also models based on $f(R, \Box R, \Box^l R, \ldots)$ deserve to be taken into account.

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