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Asymptotic normalization coefficients for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ from the peripheral α-particle transfer reactions and their astrophysical application

R. Yarmukhamedov(*), O. R. Tojiboev and S. V. Artemov

Department of Physics, National University of Uzbekistan - Tashkent, Uzbekistan

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Summary. — The results of the analysis of the α-particle transfer ${}^{12}\mathrm{C}(\mathrm{d}, {}^6\mathrm{Li})^8\mathrm{Be}$ and ${}^{12}\mathrm{Be}({}^3\mathrm{He}, {}^7\mathrm{Be})^8\mathrm{Be}$ reactions at the low energies performed within the modified DWBA have been presented. New estimates are obtained for values of the asymptotic normalization coefficients (ANCs) for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ (g.s.) and $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ ($E^* = 0.429\,\mathrm{MeV}$) as well as of the astrophysical $S_{34}(0)$ factors for the direct capture ${}^3\mathrm{He}(\alpha, \gamma)^7\mathrm{Be}$ reaction.

1. – Introduction

The direct radiative capture ${}^3\mathrm{He}(\alpha, \gamma)^7\mathrm{Be}$ reaction is one of the main links in the *pp* chain of solar hydrogen burning [1, 2]. Its rate at the stellar temperature $T_6 \sim 15\,\mathrm{K}$, corresponding to the solar Gamow's peak around $E_G \sim 22\,\mathrm{keV}$, determines how much the ${}^7\mathrm{Be}$ branch of the *pp* chain contribute to solar hydrogen burning [3].

In this paper, we present new results for the asymptotical normalization coefficients (ANCs) [4] for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ obtained within the modified DWBA (MDWBA) [5,6] from the peripheral α-particle transfer reactions. The obtained results are applied for estimation of the astrophysical S factors $S_{43}(0)$ for the direct radiative capture ${}^3\mathrm{He}(\alpha, \gamma)^7\mathrm{Be}$ reaction.

2. – Basic formulas of the modified DWBA

In the MDWBA, the calculated differential cross section at the main peak for a peripheral one-particle (a) transfer reaction $x + A \to y + B$ ($x = y + a$ and $B = A + a$)

(*) E-mail: `rakhim@inp.uz`

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is parameterized in terms of the ANCs and has the form [5, 6]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C^2_{aA;\,l_Bj_B} C^2_{ya;\,l_xj_x} \mathcal{R}_{l_xj_xl_Bj_B}(E_i, \theta;\, b_{ya;\,l_xj_x}, b_{aA;\,l_Bj_B}), \tag{1}$$

where

$$\mathcal{R}_{l_xj_xl_Bj_B}(E_i, \theta;\, b_{ya;\,l_xj_x}, b_{aA;\,l_Bj_B}) = \frac{\sigma^{\mathrm{DWBA}}_{l_xj_xl_Bj_B}(E_i, \theta;\, b_{ya;\,l_xj_x}, b_{aA;\,l_Bj_B})}{b^2_{ya;\,l_xj_x} b^2_{aA;\,l_Bj_B}}, \tag{2}$$

$\sigma^{\mathrm{DWBA}}_{l_xj_xl_Bj_B}$ is the single-particle cross section calculated in DWBA, C's are the ANCs for $y + a \to x$ and $A + a \to B$ [4]; b's are the single particle ANCs of the shell model bound-state wave functions; l_x and l_B (j_x and j_B) are the orbital (total angular) momenta of the transfer particle a in the $x(= (y + a))$ and $B(= (A + a))$ nuclei.

The peripheral character for the reaction at least in near the main peak of the angular distribution is conditioned by [5, 6]

$$\mathcal{R}_{l_xj_xl_Bj_B}(E_i, \theta;\, b_{ya;\,l_xj_x}, b_{aA;\,l_Bj_B}) = f(E_i, \theta), \tag{3}$$

as a function of the free parameters $b_{aA;\,l_Bj_B} = b_{aA;\,l_Bj_B}(r_o, a)$ and $b_{ya;\,l_xj_x} = b_{ya;\,l_xj_x}(r_o, a)$, where r_o and a are geometric parameters (radius and diffuseness) of the Woods-Saxon potential [7]. Then the ANC for $\mathrm{A} + a \to \mathrm{B}$ can be determined from the condition

$$C^2_{aA;\,l_Bj_B} = \frac{\mathrm{d}\sigma/\mathrm{d}\Omega}{C^2_{ya;\,l_xj_x} \mathcal{R}_{l_xj_xl_Bj_B}(E_i, \theta;\, b_{ya;\,l_xj_x}, b_{aA;\,l_Bj_B})} = \mathrm{const}, \tag{4}$$

which must be fulfilled for each fixed energy E_i, scattering angle θ and the $\mathcal{R}_{l_xj_xl_Bj_B}$ function from (3).

3. – Asymptotic normalization coefficients for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ and their nuclear astrophysical application

The ANC values for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ are determined from the combined analysis of the experimental differential cross sections for the ${}^{12}\mathrm{C}(d, {}^6\mathrm{Li})^8\mathrm{Be}$ and ${}^{12}\mathrm{C}({}^3\mathrm{He}, {}^7\mathrm{Be})^8\mathrm{Be}$ reactions measured in [8, 9]. The optical potentials recommended in [8, 9] were used.

The test of the peripheral character of all the reactions mentioned above has been done by means of verifying the conditions (3) and (4) and by changing the geometric parameters (radius r_o and diffuseness a) of the adopted Woods-Saxon potential, similar as it was done in [5, 6]. This potential is used for calculations of the bound-state wave functions involving in the considered reactions. Varying of the parameters of r_o and a is done within the wide intervals in respect to their standard values. In turn, such varying results in strong changing of the single-particle ANCs of the bound-state wave functions. We used several experimental θ points from the angular region of the mean peak of the angular distributions. The analysis shows that both conditions (3) and (4) are fulfilled within the experimental errors for the differential cross sections of the considered reactions and, consequently, they are peripheral at least in the angle region of near the main peak of the angular distributions. The results of the ANCs obtained for each experimental θ point from the angle regions of the main peak of the angular distributions and

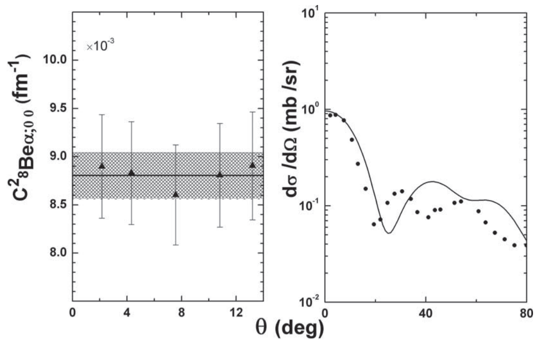

Fig. 1. – The ANC for ${}^8\mathrm{Be}+\alpha \to {}^{12}\mathrm{C}$ for each of the experimental θ (the left-hand side) and the differential cross sections of the ${}^{12}\mathrm{C}(\mathrm{d},{}^6\mathrm{Li}){}^8\mathrm{Be}$ reaction at $E_d = 28\,\mathrm{MeV}$ (the right-hand side). The solid line in the left-hand side presents the result for the weighted mean and the width of the band are the weighed uncertainty. The solid curve in the right-hand side is the result of calculation done within the MDWBA. Experimental data is taken from [8].

the weighted means are presented in the left-hand side of figs. 1 and 2. The differential cross sections calculated by using of (1) and the "post"-approximation of DWBA with the DWUCK5 code as well as the weighted means of the corresponding ANC value are plotted in the right hand side of these figures. At this, the ANC value for $\alpha+d \to {}^6\mathrm{Li}$ recommended in [2] and the weighted mean for ${}^8\mathrm{Be}+\alpha \to {}^{12}\mathrm{C}$ ($C^2_{{}^{12}\mathrm{C}\alpha;00} = 8800\pm240\,\mathrm{fm}^{-1}$) obtained in the present work from the ${}^{12}\mathrm{C}(d,{}^6\mathrm{Li}){}^8\mathrm{Be}$ analysis, which enter the right-hand side of the relation (4), were used. The results of the weighed means of the ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ and their application for obtaining the extrapolated $S_{34}(0)$ value for the direct capture ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ reaction, similar as it was done in [11] within the method proposed in [10], are presented in table I together with the results obtained by other authors. As is seen from table I, the $C^2_{\alpha{}^3\mathrm{He};\,1\,j_{\mathrm{Be}}}$ values ($j_{\mathrm{Be}} = 3/2$ for the ground state of ${}^7\mathrm{Be}$ and $j_{\mathrm{Be}} = 1/2$ for the first excited ($E^* = 0.429\,\mathrm{MeV}$) state of ${}^7\mathrm{Be}$) and the $S_{34}(0)$ value derived in the present work, confirm the independent results of [11].

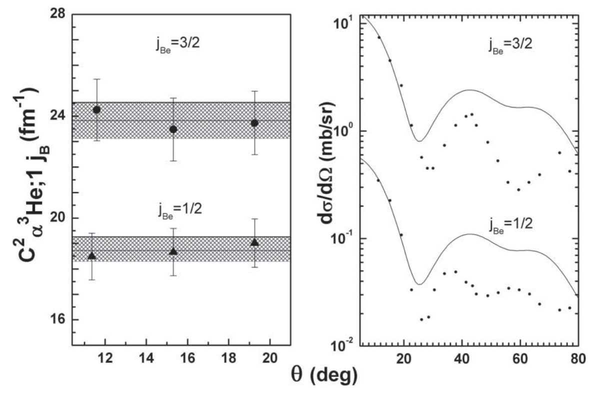

Fig. 2. – The same as in fig. 1 for the ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ and the ${}^{12}\mathrm{C}({}^3\mathrm{He},{}^7\mathrm{Be}){}^8\mathrm{Be}$ reaction at $E_{{}^3\mathrm{He}} = 41\,\mathrm{MeV}$ populating the ground ($j_B = 3/2$) and first excited ($j_B = 1/2$) states of the ${}^7\mathrm{Be}$ nucleus. Experimental data is taken from [9].

TABLE I. – *The ANCs for* $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}(E^* = 0.0; J^\pi = 3/2)$ *and* $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}(E^* = 0.429\,\mathrm{MeV}; J^\pi = 1/2)$ *and the summarized* $S_{34}(0)$ *for the direct radiative capture* ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ *reaction obtained within the different methods.*

Method	$C^2_{^3\mathrm{He}\alpha;\,1\,3/2}$ (fm^{-1})	$C^2_{^3\mathrm{He}\alpha;\,1\,1/2}$ (fm^{-1})	$S_{34}(0)$ (keV b)	Refs.
The MDWBA ${}^{12}\mathrm{C}(d,{}^6\mathrm{Li}){}^8\mathrm{Be}$ and ${}^{12}\mathrm{C}({}^3\mathrm{He},{}^7\mathrm{Be}){}^8\mathrm{Be}$ analysis	22.9 ± 0.7	18.0 ± 0.5	0.596 ± 0.017	Present work
The MTBPA ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ analysis	$23.3^{+1.0}_{-2.3}$	$15.9^{+0.6}_{-1.5}$	$0.613^{+0.026}_{-0.063}$	[11]
The R-matrix ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ analysis	14.4	14.4	0.51 ± 0.04	[12]
The compiled phenomenological approaches			$0.56 \pm 0.02(\mathrm{exp}) \pm 0.02(\mathrm{th})$	[1]
The MRGM			0.609	[13]
The ab initio type calculations			0.593	[14]

4. – Outlook

The scrupulous analysis of the peripheral α-particle reactions under consideration shows that these reactions are peripheral in the angle region near the main peak of the angular distributions and can be used as an independent source for getting the reliable information about the ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$. The obtained ANC values allow one to obtain a reliable estimation of the $S_{43}(0)$ value with the uncertainties not exceeding the modern experimental data errors. New values for the ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ and the $S_{43}(0)$ are in agreement with that obtained from the direct-capture ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ reaction by the independent way in ref. [11] and differ strongly from the values, which have been deduced by other authors with different methods.

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