IL NUOVO CIMENTO **39 C** (2016) 402
DOI 10.1393/ncc/i2016-16402-y

Colloquia: IWM-EC 2016

Code comparison of transport simulations of heavy ion collisions at low and intermediate energies

H. H. Wolter(*)
University of Munich - Garching, Germany

received 10 January 2017

Summary. — Transport simulations are an important and successful tool to extract information on the equation of state of nuclear matter away from saturation conditions from heavy ion collisions. However, at times, such calculations with seemingly similar physical input have yielded different conclusions. Therefore it is deemed important to compare transport simulations under controlled conditions, which is the objective of the Transport Simulation Code Comparison project, on which we report here. We obtain for the first time a quantitative systematic error of transport simulations. We discuss possible reasons for these deviations and further comparisons to improve the situation.

1. – Introduction

Understanding the nature of dense matter constitutes a significant scientific objective for both nuclear physics and astrophysics. In astrophysics, measurements of neutron star masses and radii have raised questions about the nuclear pressure that supports these stars against gravitational collapse into black holes. Clearly, nuclear physics has the task to provide laboratory constraints on properties of dense nucleonic matter. This requires constraining the nuclear Equation of State (EoS) which can be given as the energy per nucleon as a function of the density ρ and the asymmetry $\delta = (\rho_n - \rho_p)/\rho$ as $\epsilon(\rho, \delta) = \epsilon_0(\rho) + S(\rho)\delta^2$, where ρ_n, ρ_p, and ρ are the neutron, proton, and total densities, respectively. The function $S(\rho)$ is known as the symmetry energy, which is of particular interest recently, see *e.g.* refs. [2]. Any asymmetric system will be influenced significantly by the symmetry energy and the associated isovector potential. *E.g.* in neutron stars, the EoS of very asymmetric matter determines the relation between mass and radius.

Heavy ion collisions (HIC) have been used successfully to momentarily create and study nuclear systems from densities below to above saturation density and at finite temperature. However, since heavy ion collisions are dynamical processes, the underlying

(*) On behalf of the Code Comparison Collaboration, the members of which are given in ref. [1]

EoS must be inferred by comparing experimentally observed properties of the reaction products to the results of transport models, which have as input the EoS, the nucleon effective mass, and the in-medium nucleon collision cross-section. Transport models have been used extensively and quite successfully in the past 30 years to describe heavy ion dynamics from Fermi to relativistic energies. Thus they play a very important role in extracting physical information from HICs.

However recently, different transport models have given different predictions for physical observables with seemingly similar nuclear input. It is therefore important to understand the causes for this and possibly to reduce the differences. This is the idea of the transport code comparison project, and also to establish a kind of theoretical systematic error that quantifies the model dependence of transport predictions.

Possible reasons for the model dependence are inherent in the complexity of transport calculations. The equations of these theories are generally solved by numerical simulations. The strategies used are not always evident in simulations which interpret experimental data which often employ different physical inputs and slightly different conditions. Therefore in the code comparison project all calculations use exactly the same physical input and, as closely as possible, the same initial conditions.

The necessity of such comparisons has been recognized since a long time. There were workshops at the ECT* in Trento in 2004 [3] and 2009, which were followed by a new effort in Shanghai in 2014 [4] and several other shorter meetings. In these workshops we compared the results of HICs at low and intermediate energies. A first publication of the results has appeared in ref. [1], on which we report in this contribution.

As will be seen there are considerable differences in the results. For the first time we try to quantify these differences. We discuss possible reasons for the differences, but these are not easy to pinpoint, since in a heavy ion collisions many effects are closely entangled. Thus in order to improve the situation and to provide further insight into the properties of transport simulations, we presently continue the comparison by calculations of infinite nuclear matter in a box with periodic boundary conditions. This will be briefly discussed in the outlook.

2. – Setup of the comparison calculations

Basically two families of transport approaches are used in the study of heavy ion collisions, the Boltzmann-Vlasov type, which is formulated for the evolution of the one-body phase-space density under the influence of a mean field, and the molecular dynamics type, which is formulated in terms of nucleon coordinates and momenta under the action of a many-body Hamiltonian. Both are supplemented with a two-body collision term depending on the in-medium cross section. Here we refer to the two types of approaches as Boltzmann-Uehling-Uhlenbeck (BUU), and Quantum Molecular Dynamics (QMD) models, according to their most common representatives. A short characterization of these approaches can be found in ref. [1], where references to more detailed descriptions and to the history of transport theories in heavy ion collisions can be found. A main difference between the two approaches is in the amount of fluctuations in the representation of the phase space distribution, which can lead to different behaviour in the disintegration and fragmentation of the system. In the quantities discussed here, namely one-body observables and collision rates, these differences are not expected to have a great influence.

Essentially all transport codes, which are used presently for the interpretation of HICs, participated in the comparison, alltogether 9 BUU- and 9 QMD-type codes. Among

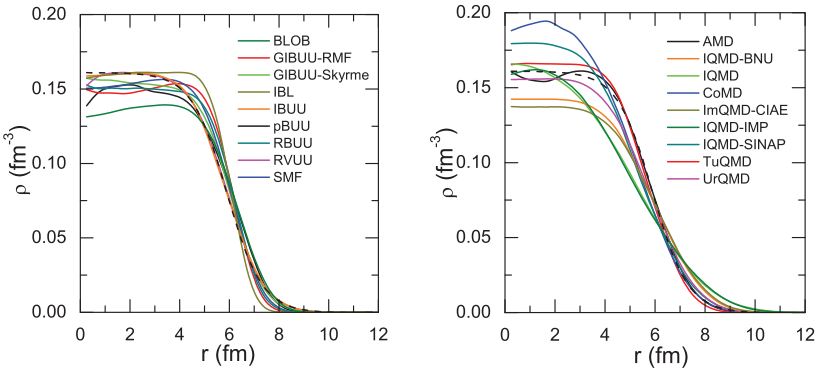

Fig. 1. – Initial density profiles for BUU-type (left panel) and QMD-type (right panel) models. The dashed curve in each panel represents the prescribed Woods-Saxon distribution.

these were non-relativistic and relativistic formulations. The particular codes are given in ref. [1] together with tables of some details of the procedures used in these codes.

The calculations were performed using the same physical input for the mean field and the cross sections, which were typical of heavy ion collisions. For the mean field a simple soft Skyrme-type functional was used for non-relativistic codes and an equivalent non-linear relativistic mean field (RMF) formulation for relativistic codes. The cross section was taken as a constant. The statistical significance, 1000 events for QMD and 10 runs with 100 test particles for BUU was similar. The procedures for mean field propagation and the treatment of the collisions, *i.e.* collision probabilities and blocking factors, were left as in the standard use of each code. We performed calculations in different "modes", namely the Vlasov mode with only mean field propagation, the Cascade mode with only collisions, and the full mode with both, in order to try to judge the separate influence of the mean field and the collisions. We performed the calculations for the collision system Au + Au at two energies typical for the present low and intermediate energy HICs, at 100 and 400 A MeV incident energy for a mid-central reaction with impact parameter 7 fm. We also performed calculations with impact parameter $b = 20$ fm, where no collsion takes place, in order to test the stability of the initialization of the collision system.

For the initialization in coordinate space a given Wood-Saxon density profile was prescribed. In momentum space the standard prescription was used to choose the momenta randomly in the local Fermi sphere corresponding to the local density. The evolution of the collision was followed up to a final time of 140 fm, and the particle motions, and the number, energy and time of the collisions were monitored. At the final time rapidity distributions and transverse flows were calculated and compared. Of all this information only a small sample of particularly interesting results can be given here, but more can be found in ref. [1].

In fig. 1 we show the intial density distributions as they are realized by the different BUU and QMD codes, where the dashed curve is the prescribed Woods-Saxon profile. It is seen that the codes succeed only approximately to reproduce it, much less so the QMD codes because of their larger fluctuations. These initial states are not necessarily eigenstates of the prescribed mean field potenial and consequently may not be stable in the evolution. This was seen in the calculation with the large impact parameter where the isolated nuclei propagate freely. In some codes the distributions start to oscillate,

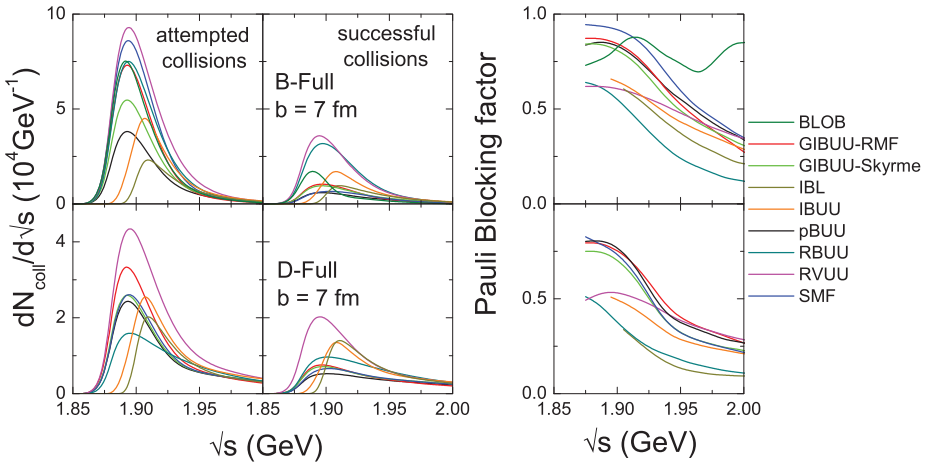

Fig. 2. – Nucleon-nucleon attempted and successful scattering numbers per 0.1 MeV and Pauli blocking factors as a function of c.m. collision energy in Au+Au collisions for 100 A MeV (upper row), and 400 A MeV (lower row) for BUU-type models.

in others they settle into other stable states (see ref. [1]). They are rather stable in those cases where the intial state was generated consistent with the mean field, *e.g.* in a Thomas-Fermi approximation. It was also seen that the instability may lead to larger or lower densities or smaller particle numbers at the time of collision, which can influence the result. Thus for a consistent calculation, and also for a more meaningful comparison, a stable initialization may be more important then an identical one.

3. – Heavy ion collision at $b = 7$ fm

In this section we discuss the comparison of the different transport codes for a "real" heavy ion collision of midcentral impact parameter. In such a reaction violent interactions take place and all aspects of a heavy ion collisions are important: initialization, mean field propagation, collision probabilities and Pauli blocking of collisions. Of the many results we discuss next the collision and blocking rates for BUU codes in fig. 2 for 100 (upper row) and 400 (lower row) A MeV incident energy as a function of the total collision energy $s = 4(m^{*2}+p^2)$. The results for QMD codes are qualitatively similar. The first column shows the attempted collisions, where the distance criterion depending on the cross section is satisfied. The second column are the successful collisions, where the final states are not Pauli blocked. The last column is the blocking rate, *i.e.* $(1 - \textit{successful}/\textit{attempted})$. The collision threshold for a mass of 938 MeV is $\sqrt{s} = 1.876$ GeV, the free values are 1.925 and 2.066 GeV for collisions at 100 and 400 A MeV, respectively. It is seen that most of the collisions happen at lower energies in the compressed phase, where the nucleons are already largely stopped. There are large differences in the attempted collsions. Some codes impose a threshold for very low energy collisions with the argument, that their effect is small and many of them are spurious. The rates at higher energies converge rather well, probably because they are early collisions, where the evolution of the reaction does not yet differ much. The blocking factors shown in the last column depend on the calculation of the occupation of the final state of the collision. They become smaller with increasing collision energy because then the final states are less blocked. Again there are large

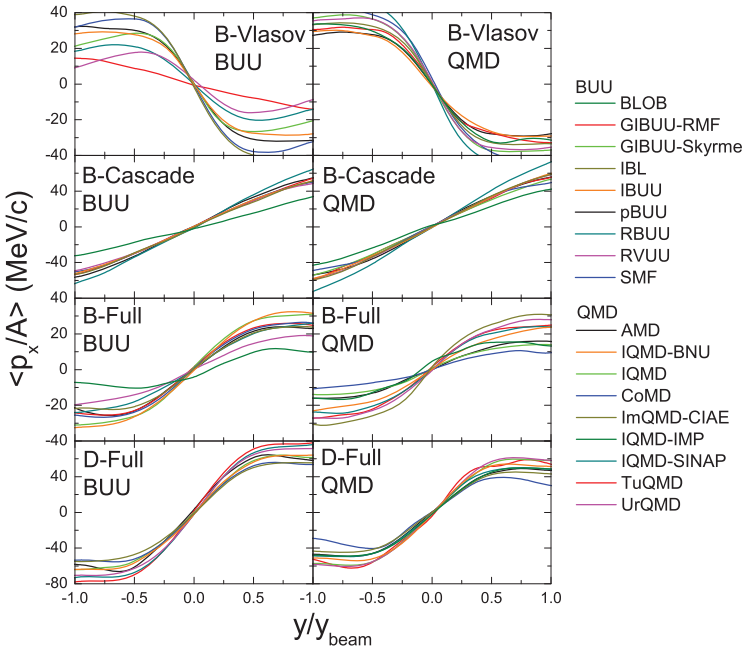

Fig. 3. – Transverse flows as a function of reduced rapidity for (from top to bottom row) Vlasov, Cascade and full modes at 100 A MeV and full mode at 400 A MeV for BUU (left) and QMD (right) models.

differences, which tend to be smaller at higher energy, however, with some exceptions. These results suggest, that a important reason for the differences in the codes is in the calculation of the blocking. This is not unexpected since the calculation of the final state occupations involves a strategy in the phase space discretization. We recall that there was no attempt to unify this part of the codes, which was used as in the normal usage.

We next discuss some observables, *i.e.* quantities which appear at the end of the collision and which can be compared to experiment (which, however, is not the aim here). Here we show in fig. 3 the average transverse flow as a function of the reduced rapidity for BUU and QMD codes. The upper three rows are for Vlasov, Cascade and full calculations at 100 A MeV, respectively, and the lower row the full calculation at 400 A MeV. These curves are often characterized by the slope at zero rapidity, often called the flow value. At 100 MeV we see that we are in the region of the balance energy, since the mean field is attractive and the flow negative, while the collisions are repulsive and lead to a positive flow. The total effect at this energy is already repulsive. At 400 A MeV both effects are acting in the same way (note change of scale). Thus at 100 A MeV one is in a particularly sensitive region, and correspondingly the flows are rather strongly divergent, while at 400 A MeV they converge much better.

The flow values give a possibility to quantify the differences between the codes. This is shown in fig. 4, for BUU codes on the left and QMD codes on the right for 100 A MeV (red triangles) and 400 A MeV (black squares). We see that QMD and BUU codes are roughly consistent within their uncertainities. The spread of the codes is in absolute value similar at both energies but relatively smaller at the higher energy. From the spread of the results in fig. 4 we derive an uncertainity of about 30% at 100 A MeV and about 13%

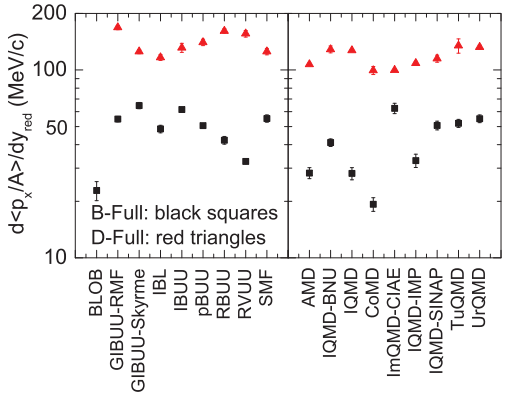

Fig. 4. – Slope parameters of the transverse flow for BUU-type (left) and QMD-type (right) models for 100 (black squares) and 400 (red triangles) A MeV. The error bars are the fitting uncertainties.

at 400 A MeV. As discussed before, the uncertainities are larger at 100 A MeV, because of the larger sensitivity because of the cancellation between the effects of the mean field and the collisions.

4. – Discussion, conclusion, and outlook

The above uncertainities represent a kind of theoretical systematic error. *I.e.*, if different codes analyse the same experimental data, the conclusions could be different up to this uncertainity. The present uncertainity is, of course, not really satisfactory, and attempts will be made to improve it. We cannot, of course, make a judgement about the correctness of the results of a particular code, *e.g.* on the basis of the deviation from the average result of the codes. The present results depend on a single observable, and different effects can influence it. A comparison in terms of several observables, as should usually be done in an analysis of an experiment, would perhaps give a stronger indication of the validity of a code.

It is not easy to attribute the differences to the specific features or strategies of a code. It appears that the differences in initialization and in the Pauli blocking factors may be significant causes, particularly at the lower energy. However, many effects are entangeled in a real heavy ion collision. *E.g.* if the collision rates are too small then there is less stopping and consequently less compression and smaller densities. Thus the mean field is probed at a different density. Thus, errors in one effect propagate into other effects, thus making it diffiult to isolate the reasons for the deviations.

A way to move forward are calculations, where the different effects can be separated better. This is the case for calculations in infinite nuclear matter, which can be realized approximately in a box with periodic boundary conditions. Here the initialization is not problematic. Calculations in the cascade mode can test separately the calculation of collision probabilites and of the Pauli blocking factors. Vlasov calculations can test, whether the dispersion relation is consistent with the underlying density functional. In many cases there exist exact analytical or numerical results, so that the results of the codes can be checked absolutely. Such calculations are presently underway.

In addition box calculations allow to compare other properties of transport codes which are important for the comparison with experiment but complicated to disentangle

in a full heavy ion collision. An example are the fluctuations in a code, which are important for the fragmentation. This can be investigated by initializing the system in the spinodal regime and observing the pattern and time behaviour of the fragmentation. The fragment production and the fractionation of isospin have, *e.g.*, been important recently in the study of the symmetry energy. In this respect a very interesting observable is the pion production, in particular the ratio of positive to negative pions, which should be strongly influenced by the symmetry energy. Pion production introduces new physics into a transport calculation in the production, propagation and decay of the Delta resonance, which has a finite width, which could then be tested separately without the whole complexity of a heavy ion collision.

To summarize, in this work we have made a comprehensive effort to obtain some measure of reliability of heavy ion transport codes, for which we give a first quantitative estimate. From the experience in this first round of comparisons it is not so easy to localize definitely the reasons for the discrepancies. This can be done with more confidance by using box calculations of infinite nuclear matter, where the effects can be separated better and where in many cases exact limits exist. In this way we hope to arrive at a kind of benchmarking of transport codes, which will help to use with increased confidence heavy ion collisions to obtain information on the equation of state and other properties of nuclear matter.

* * *

We thank the community of heavy ion practitioners for the support of this project. The present author is supported by the DFG Cluster of Excellence *Origin and Structure of the Universe*, Germany. Acknowledgements for the other members of the collaboration are given in ref. [1].

REFERENCES

[1] Xu Jun, Chen Lie-Wen, Tsang ManYee Betty, Wolter Hermann, Zhang Yingxun *et al.*, *Phys. Rev. C*, **93** (2016) 044609.
[2] Horowitz C. J. *et al.*, *J. Phys. G*, **41** (2014) 093001; *Topical Issue on Nuclear Symmetry Energy*, edited by Li B. A. *et al.*, *Eur. Phys. J. A*, **50** (2014) 9.
[3] Kolomeitsev E. E. *et al.*, *J. Phys. G*, **31** (2005) S741.
[4] Workshop website: `http://www.physics.sjtu.edu.cn/hic2014/`.