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Nuclear matter EoS including few-nucleon correlations

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Summary. — Improving a mean-field approach to the nuclear matter equation of state (EoS), few-nucleon correlations are investigated. The contribution of clusters to the thermodynamic properties is suppressed at increasing density because of Pauli blocking. Continuum correlations are implemented. Applications to heavy-ion collisions (HIC) are discussed.

1. – Correlations and light clusters in dense matter

The quantum statistical (QS) approach to few-nucleon ($A \leq 4$) correlations in warm dense nuclear matter (temperature $T < 20\,\mathrm{MeV}$, baryon density $n_B < n_{\mathrm{sat}} \approx 0.16\,\mathrm{fm}^{-3}$, asymmetry (total proton fraction) $0 < Y_p = n_p^{\mathrm{tot}}/n_B < 1$) can be given within a Green function approach [1]. The particle density $n_\tau^{\mathrm{tot}}(T, \mu_n, \mu_p)$ of neutrons ($\tau = n$) or protons ($\tau = p$) as function of T and the chemical potentials μ_τ is obtained from the single-particle spectral function (normalization condition) which is determined by the self-energy. A cluster decomposition is performed for the self-energy [2]. In quasiparticle approximation we obtain

$$n_n^{\mathrm{tot}}(T, \mu_n, \mu_p) = \frac{1}{\Omega} \sum_{A,\nu,\mathbf{P}} N f_{A,Z} \left[E_{A,\nu}(\mathbf{P}; T, \mu_n, \mu_p) \right],$$
$$n_p^{\mathrm{tot}}(T, \mu_n, \mu_p) = \frac{1}{\Omega} \sum_{A,\nu,\mathbf{P}} Z f_{A,Z} \left[E_{A,\nu}(\mathbf{P}; T, \mu_n, \mu_p) \right], \tag{1}$$

where $\mathbf{P}$ denotes the c.o.m. momentum of the cluster (or, for $A = 1$, the momentum of the nucleon). The internal quantum state ν contains the proton number Z and neutron

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number $N = A - Z$ of the cluster;

$$f_{A,Z}(E;T,\mu_n,\mu_p) = \frac{1}{\exp[(E - N\mu_n - Z\mu_p)/T] - (-1)^A} \tag{2}$$

is the Bose or Fermi distribution function for even or odd A, respectively, which depends on $\{T, \mu_n, \mu_p\}$. The $\mathbf{P}$-dependent quasiparticle energies $E_{A,\nu}(\mathbf{P};T,\mu_n,\mu_p)$ [3] are depending on the medium characterized by $\{T, \mu_n, \mu_p\}$ as discussed below.

The internal quantum states ν cover not only the possible bound states of the few-nucleon system but also the scattering states. They are implemented using the scattering phase shifts as known from the Beth-Uhlenbeck formula. Only after including the contributions of the scattering states, the correct virial expansion is obtained [4].

The normalization condition $n_\tau^{\rm tot}(T,\mu_n,\mu_p)$ is a particular form of the EoS, which relates the particle densities to the temperature and the chemical potentials. Further thermodynamic quantities and corresponding versions of the EoS are consistently introduced if a thermodynamic potential is known. For instance, the density of the Helmholtz free energy as function of $T, n_\tau^{\rm tot}$ is found after integration of the relation $\mu_\tau(T, n_n^{\rm tot}, n_p^{\rm tot})$.

2. – Quasiparticle shifts

The quasiparticle energies $E_{A,\nu}(\mathbf{P};T,\mu_n,\mu_p) = E^{(0)}_{A,\nu}(\mathbf{P}) + \Delta E^{\rm SE}_{A,\nu}(\mathbf{P}) + \Delta E^{\rm Pauli}_{A,\nu}(\mathbf{P}) + \Delta E^{\rm Coul}_{A,\nu}(\mathbf{P})$ follow from the solution of the A-nucleon in-medium Schrödinger equation

$$\begin{aligned}&[E_{\tau_1}(\mathbf{p}_1;T,\mu_n,\mu_p) + \cdots + E_{\tau_A}(\mathbf{p}_A;T,\mu_n,\mu_p) - E_{A\nu}(\mathbf{P};T,\mu_n,\mu_p)]\psi_{A\nu\mathbf{P}}(1\ldots A)\\ &+ \sum_{1'\ldots A'}\sum_{i<j}[1 - n(i;T,\mu_n,\mu_p) - n(j;T,\mu_n,\mu_p)]V(ij,i'j')\\ &\times \prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu\mathbf{P}}(1'\ldots i'\ldots j'\ldots A') = 0,\end{aligned} \tag{3}$$

which is derived from the Green function approach. The interaction $V(ij,i'j')$ contains the Coulomb interaction which is fundamental and leads to the well-known energy shifts given by the Wigner-Seitz approximation or the Debye shift. In addition, it includes also the strong part of the nucleon-nucleon interaction which is adapted to reproduce known properties of nuclear systems.

The quasiparticle energy $E_\tau(\mathbf{p};T,\mu_n,\mu_p) = \hbar^2p^2/(2m_\tau) + \Delta E^{\rm SE}_\tau(\mathbf{p};T,\mu_n,\mu_p)$ contains the single-nucleon energy shift $\Delta E_\tau{}^{\rm SE}(\mathbf{p})$. It can be taken from Skyrme or relativistic mean-field expressions, which are adapted to reproduce properties of nuclei. For instance, the RMF-DD2 approximation [5] is taken in ref. [3].

For $A \geq 2$, the interaction term occurs in eq. (3). It contains the Pauli blocking expressed by the effective occupation numbers $n(i;T,\mu_n,\mu_p)$

$$n(1) = f_1(1) + \sum_{\bar{A}=2}^{\infty}\sum_{\bar{\nu}\bar{\mathbf{P}}}\sum_{2\ldots\bar{A}}\bar{A}\, f_{\bar{A},\bar{\nu}}[E_{\bar{A},\bar{\nu}}(\bar{\mathbf{P}};T,\mu_n,\mu_p)]|\psi_{\bar{A}\bar{\nu}\bar{\mathbf{P}}}(1\ldots\bar{A})|^2, \tag{4}$$

exchange terms are neglected. The shift of the energy eigenvalues $E_{A\nu}(\mathbf{P})$ of eq. (3) caused by the phase space occupation $n(1)$ is denoted as $\Delta E^{\rm Pauli}_{A,\nu}(\mathbf{P})$.

In the low-density limit where both, the single-nucleon energy shift and the Pauli blocking, can be neglected, the solution of the A-nucleon Schrödinger equation (3) gives

the binding energies of nuclei. As in the case of the single-nucleon energy shift $\Delta E_\tau^{\rm SE}(\mathbf{p})$, we avoid a first-principle solution of the A-nucleon wave equation (3) (with some effective nucleon-nucleon interaction) but use the empirical binding energies [6].

To calculate the Pauli blocking as well as the contribution of continuum states, a modeling of the nucleon-nucleon interaction is necessary. With a simple (separable) form of the interaction which reproduces known properties of nuclei, expressions for the Pauli blocking as function of $\mathbf{P}, T, n_B, Y_p$ have been found. Simple fit formula are given in ref. [3]. The account of correlations in the medium leads to occupation numbers $n(1)$ which are different from the single-nucleon distribution function $f_{\tau_1}(\mathbf{p}_1; T, \mu_{\tau_1})$. A fit with effective temperatures and effective chemical potentials has been proposed, see [3], to approximate the actual occupation numbers $n(1)$.

For a consistent approach to the EoS, the contributions of scattering states have to be taken into account. Only then, the correct virial expansion is obtained. In particular, if bound states are dissolved because of Pauli blocking, resonances can occur which contribute to the EoS almost in the same way as a bound state. For instance, ^{8}Be appears as a resonance in the α-α continuum but can be considered like a bound state. To avoid double counting, the contribution of the continuum of scattering states must be reduced if resonances are treated as bound states. Also, the mean-field contributions which are considered to form the single-nucleon quasiparticle must be dropped from the continuum of scattering states, as shown for the generalized Beth-Uhlenbeck formula [4]. The rigorous treatment of continuum contributions to the EoS is not solved at present for clusters with arbitrary A and needs further investigations.

3. – Examples

The EoS including light elements ($A \leq 4$) has been investigated in different publications, for details see [3]. An important feature is the suppression of correlations with increasing density because of Pauli blocking. In particular, the bound states disappear near the density $n_B \approx 0.05\,\mathrm{fm}^{-3}$ what will be denoted as Mott effect. A survey over the composition as well as various thermodynamic quantities has been given in [11]. An improved treatment including the contributions of continuum correlations is presented in ref. [3].

The influence of mean-field corrections and formation of light clusters on the EoS, in particular the chemical potential as a function of temperature and baryon density, has been shown in [3] for symmetric matter. In fig. 1 we present the chemical potentials for stellar matter (β equilibrium, zero neutrino chemical potential) at $T = 10\,\mathrm{MeV}$. The parametrization of the QS approach according to [3] has been used. The effect of clustering leads to a lower chemical potential.

The composition (mass fraction $X_i = A_i \sum_P f_i[E_i(\mathbf{P})]/n_B$ of light nuclei) for stellar matter at $T = 10\,\mathrm{MeV}$ as a function of the baryon density is shown in fig. 2. The depletion of clusters because of Pauli blocking and disappearance near $n_B = 0.05\,\mathrm{fm}^{-3}$ is shown. Preliminary calculations of the mass fraction distribution $X_A = A \sum_{Z,P} f_{A,Z}[E_{A,\nu}(\mathbf{P})]/n_B$ including heavier nuclei ($A > 4$) are shown in fig. 3 for stellar matter at parameter values $T = 10\,\mathrm{MeV}$, $n_B = 0.01\,\mathrm{fm}^{-3}$ ($Y_p = 0.068$) and $0.02\,\mathrm{fm}^{-3}$ ($Y_p = 0.05$). The nuclear binding energy is approximated by the Bethe-Weizsäcker formula. The summation over the charge number Z is performed so that only the dependence on A remains. Isobars with high neutron numbers (*t vs. h*, *e.g.*) are abundant in stellar matter, as a consequence of the different chemical potentials for neutrons and protons. An opposite effect is obtained from the screening of the Coulomb

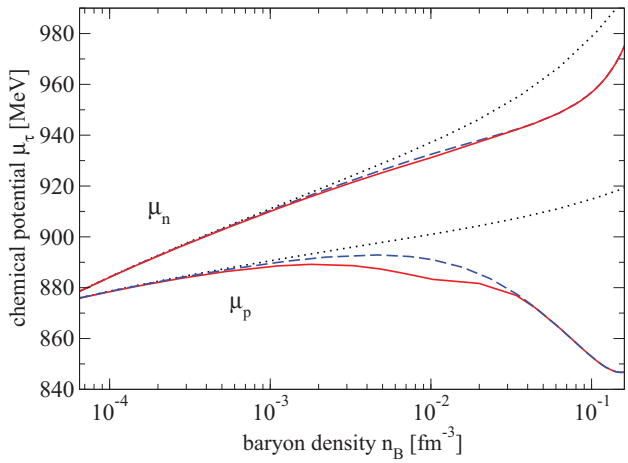

Fig. 1. – Chemical potentials of stellar matter at $T = 10\,\mathrm{MeV}$ as a function of the baryon density. The ideal fermion gas (dotted) is compared with the RMF approximation containing the mean-field shifts (dashed). The QS approach (full line) includes correlations and light clusters modified by medium effects.

interaction which is taken in Debye approximation. A strong decrease with increasing A is shown. The calculations should be improved considering the medium effects for the heavier nuclei more in detail [7]. There, results for symmetric matter are presented.

4. – Application to HIC, chemical constants

Recently [8] a laboratory test of the nuclear matter EoS has been proposed by Natowitz *et al.* analyzing the yields of light elements in heavy-ion collisions (HIC).

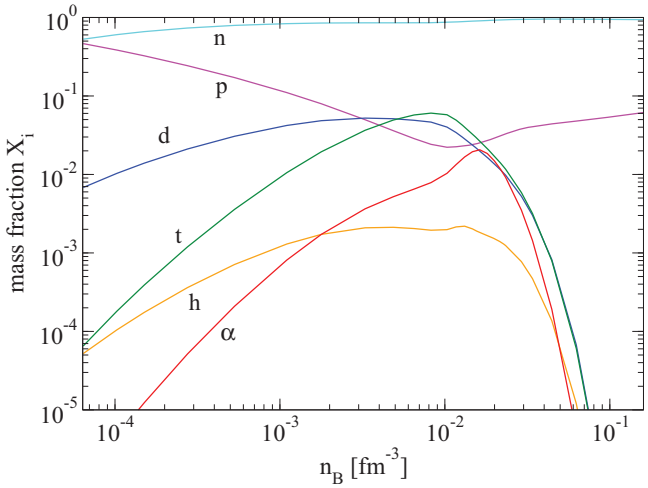

Fig. 2. – Composition of stellar matter at $T = 10\,\mathrm{MeV}$ as a function of the baryon density.

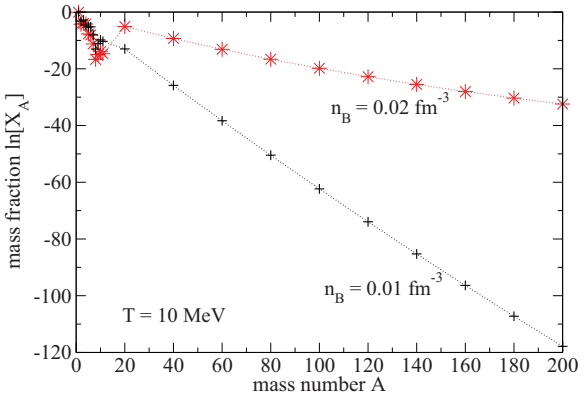

Fig. 3. – Mass fraction distribution X_A of stellar matter at $T = 10\,\mathrm{MeV}$, $n_B = 0.01\,\mathrm{fm}^{-3}$ and $0.02\,\mathrm{fm}^{-3}$, as a function of the mass number A (selected values of A are shown).

The chemical constants have been considered which are not very sensitive with respect to the asymmetry, but also to the chemical potentials which exactly cancel when the non-interacting nuclear statistical equilibrium (NSE) is assumed. The analysis of data from HIC experiments show a significant deviation from the NSE predictions. This proves that under conditions relevant for the yields of light fragments in HIC near the Fermi energy, medium corrections have to be taken into account.

A detailed analysis of theoretical models has been performed in ref. [9]. The concept of excluded volume [10] is an empirical ansatz which simulates the Pauli blocking effect. The global behavior of the in-medium modifications is well described, but details, in particular the dependence of the parameters $\mathbf{P}, T, n_B, Y_p$, cannot be reproduced. The quantum statistical (QS) approach allows for a detailed description of the in-medium modifications for few-nucleon clusters. A generalized RMF approach [11], which considers the light elements as additional degrees of freedom with shifts according to the QS approach, can also approximately reproduce the measured data.

A general problem represents the relation between the measured yields of nuclei and the composition of warm dense matter in equilibrium. HIC are highly dynamic and inhomogeneous in time and in space. Two different pictures are used to describe this process: i) The expanding fireball is described by local thermodynamic equilibrium with thermodynamic parameters $T(\mathbf{r}, t), \mu_\tau(\mathbf{r}, t)$ depending on space and time. Chemical equilibrium between the components is suspended at the freeze-out time where the reaction rates become too slow. This picture has been used in [8] where the time dependence of temperature and density is extracted using a sophisticated treatment of the measured yields. ii) Kinetic equations are used to simulate the nucleon distribution functions depending on space and time. This approach has problems with the treatment of correlations (energy conservation) and the formation of clusters which are introduced artificially.

An adequate description should consider kinetic equations for the distribution functions (Wigner functions) of all clusters which have as equilibrium solutions not the ideal Fermi gas but an appropriate approximation of the EoS. Thus, the EoS containing quasiparticle clusters (medium-modified nuclei) may be considered as a prerequisite to formulate a transport code for the nonequilibrium evolution, described by the extended

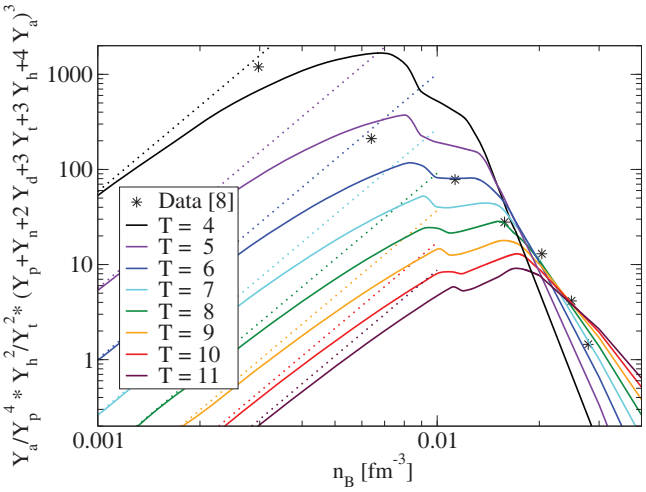

Fig. 4. – Chemical constant $\tilde{K}_\alpha$, eq. (6), as a function of density and temperature (T in MeV). Data (stars) for $T = 5, 6, 7, 8, 9, 10, 11\,\mathrm{MeV}$ (increasing density) in comparison with the NSE values (thin dotted lines) and QS calculations (bold straight lines). Taken from ref. [14].

von Neumann equation for the statistical operator $\varrho(t) = \lim_{\varepsilon\to 0}\varrho_\varepsilon(t)$ [12],

$$\frac{\partial}{\partial t}\varrho_\varepsilon(t) + \frac{i}{\hbar}\left[H, \varrho_\varepsilon(t)\right] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right). \tag{5}$$

The relevant statistical operator $\varrho_{\rm rel}(t)$ is obtained from the maximum of entropy reproducing the local, time-dependent composition with parameter values $T(\mathbf{r},t), \mu_\tau(\mathbf{r},t)$, but contains in addition the cluster distribution functions $f_{A\nu}^{\rm Wigner}(\mathbf{p},\mathbf{r},t)$ as relevant observables which allow a consistent description of kinetic and hydrodynamic processes in dense systems [13].

Even if we can define a freeze-out state (temperature and chemical potentials) which determines the main features of the composition, further reaction and decay processes will occur before the cluster yields, observed in the detectors, are established. In this context it is of interest not only the decay of excited and unbound (*e.g.*, ^{8}Be) nuclei, but what happens with the continuum correlations which are present at high densities. Future work is necessary to devise a transport theory for HIC which is compatible with the thermodynamic properties and the EoS, described in this work, as equilibrium solution.

5. – Relevant distributions

To describe the nonequilibrium evolution, it is of interest to find optimum parameter sets $\{T, n_B, Y_p\}$ for the reproduction of observed abundances of clusters, *i.e.*, to construct the relevant statistical operator $\varrho_{\rm rel}(t)$ in eq. (5). This has been done for HIC where the yields of light clusters have been used to infer the thermodynamic parameter values [8,14]. In this context the reproduction of the measured yield of light clusters within different approaches [9] is of relevance. The same intension is given by fig. 4 to infer optimum parameter values for the thermodynamic parameter T, n_B, Y_e from measured yields [14].

In particular, the quantity

$$\tilde{K}_\alpha = \frac{Y_\alpha}{Y_p^4}\frac{Y_h^2}{Y_t^2}\left(\sum_{A=1}^{4} A_i Y_i\right)^3 = \frac{n_\alpha}{n_n^2 n_p^2} n_B^3 \tag{6}$$

is shown. ($Y_n \approx Y_p Y_t / Y_h$ has been assumed to estimate the neutron yield.)

In contrast to a simple chemical equilibrium (NSE) such as the Albergo thermometer or densitometer which is connected with the ideal mixture of different components, density effects are of importance. The freeze-out parameters represent a state during the evolution where local thermodynamic equilibrium is approximately realized. The further evolution is characterized by the different cluster distribution functions and contains reaction and decay processes.

This description can also be applied to astrophysical abundances of elements [7]. Only the gross properties of elemental distribution are described by a freeze-out approach. Details are related to further reactions during cooling and expansion, forming local (with respect to the N-Z plane) deviations. Based on the cluster distribution functions $f_{A\nu}^{\mathrm{Wigner}}(\mathbf{p}, \mathbf{r}, t)$ as relevant observables, a reaction network can describe this stage of evolution of the nuclear system.

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