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Primordial gravitational waves as a promising test for inflationary models

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Summary. — Any inflationary model predicts the production of a stochastic gravitational-wave background. Such a signal includes unique information about the primordial mechanism that generate it, then representing a promising way of probing the inflationary physics. In this direction, upcoming and future experiments of direct gravitational-wave detection at small scales is expected to play a relevant role, providing new constraints on the features of the inflationary gravitational waves.

1. – Introduction

Current cosmological observations point out that, in its early stages, the universe underwent a period of accelerated expansion, called Inflation. However, several features of such a primordial phase are still unknown and unconstrained. In the direction of testing and better understanding the inflationary physics, the gravitational-wave (GW) signal associated to such a mechanisms represents a promising observable. Moreover, the present-time GW background generated during inflation, is expected to include the signatures of the thermal history of the universe they underwent from their generation up to the detector. Therefore, inflationary GW represent an exciting window on the physics of the early universe but also on the subsequent history of the universe; for a recent review see for example [1].

In this work I will briefly overview the mechanisms of GW production that can take place during inflation, and which information are included in the related signals. Then I will show how the upcoming experiments of GW detection at small scales should provide

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new limits on the features of such GWs, then giving new constraints on the related inflationary physics. From now on we will consider $c = 1$, and we will indicate with $M_{\rm pl} \equiv 1/\sqrt{8\pi G}$ the reduced Planck Mass.

2. – Gravitational-wave production during inflation

In single-field slow-roll inflation, the dynamics of the universe is driven by a scalar field, whose potential exhibits a sufficiently flat region which provides a period of slow-rolling of the field. In this stage the universe underwent an accelerated expansion described by a quasi-de Sitter spacetime. As usual for FRW spacetime, the evolution of the scale factor $a(t)$ is provided by the Hubble parameter $H(t) = \dot{a}/a$.

Interestingly, any inflationary model predicts the production of a stochastic GW background. In particular, quantum fluctuations of the gravitational field, enhanced by the accelerated expansion and pulled outside the horizon, give rise to a GW power spectrum, also called tensor power spectrum. This GW signal represents a *smoking gun* for the cosmological inflationary scenario. The equation of motion of GWs, indicated by the tensor perturbation h_{ij} is given by

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 0, \tag{1}$$

where $\mathcal{H}$ is the Hubble parameter with respect to the conformal time and the prime indicates differentiation with respect to it. From the solution of this equation, the expression for the corresponding GW power spectrum can be found. For several inflationary scenarios, it turns out that the GW power spectrum is well described by a power-law, *i.e.* by an amplitude $A_{\rm GW}$ evaluated at a fixed pivot scale k_* and a spectral index $n_{\rm GW}$:

$$P_{\rm GW}(k) = A_{\rm GW}(k_*)\left(\frac{k}{k_*}\right)^{n_{\rm GW}}. \tag{2}$$

Notice that the scale k is related to the frequency f by the following relation $f = k/2\pi a$. Interestingly, for single-field slow-roll scenario, the amplitude of the GW power spectrum turns out to reflect the energy scale of inflation, through the Hubble parameter H, while the tensor spectral index includes information about the deviation from a perfect de-Sitter background evolution. Moreover, for this inflationary scenario, $n_{\rm GW}$ is predicted to be a small negative quantity. In this case the spectrum is called *red*, *i.e.* with a lowering amplitude for higher values of k, in the opposite case it is called *blue*.

Actually, the features of GW produced by quantum fluctuations of the gravitational field are strictly related to the gravity theory which underlies the considered inflationary model. Moving from General Relativity, the equation of motion of GW, in general, turns out to be different with respect to the standard one. As a consequence, new features in the GW power spectrum are expected. For example, a deviation of the propagation speed of GW with respect to the speed of light is expected to modify the amplitude of the GW spectrum, while a massive graviton is expected to change the GW spectral index. For our purposes, it is interesting to notice that, in some scenarios based on modified gravity theories, a blue GW power spectrum is predicted; for this kind of models, see for example [2-4].

2˙1. *Gravitational-wave classical production during inflation.* – However, quantum fluctuations of the gravitational field, are not the unique mechanism by which GWs can be produced during inflation. In fact, when further fields besides the inflaton are present during inflation, these can give rise to efficient source terms in the GW equation of motion and then introduce a further production of GWs:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -\frac{4}{M_{\rm pl}}\hat{\Pi}^{lm}_{ij}\mathscr{S}_{lm}, \tag{3}$$

where $\hat{\Pi}^{lm}_{ij}$ is the transverse-traceless projector and $\mathscr{S}_{lm}$ the anisotropic stress. This mechanism is the so-called *classical* production. This phenomenon can be particularly significant, for example, in models where the inflaton field is coupled to a gauge field [5-7], or when scalar spectator fields are present [8-10]. In several cases, the classical contribution to the inflationary GW power spectrum is not well described by a simple power-law. However, it is interesting to notice, that among these contributions, some of them are predicted to present an enhancement at small scales. Differently from GWs produced by quantum fluctuations of the gravitational field, GWs sourced by this mechanisms mainly reflects the features of the fields which give rise to the source term in the GW equation of motion.

2˙2. *Gravitational-wave as a discriminant among the variety of inflationary models.* – In summary, the total inflationary GW power spectrum is provided by two contributions: one due to quantum fluctuations of the gravitational field, which is predicted by any inflationary model, and one due the classical GW production. The interesting point is that they include several information about the inflationary process and that each of them reflects different aspects of it. The picture is summarized in table I. Inflationary GWs then represent, in principle, a promising way of discriminating among the variety of inflationary models.

2˙3. *Inflationary consistency relation.* – Single-field slow-roll inflation predicts a consistency relation in which GWs are involved [11]:

$$r = -8n_{\rm GW}, \tag{4}$$

where r is the tensor-to-scalar ratio. It is interesting to notice that several other inflationary models predict the violation of such an equality, making it a powerful test for the single-field slow-roll inflation scenario. In particular, the tensor power spectra with an enhancement of the GW amplitude at small scales, clearly violate such a relation, since they are described by $n_{\rm GW} > 0$.

2˙4. *From primordial time to present time.* – Up to now we have described the GW power spectrum referring to the primordial time, when its production takes place. However, considering the subsequent thermal history of the universe, the corresponding present-time GW spectral-energy density $\Omega_{\rm GW}$ can be calculated [12]:

$$\Omega_{\rm GW}(k,\tau_0) \equiv \frac{1}{\rho_{\rm c}}\frac{{\rm d}\rho_{\rm GW}}{{\rm dln}k}\left(\frac{k}{aH}\right)^2 T(k)P_{\rm GW}(k), \tag{5}$$

where $\rho_{\rm c}$ is the critical energy density, τ is the conformal time, and $T(k)$ is the transfer function. The present-time GW spectral energy density turns out to cover a wide range of

Table I. – *Summary of GW production for several inflationary models. For an extended version and for references of each model see* [1].

GW PRODUCTION	**Discriminant**	**Specific discriminant**	**Examples of specific models**
Vacuum oscillations quantum fluctuations of the gravitational field stretched by the accelerated expansion	**theory of gravity**	General Relativity	*single-field slow-roll*
			all other models in GR
		MG/EFT approach	G-Inflation
			Potential-driven G-Inflation
			EFT approach
Classical production second-order GW generated by the presence of a source term in GW equation of motion	**source term**	vacuum inflaton fluctuations	*all models*
		fluctuations of extra scalar fields	inflaton+spectator fields
			curvaton
		gauge particle production	pseudoscalar inflaton+gauge field
			scalar infl.+pseudoscalar+gauge
		scalar particle production	scalar inflaton+ scalar field
		particle production during preheating	chaotic inflation
			hybrid inflation

frequencies: from $f \sim 10^{-16}$ Hz, where CMB experiments are sensitive, up to $f \sim 10^4$ Hz, where ground-based laser interferometers are active. The range of scales at which space-borne and ground-based laser-interferometers are sensitive is usually referred to as *small scales*. Interestingly, the present-time GW background is expected to manifestly reflect the primordial amplitude and spectral index.

3. – Current bounds and observational prospects

Up to now, we have nor direct nor indirect probes of the inflationary GWs. Present-time measurements provide only upper bounds on such a stochastic GW background. The three solid curves in fig. 1 represents different GW signal that can be expected from the inflationary mechanism. For such curves the amplitude at the CMB scales is chosen to match the current upper bounds given by CMB experiments [13].

The most stringent limit at small scales is obtained by measurements related to the Big Bang Nucleosinthesys physics [14]. Such measurements provide an upper bound on the amount of the energy density of the universe that can be in the form of GWs. Other current bounds are given by Pulsar Timing Arrays experiments [15] and the aLIGO first observational run O1 [16] (see also [17]).

In the same plot, the sensitivity curves of some upcoming and future experiments of direct GW detection are shown. Upcoming experiments sensitive to small scales are expected to be able to detect the inflationary GW background with a blue power spectrum. As is visible, in any case, these kind of experiments should provide new information with respect of the inflationary signal.

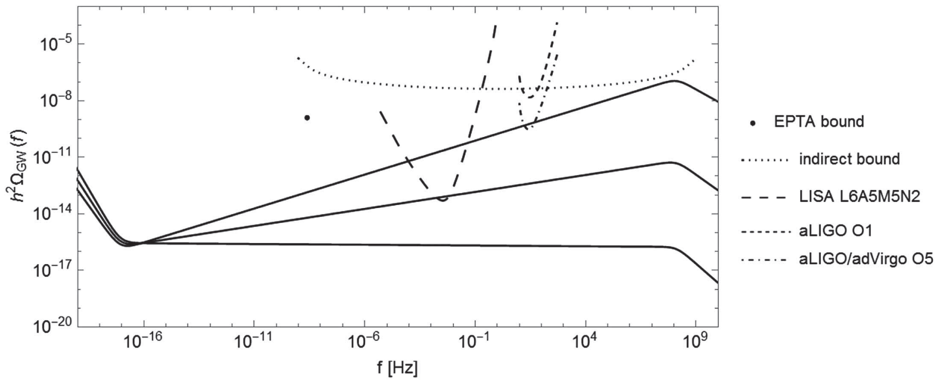

Fig. 1. – Gravitational-wave spectral energy density at present time. The three solid curves represent three possible inflationary GW signal. In all cases the amplitude at CMB scales matches the current upper bound: $r = 0.07$ for $k = 0.05\,\mathrm{Mpc}^{-1}$. From the bottom to the top the spectral indexes are respectively: $n_{\rm GW} = -r/8, n_{\rm GW} = 0.18, n_{\rm GW} = 0.36$. Other curves represent the expected sensitivity curves for the best configuration of the LISA experiment [18, 19], for the $O1$ and $O5$ observational run of the aLIGO experiment [16] (see also [17]). The dot refers to the current upper bound provided by EPTA experiment [15], while the dotted curve is the upper limit provided by [14].

4. – Role of gravitational-wave experiments at small scales in probing the inflationary physics

Measurements at different scales, clearly provide the possibility of constraining the GW spectral index. In this direction, the combination of CMB experiments and of experiments of direct GW detection at small scales are expected to provide new significant constraints on the GW spectral index. As a consequence, new bounds on the GW spectral index would give new constraints on the physics of the source of such a GW signal. Moreover they would provide the possibility of improving our capability of testing the inflationary consistency relation introduced before.

4·1. *Probing specific inflationary models, an example.* – In the following section, the analysis of a selected inflationary scenario is provided as an example to point out the role of experiments of GW detection at small scales in probing the inflationary physics.

Consider a model in which besides the inflaton φ, another scalar field σ is supposed to be present [8-10]. More precisely, we consider a system described by the following Lagrangian [10]:

$$L = \frac{1}{2}M_{pl}^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + P(X,\sigma), \tag{6}$$

where $X = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$ and P is a generic function of X and σ. Let us suppose that σ plays the role of a spectator field, that is it does not influence the background dynamics, nor its perturbation contribute to the curvature power spectrum. However, scalar perturbations of this spectator field turn out to give rise to a source term in the GW equation of motion and then to generate an extra contribution to the GW power spectrum, with respect to that due to quantum fluctuations (which is the same as for single-field slow-roll inflation).

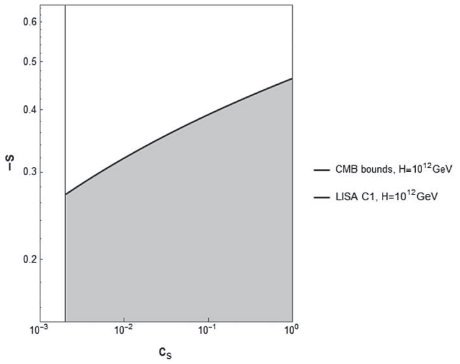

Fig. 2. – Parameter space of c_s-s at $k_* = 0.05\,\mathrm{Mpc}^{-1}$. The vertical line represents the bound provided by CMB experiments (the allowed region is on the right) [21, 13, 22], while the curve is that provided by the best configuration of the LISA experiment [18, 19]. The colored region in the right-down corner is that allowed by an eventual non-detection by the LISA detector. The limits refers to a fixed value $H = 10^{12}$ GeV.

The sourced contribution is well described by a power law [10, 20]:

$$P^{\sigma}_{\mathrm{GW}}(k) \simeq \frac{8}{15\pi c_\mathrm{s}^3}\frac{H^4}{M_\mathrm{pl}^4}\left(\frac{k}{k_*}\right)^{-4\epsilon-3s}, \tag{7}$$

where c_s is the speed of sound of the spectator field, $s \equiv \dot{c}_\mathrm{s}/Hc_\mathrm{s}$ reflects its variation with respect to time and ϵ is the usual slow-roll parameter. H and c_s are evaluated at the pivot scale k_*. The amplitude of this extra term strictly depends on the speed of sound c_s of the spectator field. In particular a speed of sound smaller than the speed of light, enhances the GW signal. Moreover, s contributes to determine the spectral index of the GW power spectrum. Therefore, for a c_s value smaller than the speed of light and for $s < 0$, the contribution to the GW signal due the spectator field can be blue, then representing an interesting signal for upcoming experiments of GW detection at small scales. The situation is summarized in fig. 2 [20]. From such a plot, it is clear how experiments at small scales should provide completely new information in such a parameter space with respect to CMB experiments, showing how such experiments represent a promising way of improving our knowledge of the inflationary physics, even in case of a non-detection of inflationary GWs.

Besides the scenario considered just above, there other inflationary models in which a significant GW production takes place, and for which an enhancement of the GW amplitude at small scales can be expected, as for example inflationary models in which a gauge particle production takes place, or inflationary scenarios built on modified gravity theories. In [20] several examples of how the inflationary physics can be probed by GW detectors at small scales, are reported, in particular with respect to the LISA experiment.

4˙2. *Testing the inflationary consistency relation.* – As said before, single-field slow-roll inflationary model predicts a consistency relation between the tensor-to-scalar ratio r and the tensor spectral index n_GW. However, many inflationary models predict a violation of such an equality, because of a non-standard contribution to the curvature perturbations or to the tensor ones; see [1] and references therein for an overview. Therefore, constraining such a consistency relation would represent a powerful test for the single-field slow-roll inflationary scenario and might unveil new hints in the direction of better modeling inflationary physics. In particular, for inflationary models in which a blue tensor power spectrum is predicted, a violation of the inflationary consistency relation is expected. Clearly, in order to test such an equality, a measurement of the GW spectral index is required. Therefore, experiments of GW detection at small scales are expected to play a crucial role, since they will open the possibility of exploiting the long lever arm between CMB scales and laser-interferometers scales. In this way, in fact, they should provide more stringent constraints on the GW spectral index with respect to the current ones.

5. – Conclusions

The production of a stochastic GW background characterizes any inflationary scenario. The features of such a signal turn out to include unique information about the primordial mechanisms that generated it and the thermal history of the universe. In particular, variations in the theory of gravity which underlies the inflationary scenario w.r.t General Relativity, or the presence of extra fields besides the inflaton, provide peculiar features into the associated GW signal. This fact confers upon to inflationary GW a discriminant power among the variety of inflationary models. In light of this, I showed, by an example, how experiments of direct GW detection should be able to put new constraints on the amplitude and spectral index of inflationary GW signal for which an enhancement of the GW amplitude at small scales is expected, and then to provide new limits in the parameter space of specific inflationary scenarios. Therefore, future and upcoming detectors of direct GW detection at small scales are expected to play an interesting role in probing inflationary physics.

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