The search for a stochastic background of gravitational waves

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Summary. — I give a short introduction about stochastic backgrounds of gravitational waves, discussing the detection methods used, current and foreseen results and activities. I comment about expectations triggered by the recent first direct detection of gravitational waves.

1. – Introduction

A stochastic background of gravitational-waves can be seen as a gravitational-wave field which evolves from an initially random configuration, or as the result of a superposition of many uncorrelated and unresolved sources. There are potentially several kinds of GW stochastic backgrounds, and accordingly with their origin they can be broadly classified as astrophysical and cosmological.

In the first case, the background's sources are astrophysical events (such as compact binaries coalescences, supernovae, rotating NS and so on) which start to be present since the beginning of stellar activity. The study of an astrophysical SB can give important information about the evolution of stellar populations. Though a large number of potential sources have been studied, it is true that the predicted amplitudes are typically too small in order to be seen with current generation detectors. There are exceptions however, as we will see.

A cosmological SB carry in principle very important information about the early Universe, the reason being that gravitational interaction is so weak (compared, for example, with the electromagnetic one) that gravitational waves could be considered decoupled since the very beginning of the universe evolution. There are several mechanism that can generate a cosmological SB, and we will sketch briefly some of them. It must be admitted that also in this case the detection could be considered problematic with current sensitivities, but once again there are interesting exceptions where the task does not seem hopeless. And the perspective for third generation earth-bound detectors and space ones like LISA are quite interesting [1].

2. – The detection of a stochastic background

The description of a stochastic background can be summarized by a mode expansion of the gravitational strain field in the frequency domain like

$$\tilde{h}_{ab}(\vec{x}, \omega) = \sum_P \int d\Omega_{\hat{n}} \, \mathcal{A}^P_{\hat{n},\omega} \, \varepsilon^P_{ab}(\hat{n}) e^{i\frac{\omega}{c}\hat{n}\cdot\vec{x}}. \tag{1}$$

Here each mode is parameterized by a polarization $P \in \{+, \times\}$ and a propagation direction $\hat{n}$, and its amplitude is represented by a stochastic variable $\mathcal{A}^P_{\hat{n},\omega}$. The statistical properties of the stochastic background depend on the generation mechanism, and can be described by the cumulants of the $\mathcal{A}$'s amplitudes.

A stochastic background can be safely considered stationary, because the typical physical time scales involved are much larger that the observation time. It is also expected to be typically Gaussian (but see sect. **4**), and in this case it is completely described by the second-order cumulants. The coupling between the gravitational strain and a detector can be represented by a *detector tensor* D^{ab}, defined in such a way that the signal is given by $h_I = D^{ab}_I h_{ab}$. As a consequence form a statistical point of view the only relevant quantity is the second order correlation $\langle \tilde{h}_{I_1} \tilde{h}_{I_2} \rangle$ between the signal of a pair of detectors.

The optimal detector can be obtained starting from the specific form of this correlation. As an example, if the stochastic background is also isotropic it comes out that the optimal statistic is given by

$$Y = \int d\omega \frac{\tilde{x}_1^*(\omega)\tilde{x}_2(\omega)\,\gamma_{12}(\omega)S_h(\omega)}{S_1(\omega)S_2(\omega)}, \tag{2}$$

where S_i is the power spectrum of the i-th detector noise, S_h the theoretical power spectrum of the stochastic background and $\tilde{x}_i$ the output of the i-th detector, which is espected to be a sum of the signal $\tilde{h}_i$ and of the noise. The function γ_{12} is called *overlap reduction function*, and is equal to the spectral coherence between the signals coupled to the two detectors. A loss of coherence is expected, since two detectors are coupled in a different way to the modes of the expansion (1), unless they are in the same place and oriented in the same way. In particular, a loss of coherence for the signal at a frequency f is expected when the separation d between the two detectors become of the same order or larger than the corresponding wavelength, $f > cd^{-1} = \tau^{-1}$. For example the separation between the two LIGO detectors if about 10^{-2} light-seconds, and the coherence is reduced by 50% around 50 Hz.

Equation (2) tell us that in order to detect a stochastic background the output of two different detectors must be correlated. The Signal-to-Noise Ratio can be defined as

$$\text{SNR}^2 \propto T \int_0^\infty S_h^2(\omega) \frac{\gamma_{12}^2(\omega)}{S_1(\omega)S_2(\omega)} d\omega \tag{3}$$

and as expected it is reduced by a loss of coherence. This means that it is good for a pair of detectors to be aligned, and in principle it should be good for them to be one near the other. However it must be taken into account that the analysis which leads to the optimal statistic (2) is based on the hypothesis that the noise of the two detectors is uncorrelated, and it is difficult to be sure about that if they are nearby. Note also the proportionality SNR $\propto \sqrt{T}$ which tell us that the probability of detection increase with the observation time, though not particularly fast.
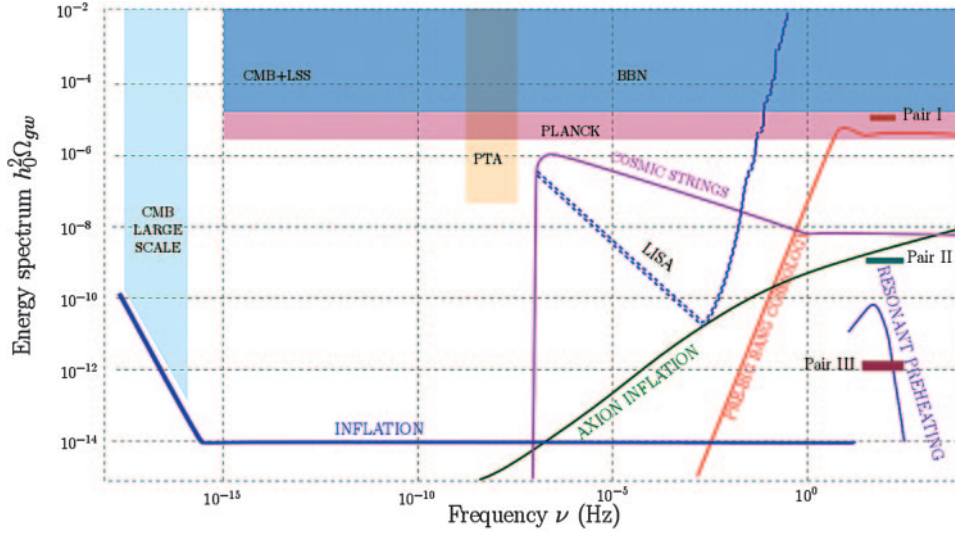
Fig. 1. – Some expected spectra of cosmological GW stochastic backgrounds, compared with upper limits and current sensitivities. Continuous lines correspond to energy spectrum predicted for a selection of production mechanism, and are described in the text. Dashed lines correspond to the sensitivity level which can be obtained using few months of data with a pair of first (LIGO, VIRGO) second (LIGO Advanced, VIRGO Advanced [2,3]) and third (ET [4]) generation detectors, and with LISA. Filled areas represent current upper limits. BBN is an integral bound obtained from the observed abundances of light elements [5]. CMB bounds are related to cosmic microwave background observations [5,6]. PTA is obtained from pulsar timing array observations [7].

3. – Sources, sensitivities and upper limits

A prediction for the amplitude of a stochastic background can be represented by its differential energy spectrum, which is defined as a energy density for logarithmic interval of frequency normalized to the critical energy density, and is connected to the observed power spectrum of the gravitational strain signal by

$$(4) \qquad h_0^2 \Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\log f} = \frac{4\pi^2 h_0^2}{3H_0^2} f^3 S_h(f).$$

Probably the most fundamental production mechanism for a cosmological GW stochastic background is the parametric amplification. Basically it can be understood by looking at the effect of the time dependence of the scale parameter a in a Friedmann-Robertson-Walker metric which describes an expanding universe. The evolution of the amplitudes of GW modes in (1) is given by an equation like

$$(5) \qquad \ddot{\mathcal{A}}_\omega + \left(\frac{\omega^2}{c^2} - \frac{\ddot{a}}{a} \right) \mathcal{A}_\omega = 0,$$

which describes an harmonic oscillator with a time-dependent parameter. The transition between two different evolution regime, for example between the inflationary epoch and

the radiation dominated one, can produce an amplification of the amplitudes. However the predicted final background predicted in the standard inflationary scenario is too small to be detected with the current and foreseen sensitivities [8] (see blue line in fig. 1).

Non standard models for universe evolution can produce more interesting results. Alternative cosmologies like pre-big bang scenarios (red line in fig. 1) can evade the CMB large anisotropy upper limit at lower frequency and the BBN-CMB integral bound at higher ones, and can produce a significant background at the frequencies of interest for earth-bound detectors [9]. In inflation models which include couplings between axion and gauge fields [10] (green line in fig. 1) the gauge field back-reaction on the inflaton can extend the inflationary epoch, in such a way that GW production at higher frequencies is increased.

During a resonant preheating phase at the end of the inflation, inflaton energy can be transferred efficiently to other particles. This mechanism can also be described by eq. (5), and the prediction for this kind of background can be interesting for third generation detectors (purple line in fig. 1). Spectrum peak depends on energy scale (which is choosen at 10 GeV in the plot, higher frequencies correspond to a larger scale).

A completely different production mechanism is connected with the presence of a dynamical network of cosmic (super)strings. String entering in the horizon can evolve dynamically, and a population of string loops can be generated via interconnections. These loops can emit gravitational waves, especially when cusps and kinks are present, and generate a stocastic background similar to the one represented in fig. 1 (magenta line) [11]. There is a large parameter space to esplore for these models, and observations started to give constraints about it.

I will not discuss here astrophysical backgrounds, and the interested reader can refer to reviews [12]. Interestingly, as a byproduct of the recent direct observation of gravitational waves [1] it has been possible to give a quantitative prediction about the expected astrophysical backgound generated by a population of binary BH-BH coalescences in the observed mass range. It comes out that such a background could be detected with the current generation of advanced detectors [13].

Until now there is no evidence of a stochastic background in the data of the LIGO-VIRGO Collaboration, however interesting upper limits exist [14, 15], which correspond to constraint on models' parameters. The analysis of the last data taken during the "first detection" run of LIGO is in progress, and at least a large improvement of these limit is expected.

4. − Conclusions and perspectives

Earth-bound GW detector measurements started to set interesting upper limits and constraints on models, especially when combined with other observations [16]. These limits are expected to improve when the data coming from the LIGO and VIRGO advanced detectors will be analyzed, and accordingly with theoretical predictions in some particular cases a detection could be possible.

When the sensitivity will further improve, for example with third generation detectors such as ET [4], which is expected to gain an order of magnitude in strain compared with advanced detectors, several more refined investigations will become of interest. In the case of a detection it will be possible to estimate parameters and disentangle contributions coming from different polarizations.

This will be particularly interesting to test for models beyond general relativity. These tipically predict additional polarizations with respect to the standard plus and cross one.

Data analysis techniques that make possible these studies are known [17] and currently applied to the available data.

It will be also possible to test for anisotropies, using techniques which are also well defined and applied in the LIGO-VIRGO Collaboration [18].

Finally, it will be possible to test non-Gaussian models [19]. Many astrophysical stochastic backgrounds can potentially be non-Gaussian, and in some cases (for example cosmic strings) this can be true for cosmological backgrounds. The reason for this is that the background can be generated by a number of events (for example compact binaries coalescences) whose signals overlap in such a way that it is not possible to separate the single contribution. But the number of overlapped events can be not very large, and in this case the central limit theorem cannot be applied. The optimal detection of these non-Gaussian backgrounds is a non-trivial problem, and interesting information about the model can be in principle extracted. This is an open issue.

In all the three cases mentioned (polarization disentanglement, study of anisotropies, study of non-Gaussianities) the availability of an extended network of detectors is crucial in order to apply the relevant data analysis techniques and to eliminate degeneracies. Advanced VIRGO will join soon the two LIGO detectors, increasing the number of detector pairs by a factor three. In the near future also KAGRA and LIGO India will join, opening good perspectives for the study of a kind of signal which is not the most easy to detect, but can reward us with a lot of very relevant physical information.

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