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Implications of Dark Matter bound states

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Summary. — We present generic formulae for computing how bound state formation affects the thermal abundance of Dark Matter with non-Abelian gauge interactions. We consider DM as a fermion 3plet (wino) or 5plet under $SU(2)_L$. In the latter case bound states raise to 14 TeV the DM mass required to reproduce the cosmological DM abundance. Furthermore, we consider DM co-annihilating with a colored particle, such as a squark or a gluino, finding that bound state effects are especially relevant in the latter case.

1. – Setup of the computation

The Boltzmann equations for the joint evolution of the dark matter (DM) and bound state number density can be reduced to the following single equation for the DM density $n_{\rm DM}$ with an effective DM annihilation cross section [1]:

$$\frac{{\rm d}Y_{\rm DM}}{{\rm d}z} = -s\frac{\langle\sigma_{\rm eff} v_{\rm rel}\rangle}{Hz}(Y_{\rm DM}^2 - Y_{\rm DM}^{\rm eq2}), \tag{1}$$

where $z = M_{\rm DM}/T$, H is the Hubble rate, $Y_{\rm DM} = n_{\rm DM}/s$ with s the entropy density and $\langle\sigma_{\rm eff} v_{\rm rel}\rangle$ is the thermally averaged effective annihilation cross section. For the simple case of a single bound state we get

$$\langle\sigma_{\rm eff} v_{\rm rel}\rangle = \langle\sigma v_{\rm rel}\rangle + {\rm BR}\langle\sigma_{\rm bsf} v_{\rm eff}\rangle, \qquad {\rm BR} = \frac{\Gamma_{\rm decay}}{\Gamma_{\rm decay} + \Gamma_{\rm break}}, \tag{2}$$

where $\langle\sigma v_{\rm eff}\rangle$ is the thermally averaged DM direct annihilation cross section, $\langle\sigma_{\rm bsf} v_{\rm eff}\rangle$ is the thermal average of the bound state formation cross section, $\langle\Gamma_{\rm break}\rangle$ is the thermal average of the rate at which bound states get broken by the interaction with the SM thermal bath and $\langle\Gamma_{\rm decay}\rangle$ is the average of the bound state decay rate into SM particles.

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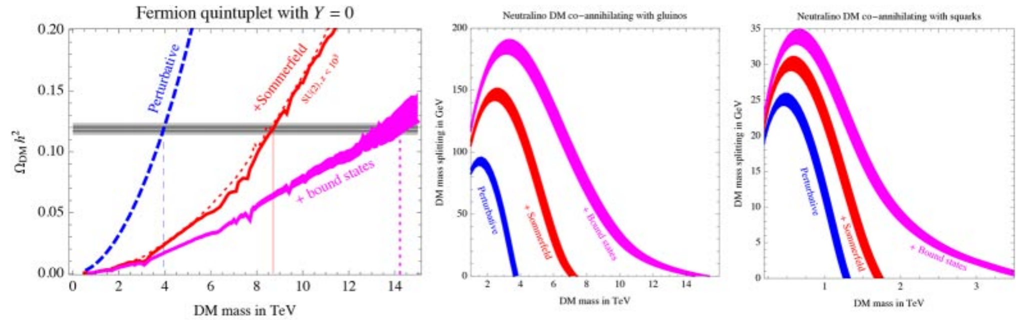

Fig. 1. – Left: relic density for a quintuplet of $SU(2)_L$ [2]. Center and right: colored bands represent the regions in the plane of mass splitting between the colored partner (gluino/squark) and the dark matter (neutralino) in which the correct relic abundance is reproduced within three standard deviations. In all the plots the computation has been performed at tree-level (blue), taking into account the Sommerfeld enhancement (red) and bound state formation (magenta).

Assuming that the global group G describing DM interactions is unbroken and that DM is a particle χ_i in the representation R of G, labeled by an index i, both the initial free state and each bound state can be decomposed into irreducible representations of G, times the remaining spin and spatial part. So the two-body DM states $\chi_i\overline{\chi}_j$ fill the representations J contained in $R\otimes\overline{R}=\sum_J J$. Each representation J is labeled by an index M. The change of basis is described by the coefficients $\mathrm{CG}^M_{ij}\equiv\langle J,M|R,i;\overline{R},j\rangle$ of the group G. For $G=SU(2)_L$ these are the usual Clebsch-Gordon coefficients. In the basis where ij is replaced by M and $i'j'$ by M', amplitudes for formation of bound states with quantum numbers $(n\ell m)$ are

$$\vec{\mathscr{A}}^{aMM'}_{p,n\ell m}=C^{aMM'}_{\mathcal{J}}\vec{\mathcal{J}}_{p,n\ell m}+C^{aMM'}_{\mathcal{T}}\vec{\mathcal{T}}_{p,n\ell m}, \tag{3}$$

where $\vec{\mathcal{J}}$ and $\vec{\mathcal{T}}$ are the overlap integrals between the initial state wave function $\phi_{p\ell}(\vec{r})$ and the wave function $\psi_{n\ell m}(\vec{r})$ of the desired bound state

$$\vec{\mathcal{J}}_{p,n\ell m}\equiv\int \mathrm{d}^3r\,\psi^*_{n\ell m}\vec{\nabla}\,\phi_p,\qquad \vec{\mathcal{T}}_{p,n\ell m}\equiv\frac{\alpha M_\chi}{2}\int \mathrm{d}^3r\,\psi^*_{n\ell m}\,\hat{r}\,e^{-M_a r}\,\phi_p, \tag{4}$$

with M_a the mass of the mediator. The group theory part has been factored out in the coefficients

$$\begin{cases} C^{aMM'}_{\mathcal{J}}\equiv\dfrac{1}{2}\,\mathrm{CG}^M_{ij}\mathrm{CG}^{M'*}_{i'j'}(T^a_{i'i}\delta_{jj'}+T^{a*}_{j'j}\delta_{ii'})=\dfrac{1}{2}\,\mathrm{Tr}\left[\mathrm{CG}^{M'}\left\{\mathrm{CG}^M,T^a\right\}\right]\\ C^{aMM'}_{\mathcal{T}}\equiv i\mathrm{CG}^M_{ij}\mathrm{CG}^{M'*}_{i'j'}(T^b_{i'i}T^c_{jj'}f^{abc})=i\,\mathrm{Tr}\left[\mathrm{CG}^{M'}\,T^b\,\mathrm{CG}^M\,T^c\right]f^{abc},\end{cases} \tag{5}$$

where the T^a are the generators of the DM interactions in the fundamental representation.

The annihilation of bound states will dominantly proceed through states with $\ell=0$. Spin 0 bound states typically annihilate in a couple of SM gauge bosons with a rate given by

$$\Gamma(B_{n0}^M \to VV) = \alpha^2 \frac{|R_{n0}(0)|^2}{F^2 M_\chi^2} \sum_{a,b} \mathrm{Tr}\left[\mathrm{CG}^M \frac{\{T^a, T^b\}}{2}\right]^2, \tag{6}$$

while spin 1 bound states dominantly decay into fermions with a rate

$$\Gamma(B_{n0}^M \to f_i \overline{f_j}) = \frac{\alpha^2}{6} \frac{|R_{n0}(0)|^2}{F^2 M_\chi^2} \sum_a |\mathrm{Tr}\left[\mathrm{CG}^M T^a\right] T^a_{\mathrm{SM}ij}|^2, \tag{7}$$

where $F = 1(2)$ for distinguishable (identical) DM particles and T^a_{SM} are the gauge generators of the considered SM fermion. In fig. 1 we show how bound states modify the DM relic density in three benchmark models.

REFERENCES

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