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Thermal processes in lava flowing in an open channel

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Summary. — In this work, cooling and dynamics of liquid lava flowing inside a channel with rectangular cross section is modeled numerically. The purpose is to evaluate the heating of the solid edges that flank the lava channel due to the flow of lava after a given time interval. Lava rheology is dependent on temperature and on strain rate by a power law function. The study couples dynamics and thermodynamics inside the channel where lava flows driven by the gravity force. The problem is thermal inside the solid edges which enclose the lava channel. Numerical tests indicate that the solution of the thermo-dynamical problem is independent of the mesh.

1. – Introduction

One of the purposes for studying the morphology of lava flows is to evaluate the consequences of an effusive eruption and the distance that the flow of hot lava can reach, with the destructive consequences in terms of lives and properties [1]. The final length of a lava flow is correlated not only to factors such as the effusion rate at the vent and the total erupted lava volume, but also to lava rheology [2]. In general, lava flows down a slope driven by gravity, resisted by the flow viscosity and affected by cooling and crust formation, and interaction with topography (see [3] for a review). Viscous dissipation can act enhancing the flow velocity locally [4-6]. The difficulties in modeling the dynamics of lava flows are due to the complex rheological behavior of lava, given the dependence, usually non-linear, of viscosity on temperature, on the crystal and bubble content and on the strain rate. The difficulties in thermal modeling of lava flows are due to the different thermal exchanges both external (surface thermal radiation, forced convection, conduction to the base) and internal (axial advection, viscous dissipation, latent heat, internal conduction) to take into account. Despite the great progresses achieved in numerical modeling, it is necessary to assume some simplification. The necessity of simplifications in the physical and mathematical assumptions for the numerical models of lava flow is described in the review paper [7]. The authors review the most relevant works on the numerical modeling of lava flows and it emerges that, due to the high complexity of the transport equations, even the numerical solution of the complete three-dimensional

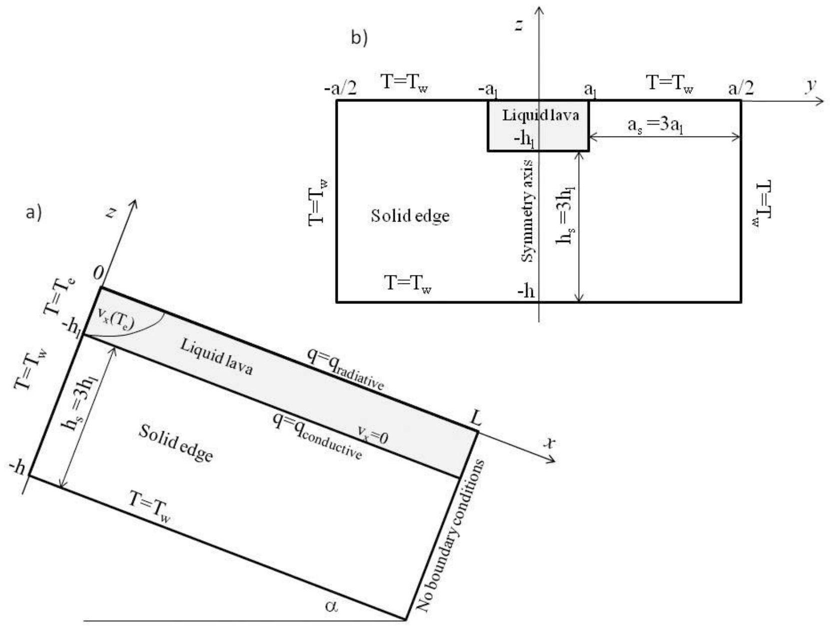

Fig. 1. – Schematic cartoon of the problem with coordinates, geometrical parameters and boundary conditions. a) x, z section; b) y, z section.

problem for real lava flows is often impossible and major efforts focus on software development of simplified models that can quickly describe the evolution of lava flow for volcanic risk assessment purposes.

The cooling process of lava flow which dominates and controls temperature at the lava flow surface is the black body radiation [8]. The cooling process which dominates the interior lava is the conduction through the ground and the levees [9]. The temperature field in lava levees has been treated for the steady state case with the purpose of giving a model which can be applied for providing information on the depth at which explosives can be placed inside the levee in order to deviate lava flow [10]. A far field thermal boundary condition has been used to model a vertical magma conduit of cylindrical cross section where the thermo-dynamical problem of the ascending magma is solved with the finite element method [5, 11]. Similarly to [5, 11], in this work, the boundary condition at the ground and at the levees is treated realistically considering a solid boundary around the lava flow with which lava can exchange heat by conduction. As described by [7], in works dealing with problems of lava flows, channel boundaries are taken at constant temperature or at constant temperature gradient. Differently, the far field thermal boundary condition allows the simplification concerning the channel solid boundaries, as constant temperature or constant heat flow, to be overcome.

2. – Mathematical problem

I consider a rectangular channel, inclined by an angle α, inside which lava flows subject only to the gravity force. The channel is surrounded by edges of solid material of thickness a_s with which it can exchange heat by conduction. The channel has width $2a_l$, thickness h_l and length L. All the computation domain has width a, thickness h

TABLE I. – *Values of the fixed model parameters.*

Parameter	Description	Value
a_l	half channel width	$1.5\,\mathrm{m}$
a	total width	$12\,\mathrm{m}$
h_l	channel thickness	$1.5\,\mathrm{m}$
h	total thickness	$6\,\mathrm{m}$
L	channel length	$100\,\mathrm{m}$
g	acceleration of gravity	$9.8\,\mathrm{m\,s^{-2}}$
c_p	specific heat capacity	$837\,\mathrm{J\,kg^{-1}\,K^{-1}}$
K	thermal conductivity	$3\,\mathrm{W\,K^{-1}\,m^{-1}}$
T_e	effusion temperature	$1050\,^{\circ}\mathrm{C}$
T_s	solidus temperature	$900\,^{\circ}\mathrm{C}$
T_w	wall temperature	$30\,^{\circ}\mathrm{C}$
k_0	rheological parameter	$1\,\mathrm{Pa\,s}^n$
p_1	rheological parameter	-18.71
p_2	rheological parameter	$33.4\cdot 10^3\,\mathrm{K}$
p_3	rheological parameter	-1.35
p_4	rheological parameter	$0.85\cdot 10^{-3}\,\mathrm{K^{-1}}$
α	channel slope	20°
ε_c	thermal emissivity	1
ρ	density	$2800\,\mathrm{kg\ m^{-3}}$
σ	Stefan constant	$5.668108\,\mathrm{W\,m^{-2}\,K^4}$
χ	thermal diffusivity	$1.28\cdot 10^{-6}\,\mathrm{m^2\,s^{-1}}$

and length L. The schematic cartoon of the geometrical parameters is in fig. 1 and the values of the parameters are listed in table I. The dynamic and thermal problem of lava flow inside a channel has already been explained and detailed [6, 12-15].

It is assumed that fluid flow is transient, laminar and subjected to the gravity force. Pressure changes are negligible with respect to the body force. The assumption on laminar flow provides that the velocity is purely axial and varies with all the coordinates $v_x(x, y, z)$. The fluid is isotropic and incompressible with constant density ρ, constant thermal conductivity K and constant specific heat c_p. The effects of viscous dissipation and of latent heat of crystallization/fusion are neglected and therefore I do not consider any internal heat source, with the aim to describe lava flows for which those effects are not significant.

Fluid rheology is modeled by a power-law viscosity function η_a

$$\eta_a(x, y, z, T) = k(T)\left[\left(\frac{\partial v_x}{\partial y}\right)^2 + \left(\frac{\partial v_x}{\partial z}\right)^2\right]^{\frac{n(T)-1}{2}}, \tag{1}$$

where v_x is the x component of velocity and both the fluid consistency k and the power law exponent n depend on temperature T. The temperature dependence of k and n is given by [16]

$$k(T) = k_0 \mathrm{e}^{p_1 + \frac{p_2}{T}}, \tag{2}$$

$$n(T) = 1 + p_3 + p_4 T, \tag{3}$$

where k_0, p_1, p_2, p_3 and p_4 are constant parameters listed in table I. The equation of motion governing the channel fluid flow in the transient state is

$$\rho\frac{\partial v_x}{\partial t} = \rho g \sin\alpha + \frac{\partial}{\partial y}\left(\eta_a\frac{\partial v_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(\eta_a\frac{\partial v_x}{\partial z}\right). \tag{4}$$

The dynamic initial condition at $t = 0$ is the steady state velocity at the effusion temperature T_e at $x = 0$ [12]. The dynamic boundary conditions are the non-slip condition at the channel walls and ground, the free surface condition at the channel surface, the steady state velocity profile at the inflow boundary as computed using the effusion temperature T_e

$$v_x(x = 0) = v_x(T_e), \tag{5}$$

$$v_x(\pm a_l, z) = 0; \qquad v_x(y, -h_l) = 0, \tag{6}$$

$$\frac{\partial v_x}{\partial y}(0, z) = 0; \qquad \frac{\partial v_x}{\partial z}(y, 0) = 0. \tag{7}$$

The time-dependent heat equation inside the channel takes into account the effect of thermal exchange by heat advection and conduction

$$\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} = \chi\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right). \tag{8}$$

The time-dependent heat equation outside the channel, in the solid boundary, is purely conductive

$$\frac{\partial T}{\partial t} = \chi\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right). \tag{9}$$

The thermal boundary conditions are the radiative heat flux q_r at the upper surface inside the channel, constant temperature T_w at the outer solid walls (such far field condition is imposed at a distance equal to three times the half-width of the channel), constant effusion temperature T_e at the vent and the symmetry of the problem with respect to the xz plane

$$T(x = 0) = T_e \text{ liquid lava}, \tag{10}$$

$$T(x = 0, y = \pm a, z = -h) = T_w \text{ solid levees}, \tag{11}$$

$$\frac{\partial T}{\partial y}(y = 0) = 0 \text{ liquid lava/solid levee}, \tag{12}$$

$$s\frac{\partial T}{\partial z}(-a_l < y < a_l, z = 0) = -\frac{q_r}{K} \text{ liquid lava}, \tag{13}$$

where $q_r = \sigma\varepsilon T_u^4$ and σ is the Stefan-Boltzmann constant, ε is the surface emissivity of lava and T_u is the temperature of the upper surface $z = 0$ (the atmospheric temperature is assumed negligible with respect to T_u).

At time $t = 0$, the liquid lava has a uniform temperature T_e, the velocity is the stationary solution of the dynamic equation with $T = T_e$ and the outer solid boundary

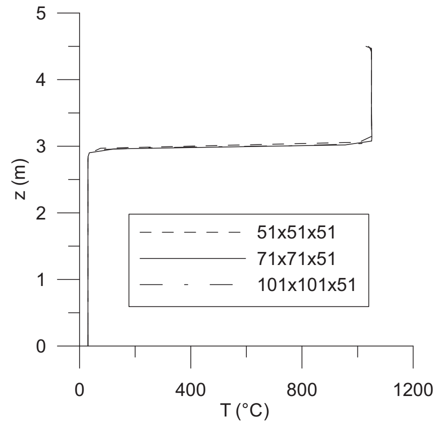

Fig. 2. – Temperature profile along the z coordinate at the center of the outflow section of the channel ($x = L, y = 0$) and at the end of the simulation ($t = 10^4$ s) for three different mesh sizes, indicated in the figure legend as product on number of control volumes for any coordinate ($y \times z \times x$).

has uniform temperature T_w. At time $t > 0$, the radiative heat flux q_r, the far field constant temperature T_w and the constant effusive temperature T_e are imposed. Since $x = L$ is the outflow boundary and both temperature and velocity need to be computed there, no boundary condition at $x = L$ is necessary.

The dynamical and thermal equations were discretized using the control volume integration method [17] using a static mesh approach. The obtained algebraic equations were solved using the classical Gauss-Seidel approach [13, 14]. Since the strong interdependence of the dynamic and thermal equations, we made use of the relaxion factor to accelerate or decelerate the convergence of the solution, as discussed by [14].

The solution of the numerical problem was tested in order to verify the independence of the mesh size. The computational problem was solved considering three grids of different sizes (101×101, 71×71, 51×51) to discretize the (y, z) section, transversal to the fluid direction. The space discretization along the x direction was fixed to 51 control volumes. The time solution is stopped at $t = 10^4$ s. The selected values of the channel geometry for the numerical test are: $\alpha = 20$ °C, $a = 6$ m, $h = 3$ m, $L = 50$ m. In fig. 2 the plot of the temperature profile along the z coordinates indicates that the solution is independent of the control volume size. As expected, the finest mesh (101×101) needs the largest number of iterations to achieve the convergence, which means a long time for calculation. In the following, for the problem with geometrical parameters listed in table I, the mesh $y \times z \times x = 71 \times 71 \times 51$ was used as the best compromise between precision and time for computation.

3. – Results and discussion

I evaluated temperature and velocity fields using the parameters in table I. The problem is illustrated in fig. 1a and b: an inclined channel inside which the hot lava flows by gravity is flanked by solid edges which cool the lava in the channel and are heated in turn. Results are obtained considering that the lava channel cools by thermal radiation on the free surface and by thermal conduction to the interfaces. The solid continuing

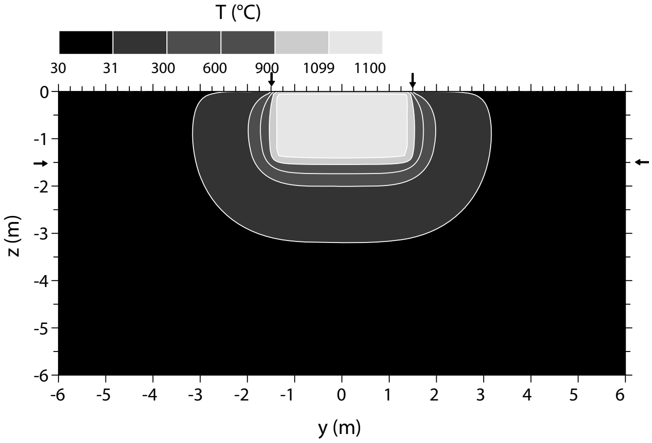

Fig. 3. – Map of isolines of temperature T (in °C) at the channel outflow cross section ($x = L$) and at the end of the simulation ($t = 10^5$ s). Black arrows indicate the lava channel levees and bottom $y = \pm a_l$ and $z = -h_l$.

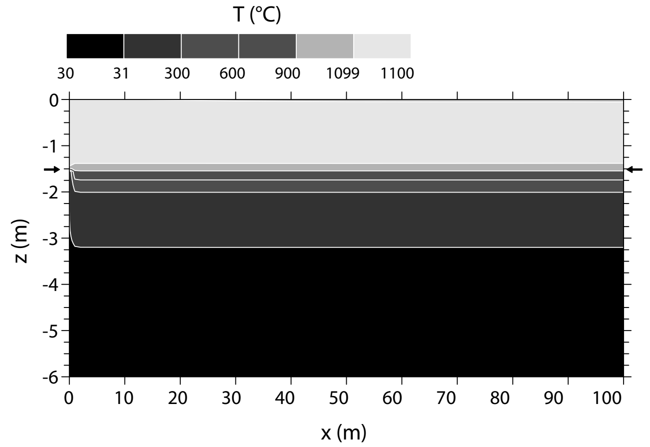

Fig. 4. – Map of isolines of temperature (in °C) at the channel vertical center section ($y = 0$) and at the end of the simulation ($t = 10^5$ s). Black arrows indicate the lava channel bottom $z = -h_l$.

rocks, in turn, heat up by thermal conduction. The problem is transient, and the first 10^5 s (approximately 1 day) of cooling of the lava flow are modeled, assuming that at $t = 0$ the whole channel has a temperature T equal to the effusion temperature T_e and the solid edges have uniform temperature T_w.

The temperature map inside the computational domain after 10^5 s (figs. 3, 4 and 5) indicates that the lava flow in the channel cools both at the edges and on the surface while the inner part is kept at the initial temperature equal to that of effusion T_e. The part of the solid body in contact with the lava channel heats up to a temperature of more than 600 °C and the heating effect, in the time interval considered, affects a solid portion of about a meter and a half thick. Figures 3 and 4 show also that the isolines of the solidus

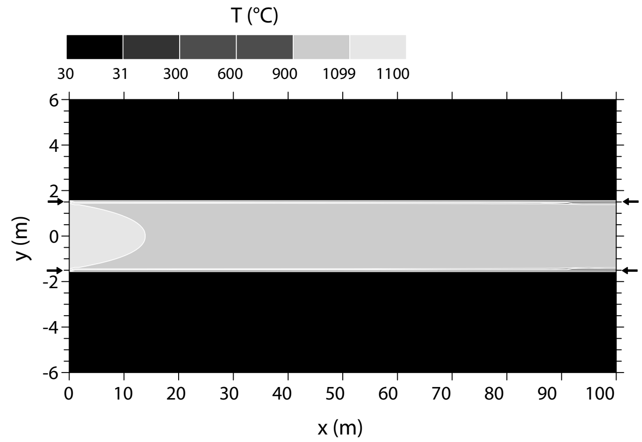

Fig. 5. – Map of isolines of temperature (in °C) at the channel upper surface ($z = 0$) and at the end of the simulation ($t = 10^5$ s). Black arrows indicate the lava channel levees $y = \pm a_l$.

temperature $T_s = 900\,^\circ\mathrm{C}$ is partially located outside the geometric boundary of the lava channel and partially located inside the lava channel. The solidus isoline delineates a geometry such that the vertices of the rectangular section cool and solidify while the sides of the rectangular section heat and melt in such a way to smooth the rectangular section facing it to an elliptical one. In order to evaluate the effects of thermal erosion on the geometry of the channel cross section it is necessary to consider the contributions of viscous dissipation and of the latent heat of melting in the heat equation (see *e.g.*, [11]).

The temperature map (fig. 5) of the upper surface of the lava channel and solid edges indicates that, for the explored length scales, the temperature of the surface is maintained at about the effusion temperature T_e at a greater distance. The advective term in the heat equation causes an increase in the lava flow velocity and a slowing of the effect of the lava cooling in the channel as the slope of the channel increases [12].

4. – Conclusion

This work aims to go beyond the classical boundary conditions at the channel walls which assume constant temperature or constant heat flow, usually adopted in works dealing with lava flow simulation [7], by considering a solid edge with which the lava flow can exchange heat by conduction. In this way, the boundary conditions are more realistic since it is not necessary to impose a constant temperature nor an arbitrary constant temperature gradient at the channel levees and at the ground. The dynamic and heat equations of liquid lava flowing inside a rectangular cross section inclined channel flanked by a thick solid edge are solved numerically by using the finite volume method [17]. The dynamic equation is solved in the liquid domain, the heat equation is solved both in the liquid and in the solid domain. The solution is tested to verify that the convergence of the numerical problem is independent of the mesh size. The results indicate that, even during the first day of flow process, the hot lava interacts with the solid edge, cooling down near the levees and heating the edge in turn. This study represents a starting point for analyzing the liquid-solid coupled problem considering the term of viscous dissipation. In fact, viscous dissipation contributes with an additional heating of the lava in the portion near the levees [6] and this can lead the solid edges to a greater heating and the channel could modify its sectional geometry due to thermal erosion, as can be observed following the geometry of the isoline of the solidus temperature in fig. 3. Considering the case study analyzed here, the channel levees can melt but the effect of thermal erosion cannot be evaluated quantitatively. The analysis of the thermal erosion and the changes in the channel morphology, which can be caused by melting of the solid edge, is beyond the purpose of this paper as it would require the adoption of a mobile boundary. Adopting a mobile boundary one can take into account that portions of the solid medium close to the lateral surfaces of the channel walls and ground pass from a conductive thermal treatment to a thermo-fluid-dynamic treatment and vice versa, portions of fluid lava in contact with the cutting edges inside the channel pass from a thermo-fluid-dynamic treatment to a conductive thermal treatment.

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