

Modeling the topside ionosphere by means of electron density values as recorded by the Swarm satellites constellation

A. PIGNALBERI

*Dipartimento di Fisica e Astronomia, Università di Bologna Alma Mater Studiorum
Bologna, Italy*

received 17 December 2017

Summary. — An empirical method to model the ionospheric topside vertical electron density profile over the European region is proposed. The method is based on electron density values recorded by Langmuir Probes on board Swarm satellites, and on $foF2$ and $hmF2$ values provided by IRI UP (International Reference Ionosphere UPdate), which is a method developed to update the IRI (International Reference Ionosphere) model relying on the assimilation of ionospheric data routinely recorded by a network of European ionosonde stations. Topside effective scale heights are calculated by fitting some definite analytical functions (α -Chapman, β -Chapman, Epstein and Exponential) through the values recorded by Swarm and the ones outputted by IRI UP, with the assumption that the effective scale height is constant in the altitude range considered. Calculated effective scale heights are then modeled as a function of the F2-layer peak characteristics, $foF2$ and $hmF2$. A statistical comparison with COSMIC/FORMOSAT-3 collected Radio Occultation profiles is carried out to assess the validity of the proposed method, and to investigate which of the considered topside profilers is the best one.

1. – Introduction

The ionosphere is the portion of the Earth's upper atmosphere where ions and electrons of thermal energy are present in quantities sufficient to affect the propagation of radio waves [1]. It is a weakly ionized medium, extending from about 60 km to 1000 km above the surface, driven primarily by the production of electrons through photoionization of the neutral thermosphere by the incident solar radiation (above all in the range of EUV and X rays). The vertical distribution of the ionospheric plasma exhibits several layers (see fig. 1). An absolute maximum in the electron density is reached for the F2-layer at an height of about 300 km. The F2-layer region and the region immediately above (the topside one) are the most important ionospheric regions from an operative point of view for telecommunication's purposes. The F2-layer maximum is characterized by means of its maximum electron density value named $NmF2$

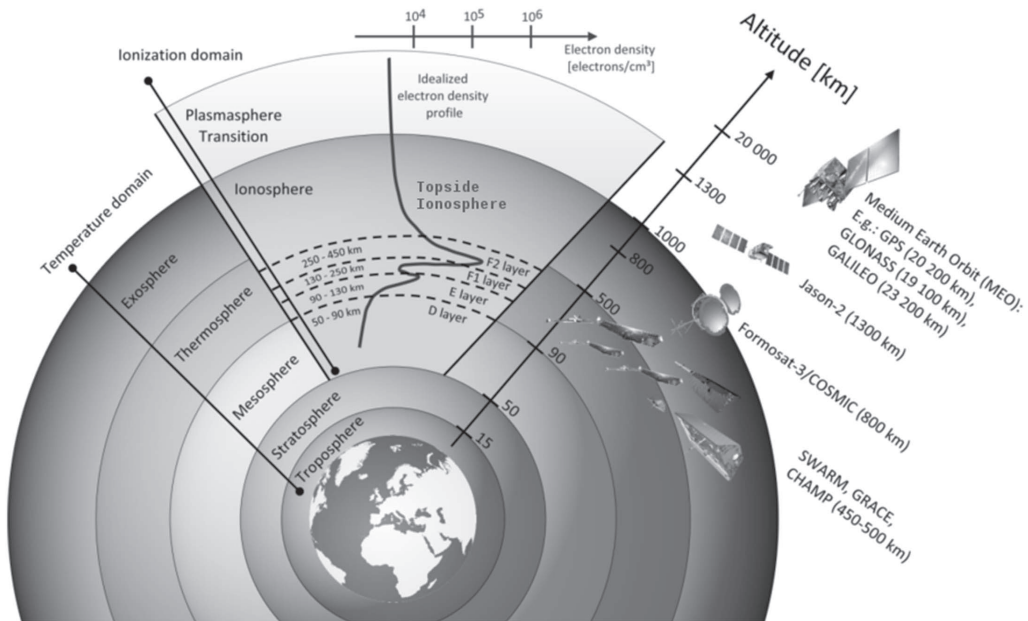


Fig. 1. – Artistic representation of the ionospheric vertical electron density profile.

(which can be expressed also as maximum reflected radio-wave frequency as $foF2$, $foF2$ [MHz] = $\sqrt{NmF2$ [el/cm³]/(1.24 · 10⁴)), and by the height at which the maximum is reached named $hmF2$. The F2-layer peak ideally divides the ionosphere in two regions: a *bottomside* region below the height for which the maximum in the electron density is reached, and a *topside* part above that. The plasma density distribution of the topside ionosphere is largely determined by field-aligned plasma flows and plasma transport processes [2] and, because of the large fraction of the *Total Electron Content* (TEC) it contains, its modeling is extremely important for telecommunication's purposes.

Knowledge of the physical and chemical state of the plasma in this region is very problematic because equipments commonly used to sound the ionosphere are not able to probe it. In fact, ground-based ionosondes can only measure the bottomside part of the vertical electron density profile, up to the height of the F2-layer peak. This task requires the use of more sophisticated and expensive techniques and equipments as: topside sounders, Radio Occultation, Incoherent Scatter Radars, and Langmuir probes on board LEO (Low Earth Orbit) satellites. Difficulties in modeling the topside part of the ionosphere are testified by the fact that often the International Reference Ionosphere (IRI) model [3] does not represent properly the real features of this part of the ionosphere [4]; as it was recently demonstrated by [5], who made a comparison between electron density values in the topside part of the ionosphere measured by Swarm satellites and calculated by IRI. The need to mathematically model the topside part of the ionosphere led many scientists to apply several analytical functions to fit the sparse information on this region. The most used analytical functions are Chapman's [6], Epstein [7], and Exponential functions. All these formulations strongly rely on a parameter called *scale height* whose definition and calculation is the most difficult task in the search of the best topside formulation. In all the aforementioned functions, the scale height controls the shape of the topside profile, thus the vertical distribution of the electron density in the topside ionosphere. From a theoretical point of view the *plasma scale height* H_p is defined as

$H_p = (k_b T_p)/(m_i g)$ [8], where k_b is the Boltzmann constant, $T_p = T_e + T_i$ is the plasma temperature (T_e and T_i are the electron and ion temperature, respectively), m_i is the ion mean molar mass, and g is the gravity acceleration. H_p is the vertical distance in which the plasma concentration changes by a factor of $e \simeq 2.71828$ in the diffusive equilibrium hypothesis. Given its dependence on the plasma temperature and composition, both varying with altitude, the plasma scale height value will obviously also vary with altitude. Thus, to apply this definition they should know the vertical distribution of plasma temperature and that of each ion constituent. The lack of this precise knowledge of the topside ionosphere physical and chemical conditions pushed to use a more practical approach based directly on electron density measurements. With regard to this, the *effective scale height*, frequently called H_m in the literature [9, 10], is the parameter that can be inferred by fitting some analytical functions to electron density values. Then, the effective scale height is a mere empirical parameter which is used to fit measured data with analytical functions in order to obtain the most reliable representation of the topside vertical electron density distribution.

The objective of this work is to provide an operational method to model the topside ionospheric vertical electron density profile. To this purpose, Swarm's Langmuir probe [11] electron density data and F2-layer peak characteristics provided by the IRI UP method [12] have been used in conjunction with several analytical functions dependent on an effective scale height parameter. Calculated effective scale heights are modeled as function of the F2-layer peak characteristics (*foF2* and *hmF2*). Finally, performances of the proposed method are evaluated carrying out a statistical comparison with COSMIC/FORMOSAT-3 collected Radio Occultation profiles.

2. – Data and method

One might think to model the topside profile using an analytical formula joining the F2-layer peak (*NmF2*, *hmF2*) to the punctual electron density value ($N(h)$, h) measured by Langmuir Probes on board Swarm above some selected ionosonde stations, by assuming a constant effective scale height H_m for the topside profile (see fig. 2). This approach has however several limitations. In [13] and [14] this approach has been pursued using simultaneous observations of ROCSAT-1 electron density values (collected at around 600 km of altitude) and ionosonde or ISR deduced F2-layer characteristics. However, selecting only satellite's passages on a definite point (the one where a ground-based measuring station is installed) heavily reduces the available data and consequently the possibility to do any spatial study on the topside effective scale height. An ionospheric model able to spatially describe the ionospheric plasma would maximize the use of the data recorded by the Swarm constellation. This task can be accomplished over the European region by the IRI UP method [12], which describes the spatial distribution of the ionospheric plasma near the F2-layer peak, that is in the lower topside region. In this way, the effective scale height can be calculated for each satellite's passage over Europe, averaging the satellite's measurements falling in each grid point of the map, which will be considered in this work with a $1^\circ \times 1^\circ$ spatial resolution. This procedure allows to take advantage of every satellite's track over the European region and not of only the ones right over a ground-based station. Carrying out this analysis for every transit of each satellite of the Swarm constellation over each grid point of the European region, a huge amount of ionospheric scale height values can be obtained. This gives the possibility to perform a robust statistical and spatial characterization of this parameter.

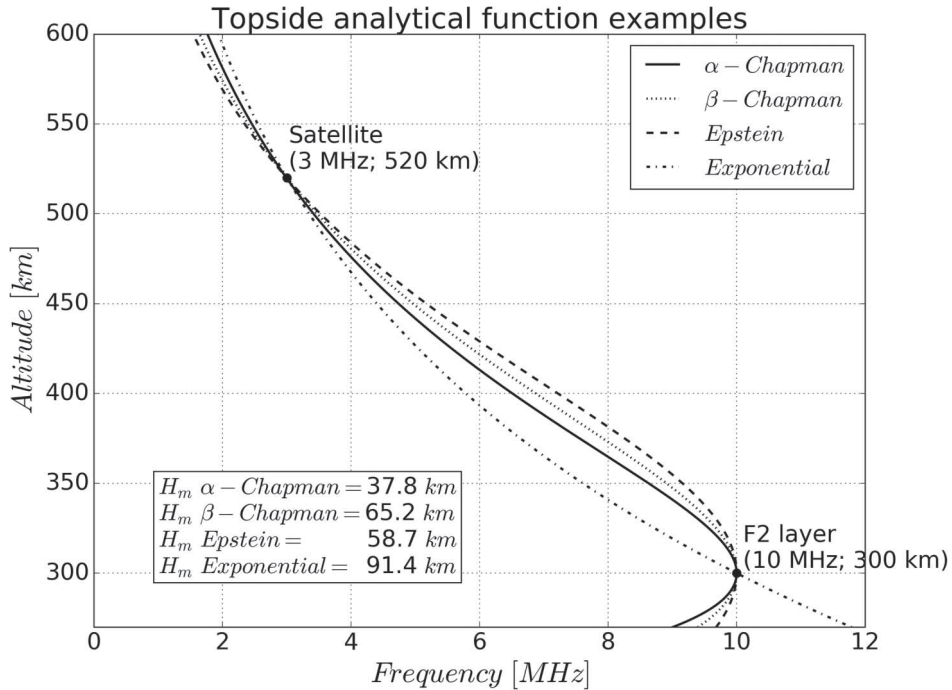


Fig. 2. – Examples of topside analytical functions calculated to meet the constrain to join the F2-layer peak point (f_oF2 , h_mF2) to the satellite point ($f(h_{\text{sat}})$, h_{sat}). The four studied topside profilers (1), (2), (3), and (4) have been used, each one with a different effective scale height.

Swarm's satellites constellation data. – Swarm is a satellite constellation launched at the end of 2013 by the European Space Agency (ESA) [15]. It is constituted by three LEO satellites in a circular near-polar orbit. Two of them (called Alpha (A) and Charlie (C)) are orbiting the Earth side-by-side at the same altitude of about 460 km (with an inclination of 87.4° , an east-west separation of 1° - 1.5° in longitude, and a maximal differential delay in orbit of approximately 10 seconds), while the third (Bravo (B)) is flying about 60 km above (with an inclination of 88°) in an orbital plane which will gradually get farther from those of the other two satellites during the mission's lifetime (9 hours in local time after 4 years). All of them make electron density measurements at 2 Hz rate by means of Langmuir probes. In particular, data collected by Swarm A and C satellites present in the Extended Set of Swarm Langmuir Probe Data dataset (SW-RN-IRF-GS-005, Rev: 1, 2016-06-23), released by S. Buchert (Swedish Institute of Space Physics) on 23 June 2016 (freely downloadable at <https://earth.esa.int/web/guest/swarm/data-access> after registration), have been used. This dataset comprises calibrated plasma density measurements collected by the Swarm satellite constellation from 12 December 2013 to 11 June 2016. Specifically, Swarm's measurements collected over the European region (from 15°W to 45°E in longitude and from 30°N to 60°N in latitude) have been considered. The near-polar orbit of satellites, the particular geometry of the constellation, and the height at which satellites fly, are particularly appropriate to study the topside ionosphere, because they provide a good spatial (both in longitude and latitude) and temporal (local time and seasonal) coverage of the European region.

IRI UP foF2 and hmF2 maps generation. – The IRI UP (International Reference Ionosphere UPdate) method [12] has the purpose to update the IRI model through the assimilation of the *foF2* and *M(3000)F2* ionospheric characteristics recorded routinely by a network of European ionosondes. Such measurements are used firstly to calculate at each ionosonde station location effective values of indices IG_{12} and R_{12} (identified as $IG_{12\text{eff}}$ and $R_{12\text{eff}}$), and secondly to generate two-dimensional European maps of these indices through the application of the Universal Kriging method [16]. The computed maps are then used as input for the IRI model to synthesize updated values of *foF2* and *hmF2* over the European region. To accomplish this task, ionospheric characteristics recorded by 14 European stations (Athens (23.5°E, 38.0°N), Chilton (0.6°W, 51.5°N), Dourbes (4.6°E, 50.1°N), El Arenosillo (6.7°W, 37.1°N), Gibilmanna (14.0°E, 37.9°N), Fairford (1.5°W, 51.7°N), Juliusruh (13.4°E, 54.6°N), Moscow (37.3°E, 55.5°N), Nicosia (33.2°E, 35.0°N), Pruhonice (14.6°E, 50.0°N), Rome (12.5°E, 41.8°N), Roquetes (0.5°E, 40.8°N), San Vito (17.8°E, 40.6°N), and Warsaw (21.1°E, 52.2°N)), have been used considering measurements carried out at minutes 00 and 30 of each hour. Ionosonde data were downloaded from the Digital Ionogram DataBASE [17] by means of the SAO Explorer software developed by the University of Massachusetts, Lowell.

Topside analytical formulation. – Four different analytical functions are used to model the topside ionospheric electron density profile, as function of both the effective scale height (H_m) and the F2-layer peak characteristics (*NmF2* and *hmF2*).

These are:

- α -Chapman:

$$(1) \quad N(h) = NmF2 \cdot \exp\left[\frac{1}{2} \left(1 - \frac{h-hmF2}{H_m} - \exp^{-\frac{h-hmF2}{H_m}}\right)\right];$$

- β -Chapman:

$$(2) \quad N(h) = NmF2 \cdot \exp\left[1 - \frac{h-hmF2}{H_m} - \exp^{-\frac{h-hmF2}{H_m}}\right];$$

- Epstein:

$$(3) \quad N(h) = 4 \cdot NmF2 \cdot \frac{\exp^{-\frac{h-hmF2}{H_m}}}{\left(1 + \exp^{-\frac{h-hmF2}{H_m}}\right)^2};$$

- Exponential:

$$(4) \quad N(h) = NmF2 \cdot \exp^{-\frac{h-hmF2}{H_m}}.$$

Among these, the most used and studied is the α -Chapman one [6, 18]. Despite being widely used, this function is not based on any theoretical consideration, because it was derived according to the simplifying hypotheses of monochromatic solar irradiance, single ion component, and, more importantly, absence of any dynamics [6]. Such hypotheses do not hold in the F2 region where the dynamics of the ions is deeply influenced by zonal

and meridional neutral winds, and by the effect of the diffusion along the geomagnetic field lines, and do not hold *a fortiori* in the topside ionosphere. For these reasons, other analytical functions were used to model the topside ionosphere. Among these, Epstein and Exponential functions meet the constraints to pass through the F2-layer peak and to monotonically decrease in the topside ionosphere. All these analytical functions are purely empirical, thus the need to find which of these can better describe the topside ionosphere. The four selected topside profilers in fig. 2, despite they have the same anchor points (one representing the F2-layer peak, and one the satellite point), display different behavior, mostly in the region immediately above the F2-layer peak. This means that different profilers are characterized by different effective scale heights. Most of the work is then devoted to the search of the profiler able to better describe the shape of the topside profile.

COSMIC/FORMOSAT-3 Radio Occultation data. – Constellation Observing System for Meteorology, Ionosphere, and Climate (COSMIC), also known as FORMOSAT-3 in Taiwan, is a constellation made by six microsattellites launched on 15 April 2006 into a circular orbit (with 72° of inclination) at about 800 km of height (gradually reached 17 months after the launch) and a separation angle of 30° in longitude between neighboring satellites [19]. The mission is a collaborative project between the National Space Organization (NSPO) in Taiwan and the University Corporation for Atmospheric Research (UCAR) in the United States. Each satellite carries a GPS radio occultation receiver able to measure the phase delay of radio waves from GPS satellites as they are occulted by the Earth’s atmosphere, allowing a precise determination of the ionospheric vertical electron density profile. Radio Occultation derived electron density profiles collected by COSMIC satellites for the same period covered by Swarm’s data (from 12 December 2013 to 11 June 2016) over the European region (from 15°W to 45°E in longitude and from 30°N to 60°N in latitude) have been used to validate our method and to decide which of the four proposed topside analytical formulations performs better. The dataset consists of 9672 radio occultation profiles.

3. – Effective scale height modeling

Effective scale height values, calculated using the procedure described in sect. 2, need to be modeled as function of some measured or modeled parameter. At this point of the analysis, effective scale height H_m is a function of the four variables

$$(5) \quad H_m = H_m(hmF2, NmF2, h_{\text{sat}}, N(h_{\text{sat}})).$$

From an operational point of view it would be better to describe H_m as a function of only $NmF2$ and $hmF2$, which are measured by ionosondes or easily modeled by several ionospheric models, while satellite’s related parameters are not routinely available. Therefore, H_m values are modeled as function of the sole F2-layer peak characteristics, $hmF2$ and $foF2$, by binning them with a bin width of 5 km and 0.25 MHz, respectively. In this way any dependence on h_{sat} and $N(h_{\text{sat}})$ is neglected

$$(6) \quad H_m = H_m(hmF2, foF2).$$

The use of $foF2$ instead of $NmF2$ is due only to numerical convenience issues, being these parameters directly related each other by means of the relation $foF2$ [MHz] = $\sqrt{NmF2$ [el/cm³]/(1.24 · 10⁴)}. In this way, for each of the four proposed topside profilers a bi-dimensional binning procedure was carried out, by selecting calculated H_m values derived from a defined couple ($foF2$, $hmF2$) of binning indices. In order to obtain a bi-dimensional map of H_m , function of $foF2$ and $hmF2$, the median of the H_m values falling in each bin was calculated. For statistical robustness, this median was calculated only when the number of values was greater than 10.

Figure 3 shows the calculated H_m bi-dimensional binning maps, after joining Swarm A and C datasets, using each of the topside profilers; the distribution of the number of values falling in each bin is also shown. Figure 3a shows that the most filled bins (the dark gray/black colored ones) are in the range [2,11] MHz for $foF2$, and [225,350] km for $hmF2$. Obviously, these are the bins for which the highest confidence level, in a statistical sense, is achieved. Figures 3b-e, depict the bi-dimensional binning maps of the effective scale height median values, for each of the considered topside analytical functions. The bin distribution reflects what is shown in fig. 3a, but the bins with a number of H_m values lower than 10 are not present. For each topside profiler, highest H_m median values are obtained for low $foF2$ values, and then they decrease for higher $foF2$ values. Generally speaking, α -Chapman derived H_m median values are characterized by the lowest values. β -Chapman and Epstein calculated H_m median values show slightly higher values compared to α -Chapman ones. Exponential H_m median values are instead doubled.

Once maps of the median effective scale height as a function of $foF2$ and $hmF2$, like those shown in fig. 3, are obtained, one can use these to model the topside profile shape using, for each specific topside profiler, the corresponding effective scale height, chosen on the base of either measured or modeled $foF2$ and $hmF2$ values. Naturally, neglecting the dependence of the effective scale height on satellite's parameters (h_{sat} , $N(h_{sat})$), imply an approximation; this is because one is trying to describe a four-dimensional mathematical function explicating only two of its four independent variables.

4. – Topside analytical formulation statistical assessment

To investigate the performance of the proposed method in modeling the ionospheric topside profile, and to asses which of the four proposed topside profilers better represents this region, a careful statistical analysis has been carried out using an independent dataset of topside profiles. Specifically, Radio Occultation derived electron density profiles collected by COSMIC/FORMOSAT-3 (as described in sect. 2) have been used as truth reference. For each of the considered 9672 COSMIC profiles, the *Root Mean Square Error* (7) and the *Normalized Root Mean Square Error* (8), between modeled topside electron density values and those measured by COSMIC (both expressed as plasma frequency, f_p , by means of f_p [MHz] = \sqrt{Ne} [el/cm³]/(1.24 · 10⁴), have been calculated for the height range from $hmF2$ to the height of Swarm's satellites (460 km)

$$(7) \quad RMSE \text{ [MHz]} = \sqrt{\frac{\sum_{i=1}^N (f_{p\text{model},i} - f_{p\text{COSMIC},i})^2}{N}},$$

$$(8) \quad NRMSE \text{ [%]} = \frac{RMSE}{f_{p\text{COSMIC}}} \cdot 100.$$

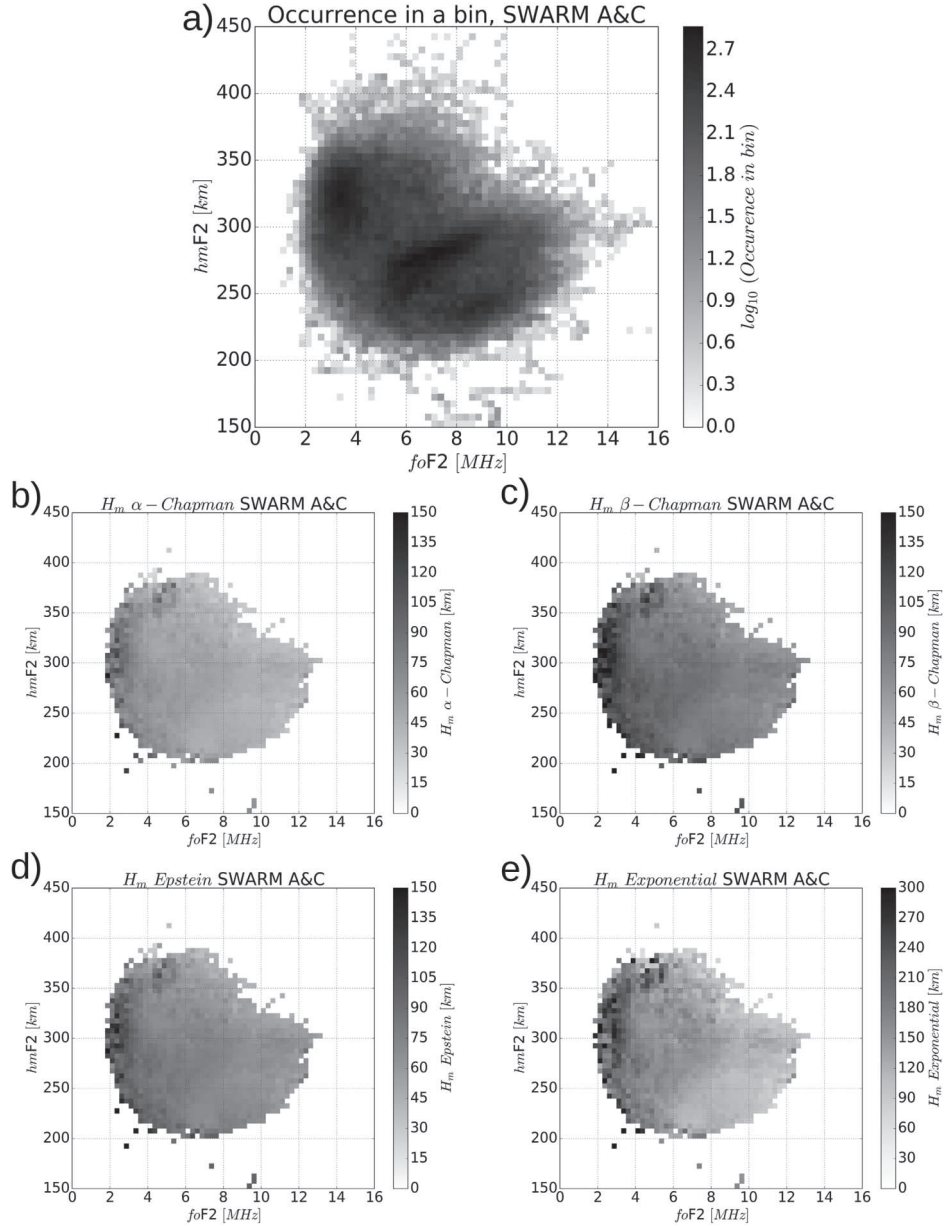


Fig. 3. – (a) Effective scale height values occurrence in a bin (logarithmic scale). Median values of the effective scale height are shown for (b) the α -Chapman topside profiler, (c) the β -Chapman topside profiler, (d) the Epstein topside profiler, and (e) the Exponential topside profiler. It is worth noting that the scale of the Exponential effective scale height is doubled. For each plot the joint dataset Swarm A&C was considered. In the panels (b), (c), (d), and (e), bins including a number of H_m values lower than 10 (the light grey colored ones in the panel (a)) have been discarded.

TABLE I. – *Statistical summary of the analysis made by comparing modeled topside profiles with those measured by COSMIC satellites.*

	α -Chapman	β -Chapman	Epstein	Exponential	IRI
			RMSE [MHz]		
Mean	0.30	0.31	0.33	0.42	0.34
Standard Deviation	0.22	0.22	0.23	0.31	0.35
			NRMSE [%]		
Mean	6.14	6.44	7.14	7.94	7.00
Standard Deviation	4.32	4.57	5.07	4.08	6.67

Modeled topside profiles have been calculated using the four profilers and considering: a) $hmF2$ and $foF2$ values measured by COSMIC; b) the effective scale height value, correspondent to values in a), given by the bi-dimensional binning map (as those shown in fig. 3) related to the definite topside profiler. A statistical summary of the analysis is shown in table I, where the mean and the standard deviation of both RMSE and NRMSE of all the 9672 analyzed topside profiles, are reported for each of the four studied topside profilers and for IRI model, for comparison. Table I points out that the α -Chapman topside profiler is the best one compared to both the other topside profilers studied and the IRI topside model, by using Swarm A&C derived effective scale heights. All the topside profilers, with the exception of the Exponential one, provide a better accuracy than IRI. These results highlight that α -Chapman, β -Chapman, and Epstein profilers can properly model the topside region immediately above the F2-layer peak, allowing a reliable description of the F2-layer shape. Table I shows also that each of the proposed topside profilers is characterized by a standard deviation lower than that associated to IRI, thus highlighting a higher precision of the proposed topside modeling method.

5. – Conclusions

In the present paper an empirical method to model the ionospheric topside profile has been presented by using both Swarm’s measured electron density values and IRI UP modeled F2-layer peak characteristics. Effective scale height values have been calculated by using four different topside profilers, and modeled as function of F2-layer peak characteristics $foF2$ and $hmF2$. A statistical analysis has been then carried out by comparing our modeled topside profiles with those measured by COSMIC satellites.

The main outcomes of this work are:

- 1) Effective scale heights median values, modeled by means of the proposed bi-dimensional binning procedure as function of only the F2-layer peak characteristics $foF2$ and $hmF2$, have the potentiality to be applied to ionosonde’s derived measurements, or to ionospheric models, like IRI for example;
- 2) α -Chapman topside profiler presents the best performance compared to the β -Chapman, Epstein, and Exponential topside profilers, and also compared to the IRI topside model, by using Swarm A&C derived effective scale heights;

- 3) All the proposed topside profilers are characterized by a standard deviation lower than that associated to IRI. This highlights a good precision of the proposed topside modeling method.

* * *

The author thanks the European Space Agency (ESA) for making Swarm data publicly available via the website <https://earth.esa.int/web/guest/swarm/data-access>, and the Swedish Institute of Space Physics in the person of S. Buchert for the calibration of Swarm Langmuir Probes electron density data used in this work. This publication uses data from 14 ionospheric observatories in Europe, made available via the public access portal of the Digital Ionogram Database (<http://ulcar.uml.edu/DIDBase/>) of the Global Ionosphere Radio Observatory in Lowell, MA. The author is indebted to the observatory directors and ionosonde operators for heavy investments of their time, effort, expertise, and funds needed to acquire and provide measurement data to academic research. The author acknowledges the IRI team for developing and maintaining the IRI model and access to the Fortran code via the IRI's website (<http://irimodel.org/>), required to run IRI UP method.

REFERENCES

- [1] ZOLESI B. and CANDLER L. R., *Ionospheric Prediction and Forecasting* (Springer Geophysics, Berlin) 2014.
- [2] RISHBETH H. and GARRIOTT O., *Introduction to Ionospheric Physics* (Academic Press, New York) 1969.
- [3] BILITZA D., ALTADILL D., ZHANG Y., MERTENS C., TRUHLIK V., RICHARDS P., MCKINNELL L. A. and REINISCH B. W., *J. Space Weather Space Clim.*, **4** (2014) A07.
- [4] BILITZA D., REINISCH B. W., RADICELLA S. M., PULINETS S., GULYAEVA T. and TRISKOVA L., *Radio Sci.*, **41** (2006) RS5S15.
- [5] PIGNALBERI A., PEZZOPANE M., TOZZI R., DE MICHELIS P. and COCO I., *Earth Planets Space*, **68** (2016) 93.
- [6] CHAPMAN S., *Proc. Phys. Soc. Lond.*, **43** (1931) 26.
- [7] RAWER K., *Adv. Space. Res.*, **8** (1988) 191.
- [8] HARGREAVES J. K., *The Solar-Terrestrial Environment* (Cambridge University Press, New York) 1992.
- [9] LIU L., WAN W., ZHANG M.-L., NING B., ZHANG S.-R. and HOLT J. M., *Ann. Geophys.*, **25** (2007) 2019.
- [10] LIU L., LE H., WAN W., SULZER M. P., LEI J. and ZHANG M.-L., *J. Geophys. Res.*, **112** (2007) A06307.
- [11] KNUDSEN D. J., BURCHILL J. K., BUCHERT S. C., ERIKSSON A. I., GILL R., WAHLUND J. E., ÅHLEN L., SMITH M. and MOFFAT B., *J. Geophys. Res. Space Phys.*, **122** (2017) 2655.
- [12] PIGNALBERI A., PEZZOPANE M., RIZZI R. and GALKIN I., *Survey Geophys.*, **39** (2018) 125.
- [13] TULASI RAM S., SU S.-Y., LIU C. H., REINISCH B. W. and MCKINNELL L.-A., *J. Geophys. Res.*, **114** (2009) A10309.
- [14] VENKATESH K., RAMA RAO P. V. S., SARANYA P. L., PRASAD D. S. V. V. D. and NIRANJAN K., *Ann. Geophys.*, **29** (2011) 1861.

- [15] FRIIS-CHRISTENSEN E., LHR H. and HULOT G., *Ann. Geophys.*, **58** (2006) 351.
- [16] KITANIDIS P. K., *Introduction to Geostatistics: Application to Hydrogeology* (Cambridge University Press, Cambridge) 1997.
- [17] REINISCH B. W. and GALKIN I. A., *Earth Planets Space*, **63** (2011) 377.
- [18] WRIGHT J. W., *J. Geophys. Res.*, **65** (1960) 185.
- [19] ANTHES R. A., ECTOR D., HUNT D. C., KUO Y., ROCKEN C., SCHREINER W. S., SOKOLOVSKIY S. V., SYNDERGAARD S., WEE T., ZENG Z., BERNHARDT P. A., DYMOND K. F., CHEN Y., LIU H., MANNING K., RANDEL W. J., TRENBERTH K. E., CUCURULL L., HEALY S. B., HO S., MCCORMICK C., MEEHAN T. K., THOMPSON D. C. and YEN N. L., *Bull. Am. Meteor. Soc.*, **89** (2008) 313.