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# Impedance matching: The UNIMI "Piano Lauree Scientifiche" (PLS) experience

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**Summary.** — The problem of impedance matching is not usually treated in Italian High School, though it is a central topic in every waves propagation phenomenon. Strictly related to the efficiency in energy transmission, a good impedance matching plays a key role in avoiding signal losses caused by undue reflections at the boundary between two different media. Impedance matching applies when dealing, as an example, with human physiology, medical diagnostic, musical instruments assembling, electronic transmission lines, seismic damage risk evaluation, etc. The PLS (Piano Lauree Scientifiche) UNIMI Team proposes to High School students and teachers, a laboratory discovery path that explores the consequences of impedance matching or not matching in everyday life. Being targeted to High School students, the discussion is not meant to be exhaustive. Instead the aim is that students will both learn how to recognize, in different physical phenomena, a common characteristic wave property, and develop citizenship awareness in situations like, as an example, the occurrence of a medical examination or the occurrence of risky natural events.

## 1. – Introduction

A first simple but fundamental example of good/bad transmission of a mechanical oscillation is given by a system of two coupled pendula. If the two strings have the same length, resonance frequencies are the same. By exciting one of the two pendulum out of equilibrium, the couple will start moving in a beating mode. On the other end, if the lengths of the strings are different, no ordered movement transmission can be seen. An equivalent pattern can be found by coupling many identical oscillators, as can be seen in the Pasco transverse wave demonstrator (1), shown in fig. 1, where 72 torsional coupled oscillators are used to approximate, in a discrete oscillators set, the transverse wave

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(1) Model 9600 series.

 

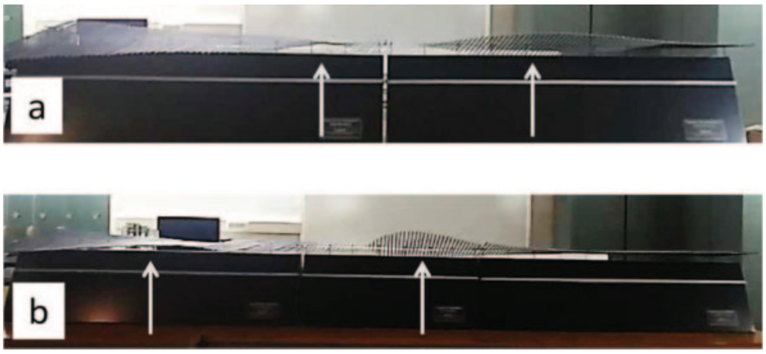


Fig. 1. – A wave coming from left side encounters: a) the boundary between two mismatched sectors (the white vertical gap in the picture), b) two matched sectors. The two arrows shows the reflected and transmitted waves.

propagation in a continuous medium. By mechanically connecting two sectors consisting of 72 oscillators each, with bars of length one half of the other (22.8 cm and 45.6 cm, respectively) with corresponding propagation velocity one double of the other, one can clearly see a strong reflected signal at the edge between sectors, of amplitude comparable to that of the transmitted signal (see top panel of fig. 1). When an impedance transformer segment is interposed, consisting in 40 transverse oscillators of lengths exponentially decreasing from 45.6 cm to 22.8 cm, the signal is very efficiently transmitted, with a small residual reflection effect, due to not perfect matching between mechanical components (see bottom panel of fig. 1).

This simple, but impressive and quite significant example, is used to trigger students to think about the many occasions during their physics course in which they have to deal with a wave crossing a boundary between two media characterized by different propagation velocity. Probably they encountered a one-dimensional signal propagating at first in a free/fixed end rope or spring, and then in two different knotted ropes or springs. They have thus learned the reflection and transmission phenomena features, but usually with no attention to the amplitude of the reflected and transmitted signals. Afterwords, they have studied the Snell's law, looking this time to the two-dimension angular dependence of the transmitted signal, but rarely stressing the fact that a reflected signal too is present and still ignoring what is the amount of the two components. In addition, aside from some short comment on the optical dispersion, with a typical reference to the rainbow phenomenon, no importance is given to the frequency dependence. With the single exception of the High School for Electronics Technicians, no Italian High School course treats the question from the point of view of a possible optimisation technique for transmitting energy from one medium to the other, which is on the contrary the main question in practical applications. Only specifically dedicated courses in Electronics show electric circuits and transmission lines structure to this extent. All other students attending our PLS Waves Laboratory are unaware of the many techniques, either originated in nature or artificially built, present in everyday life, aimed at a good quality transmission of sound, light, electric, etc. signals. Before starting to apply the previous considerations to acoustic phenomena, a short theoretical introduction is needed, supported by a classical experiment of an electromagnetic signal transmission along a cable.

## 2. – The theory in a nut shell

The idea of impedance matching originated in electronics, where in a linear circuit, a complex quantity, called impedance $Z$, representing with its real component an attenuation coefficient and, with its imaginary component, a phase coefficient, relates the flowing electrical current to a generator voltage. More generally, for a system transforming an input physical quantity $I$ in an output one $O$, the relation can be written as $O = FI$, where $F$ is defined as a transfer function. The explicit form of the transfer function depends on the physical phenomena considered, and, in addition, it could be a complicate object as a mathematical operator or a matrix. In the simplest cases and when the signal is small enough, the relation could be written in a linear approximation as $O = ZI$. The transfer function, called here, after electronics usage, the impedance $Z$, is a complex variable depending, for wave phenomena, on the signal frequency. If attenuation, or conversely phase changing, could be ignored, the impedance, for a given frequency, could be treated as a constant. It could be simply demonstrated for a deformation propagating in a stretched string (see ref. [1]), or for an acoustic plane wave field propagating in an ideal lossless fluid (see ref. [2]), that $Z$ is a real quantity, related to the propagation velocity characterizing the medium physically supporting the wave. In the case of a wave propagating in a string, the impedance is the ratio between the elastic force acting on the rope and the rope particle velocity. For an acoustic plane wave, sound pressure and fluid particle velocity are in phase, and the impedance results in the ratio between the two. Thus in both cases $Z$ is inversely related to the velocity of movement of media components. The faster they can move, the lower is the medium impedance. When a wave crosses the boundary between two different media, two coefficients quantify the amount of transmitted and reflected signal, by comparing the respective amplitude with that of the incident wave at a normal incidence. They could be expressed in terms of the wave propagation velocities $v$ in the two media or of the two media velocity or of the two corresponding impedances and are called transmittance $T$ and reflectance $R$:

$$T = \frac{2Z_1}{Z_2 + Z_1} = \frac{2v_2}{v_1 + v_2}; \quad R = \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{v_2 - v_1}{v_1 + v_2}.$$

## 3. – Electronics experiments

The term impedance originates from electronics where a fundamental students laboratory experiment consists in observing the propagation of an electromagnetic signal along a transmission line. The impedance of most commonly used transmission lines in scientific research is 50 Ω and thus every circuit component is to be matched to 50 Ω. Connecting a not terminated 2 m long line with a BNC splitter, as in the scheme a) in fig. 2, one can observe a square signal reflection with phase inversion (fig. 2 panel c)) as it would be for a deformation propagating in a fixed end rope. Conversely, no inversion is observed (fig. 2 panel b)), as it would be in a free end rope, when a short circuit is applied between internal and external line conductors. From the oscilloscope display, students can also measure the 20 ns delay between the incoming and reflected signal, corresponding to a signal traveling forth and back in a 2 m long line at a speed of 0.5 m/ns, and half the height in both transmitted signals when the incoming signal is split. This applies also to a situation more familiar to students: a loudspeaker to be connected to an amplifier. A mismatched impedance would cause the energy from the amplifier to be reflected back instead of being transmitted to the loudspeaker.

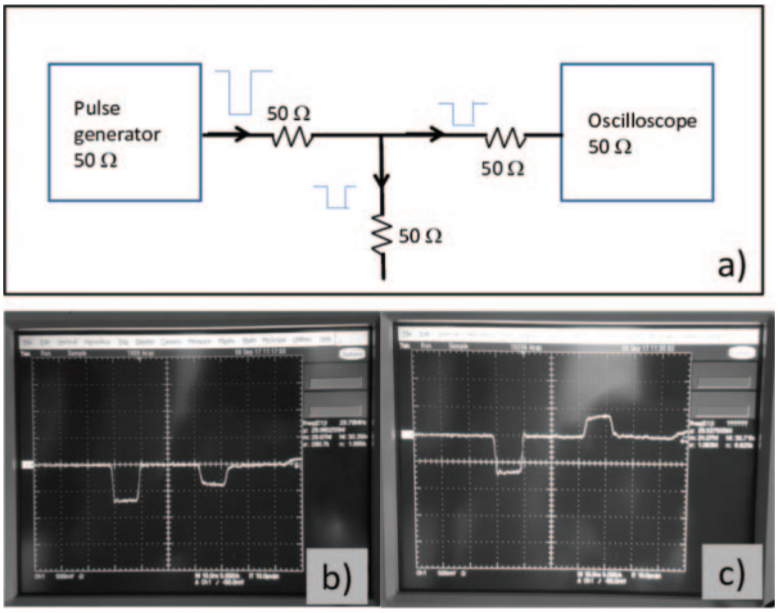


Fig. 2. – In the upper panel a) the circuit scheme. In left lower panel b) the oscilloscope display of a signal reflected in an open-end cable. In the right lower panel c) the signal reflected in a short-circuited end cable.

## 4. – Sound experiments

After the eyesight observation of two kinds of waves, mechanical the former and electromagnetic the latter, the experimental path in the PLS Laboratory follows up with the study of sound waves, that constitute another example of mechanical waves. Preliminary simple experiments are realised with a sound source and a smartphone. Many free applications for smartphone are available to measure sound intensity, analyse spectra and perform an FFT (Fast Fourier Transform). Thus these experiments may be realised by students themselves at home. We use the Spectrum Analyzer application on a Samsung S6 mobile. In fig. 3 are compared the waveform detected when the same sound source, a little bell, is ringing either in air or under water, approximatively at the same distance from the mobile microphone. A different sound loudness can be clearly perceived just by hearing. The same comparison could be made putting the source either in air or underwater and the smartphone underwater. The strong reduction in the signal transmission comes out unexpected, though the students should have already noticed in their textbook table the big factor five ratio between sound velocity in air and in water. Acoustic impedance is the measure of the medium movement response to acoustic pressure and is given by $Z = \rho$ c where $\rho$ is the medium density and $c$ the sound velocity. For a signal propagating from air into water, the expected transmission coefficient is 0.001. It does not describe the loss of energy by absorption. Since higher frequencies suffer greater absorption than lower frequencies, in order to neglect absorption disturbance, a lower-frequency source would be a better choice. Anyway, the few centimetres water depth cannot be accounted for the bell sound reduction. Due to poor experimental accuracy, the quantitative evaluation has a poor significance, but anyway, when the source

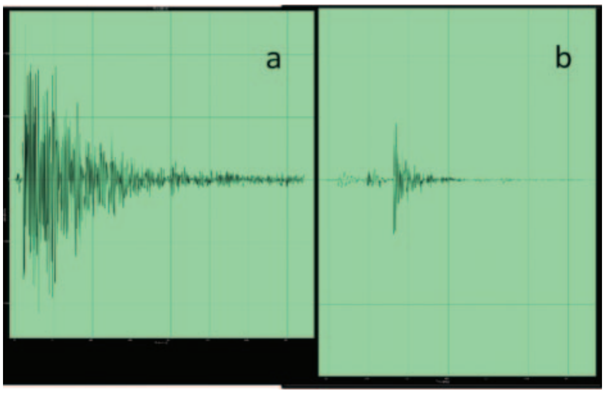


Fig. 3. – Comparison between the waveform in time domain when both the smartphone and the bell are in air (on the left) and when the bell rings underwater (on the right). The amplitude scale is arbitrary.

is underwater, the registered wave form is neatly lower than the one in air, as can be seen in fig. 3. An equivalent reduction is expected for a sound signal coming to air from underwater.

These experiments show clearly that when a good quality water/air signal transmission is needed, an impedance adapter is required. Many examples of mechanisms aimed to match acoustic impedance could be found in everyday life. As an example: the middle ear is a biological impedance adapter, developed to match the propagation of sound waves from air to cochlear fluids, where the sound is transduced to a nervous pulse.

In medical diagnostic by imaging techniques, a relevant importance assumes the echo-Doppler test, a today routine test to monitor pregnancy. Being completely harmless to mother and baby, simple small echo-Doppler scanners may be bought on Internet. They are cheap enough to be present in a school laboratory. These instruments, designed to register foetal heart beats daily at home, allows to register also an adult heart beats sound, experimenting the different transmission quality whether an adapter gel film is used or not. The gel acts removing the air interposed between the transductor probe and the patient skin, putting so the transductor in direct contact with the gel, which is a medium of impedance comparable with that of the skin. Lacking a coupling medium the 99,9% of ultrasound waves would be backscattered at the air/soft tissue interface, preventing them from entering in the patient body. The students could object that the same considerations would prevent reflection from inside body different soft tissue and in fact the reflection coefficient is very small (about 1%), but still detectable with a suitable amplification (see for example ref. [3]).

The following table I is used to show that water, skin, muscles, blood, brain, have approximately the same density and sound velocity, that implies they have the same ultrasound impedance of water ($Z_{air} = 429\,\mathrm{kg/m^2s}$, $Z_{water} = 1.43 \cdot 10^6\,\mathrm{kg/m^2s}$), whilst bones have a very different impedance, allowing an echo-Doppler test to be performed on the hearth muscle movement or on the blood flux in carotid artery but not on bones. This is also a good point to make students to reflect on the fact that high-frequency waves are less diffracted than low-frequency waves, so, in order to perform an accurate scan, we need ultrasounds.

The records of a young adult student hearth beat, with and without an interposed

TABLE I. – *Tabulated values for ultrasound velocity and density of biological materials.*

| Tissue | Ultrasound velocity (m/s) | Density ($kg/m^3$) |
|---|---|---|
| Water | 1430 | 1000 |
| Blood | 1570 | 1060 |
| Brain | 1540 | 1025 |
| Fat | 1450 | 952 |
| Muscle | 1590 | 1075 |
| Bone | 4080 | 1400 |

gel film, are shown in fig. 4, where one can size the different signal amplitude. In fact, without the gel, the sound track is reduced to background noise. The record has been taken with the low-cost Monsieur Bébé Echo Doppler probe (2.2 MHz, $< 5\,\mathrm{mW/cm^2}$), and elaborated with the free software Audacity.

Musical instruments are everywhere present in the students life, but they are usually unfamiliar with the underlying physical working principles. A musical instrument is basically an open-end reflecting tube, where the reflectance is lower as the diameter increases compared to the wavelength. The boundary sound reflection is needed in order that the tube could be the seat of a standing wave, but this implies that less sound gets out of the instrument. Flute soft sound is just due to a strong impedance mismatch at the end. In order to get a louder sound most wind musical instruments (typically brass instruments) are composed by a cylindrical structure called tube and an ending structure called horn. The sound standing wave, generated inside the cylindrical structure, must exit and propagate freely in the air. But, doing so, the modality of propagation changes from that of a plane wave propagating in a tube, to that of a spherical wave propagating in the space, with a different characteristic impedance. The role of the horn is exactly that of an impedance adapter between the two media. Moreover, the internal standing-wave pattern depends from the tube diameter. To explore further this feature, usually not explained at school, an interesting experiment could be to record the amplitude of a standing wave as a function of the position inside a two-open-end acoustic measuring tube (Kundt's tube). To detect the position of pressure nodes, a microphone is moved along the axis of the tube. If the tube diameter is small compared to the wavelength, only a plane wave perpendicular to the axis can propagate in the tube. Textbooks usually schematize these conditions by a standing wave having a pressure node and a position

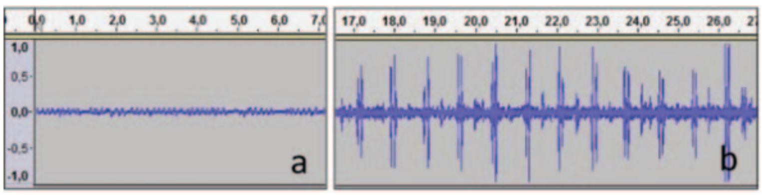


Fig. 4. – Plot of an heart beats echo-Doppler without interposing the gel (on the left) and with the gel film (on the right).

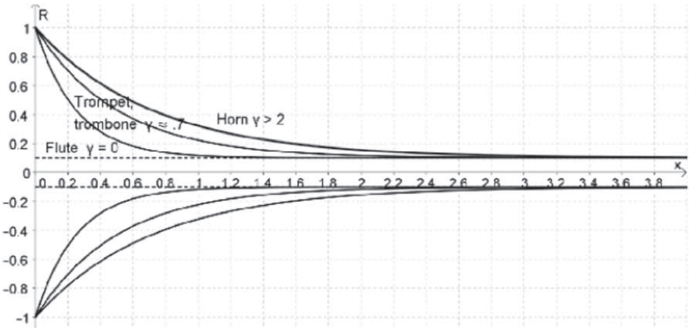


Fig. 5. – Horn shapes drawn with the software Geogebra.

anti-node exactly at the terminal plane of the tube, so the students expect to measure a wavelength exactly matching with the tube length and sound velocity. This is a very rough approximation, ignoring the need of an impedance matching between inside and outside tube propagation. In fact, as it is explained for example in ref. [2], the sound pressure node is located a bit outside the tube end, as it would be in a step-like variation of the tube diameter from a finite size to an infinite one. Differently from a wave in the rope, that cannot propagate in air, the music sound wave has to, changing from plane to spherical. A continuous variation of the cross sectional area is so needed to match different impedances: this is the aim of the musical instrument horn. Horns shapes are constrained by hydrodynamics equations. The resulting wave equation for the sound pressure is similar to that for the damped oscillator and the solution is an exponential function with a complex frequency-dependent exponent, decreasing with distance and giving rise to a spatial damping and a dispersion effect. A most common choice are exponential horns, having a radius given by $R(x) = R_0 e^{-\frac{\gamma}{2}x}$, where the exponent $\gamma$, called the flare constant, characterises different instruments types: flutes, trumpets, oboes, etc. In the above equation the horn throat is located at a given distance $d$ and the mouth at $x = 0$. Other effects derive from the system geometry: a larger bore diameter produces more intense low-frequency harmonics, while a stepped flare reflects more the low frequencies and thus radiates principally the high-frequency harmonics.

The mathematics required in the demonstration is out of reach for High School students, but they could be able anyway to draw the shape of the resulting functions with a free educational software as Geogebra. Considering just the real part of impedance, the resistive one, the students can draw the different exponential horns profiles, as shown in fig. 5. As said before, the different gamma coefficient values regulate the activation and the ratio of the instrument characteristic sound harmonic components, giving rise to a different instruments timbre.

## 5. – Oceanic sound channel

The sonar technique was developed during WW1 to detect submarines. The real situation got more complicated by another not widely known phenomenon: the presence of oceanic sound channels, a sort of sound wave guides that are produced in deep ocean waters. A big amount of sound emitted or reflected by an underwater object is trapped in the sound channel and results undetectable outside, unless a dedicated array of detectors is put inside the channel. In exchange the sound can propagate very far. A short clip from the celebrated film *The Hunt for Red October* can be shown to the student; the picture plot tells about a sonar technician that has discovered a way to detect the enemy submarine Red October using underwater acoustics. A sound channel is generated in the water column when sound speed first decreases with depth, reaches a minimum value, and then increases again. These changes in sound speed, and correspondingly in sound impedance, are the consequence of the water temperature decrease and pressure increase with depth. The maximum sound speed is found at the channel upper and lower edges and its minimum at an intermediate depth called the Sound Channel Axis. This structure acts in an analogous way to an optical fibre with a continuous impedance gradient, giving rise to a continuous sequence of refractions inside the channel and total reflections on boundaries. In fact only frequencies above a cut-off (corresponding to shortest wave lengths) can be trapped. In shallow waters, sound channels are a transitory structure. On the contrary, deep sound channels (DSC) are permanent in the oceans, permitting whales to use this channel to communicate over many tens of kilometres long distances.

## 6. – Seismic risk prevention

The previous considerations will lead us to our final and most relevant topic, regarding the consequences of seismic waves propagation. It is a common belief that seismic damages depend only on the structures buildings technique. But already in the 19th century, geologists knew that the soil structure is quite relevant too. We will see that a mechanism analogous to the ocean sound channel is present in the sedimentary soil reached by a seismic wave, potentially rising the consequences of an earthquake to a catastrophic level. It is of primary importance that the students acquire at school at least a basic awareness about seismic risk. This is regarded as a major task also for our University PLS Laboratory. Let us start with an historical overview. In December 19th, 1985, a seism of 8.1 Richter intensity stroke Mexico City, provoked 10000 casualties, destructed the historical centre (5000 buildings get damaged, 500 crumbled down) but provoked only minor damages in the outskirts. This tragedy was singled out for a detailed inquiry because, being the epicentre located at more than 400 km offshore in Pacific Ocean, the wave absorption should have prevented significant damages at such a distance. A subsequent geological survey discovered that the city historical centre was built over soft deposit of an ancient lake drained in early 1900. The amplification of seismic oscillations was due exactly to the presence of that soft soil lying over hard volcanic rocks. Following these facts geologists became aware of the need to evaluate the Local Seismic Response, in terms of amplitude, frequency spectrum and time duration of the seismic oscillations. The local amplification is evaluated as the ratio between these characteristic features measured at the given site and the ones that would have been measured if on the same location there would be an outcropping horizontal rigid rock basement, taken as a standard. A ratio greater than one means amplification. Starting from the year 2008 the Italian Law foresees the evaluation of the soil Local Seismic Response for any anti-seismic building project.

To understand how soil stratification can influence the situation, let us have a look at a table of seismic waves propagation velocities and densities (table II) at low frequency (less than 20 Hz). Data are taken from ref. [4].

Generally, the velocities increase with soil density and decrease with the degree of fluids saturation. So a typical situation that can change the frequency spectrum and the maximum horizontal acceleration is that of soft soil lying over hard bedrocks. As we can see from the table, seismic waves arriving from hard bedrock and entering soft sediment

TABLE II. – *Tabulated values of seismic wave velocity and density in different type of soils.*

| Soil type | $S$ wave velocity (m/s) | Density (kg/dm$^3$) |
|---|---|---|
| Water saturated clay | 200–800 | 2.0 |
| Dry sand | 100–500 | 1.6 |
| Dry gravel | 500–1500 | 1.6 |
| Sandstone | 800–1800 | 2.1–2.4 |
| Limestone | 2000–3300 | 2.4–2.7 |
| Marls | 2000–3000 | 2.1–2.6 |
| Dolomite | 1900–3600 | 2.5–2.9 |
| Granite | 2500–3300 | 2.5–2.7 |

layers, slow down considerably and so doing augment their amplitude and consequently the acceleration applied on surface buildings. This is called a stratigraphic amplification. The Dynamical Amplification Factor is defined as the ratio between the impedance of the bedrock substratum and that of the overlying soft soil $DAF = \frac{Z_S}{Z_B}$. Since the phenomenon is typically related to secondary surface waves with horizontal polarisation (see ref. [5]), the so-called Love's waves, the soils characteristic impedance is defined using the corresponding propagation velocities. At a typical seismic frequency of a few hertz, a 20 m soft deposit could give rise up to factor-three acceleration amplification.

In the case of strong impedance differences, both at the rock floor-sediment interface and sediment-air interface, the emerging wave is trapped in the layer. If the impedance contrast at the basement is very big, the trapping is perfect, in an analogous way to the sound channel we have seen before. If in addition, as is the case in many soils, the layer dimensions allow a resonance phenomenon to set up, the resulting amplification is catastrophic. In general only the fundamental frequency is able to resonate, since higher frequencies are dumped, but fundamental frequencies are usually compatible with seismic frequency, as we can see from the following data. The fundamental resonance frequency range of the deposits varies from 0.25 to 10 Hz, for depths ranging from 10 to 500 m and waves velocity of 400 m/s; doubling the velocity the frequencies double too. The typical seismic wave frequency varies from 0.1 to 20 Hz.

The final parameter to be taken into account is the resonance frequency of the various industrial or living structures. For buildings it ranges from 25 Hz for a small house, to 0.5 Hz for a tall tower. For a dam it could be 1.25 Hz, for a long bridge 0.2 Hz. In the Mexico City disaster the seismic wave, the sediment and the building resonance frequencies were all the same and equal to 0.5 Hz. Most of the severe damaged buildings have from 7 to 18 stories (from ref. [6]).

As a final example of very catastrophic effects due to small-scale stratigraphic variations, we can examine those observed in 1997 in Italy in the town of Cesi (MC) during a moderate earthquake ($M = 6.0$ Richter Scale). The small town has been built partly on a hill and partly on a sediment valley. At Cesi Bassa (in the valley) severe damages and collapses measuring IX on the Mercalli Scale were recorded, whilst at Cesi Villa (on the hill) only minor damages of VII on the Mercalli Scale were observed. The town two areas are separated only by a few hundred meters. The valley soft soil consists in a superficial 10 m depth of recent sand-clay deposits (waves velocity = 80–100 m/s) and in 25 m valley floor (waves velocity = 200–400 m/s). The underlying bedrock presents a velocity of 1000–2000 m/s. The big impedances difference has been responsible for the catastrophe. The dominant wave frequencies were around 5 Hz, just in the range of typical buildings resonance frequencies (ref. [7]).

## 7. – Conclusions

An experimental path to discover the role of impedance in every situation of wave-like energy transmission has been set as part of the UNIMI PLS laboratories program. Most of the experiments may be realised with cheap materials and instrumentation and are easily reproducible at school or home by the student himself. The goals of the laboratory are both to get the students acquainted with the everyday life physics and aware of the geological configuration of Italian territory as regarding seismic prevention. A very preliminary partial version of the laboratory has been realized last year and encountered curiosity and interest of teachers and students. This year the laboratory has been upgraded with new experiments and further insights, as described above.

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