

## Detecting gravitational waves with atomic sensors

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**Summary.** — The recent detections of gravitational waves (GW) by the LIGO and VIRGO Collaborations opened a new era in the fields of physics and astrophysics. The possibility to explore the Universe with a new type of messenger paves the way for the observations of a novel class of phenomena otherwise not detectable in the electromagnetic spectrum. To this end, new space-based and ground-based experiments are planned in the near future to extend the detectable frequency band and improve the sensitivity of the existing instruments. In this paper, we present the possibility of using atom interferometry for GW detection in the mHz to about 10 Hz frequency band. After a discussion about the possible GW sources to be studied, we will provide an intuitive description of the measurement principle, highlighting the differences between detectors based on two-photon and single-photon transitions. Important noise sources which are expected in such devices are briefly explained. We finally present the perspective for the realization of a future large-scale demonstrator.

### 1. – Introduction

Cold atom interferometry relies on the quantum nature of microscopic objects like atoms to create interferometric schemes where the roles of matter and light are exchanged with respect to the optical interferometers. Thanks to the fact that massive particles are much more sensitive to gravitational fields with respect to photons, inertial sensors based on atom interferometry reached astonishing precision levels, comparable to, if not better than, state of the art classical instruments [1]. Motivated by the rapid technological

progress in the field, the possibility to detect GWs with atom interferometry has received increasing attention from the scientific community. Starting from seminal works where a single atom interferometer (AI) has been considered [2], today all the modern detection strategies are inspired by a 2008 paper [3]. Here, the idea is to use two spatially displaced AIs interrogated by the same laser beams, realizing in this way a gradiometer with a long baseline. The passage of a GW modifies the laser phase imprinted on the atomic wave packet, in analogy with the mirrors employed as phase references in optical detectors. This last feature is crucial to understand the attractiveness of the method: atoms in free fall are naturally isolated from environmental vibrations while mirrors require complex seismic isolation systems. Moreover, the AI sensitivity can be maximized in a frequency region (30 mHz to 10 Hz) between the LISA and LIGO sensitivity bands, adding in this way a new window for observations. One of the major weaknesses of this approach lies in the use of two-photon Raman transitions to manipulate the atomic wave packets. As we will show later, this places an upper limit on the baseline length and therefore on the achievable sensitivity. This issue can be elegantly solved by exploiting the single-photon clock transition of alkaline-Earth atoms like Sr or Yb, as originally proposed in [4, 5]. Recent papers foresee the realization of a space-based AI detector having a very long baseline [6], whose sensitivities exceed those of existing proposals (*e.g.*, LISA), based on traditional laser interferometry.

## 2. – Science case

The large optical interferometers of Advanced LIGO [7] have demonstrated for the first time in 2015 the existence of gravitational waves, with the detection of the signal emitted by a coalescence of stellar mass binary black holes (BBH), the famous GW150914 event [8], consisting of a pair of  $36M_{\odot} + 29M_{\odot}$  objects, merging into a  $62M_{\odot}$  BH. Since this first detection, several other BBH coalescences have been published [9], and in summer 2017 a binary neutron star event has been detected [10], well localised in the sky thanks to the presence of Advanced Virgo [11] in the detectors' network, and associated with a short GRB event [12], thus confirming a long standing hypothesis about the origin of these  $\gamma$ -ray events [13].

BNS coalescences had been long considered promising sources of GWs, since the period evolution in systems like PSR1913+16 with the predicted loss of orbital energy had provided indirect confirmation of the emission mechanism [14]; furthermore, the observational evidence, by way of the pulsar phenomenon, of several binary neutron star systems in our galaxy allowed to reliably estimate an event rate [15, 16].

However, it turns out that BBH signals are more frequent, essentially because the amplitude of the GW signals scales roughly as  $h \propto M^{5/3}$ , hence a  $\sim 30 + 30M_{\odot}$  BBH system is roughly detectable  $O(10)$  farther away than a  $10 + 10M_{\odot}$  BBH system, which had always been considered as a benchmark. Since the observed volume scales with the cube of the maximum observable distance, this led to the  $O(10^3)$  rate enhancement over the benchmark that has been observed.

The lesson learnt is therefore that massive BBH systems exist, despite all possible odds against their formation, and a considerable theoretical effort is now underway to understand how these systems originate: do they result from the evolution of pairs of very massive stars? Or are they formed by way of a capture mechanism, in dense stellar environments?

**2.1. The case for searching BBH at low frequencies.** – In order to answer these questions, it is of paramount importance to characterise the mass spectrum of these systems: are BBH pairs like GW150914 the most massive we should expect? Or are there more massive systems to be detected, that we cannot just see yet?

The question is open because the maximum frequency of the gravitational waves emitted roughly scales as  $M^{-1}$ ; hence an event 10 times more massive than GW150914 would be confined at frequencies below  $\sim 30$  Hz, where the current LIGO and Virgo sensitivity is limited. Though these instruments have not reached their design sensitivity yet, to be achieved in a few years, we expect them to be essentially blind below  $\sim 10$  Hz, therefore their ability to detect and characterise events due to BBHs significantly heavier than GW150914 will remain limited.

In the long term, the ground-based Einstein Telescope [17] will push the lower frequency limit down to  $\sim 3$  Hz, thus considerably widening the range of detectable masses [18], whereas the space based LISA detector [19] will open up the mHz–Hz range to observation, making it possible to detect very massive systems, and extreme events like the infall of matter into supermassive BHs.

However we can see in fig. 1 that a gap will remain which could prevent, for instance, to directly observe the merger phase of the so-called Intermediate Mass Black Holes (IMBH), systems including BHs of  $O(10^3 M_\odot)$ . Filling this gap could be one of the purposes of an atom interferometer designed for the detection of gravitational waves; evidence for such systems could shed light on the possible existence of a ladder of BH masses, from stellar mass to supermassive ones. An absence of evidence could indicate that entirely different formation mechanisms are at play in different mass ranges.

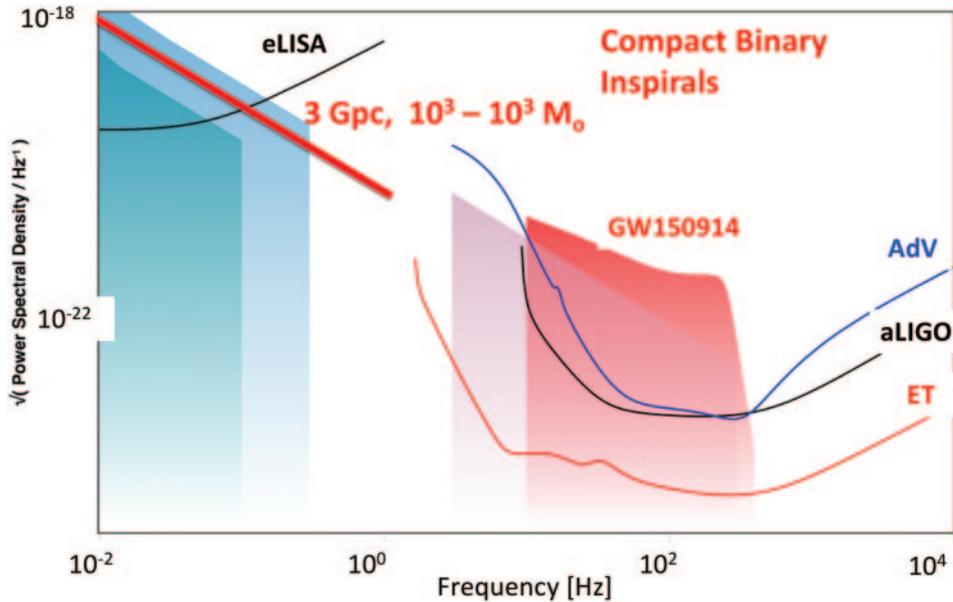


Fig. 1. – Design sensitivity of ground based detectors Advanced LIGO (aLIGO), Advanced Virgo (AdV), Einstein Telescope (ET) detectors, and of the space based detector eLISA, along with the trace of the BBH event GW150914, of an hypothetical IMBH event at 3 Gpc, and spectra of BNS events. Figure obtained using the GWplotter tool [20].

What is the sensitivity needed to do science on massive BHs? We can easily predict the strain waveforms  $h(t)$  emitted; considering that the only relevant physical parameters are the Newton constant  $G_N$  and the speed of light  $c$ , dimensional analysis imposes that

$$(1) \quad h(t, d, M_1, M_2, \mathbf{S}_1, \mathbf{S}_2) = h\left(\frac{t}{G_N \mu c^{-3}}, \frac{d}{G_N \mu c^{-2}}, \frac{M_1}{\mu}, \frac{M_2}{\mu}, \frac{\mathbf{S}_1}{G_N \mu^2 c^{-1}}, \frac{\mathbf{S}_2}{G_N \mu^2 c^{-1}}\right),$$

where  $\mathbf{S}, M$  are BH spin and mass, and  $\mu$  is an arbitrary mass scale. In a system (say)  $\lambda$  times more massive than a reference one (could be GW150914 for instance), noticing that the waveform is independent on  $\mu$ , the following scaling equivalence holds:

$$(2) \quad h(t, d, \lambda M_1, \lambda M_2, \lambda^2 \mathbf{S}_1, \lambda^2 \mathbf{S}_2) = h\left(\frac{t}{\lambda}, \frac{d}{\lambda}, M_1, M_2, \mathbf{S}_1, \mathbf{S}_2\right).$$

In the frequency domain

$$(3) \quad \hat{h}(f, d, \lambda M_1, \lambda M_2) \equiv \int_{-\infty}^{+\infty} h\left(\frac{t}{\lambda}, \frac{d}{\lambda}, M_1, M_2\right) e^{-2\pi i f t} dt \\ = \lambda \int_{-\infty}^{+\infty} h\left(\tau, \frac{d}{\lambda}, M_1, M_2\right) e^{-2\pi i \lambda f \tau} d\tau = \lambda^2 \hat{h}(\lambda f, d, M_1, M_2),$$

where in the last equation we have exploited the  $\frac{1}{d}$  dependency of the waveform on distance. Hence the frequency evolution  $f_l \rightarrow f_u$  for a system with masses  $\lambda M_{1,2}$  is mapped onto the evolution  $\lambda f_l \rightarrow \lambda f_u$  of a system with masses  $M_{1,2}$  at the same distance, including a  $\lambda^2$  factor on amplitude and  $\lambda$  scaling in the frequency. It follows a scaling by  $\lambda^{3/2}$  of the  $SNR$

$$(4) \quad SNR \equiv 2 \sqrt{\int_{f_l}^{f_u} \frac{|\hat{h}(f, d, \lambda M_1, \lambda M_2)|^2}{S_n(f)} df} = 2 \lambda^{3/2} \sqrt{\int_{\lambda f_l}^{\lambda f_u} \frac{|\hat{h}(\nu, d, M_1, M_2)|^2}{S_n(\frac{\nu}{\lambda})} d\nu}.$$

For instance, a  $10^3 + 10^3 M_\odot$  BBH ( $\lambda \simeq 30$  with respect to GW150914), located at 3 Gpc could be observed in the band 1–10 Hz with  $SNR \simeq 5$  using a detector having a noise floor at about  $10^{-21} \text{ Hz}^{-1/2}$ , thus 100 times worse than Advanced LIGO or Virgo.

**2.2. Type Ia supernovae.** – Another source of interest at low frequencies is represented by the type Ia supernova events, expected to emit neutrinos and GWs [21] at frequencies in the gap between terrestrial large optical interferometers and the space detector eLISA, as shown in fig. 2, where we plot the range of signal strengths for galactic sources.

Again, these are signals potentially accessible for a detector with sensitivities better than  $10^{-21} \text{ Hz}^{-1/2}$ , exactly as type II (core-collapse) galactic supernovae are potentially accessible to LIGO and Virgo. Type Ia supernovae are about as frequent as core-collapse ones, with a frequency in our galaxy of about one per hundred years; therefore the chances for detection are relatively modest, since the odds of detecting events in other galaxies are very unfavourable. Yet, like in the case for core-collapse supernovae, even the detection of a single event could provide a wealth of information on the detonation mechanism [21].

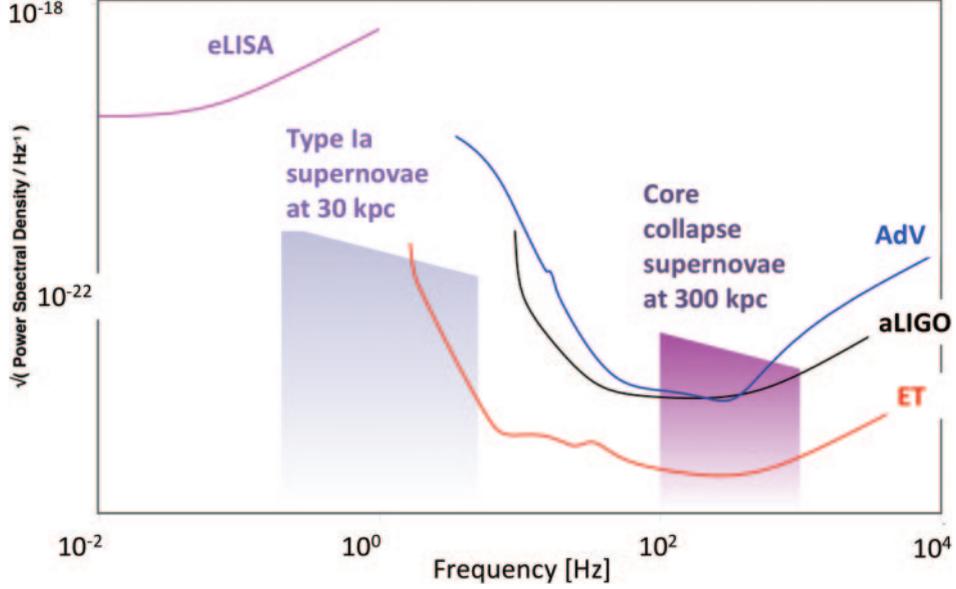


Fig. 2. – Type Ia and type II supernovae signal spectra, plotted against detector sensitivities. Figure obtained using the GWplotter tool [20].

### 3. – Atom interferometer phase shift in the presence of a GW

In this section we are going to discuss using intuitive arguments the origin of the AI phase shift induced by the presence of a GW. Firstly, we will consider the original gradiometric scheme based on two-photon transitions, presenting the leading terms of the calculated phase shift. Afterwards, we will put in evidence the main limitations of such approach, introducing thus the Sr/Yb optical clock transition as a keystone for experiments with very long baseline. Finally we will provide the expression of the sensitivity function of the instrument at the shot noise.

A Mach-Zehnder AI consists of three light pulses which respectively split, redirect and recombine the atomic wave. The diffraction process is usually realized with Raman or Bragg transitions, where the atom absorbs a photon from one beam (momentum  $\hbar k_1$ ) and stimulatedly emits a photon into the other beam (momentum  $\hbar k_2$ ). If the two beams are counterpropagating, the momentum of the atoms changes by  $\hbar(k_1 + k_2) = \hbar k_{\text{eff}} \approx 2\hbar k_2$ . In fig. 3(a), the space-time diagram of two simultaneous, spatially separated AIs is presented. The laser propagating from right to left (k-vector  $k_2$ , frequency  $\omega_2$ ), located at  $x = L$  is called “passive laser” and it is always on during the sequence. The laser propagating from left to right (k-vector  $k_1$ , frequency  $\omega_1$ , located at  $x = 0$ ) is called “control laser” and it is temporally pulsed in order to define the time at which the atom-light interaction vertices occur. We consider a GW propagating in the direction perpendicular to the plane and polarized along the laser propagation direction (x) with amplitude  $h$ , frequency  $\omega$  and initial phase  $\phi_0$ . According to the relativistic calculations carried out in [3], the leading term in the phase shift for a single AI is equal to

$$(5) \quad 4h \frac{ck_2}{\omega} \sin^2 \left( \frac{\omega T}{2} \right) \sin \left( \omega \frac{x_i - L/2}{c} \right) \sin \left( \phi_0 + \omega \frac{x_i - L/2}{c} + \omega T \right).$$

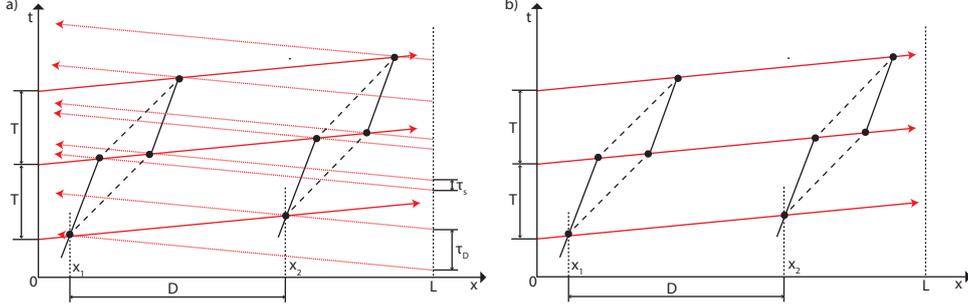


Fig. 3. – (a) Space-time diagram of two conjugated light pulse interferometers based on two-photon Raman transitions. The black and dashed lines indicate the motion of a single atom in the fundamental and excited state respectively. Control laser (solid red lines) and passive laser (dotted red lines) are also depicted. (b) Simplified space-time diagram of two conjugated light pulse interferometers based on single-photon clock transitions.

If we consider the differential phase shift of the two AIs in a regime where  $T > 1/\omega > D/c$ , we obtain the gradiometric phase

$$(6) \quad 4\hbar k_2 D \sin^2\left(\frac{\omega T}{2}\right) \sin(\phi_0 + \omega T),$$

where  $D = x_2 - x_1$ . Remarkably, when  $\omega T \sim 1$ , such last quantity is very similar to the signal detected by an optical interferometer which goes as  $\hbar k D$ . This contribution arises from the phase imparted by the laser beams on the atomic wave-function. In particular, while the phase of the control laser is common and cancels out, the passive laser accumulates a phase proportional to  $\omega_2 \tau_D$  where  $\tau_D = D/c$  is the difference in the emission time. Therefore, the GW will produce a signal proportional to  $\hbar k_2 D$ .

Another important effect to take into account is described in the following term:

$$(7) \quad 4\hbar \frac{\omega_{HF}}{\omega} \sin^2\left(\frac{\omega T}{2}\right) \sin\left(\omega \frac{x_i}{2c}\right) \sin\left(\phi_0 + \omega \frac{x_i}{2c} + \omega T\right).$$

Again, if we consider the differential phase shift in the intermediate frequency regime we obtain

$$(8) \quad 2\hbar \frac{\omega_{HF}}{c} D \sin^2\left(\frac{\omega T}{2}\right) \sin(\phi_0 + \omega T).$$

This term originates from the difference in the rest energy  $E_{HF} = \hbar\omega_{HF}$  of the two states involved in the transitions. Indeed, a GW with  $\omega \sim 1/T$  can produce an asymmetry in the time spent by the atoms in the excited state during the first and the second part of the interferometer. Such modulation, proportional to  $\hbar x_i/c$ , induces a propagation phase  $\propto E_{HF} \hbar x_i / \hbar c = \hbar\omega_{HF} x_i / c$ . In the case of Raman transitions we have  $\omega_{HF}/c \simeq 10^{-5} k_2$ , and therefore this contribution is negligible.

Finally, there are other two terms worth being discussed, both proportional to the initial velocity of the atoms  $v_L$  along  $x$ . One of them can be explained considering the distance traveled between the interferometer pulses  $v_L T$ . In the case of constant acceleration, it is well known that a phase  $k_{\text{eff}}[(x(2T) - x(T)) - (x(T) - x(0))] = k_{\text{eff}} a T^2$

is imprinted on the atomic wave-packet. Similarly, a length modulation of  $hv_L T$  between the first and the second part of the interferometer will induce a phase  $\sim k_{\text{eff}} hv_L T$ . Remarkably, this quantity can be easily rewritten as  $hl/\lambda_{dB}$  where  $l$  is the separation of the interferometer arms and  $\lambda_{dB}$  is the de Broglie wavelength of the atoms, retrieving thus the original result described in [2]. The other term can be loosely understood by considering the GW-induced time asymmetry previously discussed. Indeed, a Mach-Zehnder interferometer having a difference  $dt = hx_i/c$  in the free evolution time will show a phase  $k_{\text{eff}} v_L dt = k_{\text{eff}} v_L hx_i/c$  due to the spatial separation of the arms during the last pulse. However, the velocity terms are considerably smaller compared to the leading one, especially if one considers the AIs differential phase.

The presented gradiometric configuration has proven to be very effective against technical noise sources that commonly affect the two AIs, such as platform vibrations and laser frequency noise. Unfortunately, the phase noise coming from the passive laser is not completely common due to the delay time  $\tau_D$ . Consequently, the requirements in terms of seismic isolation and laser frequency noise become extremely hard to achieve when  $D > 1$  km, placing a limit on the baseline length [22]. This problem can be solved considering a simplified scheme based on a single laser, as depicted in fig. 3(b). Here the momentum transfer is performed by a single photon transition between two levels separated by optical energies. Atoms characterized by a long ( $\sim$  seconds) lifetime of the optically excited state like Sr or Yb can be used for such a protocol. In terms of expected phase shift, it is easy to realize that the term (6) vanishes (meaning that laser phase plays no role) while the propagation phase (8) is amplified by a factor  $10^5$ , thus becoming the leading contribution. Typically, to further enhance the sensitivity, large momentum transfer (LMT) beam splitters can be implemented by adding a secondary laser at  $x = L$  propagating in the opposite direction with respect to the primary one (see [4] for further details).

Several proposals based on the Sr clock transition have been published exploring different AI configurations and technical solutions for space experiments, where long baselines are achievable. More recently, also optical lattice clocks have been proposed for GW detection [23]. This is not surprising since, as already stressed, the GW induced phase stems only from the internal state evolution and not from the laser phase. A detailed explanation of the fundamental similarity between single photon AIs and clocks can be found in [24].

**3.1. Quantum limited strain sensitivity.** – To illustrate the potential performance of the instrument, we will evaluate the strain sensitivity curves of an AI gradiometer based on the Sr/Yb optical clock transition for different values of the main parameters at the shot noise. According to the expressions previously derived and considering a minimum sensitivity with a signal-to-noise ratio of 1, we obtain the following strain sensitivity function:

$$(9) \quad (S_h(\omega))^{1/2} = \left( \frac{1}{N_{at}} \right)^{1/2} \frac{1}{4n \frac{\omega_a}{\omega} \sin^2\left(\frac{\omega T}{2}\right) \sin\left(\frac{\omega D}{2c}\right) \sin\left(\frac{\omega D}{2c} + \omega T\right)},$$

where  $N_{at}$  is the cold atom flux (expressed in units of  $s^{-1}$ ),  $\omega_a$  the frequency of the clock transition ( $\lambda_{Sr} = 698$  nm) and  $n$  the number of momenta transferred to the atom during the diffraction process. In fig. 4 the envelope of the strain sensitivity function for various parameters of the AI gradiometer is shown. The red curve corresponds to an AI with  $N_{at} = 10^6 s^{-1}$ ,  $n = 200$ ,  $T = 1$  s and  $D = 10^6$  m. In the blue curve, the baseline

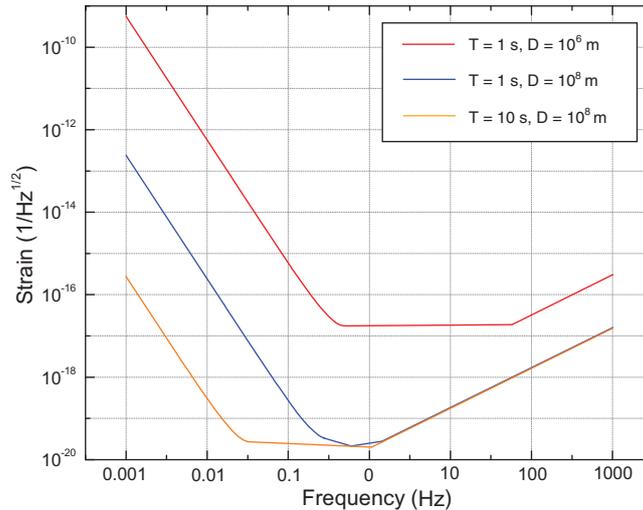


Fig. 4. – Shot-noise limited strain sensitivity curve of the AI gradiometer GW detector for different parameters of the AI.

has been increased to  $10^8$  m, while in the orange curve, additionally, the free evolution time has been enhanced to 10 s. Notably, with a longer baseline the peak sensitivity is proportionally increased at the expense of the instrument bandwidth. This is valid until  $D/c \sim T$ , whereupon the use of longer  $T$  is mandatory in order to gain further sensitivity with  $D$ .

#### 4. – Conclusion and prospects

The development of different detectors to enlarge the covered frequency band is of crucial importance for the future of Gravitational Wave astronomy. Although the performances of current ground-based laser interferometers are outstanding, they are strongly limited at frequencies below 10 Hz. GW detection with atom interferometry based on the Sr optical clock transition offers a promising alternative to traditional optical interferometry. Insensitivity to laser frequency noise allows for the possibility for space-based detectors based on a single linear baseline, thus requiring only two satellites instead of the conventional three. However, several technological challenges have still to be addressed, since matter-wave interferometry with alkaline-Earth atoms is a very recent research topic where only proof-of-principle experiments on short scale have been performed so far [25]. In the near future we foresee the development of ground-based experiments where large, vertical interferometric regions will be available to develop AIs with long free evolution time (1–3 s) and LMT beam splitters.

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