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# Cosmological phase transitions in the Standard Model with hidden scale invariance

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**Summary.** — The Standard Model of particle physics is minimally extended by a dilaton field. This field expresses the spontaneous breaking of scale invariance of an unspecified ultraviolet complete theory. While being manifestly scale invariant at the classical level, the low energy theory admits quantum scale anomaly responsible for the generation of mass scales. In addition to predicting a very small dilaton mass, this model has important consequences for the cosmology of the early universe. The electroweak phase transition can only be triggered by the QCD chiral phase transition occurring around 132 MeV. Since the QCD transition is expected to be first order in this picture, such a scenario gives rise to the production of stochastic gravitational waves with a peak frequency around $\sim 10^{-8}$ Hz.

## 1. – Introduction

Scale invariance is an appealing concept in field theory to accommodate both the origin of mass as well as the hierarchy between mass scales. Starting from a classical Lagrangian containing only dimensionless couplings, quantum fluctuations are known to break classical scale invariance and to generate an overall scale through dimensional transmutation [1]. Additional mass scales and the hierarchy between them are then induced from various combinations of the initial couplings with the overall scale.

Among different approaches, one concrete realisation proposed in [2] consists in assuming the existence of an ultraviolet complete and scale invariant theory. Without specifying the details of such a completion, it is assumed that spontaneously broken scale invariance manifests itself in the low energy regime of the theory through a dilaton field. In practice, this low energy theory can be described as a minimal extension of the Standard Model with all the dimensionfull parameters promoted to the dynamical dilaton. Phenomenological and cosmological properties of this model are presented.

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## 2. – Standard Model with hidden scale invariance

The Standard Model is considered here as an effective low energy limit of an unknown UV complete theory. The energy scale $\Lambda$ below which the approximation is valid represents a physical parameter encompassing the effects of the ultraviolet sector. In this way, the Higgs potential is written as [2]

$$V(\Phi^\dagger\Phi) = V_0(\Lambda) + \lambda(\Lambda)\left[\Phi^\dagger\Phi - v_{ew}^2(\Lambda)\right]^2 + \cdots, \tag{1}$$

where $\Phi$ is the Higgs electroweak doublet and $V_0$ the bare cosmological constant. The ellipsis indicates that higher order nonrenormalisable operators are irrelevant at low energy. In order to express the scale invariance of the fundamental theory in this regime, the mass parameters (including the cut-off $\Lambda$) are replaced by a dynamical dilaton field $\chi$ as follows:

$$\Lambda \to \Lambda\frac{\chi}{f_\chi},\ v_{ew}^2(\Lambda) \to \frac{v_{ew}^2(\chi)}{f_\chi^2}\chi^2 \equiv \frac{\xi(\chi)}{2}\chi^2,\ V_0(\Lambda) \to \frac{V_0(\chi)}{f_\chi^4}\chi^4 \equiv \frac{\rho(\chi)}{4}\chi^4, \tag{2}$$

where $f_\chi$ is the dilaton decay constant associated with the spontaneous breaking of scale symmetry. The new Higgs-dilaton potential is therefore explicitly scale invariant at the classical level(1)

$$V(\Phi^\dagger\Phi,\chi) = \lambda(\chi)\left[\Phi^\dagger\Phi - \frac{\xi(\chi)}{2}\chi^2\right]^2 + \frac{\rho(\chi)}{4}\chi^4. \tag{3}$$

In this approach, mass scales originate from quantum scale anomaly which is incorporated in the potential (3) through the $\chi$-dependence of the dimensionless couplings: $\lambda(\chi) = \lambda(\mu) + \beta_\lambda(\mu)\ln(\chi/\mu) + \beta'_\lambda(\mu)\ln^2(\chi/\mu) + \cdots$ (idem for $\xi(\chi)$ and $\rho(\chi)$). For convenience, the renormalisation scale $\mu$ is fixed at the dilaton VEV defined itself at the cut-off scale: $v_\chi = \Lambda$. The existence of this dilaton VEV actually relies on the presence, at the classical level, of a flat direction in the potential (3), along which the Coleman-Weinberg mechanism [1] takes place. This is mathematically expressed by solving the two minimisation conditions $\frac{\mathrm{d}V}{\mathrm{d}\chi}|_{\Phi=v_{ew},\chi=v_\chi} = \frac{\mathrm{d}V}{\mathrm{d}\Phi}|_{\Phi=v_{ew},\chi=v_\chi} = 0$. Taking also into account the phenomenological constraint on the cosmological constant, $V(v_{ew}, v_\chi) = 0$, this leads to [2]

$$\rho(\Lambda) = 0, \quad \beta_\rho(\Lambda) = 0, \quad \xi(\Lambda) = \frac{v_{ew}^2}{v_\chi^2}. \tag{4}$$

These equations provide a relation between dimensionless and dimensionfull parameters (dimensional transmutation) as well as the hierarchy between the electroweak and the high-energy scales. Note that the coupling $\xi(\Lambda)$ can be arbitrarily small in the technical sense [3-5].

The running mass of the dilaton is as well induced by the quantum anomaly, at two loop level [2]: $m_\chi^2(\Lambda) \simeq \frac{\beta'_\rho(\Lambda)}{4\xi(\Lambda)}v_{ew}^2$. In particular, the dilaton appears to be very light

(1) For simplicity, the dilaton decay constant is assumed here to be similar to the cut-off scale: $f_\chi \sim \Lambda$.

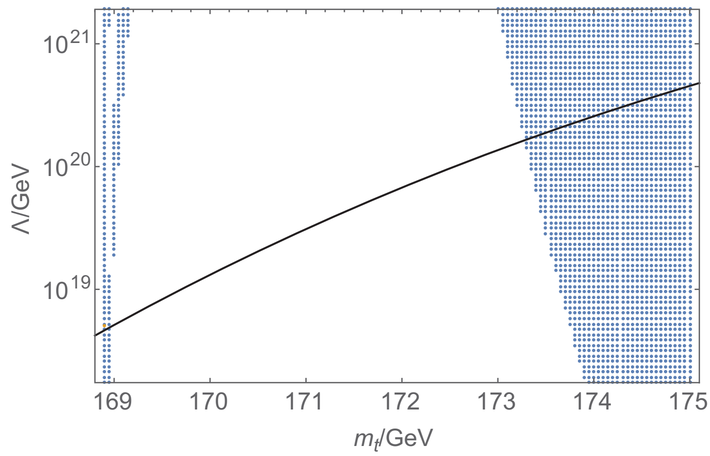


Fig. 1. – Plot of the allowed range of parameters (shaded region) with $m_\chi^2(v_{ew}) > 0$, *i.e.*, the electroweak vacuum being a minimum. The solid line displays the cut-off scale $\Lambda$ as a function of the top-quark mass $m_t$ for which the conditions in eq. (4) are satisfied.

compared to the Higgs mass $m_h^2 \simeq 2\lambda(\Lambda)v_{ew}$, namely $m_\chi^2/m_h^2 \simeq \xi(\Lambda)$. An estimate for the physical dilaton mass can then be evaluated by solving the (one-loop) renormalisation group equations starting from the high-energy scale $\Lambda$ down to the electroweak scale [2,6]. Such a solution has to satisfy the conditions (4) as well as $m_\chi^2(v_{ew}) > 0$ to ensure that the field configuration corresponds to a local minimum of the potential at low energy. The results are shown in fig. 1 which displays the allowed values of $\Lambda$ and top quark mass $m_t$ satisfying these conditions. For a cut-off at the Planck scale the dilaton mass is predicted to be $m_\chi \approx 10^{-8}\,\mathrm{eV}$.

## 3. – Cosmological phase transitions

The behaviour of the Higgs and dilaton fields during the early universe is described by the free energy density of the system, given by the finite-temperature Higgs-dilaton potential $V_T(\Phi, \chi)$. This potential can be evaluated as the sum of the zero-temperature expression (3) and one-loop thermal corrections (see review [7]). It is actually sufficient to evaluate it only along the flat direction of the classical potential, namely where the main dynamics takes place. In this case, the thermal potential can be expressed as a function of the Higgs field only. The main effect of thermal corrections can then be understood by looking at the high-temperature expansion of the potential [6]

$$V_T(h, \chi(h)) = \left[c(h)\pi^2 - \frac{\lambda(\Lambda)}{576}\right] T^4 + \frac{1}{48}\left[4\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2T^2 + \cdots, \tag{5}$$

where $h$ is the neutral component of the Higgs doublet $\Phi = (0, h/\sqrt{2})^{\mathrm{T}}$ and $c(h)$ is the number of relativistic degrees of freedom at temperature $T$. In (5), the temperature independent terms have vanished as a consequence of the flatness of the classical potential

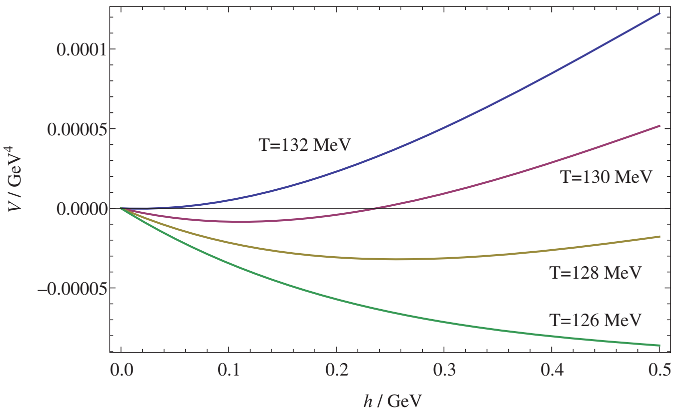


Fig. 2. – $V_T(h) - V_T(0)$ for different temperatures below the chiral phase transition.

in this direction($^2$). Since the coefficient $4\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)$ is positive (as confirmed numerically), the term proportional to $T^2$ produces a barrier between the minimum at $h = 0$ and the minimum corresponding to electroweak symmetry breaking. Taking into account that these two minima are degenerate at the classical level, $V(0,0) \approx V(v_{ew}, v_\chi)$ (see eqs. (3), (4)), and that the barrier persists until $T = 0$, the electroweak phase transition will not occur unless the universe reaches a very low temperature.

The previous analysis indicates that the temperature at which QCD interactions become strong will be reached before the electroweak transition happens. Therefore, all quarks are still massless at this epoch (since $h = 0$) and the chiral symmetry is given by $SU(6)_L \times SU(6)_R$. Once this symmetry gets spontaneously broken, quark-antiquark condensates form, according to [8]

$$\langle\bar{q}q\rangle_T = \langle\bar{q}q\rangle\left[1 - (N^2-1)\frac{T^2}{12Nf_\pi^2} - \frac{1}{2}(N^2-1)\left(\frac{T^2}{12Nf_\pi^2}\right)^2 + \cdots\right], \tag{6}$$

where $N$ is the number of massless quarks, $\langle\bar{q}q\rangle \approx -(250\,\mathrm{MeV})^3$ is the zero temperature condensate and $f_\pi \approx 93\,\mathrm{MeV}$ is the pion decay constant. As pointed out by Witten [9] and others [10], these condensates contribute linearly to the Higgs-dilaton potential through the quark-Higgs Yukawa interactions

$$V_T(h, \chi(h)) \to V_T(h, \chi(h)) + y_q\langle\bar{q}q\rangle_T\frac{h}{\sqrt{2}}, \tag{7}$$

where $y_q$ is the Yukawa coupling of the $q$-type quark. As the temperature decreases and $\langle\bar{q}q\rangle_T$ increases, this new term starts to dominate over the thermal barrier until the barrier completely disappears. The Higgs field will then smoothly rolls from $h = 0$ to $h = v_{ew}$. Namely, the chiral phase transition triggers the electroweak symmetry breaking.

($^2$) Zero-temperature one-loop corrections are not included in this expression as they are subdominant compared to thermal corrections.

This scenario has been numerically investigated with the use of the full one-loop effective thermal potential [6]. In summary, the QCD phase transition occurs at the critical temperature $T_c \approx 132\,\mathrm{MeV}$ defined by $\langle\bar{q}q\rangle_{T_c} = 0$, for $N = 6$. Then, as shown in fig. 2, the barrier in the potential (7) persists for $127 < T < 132\,\mathrm{MeV}$, but it disappears once $T \approx 127\,\mathrm{MeV}$ and the electroweak phase transition completes at that temperature.

One typical prediction of this scenario is the production of a stochastic background of gravitational waves. Although the Higgs transition is homogeneous and does not proceed through bubble nucleation, there are theoretical arguments [11] supporting that QCD chiral phase transition with $N \geq 3$ massless quark is first-order, with the production of QCD vacuum bubbles. Therefore, bubble collisions and bubble-plasma interactions will generate gravitational waves with a peak frequency $f_p \approx R_c^{-1}$ where $R_c$ is the bubble size at collision time $t_c$. Taking into account the redshift, the peak frequency today becomes $f_0 \approx f_p a(t_c)/a(t_0) \approx 10^{-8}\,\mathrm{Hz}$ for typical values of $R_c$ [6]. Such a signal can potentially be detected by pulsar timing arrays like the SKA telescope [12].

## 4. – Conclusion

Both phenomenological and cosmological properties of a minimal scale invariant extension of the Standard Model have been presented. Motivated by the origin of mass and hierarchy problem, this model introduces a new dilaton field in order to dynamically generate all the mass scales of the theory. This dilaton is predicted to be very light. Moreover, the evolution of the thermal Higgs-dilaton potential shows that the electroweak symmetry breaking can only be triggered by the QCD chiral phase transition occurring at $T \approx 132$ MeV. This QCD transition is expected to be first order and to produce a gravitational wave background with a peak frequency $\sim 10^{-8}\,\mathrm{Hz}$.

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