# Spectral anisotropies in high resolution three-dimensional simulations

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**Summary.** — We present a 3D high resolution hybrid particle-in-cells simulation of decaying Alvénic turbulence and we compare its results with an analogous simulations at MHD scales. We show that, while MHD simulations show spectral anisotropies in agreement with critical balance arguments, at ion scale magnetic spectrum is more isotropic than predicted. This suggest that intermittent small-scales structures, like reconnecting current sheets can play a fundamental role in transferring energy across kinetic scales.

## 1. – Introduction

Electromagnetic and plasma fluctuations in the solar wind show a turbulent spectrum which spans several order of magnitude [1]. At low frequency the spectral powers in the velocity, magnetic, and electric fields follow power-law which are roughly in agreement with the Kolomogrov prediction. At higher frequency, corresponding to scales of the order of the ion's kinetic scales, after a transient, the magnetic field power-law spectrum steepens following a power-law of index about $2.8-3.0$, the ion velocity decouples form the magnetic field showing an even steeper spectrum while the non-ideal terms in the genaralized Ohm's law makes the electric field power flatter than MHD scales [2]. At odds with hydrodynamic turbulence, the presence of the magnetic field **B** in plasmas introduces an asymmetry in the non linear-coupling and the turbulent cascade proceeds differently in direction parallel and perpendicular to **B**: At large (MHD) scales it has been shown both numerically (e. g. [3,4]) and empirically [5] that such spectral anisotropy is regulated by the critical balance condition [6] in which the non-linear energy transfer rate is allowed only when it is faster than the characteristic propagation timescale of the
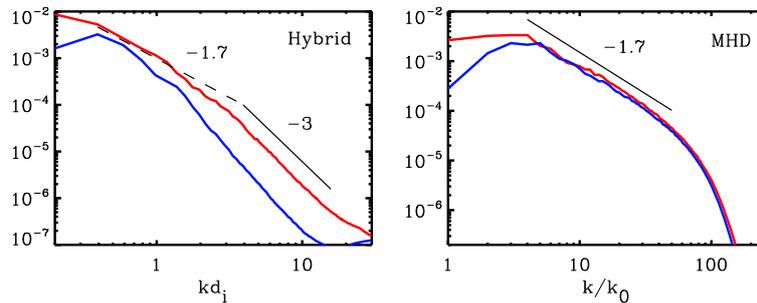
Fig. 1. – Omnidirectional Fourier power spectra of the magnetic (red) and velocity (blue) fields for hybrid (left) and MHD (right) simulations

system's proper modes (Alfèn waves in the incompressible MHD limit). In this scenario the k-vectors perpendicular to the main field are the most energetic and its power goes as $k_\perp^{-5/3}$, while in the parallel direction the power decreases faster as $k_\parallel^{-2}$, so that a spectral anisotropy is formed which increases as $k_\parallel \propto k_\perp^{2/3}$. Similar arguments have been applied at kinetic scales: in this case, assuming the relevant non-linear time scales as the electron eddy turn-over time [7] the power perpendicular to **B** should decrease as $k_\perp^{-7/3}$: the critical balance condition in the sub-ion range applied to the dispersive linear modes predicts $k_\parallel \propto k_\perp^{1/3}$ [8].

The different behaviour of the spectral anisotropy can be studied numerically and here we present a comparison between two 3D simulations of decaying turbulence: a MHD and an Hybrid PIC for large and sub-ion kinetic scales respectively. We show that, while the critical balance conjecture is observed for the MHD scale this is not true for the kinetic simulations.

## 2. – Results

We present the spectral analysis of two simulations of decaying turbulence performed with a fluid (MHD) [9,10] model and a hybrid-PIC codes [11,12]. The fluid simulation is representative of the turbulent cascade which occurs in the heliosphere at large scales (much large than the ion skin-depth and Larmor radius) while the kinetic simulation is able to capture the ion kinetic physics and the decoupling between the ion and electron motion. Both simulations have a very similar setup (details in [13-15]): a uniform plasma with a mean magnetic field $\mathbf{B_0} = B_0\hat{z}$ is initially perturbed with Alfvénic like fluctuations with an amplitude of $\delta B/B_0 \simeq 0.4$. The resolution of the simulations are both $n_x = n_y = 512$ in the perpendicular direction and $n_z = 256$ in the parallel one. Times and lengths in the hybrid code are defined in terms of the ion-cyclotron frequency $\Omega_i$ and ions skin-depth $d_i = v_A/\Omega_i$. In the MHD range the length-scale $L_0$ is arbitrary and the time is measured in term of $t_0 = L_0/v_A$. The size of the simulations are cubic box of size $L = 32d_i$ and $2\pi$ for the hybrid and MHD respectively.

The spectral analysis of the two simulations are performed at the maximum of the current activity which are $t = 40$ and $t = 2.6$ for the hybrid and MHD simulations respectively. In the MHD case, the magnetic and velocity omnidirectional spectra (i. e. mediated over all the possible directions on wavenumber) show a classical Kolmogorov-like cascade with a power spectra of about 5/3 (Fig. 1, left panel). In the hybrid simulation
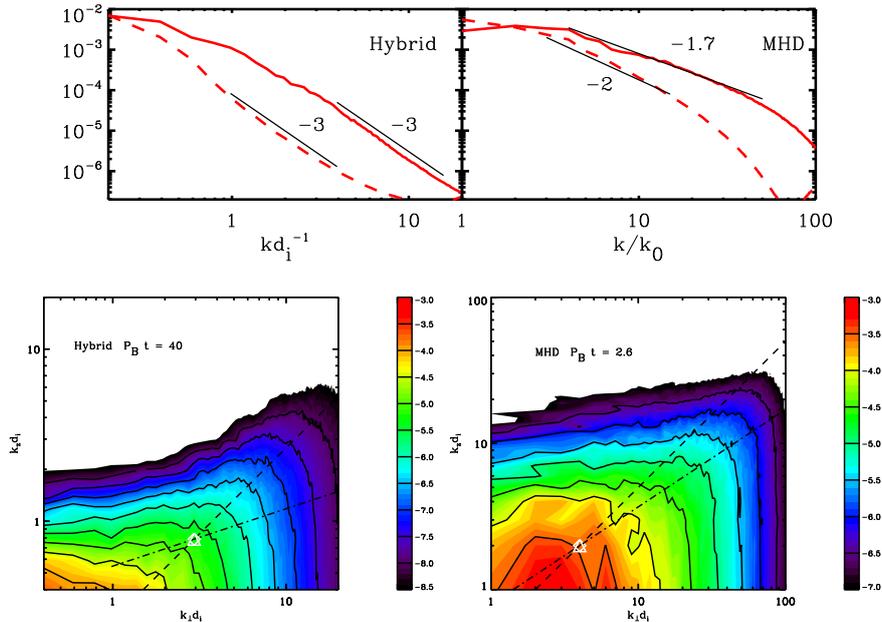
Fig. 2. – Top: 1D reduced spectra for the magnetic field power in direction perpendicular (solid) and parallel (dashed) to the main field $\mathbf{B_0}$ for the hybrid (left) and MHD (right) simulation. Bottom: 2D reduced spectra in the $(k_\parallel, k_\perp)$ plane (log-log scales) for the same fields. The dashed lines highlight the $k_\parallel \propto k_\perp$ condition in both simulations while the dash-dotted lines show the critical balance conditions ($k_\parallel \propto k_\perp^{1/3}$ in the sub-ion range and $k_\parallel \propto k_\perp^{2/3}$ at MHD scales).

(right panel of Fig. 1), a similar behaviour is observed at large scales, while at about $kd_i = 3$ the magnetic field power steepens with power index of about 3. Such result, together with a stronger decay in the velocity power is consistent both with observations [2, 16] and previous 2D and 3D hybrid simulations [17, 15].

In the MHD simulations the reduced spectra perpendicular to $\mathbf{B_0}$ becomes dominant few scales below the injection scale and follows the Kolmogorov prediction while the parallel spectrum decreases much faster with a slope closer to the one predicted by the critical balance condition (Fig. 2) . In the hybrid simulations the power in the magnetic fluctuations is still the dominant one with the parallel counterpart decreasing much more at very low wavevectors. However, above $kd_i \simeq 1$ both spectra start to follow the same power-law meaning that the spectral anisotropy stays constant at sub-ion scales, i. e. $k_\parallel \propto k_\perp$. Isocontours of the axisymmetric spectra show that, while the MHD simulations roughly follow the critical balance condition, in the sub-ion energy most of the power is confined in the region $k_\parallel \leq k_\perp$ and not in the region $k_\parallel \leq k_\perp^{1/3}$ as predicted.

## 3. – Discussion and conclusions

The comparison between the two simulations show that the spectral anisotropy at sub-ion scales is not easily interpreted in terms of the critical balance condition: the power, as in the observations, decreases significantly faster and the spectra are more isotropic than predicted. Although the faster decrease can be interpreted as the effect

of damping [18, 19], the analysis of the 3-th order structure functions in analogous $2D$ hybrid simulations suggest that, at least at moderate value of the plasma $\beta$, the sub-ion scales are essentially inertial [20]. Alternatively, the presence of small-scales intermittent structures could reduce the spectral anisotropy and produce steeper spectra [21]. Such structure could be strong current-sheets, prone to fast tearing-like instabilities (e. g. [22-24]). Numerical simulations [25, 26] suggest that those structures are a fundamental channel to feed turbulence at ion-scales.

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