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Isospin-symmetry breaking in nuclear structure

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Summary. — The breaking of the isospin symmetry is discussed from the point of view of the nuclear shell-model. The accuracy of the theoretical framework is demonstrated on the description of the isobaric multiplet splittings. The use of phenomenological versus realistic interactions is discussed. We demonstrate the relevance of the theoretical description for isospin-forbidden processes. The developed formalism is important for the calculation of isospin-symmetry breaking corrections to the weak-interaction process on nuclei, and it can provide missing information on the structure of proton-rich nuclei relevant for astrophysics applications.

1. – Introduction

The isospin symmetry is a symmetry between a proton and a neutron (or between up and down quarks at a quark level) with respect to the strong interaction. A nucleon is said to be characterized by an isospin quantum number t=1/2, similar to an ordinary spin, with neutron and proton being labeled by its projection t_3= ±1/2, respectively. The three components of the isospin operator, $\hat{\mathbf{t}}$, generate an isospin SU(2) algebra: $[\hat{t}_j, \hat{t}_k] = i\epsilon_{jkl}\hat{t}_l$. The total isospin operator for an A-nucleon system is $\hat{\mathbf{T}} = \sum_{k=1}^{A} \hat{\mathbf{t}}_k$, with $T(T+1)$ and $(N-Z)/2$ being eigenvalues of $\hat{\mathbf{T}}^2$ and $\hat{T}_3$. A charge-independent Hamiltonian would commute with $\hat{\mathbf{T}}$, giving rise to degenerate multiplets of states (J^π, T) in nuclei with the same A and $T_3 = -T, \ldots, T$, called *isobar analogue states (IAS)*.

The isospin symmetry is, however, an approximate symmetry. A dynamical breaking of the isospin symmetry can explain the splittings of the isobaric multiplets, known also as Coulomb displacement energies. In addition, modern accelerator facilities combined with advanced detection systems provide nowadays extensive and accurate data on isospin-forbidden decays, indicating that isospin is not a good quantum number anymore and there is a certain amount of isospin mixing in nuclear states.

An accurate theoretical description of the isospin-symmetry breaking is important not only for understanding the structure and decay of proton-rich nuclei. It is also crucial for the evaluation of the nuclear-structure corrections to nuclear β-decay, such as superallowed Fermi $0^+ \to 0^+$ decays, serving for the tests of the Standard Model, as well as it can be of help in applications to nuclear astrophysics.

At a nuclear level, the isospin symmetry is mainly broken by the Coulomb interaction among protons and, to a minor extent, by the proton and neutron mass difference and the presence of the charge-dependent (CD) forces of nuclear origin. Those nuclear causes can be rooted to the u and d quark mass difference and electromagnetic interactions between the quarks. Experimental evidence on the existence of CD forces of nuclear origin is quite firm. First, it is well established that there are differences in the neutron-neutron, proton-proton (with electromagnetic effects being subtracted) and neutron-proton $^1S_0$ scattering length: the difference of a_{nn} and a_{pp} is a signature of *charge-symmetry breaking* of the strong NN force, while the even larger difference between a_{np} and the average of a_{nn} and a_{pp} is known as the *charge-independence breaking* property [1].

Second, it turns out that the Coulomb force alone is not sufficient to explain the binding energy differences in mirror nuclei, starting from the lightest ^{3}H-^{3}He case up to heavy nuclei (the so-called Nolen-Schiffer anomaly [2]).

Henley and Miller [3] proposed to range two-nucleon forces into four classes according to their isospin character. Class I are charge-independent forces $\{1, \hat{\mathbf{t}}(1)\cdot\hat{\mathbf{t}}(2)\}$; class II are forces which break the charge-independence, but preserve the charge-symmetry, $\{\hat{t}_3(1)\hat{t}_3(2)\}$; class III are charge-symmetry breaking forces, which vanish in the neutron-proton system, $\{\hat{t}_3(1)+\hat{t}_3(2)\}$; and class IV are forces which do not conserve the isospin of a two-nucleon system: $\{\hat{\mathbf{t}}(1)\times\hat{\mathbf{t}}(2), \hat{t}_3(1)-\hat{t}_3(2)\}$. The following hierarchy is expected [5]: $V_I > V_{II} > V_{III} > V_{IV}$. It is important to note that already class II and III forces do violate the isospin symmetry in a many-body system ($A>2$). For example, the ordinary Coulomb interaction between protons contains terms of classes I, II and III.

Isospin-symmetry breaking NN forces have been successfully constructed and understood both with meson-exchange models [1, 4], as well as within the modern chiral effective field theory (χ-EFT) [5-7] with proper identification of various contributions. Moreover, CD three-nucleon forces have been explored in the latter approach (e.g. [6] and refs therein).

Ab-initio calculations with CD forces successfully reproduce binding-energy differences in light mirror nuclei and the expected amount of isospin-mixing in ^{8}Be [8]. To solve the nuclear many-body problem for heavier nuclei one still needs an approach requiring effective CD interactions. Numerous investigations have been recently performed within various theoretical frameworks aimed at a reliable description of the isospin-symmetry breaking: state-of-the-art shell-model calculations [9, 10, 12, 11, 13], including its no-core realization [14] and continuum-coupling extension [15], mean-field approaches and beyond (*e.g.* [16]), relativistic RPA [17], JT projected nuclear DFT [18] and others. Many of these approaches applied their results to the calculation of the isospin-symmetry breaking correction to superallowed $0^+ \to 0^+$ β-decay, providing an extensive, but diverging set of results.

The shell model, being a symmetry-conserving approach, is particularly adequate for searches of tiny isospin-symmetry breaking effects in low-energy states and transitions. Below I will focus on some recent progress obtained within the shell model.

2. – Isospin-nonconserving shell-model Hamiltonians

The starting point is a non-relativistic Hamiltonian for point-like nucleons containing nucleon kinetic energies and effective NN interactions (only two-body interactions are considered here). Being interested in the low-energy structure of medium-mass nuclei, we consider only valence nucleons in a restricted (valence) space beyond a given closed-shell core. Adding and subtracting a one-body spherically-symmetric potential (*e.g.*, a harmonic-oscillator potential), we can rewrite the Hamiltonian as a sum of an independent-particle Hamiltonian ($\hat{H}_0$) and a residual interaction ($\hat{V}$): $\hat{H}\Psi \equiv (\hat{H}_0 + \hat{V})\Psi = E\Psi$. The eigenstates Ψ_p are searched for in terms of Φ_k, an orthonormal set of eigenfunctions of $\hat{H}_0$: $\Psi_p = \sum_k C_{pk}\Phi_k$. The eigenproblem for $\hat{H}$ is thus reduced to diagonalization of the energy matrix $\langle\Phi_{k'}|\hat{H}|\Phi_k\rangle$, computed from single-particle energies of valence-space orbitals, ε_i, and two-body matrix elements (TBMEs) of the residual interaction $\hat{V}$. As a result, we get eigenvalues E_p and the corresponding sets of expansion coefficients $\{C_{pk}\}$. For a rotational invariant and charge-independent Hamiltonian $\hat{H}$, the eigenstates are characterized by the angular momentum and isospin quantum numbers ($JMTT_3$), forming thus degenerate isospin multiplets.

Consider now an isospin-nonconserving (INC) term, comprising the two-body Coulomb interaction and effective CD NN forces of classes II and III (no class IV forces are considered). Such an operator is a sum of an isoscalar, an isovector and an isotensor term:

$$\hat{V}_{INC} = \hat{V}_C + \hat{V}_{CD} = \sum_{k=0,1,2} \hat{V}^{(k)}_{INC}\,, \qquad \text{where} \quad \begin{cases} \hat{V}^{(0)}_{INC} = (v_{pp} + v_{nn} + v^{T=1}_{np})/3 \\ \hat{V}^{(1)}_{INC} = v_{pp} - v_{nn} \\ \hat{V}^{(2)}_{INC} = (v_{pp} + v_{nn})/2 - v^{T=1}_{np} \end{cases}$$

To describe the Coulomb effects of the core, we add an isovector one-body term, which gives rise to the so-called *isovector single-particle energies*, $\tilde{\varepsilon}_i = \varepsilon_{p_i} - \varepsilon_{n_i}$. In first-order perturbation theory, the splitting of the isobaric multiplets is expressed by a quadratic polynomial in T_3:

$$\langle\Psi_{TT_3}|\hat{V}_{INC}|\Psi_{TT_3}\rangle = E^{(0)}(\alpha,T) + E^{(1)}(\alpha,T)T_3 + E^{(2)}(\alpha,T)\left[3T_3^2 - T(T+1)\right]$$

This dependence is known as the *isobaric-multiplet mass equation* (IMME) [19]

$$M(\alpha,T,T_3) = a(\alpha,T) + b(\alpha,T)T_3 + c(\alpha,T)T_3^2\,,$$

where $\alpha = (A, J^\pi, \ldots)$, M is a mass excess. Experimental a, b, c coefficients can be deduced from available data on nuclear masses and spectra up to about $A = 71$ [20, 21].

Ideally, one would derive an effective Hamiltonian for valence-space calculations from a bare NN potential via a certain renormalization procedure [22]. With two-nucleon interactions only, the resulting effective interaction suffers from serious deficiencies [23]. An efficient way to remove them is to fit all TBMEs of the residual interaction or some components of the interaction (the monopole term) to selected experimental spectra of nuclei from a given model space. The Coulomb contribution is usually evaluated and subtracted from the data. Resulting phenomenological (charge-independent) interactions are therefore called realistic and they can provide high accuracy, e.g., USD [24] in the *sd* shell, KB3G [25] or GXPF1A [26] in the *pf* shell.

Knowing this difficulty, a direct way to construct an accurate INC Hamiltonian would be to add $\hat{V}_{INC}$ and $\tilde{\varepsilon}_i$ to a well-established charge-independent Hamiltonian. One needs thus at least two extra parameters, which govern isovector and isotensor components of an effective term of nuclear origin, $\hat{V}_{CD}$. Those parameters can be fixed by a fit of theoretical isovector and isotensor contributions to experimentally deduced b and c

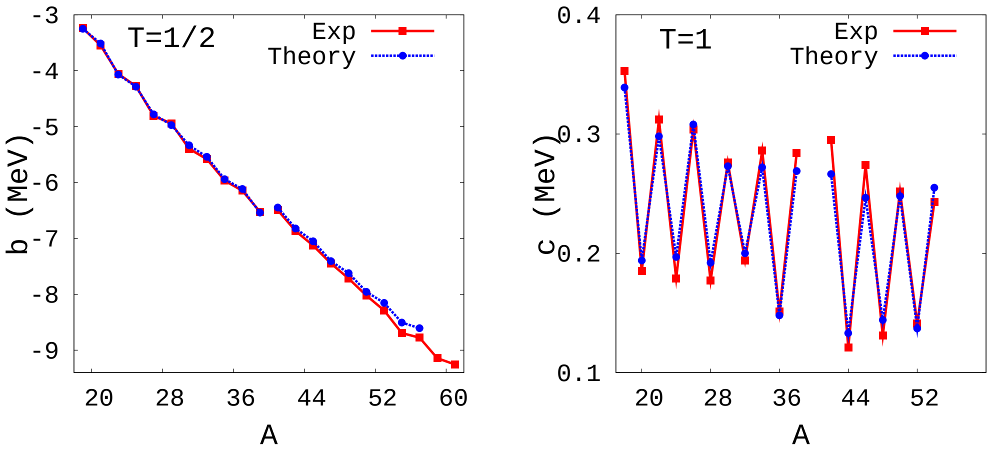

Fig. 1. – Experimental [20,28] and theoretical IMME b and c coefficients for the lowest $T = 1/2$ qnd $T = 1$ multiplets, respectively, in sd and pf shell. The sd-shell results are quoted from ref. [11], while pf-shell calculations have been performed with GX1Acd interaction [27].

IMME coefficients. This strategy was proposed in ref. [9], and re-examined recently for sd-shell [10, 11] and pf-shell and heavier nuclei [13]. Various forms for $\hat{V}_{CD}$ have been explored. The use of either a ρ-exchange Yukawa potential (with a scaled meson mass) or the $T = 1$ term of the isospin-conserving Hamiltonian in the isovector and isotensor channels leads to similar quality fits [9, 11]. Figure 1 shows the b coefficients for doublets and c coefficients from triplets as obtained from such phenomenological interactions in sd and pf shell nuclei, in comparison with the experimental values. The theory well reproduces a visible staggering of these b and c coefficients as a function of A [11]. More realistic forms of $\hat{V}_{CD}$ usually worsen the fit [10].

In ref. [13], another strategy has been chosen to model V_{CD}: two $J = 0$, $T = 1$ TBMEs in the $f_{7/2}$-orbital have been added and found to be sufficient to reproduce the staggering behavior of b and c coefficients.

The idea of modeling CD forces of nuclear origin by a few TBMEs was originally proposed in the description of the differences in the excitation energies of isobaric multiplets relative to the lowest in energy multiplet. Those quantities are known as mirror energy differences (MEDs) and triplet energy differences in $T = 1$ multiplets. MEDs considered as a function of J along an excitation band can bring detailed information on nuclear structure effects. A vary accurate description has been achieved [12, 29] in the pf shell by a phenomenological parameterization of various physical effects, such as nucleon pair alignment, changes in nuclear radius (or deformation), electromagnetic corrections to the single-particle energies, with $\hat{V}_{CD}$ being modeled by a few $J = 0$ TBMEs in isovector and isotensor channels. Moreover, MEDs have been shown [30] to depend linearly on the difference of neutron and proton radii, known as *neutron skin*, and that they strongly correlate with the $s_{1/2}$-orbital occupation.

In general, low-l orbitals, especially $s_{1/2}$-orbitals, are characterized by an extended radius and play thus a special role. In particular, it was noted that MEDs of states having higher occupation of $s_{1/2}$ are unusually large (Thomas-Erhman shift [31]). Attempts to account for this effect include an additional shift of $\tilde{\varepsilon}(s_{1/2})$ (*e.g.* ref. [32]) or some specific quenching of the TBMEs involving the $s_{1/2}$ orbital [33].

Recent progress in the NN interaction and many-body theories has led to the appearance of first microscopic effective CD Hamiltonians. First large-scale calculations for proton-rich nuclei in extended $sdf_{7/2}p_{3/2}$ and $pfg_{9/2}$ model spaces with effective NN+3N Hamiltonians, constructed on the basis of the renormalized χ-EFT potential, have been reported in ref. [34]. In ref. [35], similarly, microscopic CD pf-shell Hamltonians have been constructed from the two-body CD-Bonn, AV18 and chiral N^3LO potentials. Comparison of the IMME c coefficients as a function of J in selected pf shell nuclei with experimental data indicates too strong contribution of the charge-independence breaking terms of nuclear origin. Further theoretical investigations with microscopic CD interactions would be of great interest.

3. – Isospin-forbidden decay processes

In the shell model, isospin impurities come from the mixing of states of the same spin and parity, but different isospin. In the simplest case of a 2-level mixing, the admixture of the one state into the other, α, to first order is proportional to the ratio of the isospin-mixing matrix element and the energy difference between the two states: $\alpha \sim \langle V \rangle / \Delta E$. As the energy difference is rather difficult to predict theoretically, especially for an odd-odd nucleus, it would be desirable if the theory could predict the value of the mixing matrix element, $\langle V \rangle$. These mixing matrix elements depend strongly on the structure of the states considered and, therefore, require systematic calculations.

Experimentally, the only model-independent way to get direct information on the amount of the isospin-mixing is provided by the Fermi β-decay. Since the Fermi operator is given by the $\hat{T}_{\pm}$ components of the isospin operator, its matrix element between IAS is known to be $|M_F^0| = |\langle T, T_3 \pm 1 | \hat{T}_{\pm} | T T_3 \rangle| = \sqrt{(T+T_3)(T-T_3+1)}$. An observed depletion of the Fermi strength in the IAS or an isospin-forbidden Fermi transition to a non-analogue state would immediately report an amount of isospin-mixing. In addition, if a $T_3 > 0$ nucleus $\beta^{\pm}$ decays, then the mixing is dominantly present in the parent (daughter) nucleus and, inversely for a $T_3 < 0$ nucleus. Relatively few cases of pure Fermi non-analogue $0^+ \to 0^+$ transitions are known [36]. In the case of $J^\pi \to J^\pi$ ($J \neq 0$) transitions, a separation of the Gamow-Teller component is required. This is an experimental challenge, bringing interesting information on the isospin impurity [37,38].

Observation of other isospin-forbidden decays requires theoretical calculations of corresponding nuclear processes for extraction of the mixing probability. Isospin selection rules for electromagnetic operators may be of use. For example, the internal part of the $E1$-operator is, in lowest-order of the long wavelength approximation, of purely isovector character. Hence, $E1$ transitions between the states of the same isospin in $N = Z$ nuclei are forbidden by the isospin symmetry. The shell-model calculation of individual $E1$ transition rates is hampered by the fact that the model space should contain orbitals of different parity which could also lead to a center-of-mass motion. Given that the center-of-mass separation is only approximate, it would be difficult to give a precise estimation of the $E1$ strength. Observed enhancements of $E1$ rates in $N = Z$ nuclei or enhanced asymmetries of mirror $E1$ transitions can be related to the giant isovector monopole resonance [39]. Various possibilities to deduce the amount of the isospin mixing from electromagnetic responses have been explored [40-44].

A novel way has been proposed recently to deduce isospin mixing from the β-delayed $p\gamma$-emission [45]. As follows from the energy balance, the proton emission from the IAS (J^π, T), populated in a β-decay of a $T_3 < 0$ precursor, is forbidden by isospin symmetry (see a schematic picture in fig. 2). Observation of such processes gives evidence of the

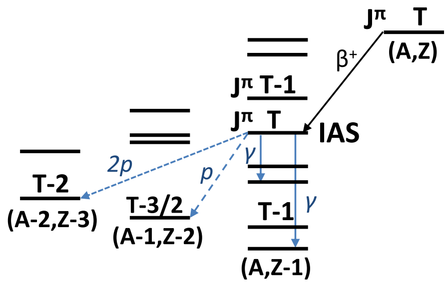

Fig. 2. – Schematic picture of β-delayed proton(s)-gamma emission from the IAS.

isospin mixing, mainly, in the IAS which is surrounded by states of another isospin, $(J^\pi, T-1)$. A large amount of mixing can be deduced from the missing Fermi strength. However, small amounts may be hidden by experimental uncertainties. If the angular momentum l of the proton is uniquely defined, an experimentally measured branching ratio of the proton to gamma decay of the IAS, $I_p^{\rm IAS}/I_\gamma^{\rm IAS}$ can provide information on the spectroscopic factor for an isospin-forbidden proton emission from the IAS, if we supplement it by the theoretical electromagnetic width, $\Gamma_\gamma^{\rm IAS}$, and the single-particle proton width, $\Gamma_{sp}^{\rm IAS}$, of the IAS:

$$S_p^{\rm IAS} = \frac{\Gamma_\gamma^{\rm IAS}}{\Gamma_{sp}^{\rm IAS}} \frac{I_p^{\rm IAS}}{I_\gamma^{\rm IAS}} \,. \tag{1}$$

If this admixture to the IAS comes from a closely-lying non-analogue state, $J^\pi, T-1$, then calculating its spectroscopic factor, $S_p^{\rm T-1}$, within the shell model we can deduce the amount of isospin mixing in the IAS: $\alpha^2 = S_p^{\rm IAS}/\, S_p^{\rm T-1}$. We remark that the same procedure is applicable to isospin-forbidden $2p$ (shown in fig. 2) or α emission from the IAS. In case of multiple transitions, one can analyze each of them separately and then cross-check the values of α. If l of the transition is not uniquely defined, we cannot unambiguously deduce spectroscopic factors, but we can still analyze the isospin mixing, if a two-level mixing model is applicable.

4. – Isospin-symmetry breaking correction to $0^+ \to 0^+$ Fermi β-decay

The superallowed Fermi β-decay between 0^+ IAS members is an important tool to verify the symmetries underlying the Standard Model of particle physics [46]. The constancy of the absolute Ft values of such transitions in various emitters would confirm the Conserved Vector Current hypothesis. If it holds, then one can deduce from the Ft value the vector coupling constant for this semi-leptonic decay, G_V. Combining G_V with the data from the purely leptonic muon decay, one can determine the absolute value of the V_{ud} matrix element of the Cabibbo-Kobayasi-Maskawa matrix.

To get the absolute Ft value from the experimental half-life $t_{1/2}$ of the transition and the decay Q value, one has to incorporate a few theoretical corrections [47]:

$$Ft^{0^+\to 0^+} \equiv ft^{0^+\to 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)} \,. \quad (2)$$

Here $K = 2\pi^3\hbar \ln 2(\hbar c)^6/(m_e c^2)^5$, while Δ_R, δ'_R, δ_{NS} are transition independent, transition dependent and nuclear-structure dependent radiative corrections and δ_C is the isospin-symmetry breaking correction. It is defined as a deviation of the squared realistic Fermi matrix element from its isospin-symmetry value $|M_F^0|$: $|M_F|^2 = |M_F^0|^2(1-\delta_C)$. The estimation of δ_C requires a nuclear-structure model which can account for the broken isospin symmetry. This is still a challenge for microscopic nuclear many-body theories. Existing predictions from various theoretical approaches largely diverge ([47, 48] and refs therein). While Δ_R provides the largest uncertainty to deduced $|V_{ud}|^2$, the largest contribution to the Ft-value uncertainty (and limitation for the CVC tests) comes from the uncertainty on δ_C, even when taken from a single approach.

The INC shell model is a well-suited tool for the δ_C calculation. Besides isospin-symmetry breaking inside the model space, described in sect. 2, one has to replace harmonic-oscillator radial wave functions by realistic spherically-symmetric wave functions from a Woods-Saxon (WS) or a Hartree-Fock (HF) potential. All ingredients involved in the calculations can be subjected to experimental verification from the data on the IMME coefficients, proton and neutron separation energies and nuclear charge radii. This opportunity greatly constrains the calculations and guaranties consistency of the results [48]. In particular, the obtained theoretical uncertainty is mostly related to the experimental uncertainty on the nuclear charge radii. There is still a dissension, however, between WS and HF results [49] which should be addressed in further studies.

5. – Astrophysics applications

A precise description of isobaric multiplet splittings is important in astrophysics applications, in particular, in the context of nucleosynthesis after novae explosions and X-ray bursts. From theoretical b coefficients and experimentally determined binding energy of a neutron-rich mirror, one can predict masses of the proton-rich partner. These ideas have been used, *e.g.* in refs. [50, 13], to map the proton drip-line towards ^{100}Sn.

In addition, precise theoretical calculations of b coefficients can be used to estimate the position of resonances, influencing radiative proton capture reaction rates [51, 32].

6. – Conclusions

In summary, we reviewed current achievements of the nuclear shell model in the description of the isospin-symmetry breaking phenomena. While sd and pf shell model spaces are well mastered and controlled, more efforts are needed for cross-shell CD interactions (such as in the $sdpf$ model space) and for studies of heavy nuclei. Important applications to weak interaction processes and astrophysical issues are in progress. Experimental data on the spectroscopy of nuclei along the $N = Z$ line and proton-rich nuclei will be of help to constrain the models.

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