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Time scales in nuclear structure and nuclear reactions of exotic nuclei

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Summary. — Two relevant time scales are introduced to describe the interplay of nuclear structure and nuclear reactions for exotic nuclei. The collision time represents the time dependence of the external field created by the target on the projectile. The excitation time represents the characteristic time dependence of the projectile degrees of freedom due to its internal Hamiltonian. The comparison of these two time scales indicate when approximate treatments of the reaction, such as the sudden approximation, implicit in the eikonal treatment, is applicable. An approach based on these time scales is used to describe recent experimental data as well as theoretical calculation involving Coulomb break-up, stripping reactions and (p, pN) reactions. It is suggested that the dependence of the stripping cross sections on the difference of binding energies of protons and neutrons may be associated to inadequacy of the eikonal approximation to describe the removal of strongly bound nucleons at intermediate energies.

1. – Introduction

Nuclear reactions can be understood as a procedure to place a nucleus to be studied (i.e. the projectile) in a time dependent external field created by other one (the target). This field is a combination of the long-range Coulomb interaction (dominant for a heavy target) and the short-range nuclear interaction (dominant for a light target). As a result of this interaction, the projectile can be excited, leading, in the case of exotic, weakly-bound nuclei, to the population of break-up states. The process of excitation depends on the magnitude of the external field as well as on its time structure. Typically, the external field is maximum at the instant of time when both nuclei are at the distance of

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closest approach corresponding to the trajectory associated to the scattering angle, and then it reduces for shorter or longer times. The characteristic time for this reduction depends on the trajectory on the proximity of the distance of closest approach, and on the range of the interaction.

For a Coulomb-dominated trajectory, the distance of closest approach can be written as

$$R(0) = a_0 \left(1 + \frac{1}{\sin\theta/2}\right) \quad ; \quad a_0 = \frac{Z_p Z_t e^2}{8\pi\epsilon_0 E}, \tag{1}$$

which is a function of the scattering angle θ and the centre of mass energy E. When nuclear forces are relevant, this expression will be modified accordingly. Note that this expression, as well as other arguments in this paper, make use of the concept of trajectory, which is meaningful provided that the De Broglie wave length of the relative motion is small compared to the length scale of the problem, which is given by the previous distance of closest approach. This is the case for the systems considered.

1˙1. *Collision time.* – In the vicinity of the distance of closest approach, the trajectory can be approximated by a straight line, perpendicular to the vector $\vec{R}(0)$, with a local velocity $v(0)$ determined by energy conservation, using relativistic or non-relativistic kinematics as required. Thus, the distance of the target in the vicinity of the distance of closest approach, as seen from the projectile, is given by $R(t) \simeq \sqrt{R(0)^2 + (v(0)t)^2}$. The field created by the target at the projectile location, assuming a spherical target, is determined by a potential

$$\hat{U}(t) = U_\lambda(R(t)) \sum_\mu [Y^*_{\lambda\mu}(\hat{R}(t))\hat{O}_{\lambda\mu}] \tag{2}$$

where $U_\lambda(R)$ is a radial form factor and $\hat{O}$ is an operator acting on the projectile states, which will be different for Coulomb and nuclear fields. The field will be strongest in the proximity of the distance of closest approach, so one can approximate the direction $\hat{R}(t)$ by $\hat{R}(0)$. Also, one can Taylor expand $U_\lambda(R(t))$ around $R(0)$, getting

$$\hat{U}(t) = \hat{U}(0) \exp\left(-\frac{1}{2}\frac{t^2}{\tau_c^2 \hbar^2}\right), \tag{3}$$

where the collision time τ_c is a magnitude with dimensions of inverse of the energy, proportional to the effective distance R_{eff} along the trajectory over which the interaction is significant, given by

$$\tau_c = \frac{R_{eff}}{\hbar v(0)} \quad ; \quad R_{eff} = \sqrt{R(0) U_\lambda(R(0))} \left(\frac{d}{dR} U_\lambda(R)|_{R=R(0)}\right)^{-1/2}. \tag{4}$$

1˙2. *Excitation time.* – The process of excitation of the projectile does not only depend on the time structure of the external field. It does also depend on the time scales of the relevant internal degrees of freedom. As an example, let us consider a projectile, which, at a given instant of time $t = 0$, due to the previous interaction with the target, is in a combination of the ground state and some excited state

$$|\psi(0)\rangle = a_g|\psi_g\rangle + a_e|\psi_e\rangle \tag{5}$$

As time evolves, the coefficients a_g and a_e can change due to the external field $\hat{U}(t)$. However, even if the external field does not act, the projectile state will change, due to the evolution in the internal Hamiltonian. After a time t, neglecting the effect of the external field, the state becomes

$$|\psi(t)\rangle = a_g e^{-ie_g t/\hbar}|\psi_g\rangle + a_e e^{-ie_e t/\hbar}|\psi_e\rangle. \tag{6}$$

This intrinsic time dependence can be identified more clearly considering the density matrix

$$\begin{aligned}\rho(t) = |\psi(t)\rangle\langle\psi(t)| &= |a_g|^2|\psi_g\rangle\langle\psi_g| + |a_e|^2|\psi_e\rangle\langle\psi_e| \\ &+ a_g^* a_e|\psi_g\rangle\langle\psi_e|e^{i(e_g-e_e)t/\hbar} + a_e^* a_g|\psi_e\rangle\langle\psi_g|e^{i(e_e-e_g)t/\hbar}.\end{aligned} \tag{7}$$

Here we see that the characteristic time scale for the excitation is given by the excitation time, with dimensions of inverse energy, and defined as

$$\tau_e = 1/|e_e - e_g|. \tag{8}$$

The interplay between collision time and excitation time can be understood considering the excitation probability, evaluated using first-order semiclassical theory. The probability amplitude for the excitation from the ground state (g) to an excited state (e) is

$$\begin{aligned}\langle e|A|g\rangle &= \frac{i}{\hbar}\int_{-\infty}^{+\infty} dt \langle e|\hat{U}(t)|g\rangle e^{i\frac{(e_e-e_g)t}{\hbar}} \\ &\simeq \frac{i}{\hbar}\langle e|\hat{U}(0)|g\rangle \int_{-\infty}^{+\infty} dt \quad \exp\left(-\frac{t^2}{2\hbar^2\tau_c^2} + i\frac{t}{\tau_e\hbar}\right) \\ &= \frac{i\tau_c\sqrt{2\pi}}{\hbar}\langle e|\hat{U}(0)|g\rangle \exp\left(-\frac{\tau_c^2}{2\tau_e^2}\right).\end{aligned} \tag{9}$$

When the collision time τ_c is small compared to the excitation time τ_e, the exponential factor is close to unity. Physically, it means that the collision is fast, in the sense that there is no time for the projectile state to be distorted during the collision. Thus, the reaction explores the density distribution of the projectile in its ground state. This condition, $\tau_c \ll \tau_e$, is implicit in the eikonal approximation, which is often used in the analysis of fragmentation reactions. However, when $\tau_c \gtrsim \tau_e$, a significant reduction of the excitation amplitude occurs, which is not accounted for in eikonal approaches. This is associated to the fact that the internal evolution of the projectile state makes the external field less effective to produce the excitation. Expression (9) contains, in a simplified manner, the essential physical ingredients of semiclassical approaches such as the equivalent photon method (EPM), which are extensively used to extract B(E1) distributions from the experimental Coulomb break-up cross sections. The EPM will be accurate for arbitrary τ_c and τ_e, provided that the field is relatively weak, so that $U(0)\tau_c \ll \hbar$, which is the case for small scattering angles and high scattering energies. However, for strong fields and large collision times, first order semiclassical approaches may not be accurate, because in that situation the projectile state can adapt to the strong, slowly varying external field. Moreover, if there is a significant distortion of the projectile, the trajectory will not be any longer the Coulomb trajectory used for undisturbed projectile. Hence, for $U(0)\tau_c \gtrsim \hbar$, one needs to perform a full quantum mechanical calculation, treating coupling to all orders to the relevant bound and unbound states.

2. – Application to Coulomb break-up cross sections of ^{11}Be

We will consider the description of the elastic, inelastic and break-up cross sections of ^{11}Be on ^{208}Pb at energies around the Coulomb barrier (32 and 39 MeV) [1]. This reaction is Coulomb-dominated and the scattering energy is small. The estimate of the collision time will depend on the scattering angle, and it ranges between 1 and 2 MeV^{-1}. In this collision, (at least) three internal degrees of freedom play a role. First, we have the excitation to the $1/2^-$ state, which is at 320 keV excitation energy. This degree of freedom has a excitation time of $\tau_e = 3$ MeV^{-1}. Second, we have the neutron-halo degree of freedom. The separation energy of the neutron is 0.5 MeV. The states of the continuum to which this neutron will be taken to when Coulomb dominated break-up occurs are around 0.5 MeV above the threshold. This implies a Q-value of about 1 MeV, and an excitation time of $\tau_e \simeq 1$ MeV^{-1}. Finally we should also consider the core deformation degree of freedom. ^{10}Be has a 2^+ excited state, at 3.3 MeV above the 0^+ ground state, which plays a relevant role in the ^{11}Be ground state. The strong Coulomb field of the target can couple these two states, inducing a rotation of the ^{10}Be core of ^{11}Be. The characteristic excitation time for this core excitation is $\tau_e = 0.3$ MeV^{-1}. The comparison of the excitation time scales and the collision time scales indicate that both are in similar ranges. This, together with the fact that we are in a strong-coupling case, as shown by the sizable reduction of the elastic cross sections [1], indicates that this situation requires a full quantum mechanical calculation, which takes into account to all orders the Coulomb as well as the nuclear couplings, and considers all the relevant continuum states. The fact that the excitation time of the core excitation is significantly smaller than the collision times is consistent with the fact that the probability of exciting the 2^+ state of the ^{10}Be is very small, as indicated by eq. (9). This was indeed confirmed experimentally in [1]. However, this does not mean that the core excitation degree of freedom can be neglected in the description of the reaction. Even if the probability of ending up, for large times, exciting 2^+ state of the ^{10}Be is very small, during the collision this state can be effectively coupled, and this effect is essential to get a consistent theoretical description of elastic, inelastic and break-up. These calculations can be carried out in the XCDCC formalism [2, 3].

The break-up of ^{11}Be on ^{208}Pb has also been studied at intermediate energies (69 MeV/u) at RIKEN [4]. In this case, the collision times are 4 times shorter than in the previous case ($\tau_c \simeq 0.25 - 0.5$ MeV^{-1}). Still, in this case these times are comparable to the excitation time of the halo degree of freedom ($\tau_e \simeq 1$ MeV^{-1}) and eikonal approaches may not be accurate. First order approaches, such as the EPM model, might be applied although they should be compared with full coupled channels calculation [3], to evaluate the effect of projectile distortion in the Coulomb field during the scattering.

3. – Application to nucleon removal reactions

Nucleon removal has been used as a workhorse to study single-particle properties of exotic nuclei, such as spectroscopic factors. Experiments typically consider scattering of exotic nuclei at intermediate energies, on targets such as ^{9}Be, and look for the removal cross sections, leading to the $A - 1$ residue in a bound state, that can be identified by gamma-ray coincidences. Systematic studies of these nucleon-removal reactions [5] have shown that the ratio between the observed cross sections and calculations (based on the shell model for the structure and a eikonal reaction theory) show a strong dependence on the asymmetry of the proton and neutron Fermi surfaces. So, the stripping of weakly bound nucleons are well described by the theory, while the stripping of strongly bound ones were overestimated. This begs the question on where lays the problem, either in the theoretical description of the structure of these very asymmetric nuclei, or in the reaction mechanism.

We can apply the present time-scale arguments for these systems. The collision time for a stripping reaction of an intermediate mass exotic nucleus with a ^{9}Be target at 90 MeV/u is characterized, following eq. (4) by an effective distance $R_{eff} \simeq 3$ fm, and a velocity $v = 0.4c$, leading to a collision time $\tau_c \simeq 0.04$ MeV^{-1}. This time should be compared with the excitation times for nucleon removal. If we consider the removal of a nucleon from a stable nucleus, the typical separation energy is 8 MeV, so the characteristic excitation time is $\tau_e \simeq 0.125$ MeV^{-1}. For an exotic nucleus close to the dripline, the binding energy of the weakly-bound nucleon (proton or neutron) is of the order of 1 MeV, and so the characteristic time of nucleon removal will be much longer, $\tau_e \simeq 1$ MeV^{-1}. However, if in a nucleus close to the proton drip line, we consider the removal of the neutron, or viceversa, the binding energy will be much larger, around 20 MeV, and the excitation time much shorter, $\tau_e \simeq 0.05$ MeV^{-1}. This excitation time is comparable to the collision time, and we may expect that the eikonal approach, which is reasonable for removal of weakly bound nucleons, will not be accurate for strongly bound nucleons. Note that the simple analytical estimate given by eq. (9) predicts a reduction on the removal cross section of $\exp(-\tau_c^2/\tau_e^2)$, which is about 50% for 20 MeV bound nucleons. This effect might be behind the large discrepancy observed between the calculated and measured cross sections for the removal of deeply-bound nucleons.

4. – Application to (p, pN) reactions

Recently, collisions of relativistic exotic nuclei ($E \sim 400$ MeV/u) on proton targets have been performed, in which all the outgoing particles are identified. The obtained cross sections, for the removal of weakly bound as well as strongly bound nuclei, have been compared to calculations [6, 7], and in this case there is no evidence of dependence on the asymmetries of the proton and neutron Fermi surfaces. Three calculations are used: the eikonal DWIA calculation [6], a Faddeev calculation [7] and a transfer to the continuum calculation [8, 9]. These calculations have their own peculiarities, but the essential difference for the present discussion is that, while the first one assumes a sudden approximation, the other two do not. For this case, the removal mechanism is a quasi-free nucleon-nucleon interaction, for which the effective distance is $R_{eff} \simeq 1$ fm. The velocity is $v = 0.71c$, leading to a collision time of $\tau_c \simeq 0.007$ MeV^{-1}. This collision time is small compared to all relevant excitation times, even the one for the strongly bound nucleons. So, we understand the agreement between the different theoretical approaches, as well as the fact that the ratios of the experimental cross sections to the theoretical calculations do not present a significant dependence on the difference of proton and neutron separation energies.

5. – Conclusions

The collision time and the excitation time are relevant time scales for a nuclear reaction. When the collision time is short compared to the excitation time, approximate treatments of the collision dynamics such the sudden approximation implicit in the eikonal approximation are justified. When the collision time is comparable or longer than the collision time, and the effect of the external field is small, first order semiclassical perturbative treatments, such as the equivalent photon method are applicable. When these conditions are not applied, as it is the case for ^{11}Be break-up on heavy targets around the barrier, a full quantum mechanical treatment is required. For ^{11}Be scattering, it is found that both halo and core degrees of freedom are relevant, and the consistent description of elastic, inelastic and break up cross sections require a continuum discretized coupled channels calculation which involves target excitation.

The dependence of the ratio of stripping cross section to theoretical values with the difference of binding energies of protons and neutrons can be related to the fact that the excitation time for strongly bound nucleons is comparable to the collision time. This may indicate that one needs to go beyond the eikonal approximation to describe stripping at intermediate energies.

The (p, pN) reactions at relativistic energies present a collision time that is very short compared with the excitation times, both of weakly and strongly bound nucleons. This is consistent with the fact that the ratio of experimental cross sections to different theoretical calculations do not present a dependence on the separation energy of protons and neutrons.

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