IL NUOVO CIMENTO **42 C** (2019) 108
DOI 10.1393/ncc/i2019-19108-8

COLLOQUIA: EuNPC 2018

Self-consistent single-particle approximation to nuclear state densities at high excitation energy

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received 5 February 2019

Summary. — We compute nuclear state densities for a number of magic and semi-magic nuclei in the usual saddle point approximation within the framework of the grand-canonical formalism in an energy range where residual two-body interactions and collective effects can reasonably be neglected. The single-particle states used in the calculations are generated in a relativistic self-consistent mean field at finite temperature based on widely adopted effective interactions. Observed limits and possible improvements of the adopted formalism are discussed.

1. – Introduction

An atomic nucleus is expected to have a finite single-particle spectrum, possibly changing with excitation energy, because it has a finite binding energy: consequently, the nuclear state density ω as a function of excitation energy U should reach a maximum and then decrease to zero at an excitation energy of the order of the nuclear binding energy. Therefore, the well-known Bethe formula based on an equispaced model of infinite single-particle states can only be a good approximation at excitation energies which are well below the maximum of the true state density. The pioneering work of Grimes and co-workers, using single-particle states from a shell model and a recursive approach to nuclear state densities in the frame of the micro-canonical (MC) ensemble [1], as well as a thermodynamic approach in the frame of the grand-canonical (GC) ensemble [2] showed that significant deviation from the Bethe formula could appear for light- and medium-mass nuclei already at excitation energies of the order of 100 MeV.

The use of a GC formalism, however, would not allow one to approach the expected maximum of the nuclear state density, if one adopts the definition of absolute temperature, T, consistent with the Boltzmann definition of entropy, $\frac{1}{T(U)} = \frac{1}{\omega(U)} \frac{d\omega(U)}{dU}$: in fact,

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T has an infinite discontinuity at the maximum of $\omega(U)$ and becomes negative beyond it, increasing to 0 with decreasing ω. Although the concept of absolute negative temperature was adopted in Ref. [2] in order to reproduce with a GC formalism the results obtained with the more rigorous MC approach, we shall not face this problem in this preliminary work. Our calculations have been carried out up to a maximum temperature of 10 MeV, even if such a high T cannot be reached by equilibrated systems formed in heavy ion reactions, where the onset of multi-fragmentation of a system entering the region of coexistence of the liquid-gas phases does not allow the temperature to exceed 7 MeV (see *e.g.* Ref. [3]). However, it has recently been suggested that future multi-MeV lasers emitting zeptosecond pulses might produce atomic nuclei in thermal equilibrium above the mentioned break-up temperature of 5-7 MeV [4]. Therefore, we think it worth exploring temperature regions not yet attained in present-day experiments.

2. – The model

The relativistic mean-field model at finite temperature that generates the single-particle states used in the present work has been described in detail in Ref. [5], to which we refer the interested reader. From the relativistic Lagrangian describing the interaction of nucleons with σ, ω and ρ meson fields one derives a complete one-body description in Dirac-Hartree-Bogoliubov approximation by means of the Gorkov formalism; the latter is formulated in terms of the nucleon self-energy Σ, corresponding to the average interaction of a nucleon with the surrounding matter, and of the pairing field Δ and its conjugate Δ^*, connected with the formation and destruction of pairs during the propagation. The calculation of the pairing fields is greatly simplified by the adopted zero-range approximation of the pairing interaction. An additional constant c_{pair} is introduced for a better reproduction of the experimental pairing gaps at zero temperature. In Ref. [5] several fundamental quantities, such as pairing energy, rms radii, nuclear spectra, binding energy, excitation energy, caloric curve and entropy were calculated as functions of temperature for both spherical and deformed even-even nuclei by using a subtraction method, proposed long ago by Bonche, Levit and Vautherin [6] in the frame of a non-relativistic Hartree-Fock approximation at finite temperature. In this method, the thermodynamic potential of an excited nucleus is obtained by subtracting from the potential of the nucleus in equilibrium with its vapour the potential of the vapour alone. For the sake of comparison, we have added in the present work similar calculations in the frame of the bound state method, which is expected to be valid at not too high temperatures, where the thermodynamic potential is simply computed by including the contributions of bound neutron states and bound and quasi-bound proton states, *i. e.* states below the Coulomb barrier. The latter method had already been used in Ref. [2], with single-particle states taken from a shell model. In order to compute nuclear state densities $\omega(U)$ in the saddle point approximation [7], we have worked out the variances and covariances of energy and particle numbers whose determinant, D, appears under the square root in the denominator of the pre-exponential factor in the formula $\omega(U) = \frac{e^S}{(2\pi)^{3/2} D^{1/2}}$. With a view to computing nuclear level densities depending on angular momentum in a more extended work, we have also derived the spin cutoff factor σ^2 versus temperature T, or, equivalently, excitation energy U.

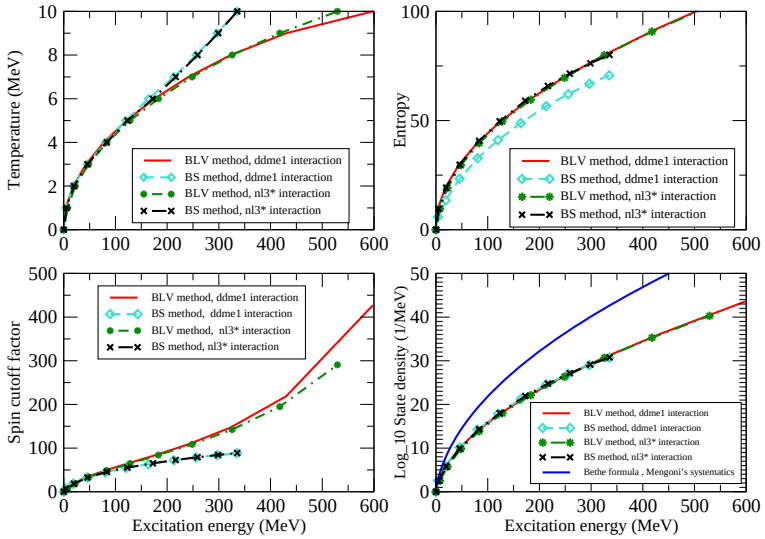

Fig. 1. – ^{60}Ni at finite temperature. Top left : T vs. U ; top right: S vs. U ; bottom left: σ^2 vs. U ; bottom right : $\log_{10}(\omega)$ vs. U.

3. – Results, comments and outlook

We have carried out calculations of binding energy, excitation energy, entropy, spin cutoff factor and state density in a range of temperatures from 0 to 10 MeV for a number of magic and semi-magic nuclei, ^{16}O, ^{40}Ca, ^{60}Ni, ^{90}Zr, ^{114}Sn and ^{208}Pb in the bound-state and subtraction methods using two very popular effective interactions, for the sake of comparison, nl3* [8] and ddme1 [9] with density-dependent nucleon-meson couplings. Fig.1 shows the U dependence of T, S, σ^2 and ω in the case of ^{60}Ni. The caloric curve, $T = T(U)$, plotted in the top left panel, shows that, at a given excitation energy, the nucleus reaches a higher temperature in the bound state method than in the subtraction method : in the former case, the maximum temperature of 10 MeV is already attained at an excitation energy of the order of 350 MeV. The curves corresponding to the bound state method thus stop much earlier than those corresponding to the subtraction method. In spite of the significant differences in temperature and entropy versus excitation energy, the state densities computed with the two methods appear almost indistinguishable on the scale of the bottom right panel. This is not true in general: significant differences appear for other nuclei under investigation, not shown here for lack of space. The state densities computed by the two methods might be fitted by Bethe formulas, albeit with level density parameters much smaller than the one obtained by the systematics of Ref. [10] based on low energy data and shown in the figure for the sake of comparison. As far as state densities are concerned, both methods used in this preliminary work have shortcomings : in the subtraction method the nucleus becomes unbound at a critical temperature between 9 and 10 MeV, but the transition has no apparent impact on the behaviour of the state density, which is non-physical, if the nucleus does not exist any more above the critical temperature; on the other hand, the bound state method neglects the contribution of unbound single-particle states of small width, the so-called Gamow states, already taken into account for a comparison with the subtraction method in

Ref. [11], albeit not in a self-consistent approach. As shown there, the contribution of Gamow states becomes more and more important with the increase of the excitation energy. We are already working on the inclusion of Gamow states in our relativistic mean-field approach by means of the complex scaling method [12], which allows one to calculate bound states and resonant states on the same footing.

In order to compare our results with low energy data, such as average neutron resonance spacings (available only for ^{208}Pb among the nuclei studied in this work), it would be necessary to go beyond the mean-field approximation, which is known to give rise to a too low effective nucleon mass and, consequently, a too low density of single-particle states at the Fermi energy in comparison with experiments. A possible solution that we are considering, at least for spherical nuclei, is the introduction of the particle-vibration coupling in the covariant formulation of Ref. [13], which has been proved efficient in bringing the calculated densities of single-particle states of magic nuclei to a satisfactory agreement with experimental data. Moreover, the relativistic random phase approximation exploited in such a formalism might give us the possibility of a consistent computation of the vibrational enhancement factor, which is not negligible in nuclear state densities at low excitation energy.

Neither of the two methods used in this preliminary work, the subtraction method and the bound state method implemented with the Gamow states, is suited to the investigation of the expected maximum of nuclear state densities, where temperature would have an infinite discontinuity, and of the region beyond it, where state densities are expected to decrease to zero, unless one accepts negative absolute temperatures, as done in Ref. [2]. In our opinion, this would be an artifice useful only to the simulation of the results of a recursive calculation of state densities in the frame of the more rigorous MC ensemble, equally employed in Ref. [2]. The non-physical assumption of negative absolute temperatures for the description of a quantum system with a bounded spectrum comes from the definition of absolute temperature consistent with the Boltzmann definition of entropy, proportional to the logarithm of the density of states of the system; it might be avoided by adopting the Gibbs entropy, proportional to the logarithm of the cumulative number of states and consistent with the general laws of thermostatistics [14].

Acknowledgments BVC acknowledges support from the INFN-Bologna, from FAPESP grant 2017/05660-0, CNPq grant 306433/2017-6 and INCT-FNA project 464898/2.

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