IL NUOVO CIMENTO **42 C** (2019) 112
DOI 10.1393/ncc/i2019-19112-0

Colloquia: EuNPC 2018

Twist-3 TMDs within a light-front model

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received 5 February 2019

Summary. — We discuss the two twist-3 Transverse Momentum Dependent parton distributions for unpolarized proton. We review their general, model independent decomposition, which follows from the QCD equations of motion. Then, we present results for the twist-3 distributions, using a light-front model for the Fock-states of three quark and three-quark plus one gluon.

1. – Introduction

Twist-3 distributions give access information on the quark-gluon correlations inside the proton, going beyond the probabilistic interpretation of the leading-twist distributions [1]. All the higher-twist Transverse Momentum Dependent parton distributions (TMDs) can be, in general, decomposed into different terms: lower-twist contributions, pure higher-twist contributions and singular terms. Most of the model calculations view the proton as a pure three-quark state, i.e. they do not have any intrinsic gluon contribution, which is necessary to calculate the pure higher-twist contributions in terms of a quark-gluon-quark correlator. Nevertheless, such quark models can have non-vanishing pure higher-twist terms generated from the quark interactions [2-4].

A step forward w.r.t. quark-model calculations has been recently discussed in Refs. [5, 6], by taking into account explicitly the contribution from gluon degrees of freedom.

In this work, after reviewing the model independent decomposition for the unpolarized twist-3 TMDs, we summarize the light-front (LF) approach of Ref. [6] to model the $3q+g$ Fock-state component of the proton state and show the model results for the twist-3 distribution $e(x)$.

2. – Decomposition

Quark's TMDs are defined from the twist expansion of the following quark-quark correlator

$$(1)\qquad \Phi^q_{ij}(x,\boldsymbol{k}_\perp,S) = \int \frac{dz^- d\boldsymbol{z}_\perp}{(2\pi)^3} e^{iz^-k^+ - i\boldsymbol{z}_\perp\cdot\boldsymbol{k}_\perp} \langle P,S|\overline{\psi}_j(0)\mathcal{W}(0;z)\psi_i(z)|P,S\rangle\,|_{z^+=0}.$$

Here and in the following, we use LF coordinates, with $v^{\pm} = \frac{1}{\sqrt{2}}\left(v^0 \pm v^3\right)$ and $\boldsymbol{v}_\perp = (v^1, v^2)$ for a generic four-vector v. In Eq. (1), $\mathcal{W}(0;z)$ is an appropriate Wilson line that connects the two quark operators, ensuring the gauge-invariance of the correlator. Up to the twist-3 level, there are three T-even unpolarized TMDs: the twist-2 and chiral-even TMD f_1, and the twist-3 TMDs e and $f^\perp$, that are, respectively, chiral-odd and chiral-even. They are defined as

$$f_1(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\Phi^q(x, \boldsymbol{k}_\perp)\gamma^+)\right], \tag{2}$$

$$\frac{M}{P^+}e(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\Phi^q(x, \boldsymbol{k}_\perp)\mathbb{I})\right], \tag{3}$$

$$\frac{k_\perp^j}{P^+}f^\perp(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\Phi^q(x, \boldsymbol{k}_\perp)\gamma^j)\right]. \tag{4}$$

If one introduces the "good" and "bad" components of the quark field as $\mathcal{P}^\pm\psi = 1/2\gamma^\mp\gamma^\pm\psi \equiv \psi_\pm$, we find that the twist-2 distribution involves only good components of the quark fields, whereas the twist-3 distributions are matrix elements of the product of a good and a bad component. It is known that the bad component is not an independent dynamical field, since it is constrained by the equation of motion at fixed light-cone time:

$$iD^+\psi_- = -\frac{\gamma^+}{2}\left(i\not{D}_\perp - m\right)\psi_+ \tag{5}$$

with D^+ being a spatial derivative. If one assumes $k^+ > 0$, Eq. (5) can be inverted in a straightforward way. Complications arise when k^+ is allowed to vanish. In particular, choosing as subtraction point 0^-, the solution of (5) is:

$$\begin{aligned}\mathcal{W}(0;z)\,\psi_-(z))|_{z^+=0} &= \mathcal{W}(0^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{z}_\perp)\psi_-(0^+, 0^-, \boldsymbol{z}_\perp)\\ &+ i\int_{0^-}^{z^-} d\zeta^-\mathcal{W}(0^-, \boldsymbol{0}_\perp; \zeta^-, \boldsymbol{z}_\perp)\frac{\gamma^+}{2}\left(i\not{D}_\perp - m\right)\psi_+(0^+, \zeta^-, \boldsymbol{z}_\perp).\end{aligned}$$

Inserting this expression in Eqs. (3) and (4), and working in the light-cone gauge $A^+ = 0$ with appropriate boundary conditions for $A_\perp(\infty^-)$, one obtains the following decompositions for the twist-3 TMDs:

$$\begin{aligned}e(x, \boldsymbol{k}_\perp) &= \frac{\delta(x)}{2M}\int\frac{d\boldsymbol{\xi}_\perp}{(2\pi)^2}e^{-i\boldsymbol{\xi}_\perp\cdot\boldsymbol{k}_\perp}\langle P|\overline{\psi}(0)\psi(\xi)|P\rangle + \tilde{e}(x, \boldsymbol{k}_\perp) + \frac{m}{xM}f_1(x, \boldsymbol{k}_\perp)\\ &+ \delta(x)\int_{-1}^{1}dy\left(\tilde{e}(y, \boldsymbol{k}_\perp) + \frac{m}{yM}f_1(y, \boldsymbol{k}_\perp)\right),\end{aligned} \tag{6}$$

$$\begin{aligned}k_\perp^j f^\perp(x, \boldsymbol{k}_\perp) &= \frac{\delta(x)}{2M}\int\frac{d\boldsymbol{\xi}_\perp}{(2\pi)^2}e^{-i\boldsymbol{\xi}_\perp\cdot\boldsymbol{k}_\perp}\langle P|\overline{\psi}(0)\gamma_\perp^j\psi(\xi)|P\rangle\\ &+ k_\perp^j\left(\tilde{f}^\perp(x, \boldsymbol{k}_\perp) + \frac{1}{x}f_1(x, \boldsymbol{k}_\perp)\right) + k_\perp^j\delta(x)\int_{-1}^{1}dy\left(\tilde{f}^\perp(y, \boldsymbol{k}_\perp) + \frac{1}{y}f_1(y, \boldsymbol{k}_\perp)\right),\end{aligned} \tag{7}$$

where the "tilde" terms correspond to the pure twist-3 contributions. They are defined as the trace of the quark-gluon-quark correlator, i.e.

$$\tilde{e}(x,\boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\Phi_{A,i}(x,\boldsymbol{k}_\perp)\sigma^{i+}\right], \tag{8}$$

$$k_\perp^j \tilde{f}^\perp(x,\boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\Phi_{A,i}(x,\boldsymbol{k}_\perp)\left(g_\perp^{ji} + i\varepsilon_\perp^{ji}\gamma^5\right)\right], \tag{9}$$

where

$$\Phi_{A,i}(x,\boldsymbol{k}_\perp) = g_s \int \frac{dz^- d\boldsymbol{z}_\perp}{(2\pi)^3} e^{iz^-k^+ - i\boldsymbol{z}_\perp\cdot\boldsymbol{k}_\perp} \langle P,S|\overline{\psi}(0)A_{\perp,i}(z)\psi(z)|P,S\rangle_{z^+=0}\,. \tag{10}$$

The singular terms in Eqs. (6) and (7) have two origins: they are related either to a matrix element that is local in the minus direction or to integrals of the twist-2 and pure twist-3 distributions. These last contributions are usually "hidden" inside the definition of the pure twist-3 terms and become explicit using the principal-value prescription in the integral that defines the quark-quark correlator [6]. In this form, it is straightforward to infer the well-known relations for the Mellin moments of the twist-3 distributions [7,9].

3. – Model

In order to calculate the pure twist-3 terms in the framework of a model based on the light-front wave functions (LFWFs), we need to include at least a Fock state with one intrinsic gluon.

$$|P,\Lambda\rangle = |P,\Lambda\rangle_{3q} + |P,\Lambda\rangle_{3q+g}\,, \tag{11}$$

with

$$|P,\Lambda\rangle_{3q} = \sum_\beta \int [Dx]_3 \Psi^\Lambda_{3q}(\beta,r)\,\varepsilon^{c_1c_2c_3}\prod_{i=1}^3 |\lambda_i,q_i,c_i,\tilde{k}_i\rangle\,, \tag{12}$$

$$|P,\Lambda\rangle_{3q+g} = \sum_\beta \int [Dx]_4 \Psi^\Lambda_{3q+g}(\beta,r)\,\varepsilon^{dc_2c_3}T^a_{d,c_1}\left(\prod_{i=1}^3 |\lambda_i,q_i,c_i,\tilde{k}_i\rangle\right)|\lambda_4,g,a,k_i\rangle\,. \tag{13}$$

In Eqs. (12) and (13), Ψ^Λ_{3q} and Ψ^Λ_{3q+g} are, respectively, the LFWFs for the $N=3$ and $N=4$ parton Fock state $\prod_{i=1}^N |\lambda_i,q_i,c_i,k_i\rangle$, with λ_i the parton LF helicity, $q_i = u,d$ and g the quark and gluon flavor index, c_i the parton color index, and k_i the parton momentum. For the argument of the LFWFs, we used a collective notation, with $\beta = (\{\lambda_i\};\{q_i\})$ and $r = \{\tilde{k}_i\}$, where $\tilde{k}_i = (k_i^+ = x_iP^+, \boldsymbol{k}_{\perp,i})$. Furthermore, the sum over the color indexes is understood, and, then, using also the sum over the flavor index, the color matrix can be saturated with the color index of the first quark only.

The LFWFs have a physical interpretation as probability amplitudes of the corresponding Fock state in the proton. The functional form for the LFWFs can be fixed by using the relations between the LFWFs and the distribution amplitudes (DAs) of the proton. The DAs can be parametrized using a polynomial expansion based on the conformal expansion, as outlined in Ref. [5]. In our work, we modified this parametrization by including a non-vanishing mass for the quarks and the gluons. We fixed the

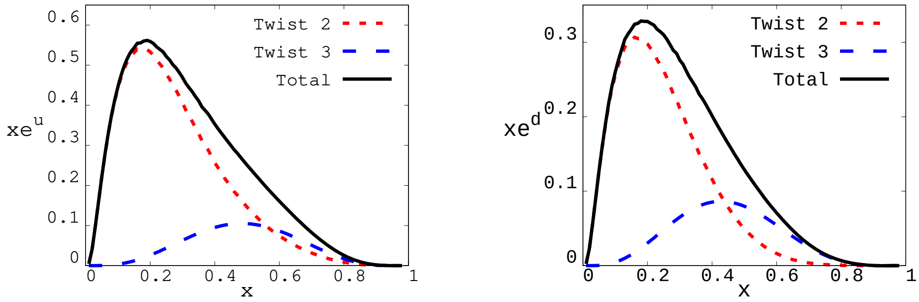

Fig. 1. – Results for the twist-3 distribution $xe(x)$ for the up (left panel) and down quark (right panel). The blue long-dashed curves are the pure twist-3 terms, the red short-dashed curves correspond to the mass term in Eq.(6), and the solid black curves show the total results.

parameters of the LFWFs by fitting the phenomenological parametrization of the twist-2 parton distribution $f_1(x)$ from Ref. [8], which provides results at the same scale of our model, i.e. $Q^2 = 1$ GeV^2. The $\tilde{e}$ distribution can be expressed as the convolution of the LFWFs for the $3q$ state and the $3q + g$ state with the same orbital angular momentum, instead $\tilde{f}^{\perp}$ is a convolution between LFWFs for the $3q$ state and the $3q + g$ state with different orbital angular momentum. Since in our model we consider only the LFWFs with zero orbital angular momentum, we can provide predictions only for $\tilde{e}$. For the explicit expression of the LFWF overlap representation we refer to [6]. The model results for $e^q(x) = \int d\boldsymbol{k}_\perp e^q(x, \boldsymbol{k}_\perp)$ are shown in Fig. 1. One notices the non-negligible size of the pure twist-3 contribution w.r.t. the mass term in Eq.(6), which is proportional to the twist-2 distribution f_1. The two contributions have a quite different x-dependence. We also note that the pure twist-3 contributions for the up and down quarks have very similar size, whereas the mass-term contribution for the up quark is approximately twice as large as the mass-term contribution for the down quark.

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This work is partially supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 647981, 3DSPIN).

REFERENCES

[1] Goeke, K., Metz, A. and Schlegel, M., *Phys. Lett. B*, **618** (2005) 90-96
[2] Avakian, H., Efremov, A.V., Schweitzer, P. and Yuan, F., *Phys. Rev. D*, **81** (2010) 074035
[3] Lorcé, C., Pasquini, B. and Schweitzer, P., *J. High Energy Phys.*, **01** (2015) 103
[4] Lorcé, C., Pasquini, B., and Schweitzer, P., , *Eur. Phys. J. C*, **76** (2016) 415
[5] Braun, V. M., Lautenschlager, T., Manashov, A. N. and Pirnay, B., *Phys. Rev. D*, **83** (2011) 094023
[6] Pasquini, B. and Rodini, S., arXiv:1806.10932
[7] Efremov, A. V. and Schweitzer, P., *J. High Energy Phys.*, **08** (2003) 006
[8] Harland-Lang, L. A., Martin, A. D., Motylinski, P. and Thorne, R. S., *Eur. Phys. J. C*, **75** (2015) 204
[9] M. Burkardt and Y. Koike., *Nucl. Phys. B*, **632** (2002) 311