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Impact of the PDFs on the Z and W lineshapes at LHC

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Summary. — The parton distribution functions (PDFs) of the proton play a role in determining the lineshape of W and Z bosons produced at the LHC. In particular, the mode of the distribution of the gauge boson virtuality gets shifted with respect to the boson mass due to the dependence of the partonic luminosity on the boson virtuality itself. This shift contributes to the systematic uncertainty on the direct measurement of the boson mass. A detailed study of the shift and of its systematic uncertainty due to the limited knowledge of the PDFs is obtained using a tree-level model of W and Z boson production in proton-proton collisions at $\sqrt{s} = 13\,\text{TeV}$. For the special case of W boson production, a Monte Carlo simulation is further used to validate the tree-level model and study the dependence of the shift on the transverse momentum of the W boson. The tree-level calculation is found to provide already a good description of the shift. The systematic uncertainty due to the PDFs is estimated to be below one MeV in the phase-space relevant for a future high-precision measurement of the W and Z boson masses at the LHC.

1. – Introduction

The Drell-Yan production of massive lepton-pairs in hadron collisions [1] has been extensively studied in the literature both as a probe of the proton structure [2] and as a tool for precise electroweak measurements. In recent years, large efforts have been devoted on understanding the role of the collinear parton density functions (PDFs) in the determination of differential distributions of leptonic variables sensitive to electroweak parameters, such as asymmetries in Z/γ production [3-5] and transverse observables in W events [6-8].

The unprecedented amount of W and Z bosons produced at the CERN Large Hadron Collider (LHC) offers new opportunities on the critical path towarards precision, but it also forces to consider sources of systematic uncertainty which may have been legitimately neglected so far, for example those related to the modeling of the virtuality of the gauge bosons [9] which would demand a control of the dilepton mass lineshape at the sub- MeV level.

At the lowest order in perturbation theory, the distribution of the virtuality Q of a gauge boson V produced at a given value of rapidity y, originates from the convolution of a relativistic Breit-Wigner with the the partonic luminosity function [10]. The latter is a function of the dimensionless parameter $\tau = Q^2/s$, where s is the square of the proton-proton center-of-mass energy. The non-trivial dependence of the partonic luminosity on τ implies a distortion of the lineshape compared to a pure Breit-Wigner. Given the narrowness of the electroweak gauge bosons width Γ_V, this effect can be treated, in first approximation, as a shift Δ_V of the mode of the distribution compared to M_V. The limited knowledge of the PDFs introduces an uncertainty on Δ_V, which contributes directly to the model uncertainty in the extraction of M_V from the dilepton mass distribution. The purpose of this work is to assess the size of this shift and of its PDF uncertainty in sight of a future high-precision measurement at the LHC.

2. – Tree level study

We first consider a simplified model of Drell-Yan production where just one Feynman diagram per quark-antiquark pair is considered, where the boson is produced in the s-channel. Besides accounting already for the bulk of the total cross section (about 80% for a 20 GeV threshold on the transverse momentum of the extra parton at $\sqrt{s} = 13\,\text{TeV}$), these diagrams are also expected to be the most sensitive to the PDF-dependent shift (see ref. [11] for details). Within the tree-level approximation, the double-differential cross section for $pp \to V(\to \ell\ell') + X$, as a function of the quark momentum fractions $x_{1,2}$, is given by

$$\text{(1)} \qquad \frac{\mathrm{d}^2\sigma_V}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{1}{N_\mathrm{C}} \sum_{ij} \left[f_i(x_1)f_j(x_2) + f_i(x_2)f_j(x_1)\right] \frac{16\pi\Gamma_V^2 \mathrm{BR}_{V\to q_i\bar{q}_j}\mathrm{BR}_{V\to\ell\ell'}}{(x_1x_2s - M_V^2)^2 + M_V^2\Gamma_V^2},$$

where N_C is the number of QCD colours, M_V and Γ_V are the mass and width of the resonance, $\mathrm{BR}_{V\to ab}$ are the relevant branching fractions, and the sum at the right-hand side runs over the different combinations of quark flavours contributing to the process under study. For simplicity, the scale-dependence is omitted from the quark PDF in the notation $f_i(x)$.

The lineshape of the boson conditional in y, $\frac{\mathrm{d}\sigma_V}{\mathrm{d}Q}(Q|y)$, can be derived from eq. (1), where $y = \frac{1}{2}\ln\frac{x_1}{x_2}$, $Q^2 = x_1x_2s$. The mode Q_0 of the lineshape can be calculated with a Taylor expansion around $Q = M_V$,

$$\text{(2)} \qquad Q_0 \approx M_V + \frac{\Gamma_V^2}{8M_V}(H_V + 1) \equiv M_V + \Delta_V,$$

where H, the first-order coefficient of the expansion averaged over the flavours, encodes the entire contribution of the PDF to the mode of the lineshape. The complete calculation is omitted here for brevity but can be found in ref. [11]. The quantity Δ_V represents the displacement of the mode Q_0 from M_V and part of it is simply due to the Jacobian factor from the transformation $(x_1, x_2) \to (y, Q^2)$, and does not depend on the PDFs. H_V is a function of rapidity y and of the mass M_V, albeit the dependence on the latter is

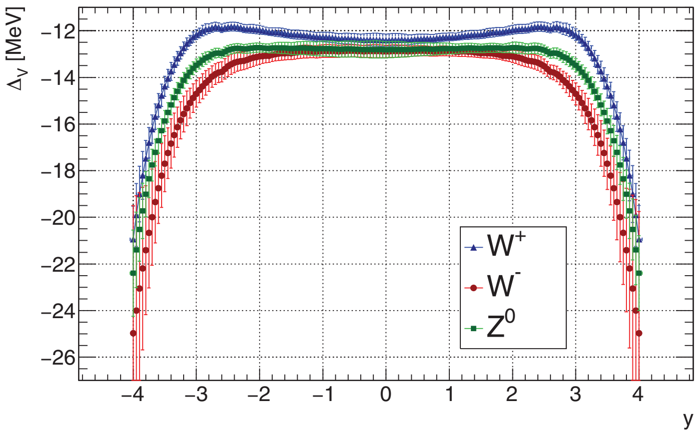


Fig. 1. – The shift Δ_V and the coefficient H_V as a function of y averaged over the various quark flavours that enter the tree-level production of $W^\pm$ and Z^0 in pp collisions at $\sqrt{s} = 13\,\text{TeV}$. On the right the equivalent scale for the correspondent H_V.

negligible in the range of experimental uncertainty on M_W ($\sim$12 MeV) and M_Z ($\sim$2 MeV) compared to the PDF uncertainties.

The shift Δ_V determined from eq. (2) is plotted in fig. 1 as a function of the boson rapidity y for $W^\pm$ and Z production. The error bars correspond to the RMS of the distribution obtained by sampling the first 100 replicas of the chosen PDF set (`NNPDF30_nlo_nf_5_pdfas` from LHAPDF libraries [12]). The relative PDF uncertainty on Δ_V is found to be in the 5% ballpark, ranging from 0.3 MeV at $|y| \sim 0$ to 1 MeV at $|y| \sim 3.5$.

3. – Monte Carlo simulation study

The tree-level calculation of sect. **2** has been validated by using a MC simulation of $pp \to V + X$ production. Besides corroborating the tree level model, the MC analysis will also allow us to extend the study to the full phase-space, which includes the contribution of other diagrams. Given the similarity between neutral- and charged-current Drell-Yan production, and the observation that W boson production, splitted by charge, can serve as a good proxy also for the Z boson (see fig. 1), the analysis has been restricted hereafter to the special case $V = W^\pm$. About 8×10^7 events in the final state $W^\pm \to \mu^\pm \nu_\mu$ have been generated using the `MG5_aMC@NLO` [13] program interfaced with `Pythia8` [14]. The dilepton mass is reconstructed using the muon momentum before QED final state radiation. The MC simulation is NLO accurate for observables inclusive in additional QCD radiation and it assumes $M_W^{\text{MC}} = 80.419$ GeV and $\Gamma_W^{\text{MC}} = 2.047$ GeV. It is expected to reproduce the tree-level prediction in the limit $q_{\text{T}} \to 0$, where q_{T} is the transverse momentum of the W boson. Indeed, in this regime the relative contribution of the tree-level $2 \to 1$ diagrams, which provide the unphysical spectrum $\mathrm{d}\sigma/q_{\text{T}} \sim \delta(q_{\text{T}})$, is enhanced compared to higher-order $2 \to 2$ diagrams. In contrast, a reduction of the shift in the large q_{T} region is expected, where gluon-initiated diagrams dominate, thus reducing the sensitivity of the partonic luminosity on τ.

3˙1. *Fit to the MC sample.* – At variance with the analytical study of sect. **2**, the shift in the MC sample has to be extracted from a statistical analysis of the dilepton mass distribution $\mathrm{d}\sigma_W^{\mathrm{MC}}/\mathrm{d}Q$. A crucial part of this task is to choose the correct functional form for $\mathrm{d}\sigma_W^{\mathrm{MC}}/\mathrm{d}Q$, capable of modelling the lineshape without introducing a bias in the estimator of Δ_V. Motivated by the tree-level study, an *ansatz* function of the same form of the second-order expansion of $\frac{\mathrm{d}\sigma_W}{\mathrm{d}Q}(Q\,|\,y)$ has been chosen:

$$\frac{\mathrm{d}\sigma_W^{\mathrm{MC}}}{\mathrm{d}Q}(Q\,|\,y) = A\frac{Q^\alpha}{(Q^2-M^2)^2+M^2\Gamma^2}\left[1+H\left(\frac{Q}{M}-1\right)+K\left(\frac{Q}{M}-1\right)^2\right]. \quad (3)$$

The choice $\alpha = 1$ defines the baseline function, which will be referred to as the *modified Breit-Wigner*. In fact, this functional form will be explicitely validated by checking that the estimator of M_W and Γ_W is consistent with the input values of the MC simulation M_W^{MC} and Γ_W^{MC}. As a further validation of this choice, two alternative instances of the parametric family of functions in eq. (3) have been considered: the *Breit-Wigner* ($\alpha = H = K = 0$, as a benchmark) and *Breit-Wigner with Jacobian* ($\alpha = 1$ and $H = K = 0$, correct in the absence of the PDF distortion). See ref. [11] for more details about these functions.

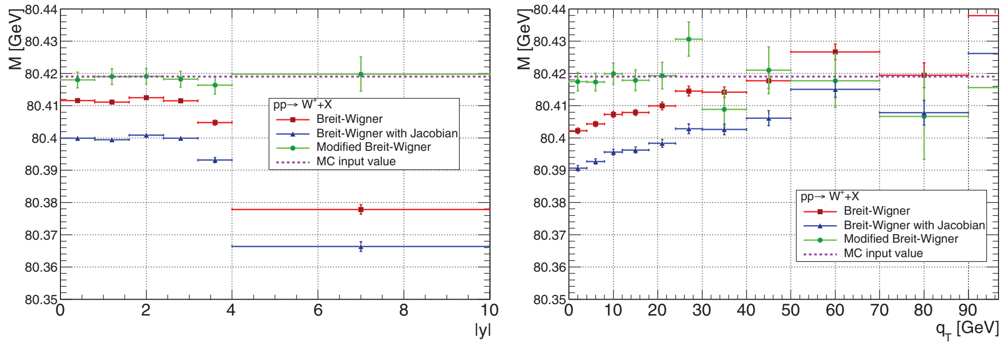
Three statistical analyses of the simulated events have been performed: an inclusive fit in the phase-space of the W boson (to benchmark the different fit functions with the largest possible statistical precision), a differential analysis in the W boson rapidity y (to reproduce qualitatively the y-dependence from the tree-level model, diluted by non-vanishing q_T events) and a differential analysis in q_{T} and inclusive in y (to validate quantitatively the model of sect. **2**).

In the inclusive analysis both the quality of the fit (in term of χ^2) and the parameter estimation (M_W, Γ_W) improves dramatically when using the modified Breit-Wigner, and the latter is the only one capable to reproduce the injected M_W value within 1σ.

The best-fit values of M_W from the differential analysis in the W boson rapidity are reported in fig. 2(a) for the W^+ sample. The W^- sample figure is omitted for brevity, but it shows very similar results. The Breit-Wigner fit underestimates M_W all over the rapidity spectrum, as also observed in the inclusive analysis. The same applies to the Breit-Wigner with Jacobian function. For the latter, the discrepancy is even more pronounced. Indeed, the Jacobian factor contributes via a positive bias to the peak position. By neglecting the PDF term, which pulls in the opposite direction, the estimator of M_W is thus shifted to even lower values compared to M_W. The modified Breit-Wigner function correctly reproduces the input value M_W^{MC} in all bins of $|y|$, including the high $|y|$ regimes, where the alternative functions perform rather poorly.

Finally, the results of the analysis differential in the W boson transverse momentum are shown in fig. 2(b). The modified Breit-Wigner is seen to correctly reproduce the input mass value for all bins of q_{T}, whereas the two alternative functions disagree, especially at low transverse momenta. Above $q_{\mathrm{T}} = 40\,\mathrm{GeV}$ the statistical error is too large to discriminate among the models.

3˙2. *Extraction of* Δ_W. – Since the fit reproduces well the true values for M_W and Γ_W, we fix the values of M and Γ to the MC input values in eq. (2) and repeat the fit with A, H, K as the only free parameters. The fitted values of Δ_W for the differential analyses are shown in fig. 3 in bins of $|y|$ and q_{T}, separately for W^+ and W^-. The variation of Δ_W with the boson rapidity is shown in fig. 3(a). It agrees well with the tree-level expectation



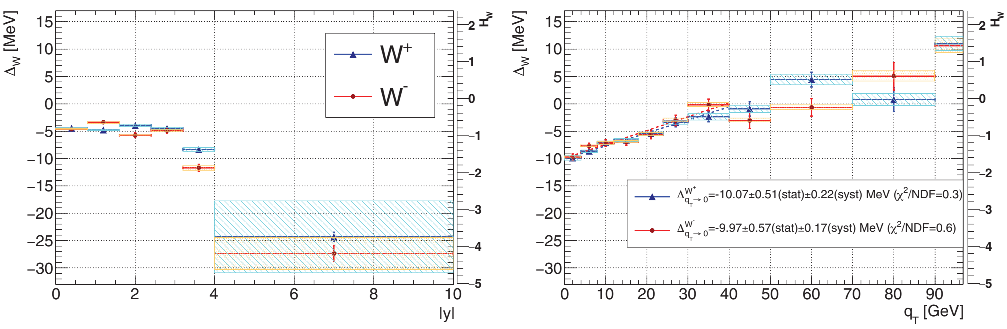
(a) (b)

Fig. 2. – The best-fit value of M_W using the Breit-Wigner (red), Breit-Wigner with Jacobian (blue), and modified Breit Wigner (green), in bins of $|y|$ (left) and q_T (right), for the simulated W^+ sample. The dotted line corresponds to the input value of M_W^{MC}. A similar result is obtained for the W^- sample.

of a flat shift in the central rapidity region followed by a rapid decrease at larger rapidity values. However, the shift in the central region is found to be smaller by a factor of about two, and a similar result is obtained for the inclusive results. Such difference has been interpreted as the result of the dilution from higher-order diagrams. Indeed, in the limit $q_\mathrm{T} \to 0$, the measured shift gets closer to the tree-level result as shown by fig. 3(b), while it vanishes for q_T in excess of about 40 GeV. A simple linear extrapolation to $q_\mathrm{T} \to 0$ yields limiting values of

$$(4) \qquad (q_T \to 0 \text{ extrapolation}) \qquad \begin{aligned} \Delta_{W^+} &= -10.1 \pm 0.5\,(\text{stat.}) \pm 0.2\,(\text{PDF})\ \text{MeV}, \\ \Delta_{W^-} &= -10.0 \pm 0.6\,(\text{stat.}) \pm 0.2\,(\text{PDF})\ \text{MeV}. \end{aligned}$$

The first uncertainty is statistical-only while the second is the estimation of the systematic uncertainty from the PDFs. The PDF uncertainty is estimated from the RMS of the



(a) (b)

Fig. 3. – The shift $\Delta_{W^\pm}$ in bins of the W boson rapidity y (left) and transverse momentum q_T (right). For the latter, a linear fit in the range $[0, 40]$ MeV is performed to extrapolate the result to $q_\mathrm{T} \to 0$. The shaded boxes correspond to the PDF systematic uncertainty, as described in the text. On the right side of each plot, the equivalent scale for the H_W parameter is reported.

shifts determined using the first 100 replicas, as described in sect. **2**. In these fits the parameters M and Γ have been left free, since the uncertainty on the PDFs would be otherwise over-constrained by the imposed knowledge on the mass and the width of the resonance.

Although reasonably close to the tree-level expectation, this result still disagrees with it by roughly 30%. This residual difference is interpreted as a pure next-to-leading-order correction to the leading-order prediction, stemming from collinear gluon emission and from gluon-initiated diagrams which contribute to the small-q_{T} regime. The relative PDF uncertainty is found to agree reasonably well with the expectation from the tree-level model averaged over the W boson rapidity. As a cross-check of this result, the fit has been repeated afterwards, varying the fit range symmetrically by $\pm 10\%$. The results for Δ_W are stable, with a maximum discrepancy of 5%, which is within the uncertainty of the parameter. The fit has been also repeated after changing the renormalization and factorization scales in the matrix elements of the MC simulation by factors of 0.5 and 2, respectively. The results are again found to be stable within the PDF uncertainty.

4. – Conclusions

The impact of the PDFs on the lineshape of W and Z bosons at the LHC has been investigated. Given the narrow width of the electroweak gauge bosons, the PDF impact can be treated, to a first approximation, as a displacement Δ_V of the mode of the dilepton mass spectrum from the boson mass M_V. The origin of such shift has been traced back to the dependence of the partonic luminosity on the virtuality Q of the gauge boson. This effect is automatically accounted for by Monte Carlo simulation of $pp \to V + X$ events. However, an uncertainty on the proton structure contributes directly to the systematic uncertainty in the extraction of M_V from the kinematics of the dilepton final state. This effect has been first studied analytically using a tree-level model of Drell-Yan production and then validated by a statistical analysis of a MC simulated sample. The tree-level calculation agrees reasonably well with the MC study in the phase-space where the two are expected to be comparable. The results of this study prove that the PDF uncertainty on Δ_V is below one MeV all over the phase space relevant for a potential mass measurement at the LHC.

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