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Geometrical models with Lorentz invariance violation and neutrino oscillations

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Summary. — The search for Lorentz Invariance Violation (LIV) is strictly connected with the high-energy structure of space-time. In the model we developed, LIV is introduced from modified dispersion relations (MDR) and a metric structure is preserved in Finsler geometry. The MDR corrections are represented by homogenous functions, introducing LIV sources of kinematical origin, without any new interaction. The Standard Model symmetries and the space isotropy are preserved. Every particle experiences its own limit velocity, as a function of its momentum, and the modifications of the neutrino propagation equations induce additional corrections to the standard flavor oscillation pattern. Their impact on the oscillation probabilities is analyzed for many cases of phenomenological relevance.

1. – Lorentz Invariance Violation and "geometrical" models

In the literature one can find many attempts to find signals of potential violation of Lorentz invariance, which could be due to quantum effects, that might manifest themselves at very high energies and have also been advocated in the recent past as possible explanations of apparent anomalies in the very high energy cosmic rays data. The Lorentz Invariance Violation (LIV) hypothesis was introduced also in neutrino physics in 1999 [1]. In our HMSR (Homogeneously Modified Special Relativity) model, we started [2, 3] by modified dispersion relations (MDR), in which LIV corrections are homogeneous functions (depending on $\frac{|\vec{p}|}{E}$, if we assume space isotropy): $E_i^2 - (1 - f(\frac{|\vec{p_i}|}{E_i}))|\vec{p}_i|^2 = m_i^2$. The index i refers to the particle species and $\lim_{|\vec{p}|\to\infty} f(p) = \epsilon \ll 1$. A metric structure is

(*) Contribution presented by V. Antonelli.

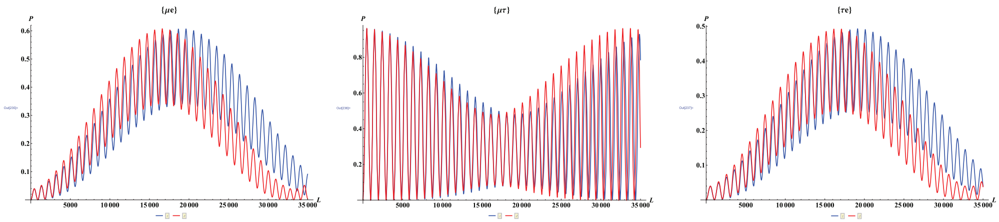

Fig. 1. – Comparison between the probabilities (in the order from left to right: $\nu_e \to \nu_\mu$, $\nu_\mu \to \nu_\tau$, $\nu_e \to \nu_\tau$) as functions of the baseline, for $E_\nu = 1\,\text{GeV}$ in the "standard" theory (red curves) and the corresponding ones in presence of LIV (blue), for LIV coefficients $\Delta f_{32} = \Delta f_{21} = 1 \times 10^{-23}$.

preserved in curved Finsler space-time and the tensor metric $\tilde{g}(p)^{\mu,\nu}$ is given by

$$\tilde{g}(p)^{\mu,\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -(1-f(p)) & 0 & 0 \\ 0 & 0 & -(1-f(p)) & 0 \\ 0 & 0 & 0 & -(1-f(p)) \end{pmatrix}. \tag{1}$$

A particle with velocity $\vec{v}_i$ propagates in its own Finsler space-time, whose curvature depends on p_i and, in order to study the interaction with other particles, one has to consider the projections on a common flat Minkowski space-time.

2. – Standard Model extension in HMSR and Neutrino Oscillation

As shown in [3], the MDR isometries define "modified" Lorentz transformations, under which the new Mandelstam variables are invariant. LIV corrections in the "modified" Dirac matrices compensate with the analogous ones of the spinorial wave functions. The $SU(3) \times SU(2) \times U(1)$ and CPT symmetries are preserved and we built a minimal extension of elementary particle Standard Model in presence of isotropic LIV. In [2] we studied the LIV impact on neutrino flavor oscillations, both with a perturbative Hamiltonian approach and with the direct calculation of $P_{\alpha,\beta}$, the modified oscillation probabilities from ν_α to ν_β. In the ultrarelativistic approximation additional terms appear,

$$\begin{aligned} P_{\alpha,\beta} = \delta_{\alpha,\beta} - 4 \sum_{i>j} \mathrm{Re}(U_{\alpha,i} U^*_{\beta,i} U^*_{\alpha,j} U_{\beta,j} \sin^2(\Delta\phi_{ij})) \\ +2\,\mathrm{Im}(U_{\alpha,i} U^*_{\beta,i} U^*_{\alpha,j} U_{\beta,j} \sin^2(\Delta\phi_{ij})), \end{aligned}$$

with $\Delta\phi_{ij} = 2\frac{\Delta m^2_{ij}\,L}{2E} - \frac{f_i - f_j}{2} LE$ and f_i and f_j are the LIV coefficients for i, j neutrino mass eigenstates. The LIV-induced corrections to the oscillation probabilities are present only if the f_i coefficients are different for different mass eigenstates ($\Delta f_{ij} = f_i - f_j \neq 0$). These modifications of the standard oscillation pattern are small, but, anyhow, significant, because they are proportional to $L \times E$, instead of L/E. We computed the 3 active neutrino flavor transition probabilities. In fig. 1, we report the comparison between the probabilities (in the order from left to right: $\nu_e \to \nu_\mu$, $\nu_\mu \to \nu_\tau$, $\nu_e \to \nu_\tau$) as functions of the baseline, for $E_\nu = 1\,\text{GeV}$, in the "standard" theory (red curves) and the corresponding ones in the presence of LIV (blue), for LIV coefficients $\Delta f_{32} = \Delta f_{21} = 1 \times 10^{-23}$.

Even restricting to much lower Δf_{ij} (around 10^{-27}), corresponding to the limit imposed by SuperKamiokande data for more general models, the transition probabilities are significantly modified for energies around 100 GeV or higher. A complete phenomenological analysis is in progress for the different relevant experimental situations: the detection at neutrino telescopes (ANTARES, KM3NeT, IceCube) of high-energy neutrinos (between TeV and PeV), the Auger study of cosmic ν with $E >$ EeV and the potential analysis of high-energy atmospheric neutrinos at the JUNO experiment.

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