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The universe: A book written in the mathematical —and the programming— language

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Summary. — This paper concerns the pronounced shift that occurred, in the twentieth century, from the formulation of scientific laws with limited computational capabilities —a distinctive trend of the early and late modern periods— to the formulation *and* the massive computation that we can now perform with the aid of computer technologies. This change altered our relationship with science, and thus our ambitions with nature. What seems clear is that the universe —the "grand book" that "stands continually open to our gaze"— is not solely written with "triangles, circles, and other geometric figures", as Galileo Galilei put it, but also with languages whose characters consist of data, processes, structures and other constructs of computer programs without which we would now wander about "in a dark labyrinth".

1. – Introduction

Computer simulations in natural science are routinely used to study the evolution, through time, of mathematical models describing complex dynamical systems. Studying the time evolution of a mathematical model means "to see it in action", *i.e.*, to continuously solve its underlying equations as conditions change over time. Nowadays, computer simulations take place through high-performance computers with which it is possible to compute sets of equations that would be practically unapproachable with traditional analytic methods. Examples of such systems are as varied as finite-temperature many-body fermion systems, the dynamics of biophysical molecules, or distortions in large, welded structures.

Computer simulations in natural science can be divided into two major categories: fundamental and *ad hoc* computer simulations([1]). Fundamental computer simulations

([1]) Computer simulations are routinely used also in formal and social sciences. "Fundamental computer simulations" and "*ad hoc* computer simulations" are not officially recognized terms. See also [1]. The reader may find it interesting to consider [2] and [3].

start with the fundamental laws of science and translate them into codes to be executed by computers, thus producing an imitation, based on the solution of those laws, of the temporal evolution of the systems under investigation, as well as their statistical properties [3]. *Ad hoc* computer simulations, on the other hand, heuristically fabricate mathematical models on the basis of available phenomenological data, but not necessarily based on a systematized and fundamental understanding of the laws governing those systems [4]. Such mathematical models are written in computer codes and used to mimic the observed behavior of the dynamical system at hand.

Both fundamental and *ad hoc* computer simulations are essential components of contemporary scientific, technological and medical research. Occasionally, the two kinds of simulations are used conjointly, yielding a mixed approach. Yet the two dramatically differ in their epistemic nature and range of uses. While *ad hoc* computer simulations are more commonly used in the applied sciences and in medical practice, fundamental computer simulations are part of a discourse in fundamental scientific research that finds its roots in the early modern period. In the following pages, I will comment on this fact while illustrating specific features of the remarkable shift from the formulation of scientific laws, with the relatively limited computational capabilities of the early and late modern periods, to the formulation and massive computations that we can now perform by means of contemporary computer technologies [4].

2. – The glories of pre-contemporary science

The act of formulating represents a crucial moment in the reasoned synthesis that we now refer to as the scientific understanding of nature. On an intuitive level, "to formulate" means "to shape" or enunciate, possibly in a concise and technical way, the fundamental characteristics of a certain entity or phenomenon, which can be actual or postulated, natural or artificial, realistic or idealized(2). In science we can formulate ideas, explanations, descriptions, hypotheses, etc. In natural science, formulations deal, directly or indirectly, with the laws of nature. A law of nature may be regarded as a "stated regularity in the relations or order of phenomena in the world that holds, under a stipulated set of conditions, either universally or in a stated proportion of instances"(3).

In the course of history, human beings have tackled laws of nature from a variety of perspectives, including the religious and the philosophical. Since the early modern period, laws of nature have also been approached by means of a procedure, now called the scientific method, grounded in reproducible experiments —distinct from simple experiences— and in mathematical reasoning(4). From this perspective, experiments are used to collect data from the natural world and to induce generalizations of the patterns that scientists can observe in nature. Scientific experiments and scientific experiences can

(2) "To formulate" derives from the Latin *forma*, which means "shape". See the *Online Etymology Dictionary*: https://www.etymonline.com/word/form?ref=etymonline_crossreference#etymonline_v_11807.
(3) See "Law of nature," in *Encyclopædia Britannica*: https://www.britannica.com/topic/law-of-nature. It might be interesting to discuss the extent to which the adjective "stated" is truly necessary in this definition.
(4) A scientific experience might be an attentive observation or a measurement; an experiment is a special kind of an experience, or a structured set of experiences, aiming at collecting data to address a certain conceptual puzzle, *e.g.*, accepting or rejecting a falsifiable hypothesis.

also be used deductively to test scientific theories or to put theory into practice. Whatever the case, scientific formulations give a mathematical expression to quantities and/or their relations; in them, each symbol is clearly defined and nothing is left unspecified([5]).

One of the seminal devisers of this approach was Galileo Galilei (1564–1642), who shook the foundations of natural philosophy, paving the way for what we now call modern science([6]). When questioning why Galileo is considered so fundamentally important to the history of science, a variety of answers are often given, all of which are somewhat equivocal. Here are some common examples: Galileo intuited the connection between nature and mathematics; he was the first to rely on data obtained from experiments and to advocate for the superiority of experiments over philosophical reasoning. Furthermore, Galileo is considered the father of modern observational astronomy, the father of the experimental method, and the person who separated the study of nature from philosophical authorities and from religion. And yet, each of the foregoing descriptions is somewhat problematic. Because this topic creates the intellectual background for the modern history of scientific formulations, it is worth clarifying the aforementioned statements.

Without a doubt Galileo had embarked on a systematic use of mathematics to approach the study of the natural world. However, astronomy, as a well-defined branch of natural philosophy, had already become substantially mathematized since ancient times([7]). An example is Babylonian astronomy, which became mathematical in the second half of the first millennium BCE [6]. Among their investigations, the Babylonians maintained detailed records of the wandering stars (*i.e.*, the planets), studied their trajectories and their risings and settings, while also investigating lunar and solar eclipses([8]). For centuries, they used refined arithmetic to compile lunar and planetary ephemerides. Eventually, and presumably between 350 BCE and 50 BCE, they occasionally employed geometry as well [9]. The Greek Eratosthenes (c. 276 BCE–c. 194 BCE) used geometric methods on observational data to calculate the circumference of the Earth. Data collected by the Babylonians was eventually used by Hipparchus (fl. 2nd century BCE), who succeeded in establishing himself as a key figure in the studies of lunar motion and the ecliptic, as well as discovering the precession of the equinoxes. Ptolemy (c. 100–c. 170) developed a sophisticated mathematical geocentric model that could account for observational data, including the puzzling anomalies represented by the apparent ret-

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([5]) Mathematics contributes to conferring a distinctive technicality to scientific formulations. Scientific formulations can also be verbal, as may happen with the enunciation of a scientific principle (*e.g.*, the Pauli exclusion principle), or visual, as in graphs or visual models. Scientific formulations can also take place through programming languages, as we shall see later on.

([6]) The term "science" derives from the Latin *scientia*, which means "knowledge"; see the *Online Etymology Dictionary*: `https://www.etymonline.com/search?q=Science`. Together with "scientist", "scientific method", "physicist", etc., the term "science", in the way we now understand it, came into common use in the nineteenth century. Of particular note in the development of the modern usage are the contributions made by William Whewell (1794–1866). Galileo, as a matter of fact, still referred to (natural) philosophy, and considered himself a philosopher and a mathematician. Before and during Galileo's time, "science" referred either to a type of knowledge that implied certainty [28] or that required specialized skills. Significant from this last point of view was Leonardo da Vinci (1452–1519) who referred to the "science of painting".

([7]) I do not intend to sketch here a comprehensive picture of the historical development of mathematical astronomy. The reader may refer to [5].

([8]) I want to thank an anonymous reviewer who shared with me some of the information elaborated in this paragraph. See also [7, 8].

rograde and prograde motion of the planets. Notably rich in examples is the Chinese experience, especially when we consider the early history of calendar making, as well as the *Zhoubi*, which appeared in the first century BCE or CE [10, 11]. Also worth mentioning is the development of quadratic interpolation by Liu Zhuo (544–608 or 610) and the development of template tables and computational practices in calendric astronomy during the Ming period [12].

Moving away from astronomy, in the age just before Galileo, a few mathematicians operating on the Italian peninsula started to apply mathematical methods to terrestrial natural phenomena [13](9). An example is Niccolò Tartaglia (c. 1499–1557) and his *La Nova Scientia* (Venice, 1537), a mathematical study of the motion of projectiles set through "means of geometric and algebraic arguments" [16]. Christopher Clavius (1537–1612) is another remarkable example [13, 17]. In other words, Galileo was not the first person to intuit the connection between physics and mathematics.

We must also take care when referring to the superiority of sense experience over philosophical reasoning. Although empiricism has its earliest roots in ancient times —as we can see, for example, in the case of Aristotle (384 BCE–322 BCE)— modern empiricism started to gain prominence with Sir Francis Bacon (1561–1626) and continued to develop with Thomas Hobbes (1588–1679), John Locke (1632–1704), George Berkeley (1685–1753) and eventually David Hume (1711–76). Yet, as stressed, one thing is an experience and another is an experiment (see footnote (4)). This point notwithstanding, glimpses of what we now call scientific experiments appeared much earlier than Galileo's day. Particularly remarkable is the case of Archimedes (c. 287 BCE–212/211 BCE), who provided evidence of theory-driven experimentation [18, 19]. We can also mention Galen (129–c. 216) who, working on completely different types of problems, performed experiments on animal bodies to determine the functions of specific organs [20]. Also worth considering are Ptolemy's optical studies, which we partly know through a twelfth-century Latin translation of a lost Arabic text, and which are supposed to derive from empirically driven experiments [21,22]. Closer to Galileo's day, Simon Stevin (1548–1620) performed an experiment on falling bodies —the Delft tower experiment, described in his *De Beghinselen der Weeghconst* (Leiden, 1586). Much discussed is Ibn al-Haytham (c. 965–c. 1040), Latinized as Alhazen, who is thought to have pioneered, in the Islamic Golden Age, experiments that might be framed within an approach that resembles the scientific method [23].

The assertion, which belongs to Charles Singer (1876–1960), that Galileo is the father of modern observational astronomy may also be equivocal [24]. Although I tend to agree with Singer, and mostly because of the development and use of the telescope, I wonder on what objective ground we may unambiguously justify such an assertion. As a matter of fact, Nicolaus Copernicus (1473–1543) is also a legitimate candidate, as is Erasmus Reinhold (1511–53) who, with his *Prutenicae Tabulae* (Tübingen, 1551), played a central role in the development of the Gregorian calendar. Along the same lines, Johann Bayer's (1572–1625) *Uranometria* (Augsburg, 1603) contains star charts that constituted the first modern atlas representing the entire celestial sphere as it is visible to the naked eye. Yet perhaps the most prominent case is represented by Johannes Kepler (1571–1630): the *Tabulae Rudolphinae* (Ulm, 1627) include planetary tables and an astronomical catalog that resulted from observations, initiated by Tycho Brahe (1546–1601), to compile the

(9) In addition, the reader may find it interesting to delve into the concept of "mixed mathematics", an expression that can be traced back as far as [14]; see [15].

coordinates describing more than a thousand stars and to provide information to locate the planets that were known at that time. This work represented one of the earliest, most rigorous and influential contributions to modern observational astronomy. It also played a role in the compilation of the *Chongzhen* calendar, which survived —with some modifications— the transition from the Ming to the Qing dynasties. The production of this lunisolar calendar, eventually renamed the *Shixian* calendar, saw the collaborative effort of European Jesuits and Chinese scholars. Most importantly among them were Johannes Schreck (1576–1630), Xu Guangqi (1562–1633) and Johann Adam Schall von Bell (1591–1666), as well as Li Zhizao (1565–1630), Li Tianjing (1579–1659) and Giacomo Rho (1592–1638) [25]. Why exactly, then, should we grant the paternity of modern observational astronomy to Galileo?

And yet, another issue arises when investigating the idea that Galileo is the father of the experimental method. Some of the earliest influential contributions, in a modern perspective, to the establishment of an experimental method of knowledge were made by Bacon in his *Novum Organum* (London, 1620), a treatise published as the second part of his *Instauratio Magna*, in an attempt to supplant Aristotle's multipart work on logic, the *Organon*. Bacon's contribution was an effective attack on Aristotelianism, and it represented the possibility of opening up new horizons, going beyond the Pillars of Hercules eloquently depicted on the cover of the *Instauratio Magna*. It must be clear, however, that Aristotelianism had been discussed and disputed since the early stages of its consistent introduction to Europe, a process which took place, through the Arab world, during the Renaissance of the twelfth century([10]). An example from this perspective is Jean Buridan (c.1300–c.1358) who contributed to the establishment, from the European outlook, of the theory of the impetus, which was originally developed by John Philoponus (fl. 6th century), a Byzantine scholar from Alexandria [27]. Criticism of Aristotelianism, from various perspectives, intensified in the years before Galileo's rise to prominence, as we can see in the work of Paracelsus (1493–1541), Bernardino Telesio (1509–88), Francesco Patrizi (1529–97), William Gilbert (1544–1603), Giordano Bruno (1548–1600), Bacon, Tommaso Campanella (1568–1639), and Kepler, just to name a few [28]. Bruno, in particular, offers a significant example of a thinker who contributed to the history of free thought and who tried to break free of the control of religious authority. Why, then, is Galileo so fundamentally important to the history of science and, in particular, in the history of the mathematical formulation of the laws of nature?

3. – Galileo and the early modern roots of scientific formulations

The mathematical tools used to shape astronomical studies by the Babylonians, as well as the Greeks at least since Eudoxus of Cnidus (c. 395–390 BCE–c. 342–337 BCE), fall within the realms of arithmetic and synthetic geometry —with the Babylonians using primarily arithmetic and the latter using mainly geometric methods [29]. The advanced approaches employed by both civilizations were essentially used for measurements and for making predictions of major astronomical objects and phenomena. Galileo, for his part, had the groundbreaking intuition that the laws of nature are written in the mathematical language. This represented quite a different perspective, and quite a remarkable one. For Galileo, mathematics was not simply a tool to aid the description of certain natural

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([10]) Before the twelfth century, fragments of Aristotle's works were known in Western Europe, as in the case of his *Organon*, which was partly translated and commented upon by Boethius (c. 475–c. 526). See also [26].

phenomena: from his point of view, nature *is* mathematical and we must thus employ mathematics if we want to understand it:

> *Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth* [30].

In other words, Galileo intuited a fundamentally different relationship between mathematics and nature, and turned mathematics into an indispensable tool of precise, or "divine", knowledge [31]. While God knows the totality of the laws of nature, humans can match God in understanding a limited number of them: yet, we can attain a divine level of knowledge of such laws if we use mathematics. This is what Galileo posited. By learning the characters of the mathematical language we can learn how to read the book of nature and effectively write, *i.e.*, formulate, its own laws. In his essence, Galileo contributed to mathematizing natural philosophy at large, going beyond astronomy and successfully entering the realms of physics by means of "sense experiences" and "necessary demonstrations" ([11]). For most of premodern history, unlike astronomy, physics was essentially qualitative and speculative. Aristotle's *Physics* (IV century BCE) is an example of this trend. However, to obtain reliable, clear and precise data useful for formulating scientific laws, we must rely on experiments and get rid of what Galileo called natural impediments, *e.g.*, friction, imperfections, interferences, etc. By looking beyond these natural impediments, and by using mathematical reasoning, it is possible to land upon scientific laws. These laws have specific characteristics such as being objective, as well as being testable through experiments that, in turn, must be reproducible.

Bacon, despite his status as a triggering force in the genesis of modern empiricism, kept right on philosophizing. In addition, mathematics was missing from his experimental method. At the same time, while mathematics was not missing from Tartaglia's approach, the latter did not follow a systematized experimental method, but remained rather Aristotelian in his own observations, deductive in his reasoning, wedded to a "mixture of ancient and novel ideas uninfluenced by medieval speculations on motion, impetus, or the science of weights" [32]. Galileo, in a sense, contributed the missing link between Bacon and Tartaglia, going beyond ballistics to start mathematizing the whole of natural philosophy, starting with an experimental foothold. Thus, he effectively showed how to use the "new organon" and played a prominent role in systematizing a new effective method of investigating nature. The Galilean method, which was experimental and mathematical, developed into the approach eventually known as the scientific method. Because of the indissoluble mixture of experiments and mathematical reasoning that he advocated for, Galileo was, at the same time, an empiricist and a rationalist, and a trigger for the glorious jump from natural philosophy to modern science. Alhazen

([11]) At least up to the time of Galileo, astronomy, physics and mechanics were conceived as separate disciplines, with mechanics dealing with the construction and functioning of machines [28]. To better frame the meaning of "sense experiences" and "necessary demonstrations" in their intellectual context, consider Drake [30].

represented glimpses of something that, with the advent of Galilean method, started to become shared, in a continuous and systematic way, by a large community of scholars. Galileo, in fact, succeeded in making it widely known and accepted —to the point that we continue to use it today [33]. Such achievements had been possible also because of Galileo's visibility, evident both in his skills as a writer and in the dramatic confrontation that he had with the Catholic Church, a circumstance that eventually shook the entire *Respublica literaria*, eventually reverberating through various European learned academies and societies.

As far as Aristotelian natural philosophy was concerned, Galileo did more than advance a criticism; he helped initiate the process that led to its successful dismantling. With the telescope, in particular, he observed an imperfect and corruptible Moon (rather different from the one described by the Aristotelians) and started collecting evidence in support of the heliocentric system([12]). This model had already appeared in ancient history, as in the striking case of Aristarchus of Samos (c. 310 BCE–c. 230 BCE). However, the first convincing evidence for the Earth's revolution and rotation only began to appear in the eighteenth century. James Bradley (1693–1762) and Friedrich Bessel (1784–1846) were leading figures in studies of the Earth's revolution; Giovanni Battista Guglielmini (1760–1817), Johann Benzenberg (1777–1846) and, most importantly, Léon Foucault (1819–68) contributed to proving the Earth's rotational motion.

When it comes to Singer, I propose that we should better say that Galileo was the first modern experimental astrophysicist, a designation related especially to the telescope that he developed (out of an existing Dutch technology), which he used, as a scientific instrument, to support what was back then the heliocentric hypothesis. He also contributed appealing suggestions that the physics of the Earth is the same one governing cosmic phenomena, and that the cosmos is not any more special than our terrestrial dimension. Galileo played a major role in the establishment of the telescope as a reliable instrument for augmented and reproducible experimental observations, expanding the modern astronomical observations that Brahe and Kepler successfully initiated before him. By doing so, he launched the creation of an experimental theory of the solar system.

Even if Galileo wrote in impactful ways about the mathematical essence of the laws of nature, he clearly did not formulate scientific laws in the way we now do it. In his writings we can find elaborate geometric constructions, as well as arithmetical and verbal formulations. Galileo's inclined-plane experiments illustrate the way he would formulate the laws of nature [34]. Of further significance is the case of the mast experiment, as well as his famed boat thought experiment published in the *Dialogo Sopra i Due Massimi Sistemi del Mondo* (Florence, 1632). Both experiments are related to the principle of Galilean relativity. The property of isochronism of simple pendulums, the law of free fall, the law of parabolic trajectories and the principle of inertia all provide additional examples and appeared in the *Discorsi e Dimostrazioni Matematiche* (Leiden, 1638). With Galileo, mathematical notation had already entered into what we now call its symbolic stage, which would find new impetus with the development of analytic geometry and infinitesimal calculus. The protagonists of these developments were René Descartes (1596–1650), on one hand, and Sir Isaac Newton (1642–1727 OS) and Gottfried Wilhelm Leibniz (1646–1716), on the other. Eventually, Daniel Bernoulli (1700–82), Leonhard Euler (1707–83), and Joseph-Louis Lagrange (1736–1813) were among the relevant figures

([12]) Remarkable are Galileo's observations of the phases of Venus and his observations of the four largest moons of Jupiter.

who would shape the development of scientific formulations in the period that saw the demise of natural philosophy and the rise of natural science. Yet, what modern natural philosophers, and eventually pre-contemporary scientists, could do with their mathematical formulations was essentially limited. The missing ingredient was the substantial computational capabilities needed to tackle nature in the whole of its complexity, with all its impediments.

4. – Contemporary computation: New paths in scientific inquiry

The verb "to compute" derives from the Latin *computare*, which means "to count" ([13]). To compute means to carry out a precise series of mathematical tasks (*e.g.*, arithmetical, geometric, logical, etc.) as part of an algorithm. Despite being highly refined in tackling and elaborating problems in natural science, the human mind in most cases is not, computationally speaking, powerful enough to solve, numerically or otherwise, all the complex equations needed to describe the time evolution of most real, dynamical systems [35]. Nor is the human mind capable of the volume of computation required to analyze the masses of data (*e.g.*, big data) generated on a daily basis in most sciences. This is why human beings devised various analytic techniques as well as artificial devices to aid calculation and computation([14]).

With the term "computer" we should not necessarily think of the digital electronic computers that we now use on a daily basis. An astrolabe, for example, can be regarded as an analog mechanical computer. In modern times, computing technologies saw substantial developments. Remarkable were the contributions made by Wilhelm Schickard (1592–1635), Blaise Pascal (1623–62), Leibniz, and even more substantially by Charles Babbage (1791–1871), a pioneer in the development of automatic digital computers [36]. These devices, originally, were essentially employed for calculations. We now use computers in a more substantial way: to see complex mathematical models in action, the essence of today's computer simulations([15]). By means of computer simulations, we can tackle real dynamical systems and effectively solve the complex set of equations that describe them. Computer simulations provide an effective tool for handling scientific formulations in a new empowered way and for making reliable predictions in the behavior of such complex systems.

As stated earlier, the fundamental laws of science are, in most cases, the result of generalizations obtained by ridding ourselves of natural impediments [37]. Yet real sys-

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([13]) See the *Online Etymology Dictionary*: `https://www.etymonline.com/word/compute#etymonline_v_17291`.

([14]) I find it useful to distinguish the meanings of two words that are synonymous in the common language: "calculation" and "computation". I am not offering exhaustive definitions here, but rather proposing a distinction that I consider to be useful. While the former concerns the implementation of numerical tasks (to be carried out, for example, with the human mind or a calculator), the latter should refer to the algorithmic implementation of possibly diverse mathematical tasks that we can carry out with the human mind or a computer. That said, computation is implemented, in a digital computer, as binary arithmetic. Yet, a binary non-programmable calculator is not a binary computer. Computation has controlling rules that interact with input and ongoing binary implementation to determine the output. These rules are called the "program" and can be modified without changing the underlying machine.

([15]) From now on, when I write "computers" and "computer simulations", I will be referring to today's computers and computer simulations.

tems are not impediment-free, and they require complex mathematics to be described and comprehended. For most of the modern era, natural philosophers and scientists, as well as applied mathematicians and engineers, did not have the technological means to decisively cope with this issue [38]. The problem, however, was not only linked to natural impediments: even a large variety of impediment-free systems can become very complex; think, for example, of the three-body problem, with all its possible idealizations. In other words, during the time leading up to the twentieth century, scientists devoted a large portion of their energy to formulations, embarking on more or less limited computations. Formulating was one of the highest intellectual targets to aim at. The task of numerically solving equations and applying them to real systems was mostly reserved for engineers and applied scientists; it was practical labor. Moreover, formulating even became a sort of aesthetic mission. Scientific formulas were not just distillates of intellectual creativity; they were also —and still are— a distillate of form and elegance: a blackboard full of scientific formulas, in particular, might even be aesthetically fulfilling, a work of art to contemplate([16]).

From 1956 to 2015, there has been a one trillion-fold increase in computer performance, measured in floating-point operations per second [39]. This circumstance has provided us with new perspectives. In addition to formulating, which of course remains an essential asset in scientific pursuit, the bulk of today's research is based on the massive solution to complex equations, on the categorization of different types of solutions, and on the development of more effective strategies for improving our computational capabilities, especially through computer simulations. I must stress here that a computer simulation *is not*, by any means, a scientific experiment and *might not* act as a substitute for a scientific experiment. Through an experiment, we interrogate nature; through simulations, we interrogate mathematical models. Yet, in the end, we must always adhere to experimental evidence, which guides the overall scientific pursuit. Similarly, a computer is not a scientific instrument, though some scientific instruments may well be computerized. A scientific instrument is a device we use to run experiments; a computer is a device used to aid computation([17]). While computers and computer simulations have existed before contemporary history (*e.g.*, through an astrolabe and its workings), computer simulations have now become a necessity in virtually any major scientific research program. Unlike in pre-contemporary science, it is now basically impossible to initiate any substantial scientific research without proper computational capabilities and the use of computer simulations. This is why we should talk of "necessary simulations": today, simulations are as much needed as "necessary demonstrations" were needed within the Galilean outlook [4].

Computers and computer simulations have also impinged on the traditional bound-

([16]) If we search for the "most beautiful" scientific formulas on the internet, we will find a huge amount of material. Perhaps "most beautiful" of all is the Dirac equation (in natural units), which sits at the top of a poll conducted in 2016 by the BBC. Source: `http://www.bbc.com/earth/story/20160120-you-decide-what-is-the-most-beautiful-equation-ever-written`.

([17]) There is an interesting analogy, though, between computers and computer simulations on one hand, and scientific instruments and experiments on the other. When computer simulations are compared with the behavior of real dynamical systems, useful information may be obtained to improve the underlying computer code or mathematical model. Similarly, scientific instruments may provide insights to further a working hypothesis or a mathematical model in an attempt to refine them and better align them with the experimental evidence.

aries among scientific disciplines: today, we have composite groups of all kinds of scientists, working side by side from an interdisciplinary perspective. These experts also work in close contact with professionals and scholars from disciplines that have been traditionally detached from the modern sciences, such as various branches of the humanities. In other words, computers are influencing the disciplinary shape of all human knowledge as we know it. In addition, some disciplines that were, before the twentieth century, considered not genuinely scientific are now acquiring a somewhat scientific taste thanks to computer technologies. This is what happens within digital humanities, theoretical linguistics, pharmaceutical chemistry, or within computing and quantitative methods in history. Interestingly, with the rise of computer technologies in the twentieth century, we have also witnessed the rise of new disciplines within the formal sciences (*e.g.*, artificial intelligence, machine learning, information theory, etc.) as well as new formal languages at our disposal. Computer codes, which result from the use of programming languages, can certainly be considered a type of scientific formulation([18]).

In contemporary science, then, scientific formulations can also take place through programming languages, which can be used both as a way to formulate scientific concepts and to implement massive computations. Following in Galileo's footsteps, contemporary scientists may well say the laws of nature are written in the language of mathematics; yet, we now create our own quasi-divine understanding of nature by mastering both mathematical *and* programming languages. While the characters of the mathematical language, from Galileo's perspective, are "triangles, circles, and other geometric figures", those of computers are data, processes, structures and other constructs of computer programs, without which we would now wander about "in a dark labyrinth" [30]([19]). This does not mean that Galileo was wrong or that we should try to correct him. What Galileo represents is a monumental, and still very topical, contribution, but we now understand that nature is a complex mixture of different essences: it's not solely mathematical but is also extensively computerizable. After the rise of computing in the twentieth century, it would not be possible to proceed further without the opportunities offered by programming languages in reading, comprehending, and in effectively *predicting*, through computer simulations, the content of the book of nature that "stands continually open to our gaze" [30].

The inseparable tasks of today's scientists are that of formulating and setting up massive computations. Within this trend, scientists interface with nature in a different way, being able to handle complexity and predictability in an unprecedented manner. In addition, if in the past natural impediments were oftentimes intentionally evaded, now scientists can develop a deep engagement with them, unveiling laws and properties that would be substantially inaccessible otherwise([20]). We can now work out virtually all details composing such systems, reformulating, in other words, our relationship with science, and also our ambitions with nature. As opposed to pre-contemporary science,

([18]) To this extent, we should recall that Fortran, which is an acronym of *Formula Translation*, was developed by John Backus (1924–2007) and IBM as a scientific programming language to translate mathematical formulas. FORTRAN —originally written in uppercase letters— was released in 1957.

([19]) Data, processes and structures are basic constructs of every computer program, essentially abstracted from the specific underlying hardware. While data is the input and output of a computer program, process indicates what happens to the input data to produce a specific output, and structure determines the interpretation of input and output data.

([20]) This trend is particularly evident within the approach of *ad hoc* computer simulations.

we can now "command nature", by obeying it, at a more complex level of detail [40].

Are we allowed then to think that computer technologies have led us, or are they in the process of leading us, to a completely new approach to nature? Given all the dramatic changes we have gone through, and the novelties we have introduced, are today's scientists still "scientific"? The doubt is legitimate and, although we cannot exclude that in the future we might create a genuinely new approach to nature, it does not seem that, up to now, we have created another jump as remarkable as the one that took place during the shift from Aristotelian philosophy to the inception of early modern science. We keep running on the tracks of modern science, just by different means. The essence of today's scientific practice is still based on experiments, and on the exploitation of formal languages to elaborate and explain experimental data([21]). To this end, we now use both mathematical and programming languages, and even if computer simulations have provided us with great computational and predictive capabilities, the final word must always be experimental, exactly as it was for Galileo and for all the natural philosophers and scientists who followed him. Today's scientists remain quintessentially Galilean, probably even more so than most late modern scientists. While the latter, in fact, were interested in producing general (and elegant) formulations, most of today's scientists have left the podium of the "intellectual formulators", working instead within real dynamical systems —with all their attendant imperfections— and directly involving themselves in extensive computations.

5. – Conclusions

The entry of computers into contemporary scientific research has caused a dramatic shift from a period characterized by substantial scientific formulations with limited computations to a new era of formulations *and* massive computations. This shift has affected (and continues to affect) the scientific *modus operandi*, as well as the overall structure of scientific disciplines and their communities. Computers have introduced new elements to the traditional scientific method, such as "necessary simulations" as well as improved computational capabilities. While in the past, scientific formulations stemmed from experiments and mathematical reasoning, we now understand that experiments and mathematics alone are not enough to continue satisfying our scientific needs. Today, we also need computer simulations —clearly distinct from scientific experiments— and their consistent introduction has marked a profound shift in the way we interface with nature. Before the twentieth century, scientists essentially aimed at general laws that could describe natural phenomena. Given the limited pre-contemporary computational capabilities, such formulations could not be used to the fullest of their potentials. Nowadays, we can run on the tracks of science by also using computers and writing computer

([21]) From a general perspective, programming and mathematical languages are two different kinds of formal languages. As a matter of fact, one can write a computer code that, in itself, has nothing to do with mathematics. That said, in natural science most computer codes are translations, through various programming languages, of mathematical models. Within this outlook, one might say that programming languages are a complement to, or an extension of, the mathematical language. Yet a fundamental difference remains: programming languages, unlike mathematics, are explicitly designed to be manipulated by computers, not directly by the human mind.

codes that can yield promising results in a wide variety of cases([22]). If we cannot exactly implement these codes yet, it is probably just a matter of time, as computer technologies continue to develop apace [41]. Yet, plenty of queries remain, questions that our creative minds can formulate —running faster than computers— from which we still seem far from any possibility of an answer.

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([22]) With the introduction of computer technologies also the artistic beauty of scientific formulations has altered its connotations. Aesthetic canons for computer codes boast a different kind of beauty. Some colleagues suggested that the implementation, in the Haskell language, of the powerset function is a “beautiful” example of a computer program.

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