

## Applications of Bayesian framework to optimal observational design strategies

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**Summary.** — This paper is focused on the optimization of ESPRESSO Radial Velocity follow-up of transiting exoplanets detected by the satellite TESS. We compare three different types of batch scheduling: one random scheduler and two types of uniform-in-phase schedulers: one myopic, which selects targets one-at-a-time, and one non-myopic that efficaciously explores all the possible combinations between stars to be observed and available time slots.

### 1. – Introduction

The hunt and the study of extrasolar planets have become one of the hot topics in astronomical research. Indeed many programs both from the ground and space missions are ongoing or foreseen [1]. Nowadays the research in exoplanetary science aiming to answer the many questions about the formation and evolution of exoplanets. In this perspective, transiting exoplanets are of special interest due to the wealth of data made available by their particular orbital configuration. Considering that many collaborations are planning to devote an important amount of their Guaranteed Time Observations (GTO) for the Radial Velocity (RV) follow-up of the TESS (Transiting Exoplanet Survey Satellite, *e.g.*, [2]) objects of interest (TOIs), it is clear that this observational effort must be planned in the most efficient possible way. For instance the ESPRESSO Collaboration [3] plans to devote around 32% of its GTO for TOIs follow-up [3]. In light of this, we focused our work to find optimal strategies for the follow-up of TOIs by adopting the theory of Bayesian Optimal experimental design. By adapting for the purpose of

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this work the Approximate Coordinate Exchange (ACE) [4], we are able to find optimal designs for complex and realistic problems with reasonable computational time.

## 2. – Simulations

The ESPRESSO GTO consists of 273 nights during 4 years, and began on the 1st of October 2018<sup>(1)</sup>. Mimic the ESPRESSO GTO distribution in ESO periods 102 and 103<sup>(2)</sup>, we simulated the scheduling of 10 different ESPRESSO TOI follow-up, from the 1st of October 2019 until the 30th of September 2022. Each of these 10 different distributions consists of 1102 observing slots. This number of time slots is very close to the fraction (*i.e.*, 30%) that can be used for ESPRESSO GTO TOI follow-up. A sample of stars for possible ESPRESSO follow-up observations was pre-selected among those stars considered in the catalogue [5] by demanding: a declination in the interval  $[-80^\circ, +30^\circ]$ , an effective temperature,  $T_{eff}$ , in the interval  $[4000, 6000]$  K, and high surface gravity,  $\log g > 4.0$ . We then included in our final sample the 50 brightest stars among those pre-selected with at least one orbiting planet with a radius below  $4R_\oplus$ , 3 detected transits and a transit signal-to-noise greater than 10. In the publicly data available from catalogue [5] only planets that transit are identified. But, in order to generate realistic simulations of a RV time series we added extra orbiting planets to each star, non-detectable by TESS. We used for such purpose the occurrence rates published in [6] and [7]. Their orbital eccentricities,  $e$ , were then randomly drawn from a Beta distribution with parameters  $\alpha = 1.03$  and  $\beta = 13.6$  following [8]. Finally, the mean anomaly  $M_0$  at the time  $t_0$  (when we start our scheduler), and the argument of periastron,  $\omega$ , were randomly drawn from a uniform distribution between 0 and  $2\pi$ . We ended up with 50 extra planets, distributed across 35 systems. Once we have obtained our sample of transiting and non-transiting planets and have drawn 10 different ESPRESSO TOI follow-up we have proceeded to the simulation of the RV time series associated with each star using the following model:

$$\begin{aligned}
 (1) \quad & v_r(t) = v_{\text{sys}} + \sum_{i=1}^{n_p} v_{r,i}(t) + \epsilon(t), \\
 (2) \quad & v_{r,i}(t) = K_i \{ \cos[\phi_i(t) + \omega_i] + e_i \cos(\omega_i) \}, \\
 (3) \quad & \epsilon(t) \sim N \left( 0, \sqrt{\sigma_{\text{act}}^2 + \sigma_{\text{ph}}^2 + \sigma_{\text{ins}}^2} \right),
 \end{aligned}$$

where  $n_p$  is the number of planets orbiting the star,  $K_i$  is the RV semi-amplitude,  $\omega_i$  is the argument of periastron,  $e_i$  is the orbital eccentricity, and  $\phi_i(t)$  is the true anomaly as a function of time,  $t$ , calculated from the other orbital parameters, all with respect to planet  $i$ . The noise  $\epsilon(t)$  that affects the RV measurements takes the form of a Gaussian white noise, with a variance equal to the sum of the variances associated with each of the following three noise components: the stellar activity  $\sigma_{\text{act}}$ , the photon-noise  $\sigma_{\text{ph}}$  and the instrumental-noise  $\sigma_{\text{ins}}$ , which we assumed to be about 0.1 m/s [9]. We also associated to every star a systemic RV relative to the centre of mass of the system,  $v_{\text{sys}}$ , drawn from a random uniform distribution between  $-100$  to  $100$  m/s, roughly the observed range for stars in the solar neighbourhood [10].

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<sup>(1)</sup> [https://www.eso.org/sci/observing/policies/gto\\_policy.html](https://www.eso.org/sci/observing/policies/gto_policy.html)

<sup>(2)</sup> Telescope time for ESO telescopes is allocated twice a year in periods of 6 months.

### 3. – Scheduling strategies

We will compare three distinct scheduling strategies. Two of them, labelled A, are myopic, *i.e.*, the best schedule is defined sequentially in time. In strategy  $A_1$ , the star chosen to be observed at any given time is randomly drawn from all stars in the sample which can be observed at that time, at an airmass equal or smaller than 2, and with a Moon separation greater than 30 degree. In strategy  $A_2$ , in addition to fulfill the observational constraints used in the previous case, we also want the sampling of the RV phase-curves of the known transiting planets to be as uniform as possible. This is achieved through the maximization of the following objective function, able of evaluate the overall dispersion of points in a given interval

$$(4) \quad f(\{x_i\}) \equiv \left\{ \sum_{i=1}^{1102} [d(x_i)]^{-2} \right\}^{-1/2},$$

where  $d(x_i)$  is the time distance between the observation  $x_i$  and its nearest neighbour (including across the orbital phase-curve boundary), as a fraction of the orbital period of the transiting planet targeted by the observation. In the context of strategy  $A_2$ , the best schedule is then also constructed sequentially in time. The third strategy, labelled B, is non-myopic. In this case, the aim is to compare all possible schedules, across the full time-span of 3 years, and then choose that which maximizes the objective function,  $f(\{x_i\})$ . In this last case, it corresponds to finding the schedule that maximizes the sum over all stars of the minimum time distance, normalized as a fraction of the orbital periods of the known transiting planets around each star, between any observation and all others of the same star. Given the large number of time slots available for scheduling and the fact that the stars considered are observable during most of any given year, the number of possible scheduling configurations is huge. Therefore, it is impossible to compare the values the objective function takes for all such configurations. For that reason, we used the `acebayes` R package<sup>(3)</sup> [4] to find the schedule that maximizes the objective function. This is done via an approximate coordinate exchange (ACE) algorithm, where a sequence of conditional one-dimensional optimisation steps are used, as described in [4].

### 4. – Results and discussion

To perform Bayesian statistical analysis of all simulated RV datasets we used the open-source software `kima`<sup>(4)</sup> [11]. In order to compare the results obtained, we define the following quantities, with respect to some planet characteristic  $X$ , and to a given simulation: absolute bias,  $\mathbf{E}[X] - X_{\text{true}}$ , relative bias  $(\mathbf{E}[X] - X_{\text{true}})/X_{\text{true}}$ , absolute accuracy  $|\mathbf{E}[X] - X_{\text{true}}|$ , relative accuracy  $|\mathbf{E}[X] - X_{\text{true}}|/X_{\text{true}}$ , absolute precision  $\sigma_X$ , and relative precision  $\sigma_X/\mathbf{E}[X]$ . Where  $X_{\text{true}}$ ,  $\mathbf{E}[X]$  and  $\sigma_X$  represent, respectively, the true, expected value and standard deviation of  $X$ . In table I, the absolute and relative bias, accuracy and precision with which,  $K$ ,  $e$ ,  $M$  and  $P$ , are recovered, averaged over all transiting and non-transiting planets and simulations, is shown for the three strategies. Overall, the non-myopic strategy, B, recovers more information about the true values of  $K$  and  $M$  for the transiting planets. In comparison, the strategy  $A_2$ , leads to somewhat

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<sup>(3)</sup> <https://cran.r-project.org/web/packages/acebayes>

<sup>(4)</sup> <https://github.com/j-faria/kima>

TABLE I. – In the upper panel the absolute and relative bias are shown, accuracy and precision with which  $K$ ,  $e$  and mass,  $M$ , are recovered, averaged over all transiting planets and simulations, for the three strategies. The same quantities, as well as the orbital period,  $P$ , are provided in the lower panel with respect to all detected non-transiting planets.

Strategy	Parameter	Absolute			Relative		
		Bias	Accuracy	Precision	Bias	Accuracy	Precision
$A_1$	$K$	$0.25 \pm 0.36$	$0.52 \pm 0.32$	$0.90 \pm 0.54$	$0.12 \pm 0.21$	$0.21 \pm 0.20$	$0.31 \pm 0.21$
	$e$	$0.10 \pm 0.08$	$0.11 \pm 0.06$	$0.11 \pm 0.05$	$14.39 \pm 44.04$	$14.46 \pm 44.01$	$0.70 \pm 0.10$
	$M$	$0.40 \pm 0.90$	$1.43 \pm 0.85$	$2.27 \pm 1.59$	$0.07 \pm 0.17$	$0.19 \pm 0.17$	$0.27 \pm 0.15$
$A_2$	$K$	$0.11 \pm 0.21$	$0.44 \pm 0.18$	$0.63 \pm 0.21$	$0.06 \pm 0.15$	$0.18 \pm 0.14$	$0.26 \pm 0.21$
	$e$	$0.08 \pm 0.08$	$0.10 \pm 0.06$	$0.10 \pm 0.04$	$14.71 \pm 46.15$	$14.80 \pm 46.512$	$0.70 \pm 0.10$
	$M$	$0.08 \pm 0.63$	$1.26 \pm 0.64$	$1.67 \pm 0.67$	$0.03 \pm 0.12$	$0.17 \pm 0.12$	$0.23 \pm 0.14$
$B$	$K$	$0.05 \pm 0.19$	$0.39 \pm 0.13$	$0.58 \pm 0.19$	$0.05 \pm 0.14$	$0.16 \pm 0.13$	$0.25 \pm 0.20$
	$e$	$0.08 \pm 0.07$	$0.09 \pm 0.06$	$0.10 \pm 0.04$	$13.64 \pm 39.94$	$13.48 \pm 39.92$	$0.69 \pm 0.11$
	$M$	$-0.05 \pm 0.55$	$1.15 \pm 0.50$	$1.56 \pm 0.61$	$0.01 \pm 0.11$	$0.15 \pm 0.11$	$0.22 \pm 0.14$

  

Strategy	Parameter	Absolute			Relative		
		Bias	Accuracy	Precision	Bias	Accuracy	Precision
$A_1$	$K$	$-8.56 \pm 25.04$	$11.97 \pm 23.64$	$10.04 \pm 11.64$	$-0.01 \pm 0.28$	$0.19 \pm 0.22$	$0.24 \pm 0.15$
	$e$	$0.04 \pm 0.06$	$0.05 \pm 0.05$	$0.09 \pm 0.05$	$3.32 \pm 5.62$	$3.41 \pm 5.57$	$0.87 \pm 0.30$
	$P$	$56.87 \pm 568.78$	$352.88 \pm 451.90$	$626.04 \pm 467.78$	$0.15 \pm 0.37$	$0.26 \pm 0.31$	$0.78 \pm 1.07$
	$M$	$-180.26 \pm 654.75$	$306.02 \pm 606.78$	$257.13 \pm 356.90$	$0.01 \pm 0.35$	$10.04 \pm 20.02$	$0.32 \pm 0.19$
$A_2$	$K$	$-5.96 \pm 21.12$	$9.32 \pm 19.87$	$8.08 \pm 9.88$	$0.24 \pm 0.67$	$0.38 \pm 0.60$	$0.28 \pm 0.23$
	$e$	$0.04 \pm 0.07$	$0.06 \pm 0.06$	$0.08 \pm 0.05$	$2.79 \pm 4.44$	$2.91 \pm 4.37$	$0.87 \pm 0.34$
	$P$	$257.82 \pm 729.02$	$481.58 \pm 605.37$	$701.87 \pm 722.12$	$0.20 \pm 0.37$	$0.28 \pm 0.31$	$0.88 \pm 2.24$
	$M$	$-128.34 \pm 556.24$	$244.17 \pm 516.14$	$214.77 \pm 324.32$	$0.32 \pm 0.84$	$22.29 \pm 39.88$	$0.36 \pm 0.26$
$B$	$K$	$-6.27 \pm 21.98$	$10.03 \pm 20.55$	$8.50 \pm 10.39$	$0.19 \pm 0.55$	$0.35 \pm 0.46$	$0.26 \pm 0.20$
	$e$	$0.05 \pm 0.06$	$0.06 \pm 0.05$	$0.08 \pm 0.05$	$3.39 \pm 5.37$	$3.48 \pm 5.31$	$0.87 \pm 0.36$
	$P$	$268.97 \pm 853.49$	$518.99 \pm 731.88$	$759.89 \pm 702.79$	$0.22 \pm 0.44$	$0.31 \pm 0.37$	$1.04 \pm 1.99$
	$M$	$-137.18 \pm 584.83$	$263.35 \pm 540.07$	$220.32 \pm 346.71$	$0.27 \pm 0.73$	$23.90 \pm 48.20$	$0.34 \pm 0.22$

more biased values. We found the myopic strategies lead to a biased estimation (on average around 3% to 7%) of the mass of the simulated TOIs. In contrast, the non-myopic strategy is able to provide an unbiased (about 1%) measurement of the masses, while keeping the relative accuracy and precision around 15% and 22%, respectively. Averaging over the 10 simulations per strategy, a total of  $9.8 \pm 0.6$ ,  $9.8 \pm 0.8$  and  $10.6 \pm 1.2$  non-transiting planets are detected using strategies  $A_1$ ,  $A_2$  and  $B$ , respectively, out of the 50 that we simulated orbiting our sample of stars.

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