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Evidence of superfluidity in a dipolar supersolid

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Summary. — The elusive supersolid, an exotic quantum phase that combines the properties of a crystalline solid and those of a superfluid, has been recently discovered in a system of Bose-condensed strongly magnetic atoms. In this paper, I report on an experiment that directly demonstrates the superfluid behavior of the dipolar supersolid measuring its anomalous response to a rotational excitation. I complement the illustration of the experimental results with a review and an original interpretation of a seminal model due to Leggett.

1. – Introduction

The supersolid is a fascinating quantum state of matter, in which the same particles that form a crystalline lattice are responsible for the coherent flow of mass, typical of superfluids. After 50 years of both experimental and theoretical efforts, the supersolid phase has been observed in a quantum gas of strongly magnetic atoms [1]. In this system, starting from a Bose-Einstein condensate (BEC) and increasing the strength of dipolar interactions relative to contact ones, a density modulation spontaneously forms, while preserving phase coherence between density peaks (also called "droplets"). The presence of sound modes in good agreement with hydrodynamic predictions provided indirect evidence for the new phase of matter to be superfluid [2]. In a later work [3], which has been the subject of my Master Thesis [4], we have observed the smoking-gun proof of superfluidity: the quenching of the moment of inertia compared to the classical value. Here I focus on this work.

Before describing the experimental results, I want to discuss the seminal theoretical work by Leggett, dating back to 1970, on a supersolid under rotation [5]. Although the model of a supersolid employed by Leggett differs in many aspects from ours, this work is very useful to get an intuitive understanding of the behavior of a rotating supersolid.

2. – Leggett's model: Can a solid be superfluid?

2·1. *Standard superfluids.* – Let us begin with a brief reminder of the peculiar rotational properties of standard superfluids, such as liquid helium or atomic BECs. Superfluids are described by a macroscopic wavefunction Ψ which obeys hydrodynamic

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equations. Contrary to classical fluids, the velocity field of superfluids must satisfy the relation

$$\mathbf{v} = \frac{\hbar}{m}\nabla\varphi, \tag{1}$$

where φ is the phase of Ψ and m the particle mass. Condition (1) means that the superfluid motion must always be irrotational, *i.e.*, $\nabla \wedge \mathbf{v} = 0$. This severely limits the possible motions of a superfluid compared to a classical system. For example, in a cylindrical container rotating at angular velocity ω, a classical system rotates with velocity ωr, with r the distance from the center. On the other hand, a superfluid cannot acquire any angular momentum from the container, since the rotational motion would violate the irrotational condition (at higher angular velocities, angular momentum can enter the system through the formation of quantized vortices, but we will not discuss this topic here). Since the moment of inertia is $I = \langle L\rangle/\omega$, with $\langle L\rangle$ the averaged angular momentum, standard superfluids have $I = 0$.

2·2. *Supersolids*. – In his 1970 paper, Leggett wondered to what extent a crystalline solid can show superfluid effects in the form discussed in the previous paragraph. In other words, the question is: if we have a superfluid crystalline lattice, instead of a homogenous superfluid, how much angular momentum can we get from a rotating container? Let us consider an annular geometry with radius R, so that we have a 1D system with periodic boundary conditions. A point on the annulus is specified by the coordinate x, which can vary from 0 to $2\pi R$. Since we need a superfluid, we assume the existence of a macroscopic wavefunction $\Psi(x)$. We take the ground-state density $\rho(x) = |\Psi(x)|^2$ as a given periodic function with lattice period λ. The other degree of freedom of $\Psi(x)$ is its phase, $\varphi(x)$, which determines the velocity field through eq. (1). To find the phase of the ground state, we minimize the kinetic energy due to the rotation $\langle\mathcal{H}_{kin}\rangle = \hbar^2/(2m)\int\left[\nabla^2\varphi(x)\right]\rho(x)\,\mathrm{d}x$, subject to the proper boundary condition (see [4], chapter 1 for a detailed discussion). From the phase $\varphi(x)$ we can easily calculate the velocity field and the result is

$$v(x) = \omega R\left(1 - \frac{k}{\rho(x)}\right), \tag{2}$$

which is plotted in fig. 1(A). The constant is $k = (\lambda^{-1}\int_0^\lambda \mathrm{d}x'/\rho(x'))^{-1}$. From eq. (2) we get a simple interpretation of the moment of inertia. A homogenous superfluid, $\rho(x) = const$, does not move in the rotating container, and hence its moment of inertia is zero. On the other hand, in the supersolid the density maxima are dragged by the container and move quasi-classically, increasing I. However, to satisfy the irrotational condition, the density minima move in the opposite direction and lower I compared to a classical system. Using eq. (2) and the definition of the moment of inertia we obtain $I = I_c(1 - f_s)$, where I_c is the classical moment of inertia of a system with density $\rho(x)$. The variational approach gives for f_s, the so-called superfluid fraction, an upper bound

$$f_s \leq \left(\frac{1}{\lambda}\int_0^\lambda \frac{\mathrm{d}x}{\rho(x)/\bar{\rho}}\right)^{-1}, \tag{3}$$

where $\bar{\rho}$ is the mean density over the annulus. The most interesting feature is that, as we can see from eq. (3), the sub-unity superfluid fraction arises from the broken translational symmetry, and not from thermal effects, as it happens for standard superfluids.

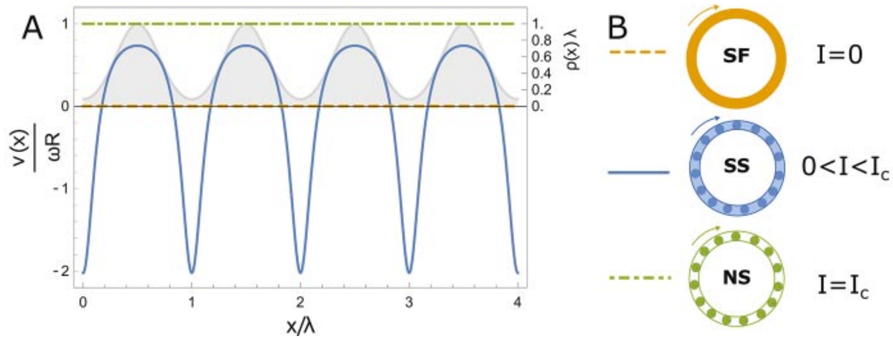

Fig. 1. – Rotation of different states of matter. (A) Velocity field of a supersolid (thick line) and a superfluid (dashed line) from eq. (2), and of a normal solid (dash-dotted line). The supersolid density is taken to be, as an example, the sum of four overlapping gaussians (shown in gray). (B) Sketch of the three cases: homogeneous superfluid (SF), supersolid (SS), normal solid (NS).

3. – Rotation of a dipolar supersolid

Our supersolid is composed of about 4×10^4 atoms, trapped by optical methods in an elliptical geometry in the xy-plane and with strong confinement in the z-direction, along which the atomic dipoles are aligned. The supersolid is made of just 4 droplets in a 1D configuration (fig. 2(D)). To study the rotational properties of such a finite-sized system, we excite a peculiar mode known for BECs, the scissors mode. It consists of an oscillation of the atoms around the main axis of the trap (fig. 2(A)). A useful link exists between the frequency of the oscillation ω_{sc} and the moment of inertia of the system [6]

$$\frac{I}{I_c} \propto \frac{(\omega_y^2 + \omega_x^2)}{\omega_{sc}^2}. \tag{4}$$

Here, ω_x and ω_y are the trapping frequencies in the xy plane and the proportionality constant quantifies the anisotropy of the system. By measuring ω_{sc} we can determine, through eq. (4), the moment of inertia. We do so for different values of the adimensional strength of the dipolar interaction, ϵ_{dd}, which we can tune exploiting Feshbach resonances. The results of both ω_{sc} and I/I_c are shown in fig. 2(B), (C), together with the theoretical points from [7]. For low values of ϵ_{dd}, in the BEC regime, the result is consistent with previous works on standard superfluids [6, 8]. The moment of inertia is finite due to the anisotropy of the trap: it would be zero in a cylindrical trap, as in sect. **2**˙1. Increasing the value of ϵ_{dd}, we cross the transition and form the supersolid. The moment of inertia increases due to the formation of the density modulation, but is still well below the classical value. This observation directly demonstrates the superfluid behavior of the dipolar supersolid. For even larger values of ϵ_{dd}, the overlap between the droplets decreases and I approaches I_c. From the experimental moment of inertia, we estimate the superfluid fraction f_s of the supersolid generalizing its definition to a non-cylindrical geometry [3]. We find $f_s \sim 0.9$ with an error bar which keeps from claiming the observation of a sub-unity f_s. We suggest that the two main contibutions to the measured f_s are a Leggett-type mechanism of mass transport along the lattice direction and the effect of superfluidity on the rotation of each droplet around its center of mass, which isn't accounted for in the 1D model. We estimate the first one to be $f_s \sim 0.3$ using eq. (3) and the 1D density of fig. 2(D), while the second one $f_s \sim 0.5$ with a simple semi-classical model.

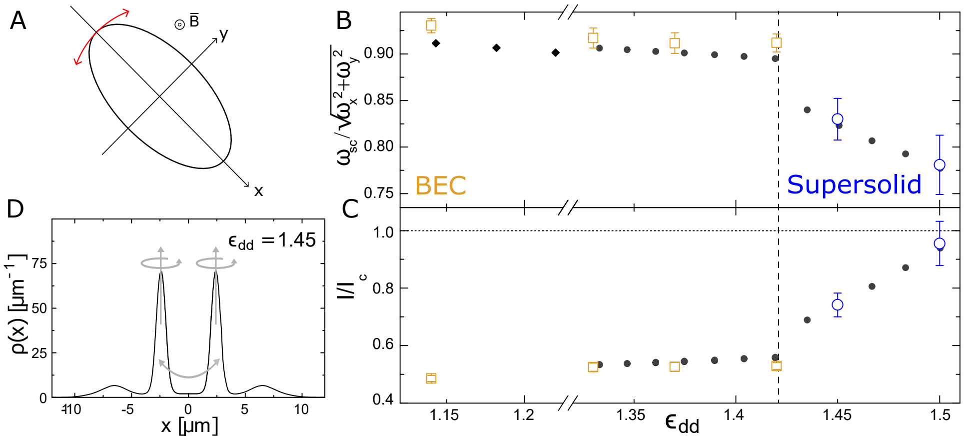

Fig. 2. – Experimental results, adapted from [3]. (A) Sketch of the scissors oscillation of the atoms (ellipse) around the main axis of the trap. (B),(C) Experimental scissors frequencies and moment of inertia from eq. (4) for both BEC (open squares) and supersolid (open circles), together with theoretical simulations (black points) [7], as a function of the interaction parameter ϵ_{dd}. The vertical dashed line indicates the transition between the two phases. (D) 1D cut along the main trap axis of the supersolid density simulated with the experimental parameters, from [7]. Gray arrows indicate the two main contributions to f_s in the scissors mode: Leggett's transport mechanism between droplets and rotation of the single droplets around their own axes.

To conclude, we have directly observed the superfluid behavior of the novel supersolid. The Leggett mechanism likely has a relevant role in the experimental system, but more studies are needed to clarify the connection, as well as to detect a sub-unity superfluid fraction.

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