

On the detailed analysis of the Table I of Ramanujan's work "Modular equations and approximations to π ". New possible mathematical connections with the Cosmological Constant values and some topics of String Theory.

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Abstract

In this paper, we analyze the Table I of Ramanujan's work "Modular equations and approximations to π ". New possible mathematical connections with the Cosmological Constant values and some topics of String Theory.

I want to dedicate this work to Prof. George E. Andrew, always available and very kind for any clarification and who always encourages me to go forward in my research

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Srinivasa Ramanujan (1887-1920)



<https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/>

Vesuvius landscape with gorse – Naples



<https://www.pinterest.it/pin/95068242114589901/>

From:

New Quantum Structure of the Space-Time - Norma G. SANCHEZ -
arXiv:1910.13382v1 [physics.gen-ph] 28 Oct 2019

With regard the Dark Energy and Cosmological constant, we have:

The classical Universe today U_Λ is precisely a *classical dilute gravity vacuum dominated by voids and supervoids as shown by observations: The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q, Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual Equations.*

$$\Lambda = 3H^2 = \lambda_P (H / h_P)^2 = \lambda_P (l_P / L_H)^2 \\ = (2.846 \pm 0.076) 10^{-122} m_P^2$$

$$\Lambda_Q = 3H_Q^2 = \lambda_P (h_P / H)^2 = \lambda_P (L_H / l_P)^2 \\ = (0.3516 \pm 0.094) 10^{122} m_P^2$$

$$\Lambda_Q \Lambda = \lambda_P^2, \quad \lambda_P = 3 h_P^2.$$

The quantum dual value Λ_Q is precisely the quantum value from particle physics:

$$\rho_Q = \rho_P (\Lambda_Q / \lambda_P) = \rho_P^2 / \rho_\Lambda = 10^{122} \rho_P$$

Fundamental are the following results: $2.846 * 10^{-122}$ and $0.3516 * 10^{122}$

$$\Lambda = 3H^2 = \lambda_p (H / h_p)^2 = \lambda_p (l_p / L_H)^2 \\ = (2.846 \pm 0.076) 10^{-122} m_p^2$$

$$\Lambda_Q = 3H_Q^2 = \lambda_p (h_p / H)^2 = \lambda_p (L_H / l_p)^2 \\ = (0.3516 \pm 0.094) 10^{122} m_p^2$$

From:

Modular equations and approximations to π – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

TABLE I

$$g_{62} + \frac{1}{g_{62}} = \frac{1}{2} \{ \sqrt{(1 + \sqrt{2})} + \sqrt{(9 + 5\sqrt{2})} \},$$

$$G_{65}^2 = \sqrt{\left\{ \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3 + \sqrt{13}}{2} \right) \right\}} \left\{ \sqrt{\left(\frac{1 + \sqrt{65}}{8} \right)} + \sqrt{\left(\frac{9 + \sqrt{65}}{8} \right)} \right\},$$

$$g_{66}^2 = \sqrt{(\sqrt{2} + \sqrt{3})(7\sqrt{2} + 3\sqrt{11})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{7 + \sqrt{33}}{8} \right)} + \sqrt{\left(\frac{\sqrt{33} - 1}{8} \right)} \right\},$$

$$G_{69}^2 = (3\sqrt{3} + \sqrt{23})^{\frac{1}{4}} \left(\frac{5 + \sqrt{23}}{4} \right)^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{6 + 3\sqrt{3}}{4} \right)} + \sqrt{\left(\frac{2 + 3\sqrt{3}}{4} \right)} \right\},$$

$$G_{77}^2 = \left\{ \frac{1}{2}(\sqrt{7} + \sqrt{11})(8 + 3\sqrt{7}) \right\}^{\frac{1}{4}} \left\{ \sqrt{\left(\frac{6 + \sqrt{11}}{4} \right)} + \sqrt{\left(\frac{2 + \sqrt{11}}{4} \right)} \right\},$$

$$G_{81}^3 = \frac{(2\sqrt{3} + 2)^{\frac{1}{3}} + 1}{(2\sqrt{3} - 2)^{\frac{1}{3}} - 1},$$

We obtain:

$$\frac{1}{2} \{ \sqrt{(1 + \sqrt{2})} + \sqrt{(9 + 5\sqrt{2})} \},$$

$$1/2 * [\text{sqrt}(1 + \text{sqrt}(2)) + \text{sqrt}(9 + 5\text{sqrt}(2))]$$

Input

$$\frac{1}{2} \left(\sqrt{1 + \sqrt{2}} + \sqrt{9 + 5\sqrt{2}} \right)$$

Decimal approximation

2.7813238039205475596483625188461181855042766742963763507332949901

...

2.781323803920547....

Alternate forms

$$\frac{1}{2} \left(\sqrt{1 + \sqrt{2}} + \sqrt{9 + 5\sqrt{2}} \right)$$

$$\sqrt{\frac{1}{2} \left(5 + 3\sqrt{2} + \sqrt{19 + 14\sqrt{2}} \right)}$$

$$\frac{\sqrt{2(9 - 5\sqrt{2})} + 2\sqrt{9 - \sqrt{31}} + \sqrt{2} \left(2\sqrt{1 + \sqrt{2}} + \frac{\sqrt{10 + 9\sqrt{2}}}{\sqrt[4]{2}} \right)}{4\sqrt{2}}$$

Minimal polynomial

$$x^8 - 10x^6 + 19x^4 - 12x^2 + 4$$

Expanded form

$$\frac{\sqrt{1 + \sqrt{2}}}{2} + \frac{1}{2} \sqrt{9 + 5\sqrt{2}}$$

From:

$$: \sqrt{\left\{ \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3 + \sqrt{13}}{2} \right) \right\} \left\{ \sqrt{\left(\frac{1 + \sqrt{65}}{8} \right)} + \sqrt{\left(\frac{9 + \sqrt{65}}{8} \right)} \right\}},$$

$$\text{Sqrt}(\left(\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{3+\sqrt{13}}{2}\right)\right)\left(\sqrt{\left(\frac{1+\sqrt{65}}{8}\right)}+\sqrt{\left(\frac{9+\sqrt{65}}{8}\right)}\right))$$

Input

$$\sqrt{\left(\frac{1}{2}(1 + \sqrt{5})\right)\left(\frac{1}{2}(3 + \sqrt{13})\right) \left(\sqrt{\frac{1}{8}(1 + \sqrt{65})} + \sqrt{\frac{1}{8}(9 + \sqrt{65})}\right)}$$

Result

$$\frac{1}{2} \sqrt{(1 + \sqrt{5})(3 + \sqrt{13})} \left(\frac{1}{2} \sqrt{\frac{1}{2}(1 + \sqrt{65})} + \frac{1}{2} \sqrt{\frac{1}{2}(9 + \sqrt{65})} \right)$$

Decimal approximation

5.8364372603724419137911559828280153147254752692005302828075165079

...

5.8364372603724....

Alternate forms

$$\frac{1}{8} \sqrt{(1 + \sqrt{5})(3 + \sqrt{13})} \left(\sqrt{2(1 + \sqrt{65})} + \sqrt{2(9 + \sqrt{65})} \right)$$

$$\frac{1}{4} \sqrt{\frac{1}{2}(1 + \sqrt{5})(3 + \sqrt{13})} \left(\sqrt{1 + \sqrt{65}} + \sqrt{9 + \sqrt{65}} \right)$$

$$\left(\frac{1}{8}\sqrt{1-8i} + \frac{1}{8}\sqrt{1+8i} + \frac{\sqrt{5}}{8} + \frac{\sqrt{13}}{8}\right)\sqrt{(1+\sqrt{5})(3+\sqrt{13})}$$

Minimal polynomial

$$x^8 - 8x^7 + 12x^6 + 8x^5 - 27x^4 + 8x^3 + 12x^2 - 8x + 1$$

Expanded forms

$$\frac{1}{4}\left(\sqrt{34+8\sqrt{5}+8\sqrt{13}+2\sqrt{65}} + \sqrt{46+20\sqrt{5}+12\sqrt{13}+6\sqrt{65}}\right)$$

$$\frac{1}{4}\sqrt{\frac{1}{2}(1+\sqrt{5})(3+\sqrt{13})(1+\sqrt{65})} + \frac{1}{4}\sqrt{\frac{1}{2}(1+\sqrt{5})(3+\sqrt{13})(9+\sqrt{65})}$$

From:

$$\sqrt{(\sqrt{2}+\sqrt{3})(7\sqrt{2}+3\sqrt{11})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{7+\sqrt{33}}{8}\right)} + \sqrt{\left(\frac{\sqrt{33}-1}{8}\right)} \right\},$$

$$\text{Sqrt}(\text{sqrt}2+\text{sqrt}3) (7\text{sqrt}2+3\text{sqrt}11)^{(1/6)} ((\text{sqrt}((7+\text{sqrt}33)/8))+\text{sqrt}((\text{sqrt}33 - 1)/8)))$$

Input

$$\sqrt{\sqrt{2}+\sqrt{3}} \sqrt[6]{7\sqrt{2}+3\sqrt{11}} \left(\sqrt{\frac{1}{8}(7+\sqrt{33})} + \sqrt{\frac{1}{8}(\sqrt{33}-1)} \right)$$

Result

$$\sqrt{\sqrt{2} + \sqrt{3}} \sqrt[6]{7\sqrt{2} + 3\sqrt{11}} \left(\frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{33} - 1)} + \frac{1}{2} \sqrt{\frac{1}{2}(7 + \sqrt{33})} \right)$$

Decimal approximation

5.9316041484156843631215221854067364890013778319170782799968092734

...

5.93160414841568....

Alternate forms

$$\frac{1}{4} \sqrt{\sqrt{2} + \sqrt{3}} \sqrt[6]{7\sqrt{2} + 3\sqrt{11}} \left(\sqrt{2(\sqrt{33} - 1)} + \sqrt{2(7 + \sqrt{33})} \right)$$

$$\frac{1}{2} \sqrt[4]{5 + 2\sqrt{6}} \sqrt[12]{197 + 42\sqrt{22}} \sqrt{3 + \sqrt{33} + \sqrt{26 + 6\sqrt{33}}}$$

| | |
|------------------------------------------------------------------------------------------------------|------------|
| root of $x^8 - 204x^7 - 972x^6 - 1716x^5 - 2410x^4 - 1716x^3 - 972x^2 - 204x + 1$ near $x = 208.697$ | $^{(1/3)}$ |
|------------------------------------------------------------------------------------------------------|------------|

Minimal polynomial

$$x^{24} - 204x^{21} - 972x^{18} - 1716x^{15} - 2410x^{12} - 1716x^9 - 972x^6 - 204x^3 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt[6]{7\sqrt{2} + 3\sqrt{11}} \left(\sqrt{-\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} + \frac{3\sqrt{11}}{2} + \sqrt{\frac{33}{2}}} + \sqrt{\frac{7}{\sqrt{2}} + \frac{7\sqrt{3}}{2} + \frac{3\sqrt{11}}{2} + \sqrt{\frac{33}{2}}} \right)$$

$$\frac{\sqrt[6]{7\sqrt{2} + 3\sqrt{11}} \sqrt{(\sqrt{2} + \sqrt{3})(\sqrt{33} - 1)}}{2\sqrt{2}} + \frac{\sqrt[6]{7\sqrt{2} + 3\sqrt{11}} \sqrt{(\sqrt{2} + \sqrt{3})(7 + \sqrt{33})}}{2\sqrt{2}}$$

From:

$$(3\sqrt{3} + \sqrt{23})^{\frac{1}{4}} \left(\frac{5 + \sqrt{23}}{4}\right)^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{6 + 3\sqrt{3}}{4}\right)} + \sqrt{\left(\frac{2 + 3\sqrt{3}}{4}\right)} \right\},$$

$$(3\sqrt{3} + \sqrt{23})^{0.25} (1/4 * 5 + \sqrt{23})^{(1/6)} ((\sqrt{((6 + 3\sqrt{3})/4)} + \sqrt{((2 + 3\sqrt{3})/4)}))$$

Input

$$(3\sqrt{3} + \sqrt{23})^{0.25} \sqrt[6]{\frac{1}{4}(5 + \sqrt{23})} \left(\sqrt{\frac{1}{4}(6 + 3\sqrt{3})} + \sqrt{\frac{1}{4}(2 + 3\sqrt{3})} \right)$$

Result

6.2220252193329289445223139953386056692521793524912615960732461473

...

6.2220252193329....

From:

$$\left\{ \frac{1}{2}(\sqrt{7} + \sqrt{11})(8 + 3\sqrt{7}) \right\}^{\frac{1}{4}} \left\{ \sqrt{\left(\frac{6 + \sqrt{11}}{4}\right)} + \sqrt{\left(\frac{2 + \sqrt{11}}{4}\right)} \right\},$$

$$\left(\frac{1}{2}(\sqrt{7} + \sqrt{11})(8 + 3\sqrt{7}) \right)^{0.25} \left(\sqrt{\left(\frac{6 + \sqrt{11}}{4}\right)} + \sqrt{\left(\frac{2 + \sqrt{11}}{4}\right)} \right)$$

Input

$$\left(\frac{1}{2}(\sqrt{7} + \sqrt{11})(8 + 3\sqrt{7}) \right)^{0.25} \left(\sqrt{\frac{1}{4}(6 + \sqrt{11})} + \sqrt{\frac{1}{4}(2 + \sqrt{11})} \right)$$

Result

7.0336566102533820668630429692660792210320262042164304387709010049

...

7.033656610253....

From:

$$\frac{(2\sqrt{3} + 2)^{\frac{1}{3}} + 1}{(2\sqrt{3} - 2)^{\frac{1}{3}} - 1},$$

$$\left((2\sqrt{3} + 2)^{\frac{1}{3}} + 1 \right) / \left((2\sqrt{3} - 2)^{\frac{1}{3}} - 1 \right)$$

Input

$$\frac{\sqrt[3]{2\sqrt{3} + 2} + 1}{\sqrt[3]{2\sqrt{3} - 2} - 1}$$

Decimal approximation

20.377499977251773758491214090560486670678090121282523865667554079

...

20.37749997725....

Alternate forms

$$\frac{\sqrt[3]{2+2\sqrt{3}}}{\sqrt[3]{2\sqrt{3}-2}-1} + \frac{1}{\sqrt[3]{2\sqrt{3}-2}-1}$$

$$\frac{1 + \sqrt[3]{2(1+\sqrt{3})}}{\sqrt[3]{2(\sqrt{3}-1)} - 1}$$

root of $x^6 - 18x^5 - 45x^4 - 68x^3 - 45x^2 - 18x + 1$ near $x = 20.3775$

Minimal polynomial

$$x^6 - 18x^5 - 45x^4 - 68x^3 - 45x^2 - 18x + 1$$

From the sum of the above results, we obtain:

$$(2.781323803920547 + 5.8364372603724 + 5.93160414841568 + 6.2220252193329 + 7.033656610253 + 20.37749997725)$$

Input interpretation

$$2.781323803920547 + 5.8364372603724 + 5.93160414841568 + 6.2220252193329 + 7.033656610253 + 20.37749997725$$

Result

48.182547019544527

48.182547019544527

From which:

$$(2.781323803920547 + 5.8364372603724 + 5.93160414841568 + 6.2220252193329 + 7.033656610253 + 20.37749997725)^{1/(3+(\sqrt{29}-5))}$$

where

$$\sqrt{29} - 5 \approx 0.38516480$$

Input interpretation

$$(2.781323803920547 + 5.8364372603724 + 5.93160414841568 + 6.2220252193329 + 7.033656610253 + 20.37749997725)^{\left(\frac{1}{3 + (\sqrt{29} - 5)}\right)}$$

Result

3.141497564718...

3.141497564718.... $\approx \pi$

Multiplying the various results, we obtain:

$$(2.781323 * 5.836437 * 5.931604 * 6.222025 * 7.033656 * 20.377499)^{1/10} + ((2 \log(3))/3^4)$$

Input interpretation

$$\sqrt[10]{2.781323 \times 5.836437 \times 5.931604 \times 6.222025 \times 7.033656 \times 20.377499} + \frac{2 \log(3)}{3^4}$$

$\log(x)$ is the natural logarithm

Result

3.1415916511053978047750264704704836995957104773679259231156236466

...

3.1415916511.... $\approx \pi$

Alternative representations

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$\sqrt[10]{85868.7} + \frac{2 \log_e(3)}{3^4}$$

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$\sqrt[10]{85868.7} + \frac{2 \log(a) \log_a(3)}{3^4}$$

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$\sqrt[10]{85868.7} + \frac{4 \coth^{-1}(2)}{3^4}$$

Series representations

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$3.11447 + 0.0246914 \log(2) - 0.0246914 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k}{k}$$

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$3.11447 + \frac{4}{81} i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \frac{2 \log(x)}{81} - \frac{2}{81} \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$3.11447 + \frac{2}{81} \left\lfloor \frac{\arg(3 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \frac{2 \log(z_0)}{81} +$$

$$\frac{2}{81} \left\lfloor \frac{\arg(3 - z_0)}{2\pi} \right\rfloor \log(z_0) - \frac{2}{81} \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}$$

Integral representations

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$3.11447 + 0.0246914 \int_1^3 \frac{1}{t} dt$$

$$\sqrt[10]{2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775} + \frac{2 \log(3)}{3^4} =$$

$$3.11447 + \frac{1}{81 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$\frac{1}{3} \ln(2.781323 * 5.836437 * 5.931604 * 6.222025 * 7.033656 * 20.377499) -$$

$$\left(\frac{1}{3} \left(\left(\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\left(\frac{105+5\sqrt{505}}{8} \right)^3} \right)^{1/14} \right) + \phi + \zeta(2) \right) - 1 \right)$$

where

$$\frac{1}{3} \left(\phi + \left(\frac{1}{2} \sqrt{\frac{1}{2} (105 + 5\sqrt{505})} + \frac{1}{2} \sqrt{\frac{1}{2} (113 + 5\sqrt{505})} \right)^{3/14} + \frac{\pi^2}{6} \right) - 1$$

0.6395842014676220030987838542575903154991090651644197533701212155

...

value that is the mean between $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$, the value of golden ratio 1.61803398... and the 14th root of the Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578..., i.e. 1.6395842014..., minus 1

Input interpretation

$$\frac{1}{3} \log(2.781323 \times 5.836437 \times 5.931604 \times 6.222025 \times 7.033656 \times 20.377499) - \left(\frac{1}{3} \left(\sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3 + \phi + \zeta(2) \right) - 1 \right)$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result

3.1472742095601914382035073953983339724975463043107755529103047041

...

3.14727420956.... $\approx \pi$

Alternative representations

$$\begin{aligned} & \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\ & \left(\frac{1}{3} \left(\sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3 + \phi + \zeta(2) \right) - 1 \right) = \\ & 1 + \frac{\log(85868.7)}{3} + \\ & \frac{1}{3} \left(-\phi - \sqrt[14]{ \left(\sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} \right)^3 - \zeta(2, 1) \right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\ & \left(\frac{1}{3} \left(\sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3 + \phi + \zeta(2) \right) - 1 \right) = \\ & 1 + \frac{\log(85868.7)}{3} + \\ & \frac{1}{3} \left(-\phi - \sqrt[14]{ \left(\sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} \right)^3 - \frac{\zeta(2, \frac{1}{2})}{3} \right) \end{aligned}$$

$$\frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) -$$

$$\left(\frac{1}{3} \left[\sqrt[14]{\left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3} + \phi + \zeta(2) \right] - 1 \right) =$$

$$1 + \frac{\log_e(85868.7)}{3} +$$

$$\frac{1}{3} \left(-\phi - \sqrt[14]{\left(\sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} \right)^3} - \zeta(2, 1) \right)$$

$\zeta(s, a)$ is the generalized Riemann zeta function

Series representations

$$\frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) -$$

$$\left(\frac{1}{3} \left[\sqrt[14]{\left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3} + \phi + \zeta(2) \right] - 1 \right) =$$

$$\frac{1}{3} \left(3 - \phi + \log(85867.7) + 3 \sum_{k=1}^{\infty} -\frac{1 + (-0.0000116458)^k k}{3 k^2} - \right.$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k x^{-k} \left(-\frac{1}{2} \right)_k \left(\exp \left(i \pi \left[\frac{\arg(-x + \frac{1}{8} (113 + 5 \sqrt{505}))}{2 \pi} \right] \right) \right. \right.$$

$$\left. \left(\frac{113}{8} - x + \frac{5 \sqrt{505}}{8} \right)^k + \right.$$

$$\left. \exp \left(i \pi \left[\frac{\arg(-x + \frac{5}{8} (21 + \sqrt{505}))}{2 \pi} \right] \right) \right.$$

$$\left. \left(-x + \frac{5}{8} (21 + \sqrt{505}) \right)^k \right)$$

$$\left. \sqrt{x} \right)^3 \wedge (1/14) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\
& \left(\frac{1}{3} \left[{}^{14}\sqrt{\left(\sqrt{\frac{1}{8}(113 + 5\sqrt{505})} + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})} \right)^3} + \phi + \zeta(2) \right] - 1 \right) = \\
& \frac{1}{3} \left(3 - \phi - \exp\left(\sum_{k=1}^{\infty} \frac{P(2k)}{k} \right) + \log(85867.7) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-11.3606k}}{k} - \right. \\
& \left. \left(\sqrt{x}^{-3} \left(\sum_{k=0}^{\infty} \frac{1}{k!} x^{-k} \left(-\frac{1}{2} \right)_k \left(-\frac{1}{8} \right)^k \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{113}{8} - x + \frac{5\sqrt{505}}{8} \right)}{2\pi} \right\rfloor \right) \right) \right) \right. \\
& \quad \left. \left((113 - 8x + 5\sqrt{505})^k + (-1)^k \right) \right. \\
& \quad \left. \exp\left(i\pi \left\lfloor \frac{\arg\left(-x + \frac{5}{8}(21 + \sqrt{505}) \right)}{2\pi} \right\rfloor \right) \right) \right. \\
& \quad \left. \left(-x + \frac{5}{8}(21 + \sqrt{505}) \right)^k \right) \right) \right) \wedge \\
& \left. (1/14) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

Integral representations

$$\begin{aligned}
& \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\
& \left(\frac{1}{3} \left[{}^{14}\sqrt{\left(\sqrt{\frac{1}{8}(113 + 5\sqrt{505})} + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})} \right)^3} + \phi + \zeta(2) \right] - 1 \right) = \\
& 1 - \frac{\phi}{3} - \frac{1}{6 \times 0!} \int_0^1 \frac{\log^2(1-t)}{t^2} dt + \frac{\log(85868.7)}{3} - \\
& \frac{1}{3} {}^{14}\sqrt{\left(\sqrt{\frac{5}{8}(21 + \sqrt{505})} + \sqrt{\frac{1}{8}(113 + 5\sqrt{505})} \right)^3}
\end{aligned}$$

$$\begin{aligned} & \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\ & \left(\frac{1}{3} \left[{}^{14}\sqrt{\left(\sqrt{\frac{1}{8}} (113 + 5 \sqrt{505}) + \sqrt{\frac{1}{8}} (105 + 5 \sqrt{505}) \right)^3 + \phi + \zeta(2)} \right] - 1 \right) = \\ & 1 - \frac{\phi}{3} + \int_1^{85868.7} \left(\frac{1}{3t} - \frac{14311.3 \log^2(1.00001 - 0.0000116458t)}{(-1+t)^2 0!} \right) dt - \\ & \frac{1}{3} {}^{14}\sqrt{\left(\sqrt{\frac{1}{8}} (105 + 5 \sqrt{505}) + \sqrt{\frac{1}{8}} (113 + 5 \sqrt{505}) \right)^3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{3} \log(2.78132 \times 5.83644 \times 5.9316 \times 6.22203 \times 7.03366 \times 20.3775) - \\ & \left(\frac{1}{3} \left[{}^{14}\sqrt{\left(\sqrt{\frac{1}{8}} (113 + 5 \sqrt{505}) + \sqrt{\frac{1}{8}} (105 + 5 \sqrt{505}) \right)^3 + \phi + \zeta(2)} \right] - 1 \right) = \\ & - \frac{1}{18 \times 0!} \left(-18 \times 0! + 6 \phi 0! - 6 \times 0! \int_1^{85868.7} \frac{1}{t} dt + \pi^2 \int_0^1 E_0(x) dx + \right. \\ & \left. 6 \times 0! {}^{14}\sqrt{\left(\sqrt{\frac{5}{8}} (21 + \sqrt{505}) + \sqrt{\frac{1}{8}} (113 + 5 \sqrt{505}) \right)^3} \right) \end{aligned}$$

$E_n(x)$ is the Euler polynomial

$$1/(5+0.96254)*(-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499))$$

Input interpretation

$$\frac{1}{5 + 0.96254} (-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499))$$

Result

1.6449321933270049341388066159723876066240226480660926383722373350

...

1.644932193327... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$ (trace of the instanton shape)

From which:

$((1/(5+0.96254)*(-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499))))^{15-18+1/3}$

Input interpretation

$$\left(\frac{1}{5 + 0.96254} (-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499)) \right)^{15} - 18 + \frac{1}{3}$$

Result

1729.0401108083420773024229931784549134139279488721618046986907043

...

1729.0401108....

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$(1/27(((1/(5+0.96254)*(-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499))))^{15-18-1/\sqrt{2}}))^2$

Input interpretation

$$\left(\frac{1}{27} \left(\left(\frac{1}{5 + 0.96254} (-(-2.781323 - 5.836437 + 5.931604 + 6.222025 + 7.033656 - 20.377499)) \right)^{15} - 18 - \frac{1}{\sqrt{2}} \right) \right)^2$$

Result

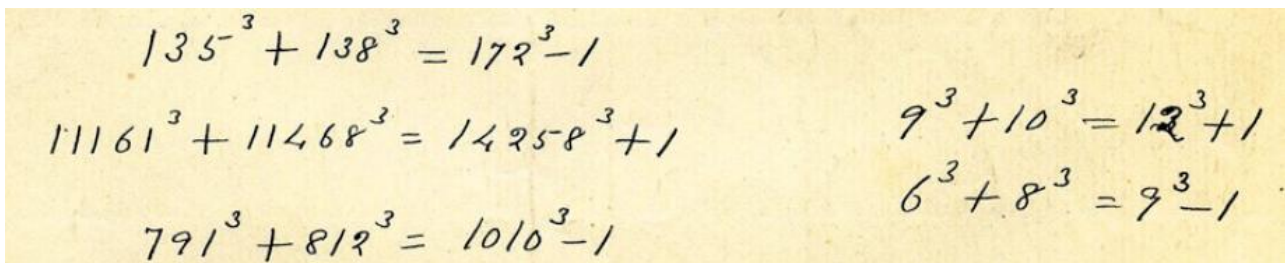
4096.00...

$$4096.00\dots \approx 64^2$$

We obtain also:

$$\frac{1}{2}(11468+14258-1010-812-1729-216)/((299792458(\frac{1}{2.781323} * \frac{1}{5.836437} * \frac{1}{5.931604} * \frac{1}{6.222025} * \frac{1}{7.033656} * \frac{1}{20.377499})))$$

where $299792458 = c$ speed of light and all the numbers highlighted in red can be to obtain easily from the following Ramanujan taxicab numbers:



Handwritten Ramanujan taxicab numbers on aged paper:

$$135^3 + 138^3 = 172^3 - 1$$
$$11161^3 + 11468^3 = 14258^3 + 1$$
$$791^3 + 812^3 = 1010^3 - 1$$
$$9^3 + 10^3 = 12^3 + 1$$
$$6^3 + 8^3 = 9^3 - 1$$

Input interpretation

$$\frac{1}{2} \times \frac{11468 + 14258 - 1010 - 812 - 1729 - 216}{299792458 \left(\frac{1}{2.781323} \times \frac{1}{5.836437} \times \frac{1}{5.931604} \times \frac{1}{6.222025} \times \frac{1}{7.033656} \times \frac{1}{20.377499} \right)}$$

Result

3.1448287061761827715869738323018119894409044806590831581226769887

...

$$3.14482870617\dots \approx \pi$$

We have that:

$$g_{90} = \{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})\}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{3 + \sqrt{6}}{4}\right)} + \sqrt{\left(\frac{\sqrt{6} - 1}{4}\right)} \right\},$$

$$g_{94} + \frac{1}{g_{94}} = \frac{1}{2} \{ \sqrt{7 + \sqrt{2}} + \sqrt{7 + 5\sqrt{2}} \},$$

$$g_{98} + \frac{1}{g_{98}} = \frac{1}{2} \{ \sqrt{2} + \sqrt{14 + 4\sqrt{14}} \},$$

$$g_{114}^2 = \sqrt{(\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{19})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{23 + 3\sqrt{57}}{8}\right)} + \sqrt{\left(\frac{15 + 3\sqrt{57}}{8}\right)} \right\},$$

From:

$$\{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})\}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{3 + \sqrt{6}}{4}\right)} + \sqrt{\left(\frac{\sqrt{6} - 1}{4}\right)} \right\},$$

$$((2+\sqrt{5})(\sqrt{5}+\sqrt{6}))^{(1/6)} [(1/4*(3+\sqrt{6}))^{0.5}+(1/4*(\sqrt{6} - 1))^{0.5}]$$

Input

$$\sqrt[6]{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})} \left(\sqrt{\frac{1}{4}(3 + \sqrt{6})} + \sqrt{\frac{1}{4}(\sqrt{6} - 1)} \right)$$

Exact result

$$\sqrt[6]{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})} \left(\frac{1}{2} \sqrt{\sqrt{6} - 1} + \frac{\sqrt{3 + \sqrt{6}}}{2} \right)$$

Decimal approximation

2.9111166557745929627683335068014893928756125470088426388712758640

...

2.911116655774....

Alternate forms

$$\frac{1}{2} \sqrt[6]{5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{30}} \left(\sqrt{\sqrt{6} - 1} + \sqrt{3 + \sqrt{6}} \right)$$

$$\sqrt[6]{\begin{array}{l} \text{root of } x^8 - 608x^7 - 388x^6 + 544x^5 - 506x^4 - 544x^3 - 388x^2 + 608x + 1 \\ \text{near } x = 608.636 \end{array}}$$

Minimal polynomial

$$x^{48} - 608x^{42} - 388x^{36} + 544x^{30} - 506x^{24} - 544x^{18} - 388x^{12} + 608x^6 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt{\sqrt{6} - 1} \sqrt[6]{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})} + \frac{1}{2} \sqrt{3 + \sqrt{6}} \sqrt[6]{(2 + \sqrt{5})(\sqrt{5} + \sqrt{6})}$$

$$\frac{1}{2} \sqrt{\sqrt{6} - 1} \sqrt[6]{5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{30}} + \frac{1}{2} \sqrt{3 + \sqrt{6}} \sqrt[6]{5 + 2\sqrt{5} + 2\sqrt{6} + \sqrt{30}}$$

From:

$$\frac{1}{2} \left\{ \sqrt{(7 + \sqrt{2})} + \sqrt{(7 + 5\sqrt{2})} \right\},$$

$$1/2 [(7+\sqrt{2})^{0.5} + (7+5\sqrt{2})^{0.5}]$$

Input

$$\frac{1}{2} \left(\sqrt{7 + \sqrt{2}} + \sqrt{7 + 5\sqrt{2}} \right)$$

Decimal approximation

3.3259342931326123801433172293996761089308505243459596578805645089

...

3.32593429313....

Alternate forms

$$\sqrt{\frac{1}{2} \left(7 + 3\sqrt{2} + \sqrt{59 + 42\sqrt{2}} \right)}$$

$$\frac{1}{2} \left(\sqrt{\frac{7-i}{2} - \frac{i}{2}} + \sqrt{\frac{7+i}{2} + \frac{i}{2}} + \sqrt{7+\sqrt{2}} \right)$$

$$\frac{1}{2} \sqrt{\frac{7-i}{2} - \frac{i}{2}} + \frac{1}{2} \sqrt{\frac{7+i}{2} + \frac{i}{2}} + \frac{\sqrt{7+\sqrt{2}}}{2}$$

Minimal polynomial

$$x^8 - 14x^6 + 35x^4 - 28x^2 + 4$$

Expanded form

$$\frac{\sqrt{7+\sqrt{2}}}{2} + \frac{1}{2} \sqrt{7+5\sqrt{2}}$$

From:

$$\frac{1}{2}\{\sqrt{2} + \sqrt{(14 + 4\sqrt{14})}\},$$

$$1/2 [(\text{sqrt}2) + (14+4\text{sqrt}14)^{0.5}]$$

Input

$$\frac{1}{2}\left(\sqrt{2} + \sqrt{14 + 4\sqrt{14}}\right)$$

Decimal approximation

3.3981395544492481259506600332389876731961163251426068708034394154

...

3.39813955444....

Alternate forms

$$\frac{1}{\sqrt{2}} + \sqrt{\frac{7}{2} + \sqrt{14}}$$

$$\frac{1 + \sqrt{7 + 2\sqrt{14}}}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} + \sqrt{\frac{1}{2}(7 + 2\sqrt{14})}$$

Minimal polynomial

$$x^8 - 16x^6 + 54x^4 - 32x^2 + 25$$

Expanded form

$$\frac{1}{\sqrt{2}} + \frac{1}{2} \sqrt{14 + 4\sqrt{14}}$$

From:

$$\sqrt{(\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{19})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{23 + 3\sqrt{57}}{8}\right)} + \sqrt{\left(\frac{15 + 3\sqrt{57}}{8}\right)} \right\},$$

$$\begin{aligned} & (\sqrt{2} + \sqrt{3})^{0.5} (3\sqrt{2} + \sqrt{19})^{1/6} \\ & [(1/8 * (23 + 3\sqrt{57}))^{0.5} + (1/8 * (15 + 3\sqrt{57}))^{0.5}] \end{aligned}$$

Input

$$\sqrt{\sqrt{2} + \sqrt{3}} \sqrt[6]{3\sqrt{2} + \sqrt{19}} \left(\sqrt{\frac{1}{8}(23 + 3\sqrt{57})} + \sqrt{\frac{1}{8}(15 + 3\sqrt{57})} \right)$$

Exact result

$$\sqrt{\sqrt{2} + \sqrt{3}} \sqrt[6]{3\sqrt{2} + \sqrt{19}} \left(\frac{1}{2} \sqrt{\frac{1}{2}(15 + 3\sqrt{57})} + \frac{1}{2} \sqrt{\frac{1}{2}(23 + 3\sqrt{57})} \right)$$

Decimal approximation

11.573048152106843872312344073975316163654599436688727342946157522

...

11.5730481521....

Alternate forms

$$\frac{1}{4} \sqrt[4]{5+2\sqrt{6}} \sqrt[12]{37+6\sqrt{38}} \left(3\sqrt{3} + \sqrt{19} + \sqrt{6(5+\sqrt{57})} \right)$$

$$\frac{1}{4} \sqrt{\sqrt{2} + \sqrt{3}} \sqrt[6]{3\sqrt{2} + \sqrt{19}} \left(\sqrt{6(5+\sqrt{57})} + \sqrt{46+6\sqrt{57}} \right)$$

$$\sqrt[3]{\text{root of } x^8 - 1548x^7 - 3180x^6 + 24588x^5 - 47914x^4 + 24588x^3 - 3180x^2 - 1548x + 1 \text{ near } x = 1550.04}$$

Minimal polynomial

$$x^{24} - 1548x^{21} - 3180x^{18} + 24588x^{15} - 47914x^{12} + 24588x^9 - 3180x^6 - 1548x^3 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt[6]{3\sqrt{2} + \sqrt{19}} \left(\sqrt{\frac{15}{\sqrt{2}} + \frac{15\sqrt{3}}{2} + \frac{9\sqrt{19}}{2} + 3\sqrt{\frac{57}{2}}} + \sqrt{\frac{23}{\sqrt{2}} + \frac{23\sqrt{3}}{2} + \frac{9\sqrt{19}}{2} + 3\sqrt{\frac{57}{2}}} \right)$$

$$\frac{\sqrt[6]{3\sqrt{2} + \sqrt{19}} \sqrt{(\sqrt{2} + \sqrt{3})(15 + 3\sqrt{57})}}{2\sqrt{2}} + \frac{\sqrt[6]{3\sqrt{2} + \sqrt{19}} \sqrt{(\sqrt{2} + \sqrt{3})(23 + 3\sqrt{57})}}{2\sqrt{2}}$$

Dividing the second by the first expression and adding 2, we obtain:

$$\frac{((1/2 [(7+\sqrt{2})^{0.5} + (7+5\sqrt{2})^{0.5}]))}{(((2+\sqrt{5})(\sqrt{5}+\sqrt{6}))^{(1/6)})} \frac{1}{[(1/4*(3+\sqrt{6}))^{0.5}+(1/4*(\sqrt{6}-1))^{0.5}]} + 2$$

Input

$$\frac{\frac{1}{2} \left(\sqrt{7+\sqrt{2}} + \sqrt{7+5\sqrt{2}} \right)}{\sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\sqrt{\frac{1}{4}(3+\sqrt{6})} + \sqrt{\frac{1}{4}(\sqrt{6}-1)} \right)} + 2$$

Exact result

$$2 + \frac{\sqrt{7+\sqrt{2}} + \sqrt{7+5\sqrt{2}}}{2 \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\frac{1}{2} \sqrt{\sqrt{6}-1} + \frac{\sqrt{3+\sqrt{6}}}{2} \right)}$$

Decimal approximation

3.1424943368501473902001770286168883802683042701077234897369743160

...

3.14249433685.... $\approx \pi$

Alternate forms

$$2 + \left(\frac{\left(\text{root of } x^4 - 14x^2 + 50 \text{ near } x = 2.65246 - 0.188504i \right) + \left(\text{root of } x^4 - 14x^2 + 50 \text{ near } x = 2.65246 + 0.188504i \right) + \left(\text{root of } x^4 - 28x^2 + 188 \text{ near } x = 4.10225 \right)}{\left(\sqrt{2} \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\left(\text{root of } x^4 - 6x^2 + 3 \text{ near } x = 2.33441 \right) + \left(\text{root of } x^4 + 2x^2 - 5 \text{ near } x = 1.20395 \right) \right) \right)}$$

$$\left(\sqrt{7+\sqrt{2}} + \sqrt{7+5\sqrt{2}} + 2\sqrt{\sqrt{6}-1} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}} + \right. \\ \left. 2\sqrt{3+\sqrt{6}} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}} \right) / \\ \left(\sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}} \left(\sqrt{\sqrt{6}-1} + \sqrt{3+\sqrt{6}} \right) \right)$$

$$\left(\begin{array}{l} \text{root of } x^4 - 14x^2 + 50 \text{ near } x = 2.65246 - 0.188504i + \\ \text{root of } x^4 - 14x^2 + 50 \text{ near } x = 2.65246 + 0.188504i + \\ \text{root of } x^4 - 28x^2 + 188 \text{ near } x = 4.10225 + \\ 2\sqrt{2} \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \text{root of } x^4 - 6x^2 + 3 \text{ near } x = 2.33441 + \\ 2\sqrt{2} \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \\ \text{root of } x^4 + 2x^2 - 5 \text{ near } x = 1.20395 \end{array} \right) / \\ \left(\sqrt{2} \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\text{root of } x^4 - 6x^2 + 3 \text{ near } x = 2.33441 + \right. \right. \\ \left. \left. \text{root of } x^4 + 2x^2 - 5 \text{ near } x = 1.20395 \right) \right)$$

Expanded forms

$$2 + \frac{\sqrt{7+\sqrt{2}}}{2 \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\frac{1}{2} \sqrt{\sqrt{6}-1} + \frac{\sqrt{3+\sqrt{6}}}{2} \right)} + \\ \frac{\sqrt{7+5\sqrt{2}}}{2 \sqrt[6]{(2+\sqrt{5})(\sqrt{5}+\sqrt{6})} \left(\frac{1}{2} \sqrt{\sqrt{6}-1} + \frac{\sqrt{3+\sqrt{6}}}{2} \right)}$$

2+

$$\frac{\sqrt{7+\sqrt{2}}}{\sqrt{\sqrt{6}-1} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}} + \sqrt{3+\sqrt{6}} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}}}$$

$$+$$

$$\frac{\sqrt{7+5\sqrt{2}}}{\sqrt{\sqrt{6}-1} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}} + \sqrt{3+\sqrt{6}} \sqrt[6]{5+2\sqrt{5}+2\sqrt{6}+\sqrt{30}}}$$

In conclusion, from the various results, after some calculations, we obtain:

$$\begin{aligned} &(((3.32593429313) * 2.911116655774 * ((11.5730481521 / (3.39813955444 + 3.3259342 \\ &9313) * 1/2.911116655774)))) - 0.9270108 - \\ &((\sqrt{((113+5\sqrt{505})/8)} + \sqrt{((105+5\sqrt{505})/8)})^3)^{1/14} \end{aligned}$$

Input interpretation

$$\begin{aligned} &3.32593429313 \times 2.911116655774 \\ &\left(\frac{11.5730481521}{3.39813955444 + 3.32593429313} \times \frac{1}{2.911116655774} \right) - \\ &0.9270108 - \sqrt[14]{\left(\sqrt{\frac{1}{8}(113 + 5\sqrt{505})} + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})} \right)^3} \end{aligned}$$

Result

3.1415911760434952274826374031214729294618082485247885964116134105

...

3.141591176..... $\approx \pi$

From which, after some calculations, we obtain:

$$\begin{aligned} &(1/6(((3.32593429313) * 2.911116655774 * ((11.5730481521 / (3.39813955444 + 3.3259 \\ &3429313) * 1/2.911116655774)))) - 0.9270108 - \\ &((\sqrt{((113+5\sqrt{505})/8)} + \sqrt{((105+5\sqrt{505})/8)})^3)^{1/14})^2)^{15-18+1/\pi} \end{aligned}$$

Input interpretation

$$\left(\frac{1}{6} \left(3.32593429313 \times 2.911116655774 \left(\frac{11.5730481521}{3.39813955444 + 3.32593429313} \times \frac{1}{2.911116655774} \right) - 0.9270108 - \sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3 } \right)^2 \right)^{15} - 18 + \frac{1}{\pi}$$

Result

1729.03...

1729.03.....

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$\left(\frac{1}{6} \left((3.325934293130000 \times 2.9111166557740000 \times 11.57304815210000) / \frac{((3.398139554440000 + 3.325934293130000) \times 2.9111166557740000) - 0.927011 - \sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3 } \right)^2 \right)^{15} - 18 + \frac{1}{\pi} = -18 + \frac{1}{\pi} + \frac{1}{470184984576} \left(4.79738 - \left(\sum_{k=0}^{\infty} \binom{1}{2k} \left(8^k (97 + 5\sqrt{505})^{-k} \sqrt{\frac{97}{8} + \frac{5\sqrt{505}}{8}} + \left(\frac{8}{5}\right)^k (21 + \sqrt{505})^{-k} \sqrt{\frac{5}{8} (21 + \sqrt{505})} \right) \right)^3 \right)^{30}$$

$$\begin{aligned}
& \left(\frac{1}{6} \left((3.325934293130000 \times 2.9111166557740000 \times 11.57304815210000) / \right. \right. \\
& \quad \left. \left. \frac{((3.398139554440000 + 3.325934293130000) \right. \right. \\
& \quad \left. \left. 2.9111166557740000) - 0.927011 - \right. \right. \\
& \quad \left. \left. \sqrt[14]{ \left(\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3 \right)^2 \right)^{15} - \right. \\
& \quad \left. 18 + \frac{1}{\pi} = -18 + \frac{1}{\pi} + \frac{1}{470184984576} \right. \\
& \quad \left. \left(4.79738 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{1}{2}\right)_k \left((-8)^k (97 + 5\sqrt{505})^{-k} \sqrt{\frac{97}{8} + \frac{5\sqrt{505}}{8}} + \left(-\frac{8}{5}\right)^k \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. (21 + \sqrt{505})^{-k} \sqrt{\frac{5}{8} (21 + \sqrt{505})} \right)^3 \right)^{\wedge (1/14)} \right)^{30} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{6} \left((3.325934293130000 \times 2.9111166557740000 \times 11.57304815210000) / \right. \right. \\
& \quad \left. \left. \frac{((3.398139554440000 + 3.325934293130000) \right. \right. \\
& \quad \left. \left. 2.9111166557740000) - 0.927011 - \right. \right. \\
& \quad \left. \left. \sqrt[14]{ \left(\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3 \right)^2 \right)^{15} - \right. \\
& \quad \left. 18 + \frac{1}{\pi} = -18 + \frac{1}{\pi} + \frac{1}{470184984576} \right. \\
& \quad \left. \left(4.79738 - \left(\sqrt{z_0}^3 \left(\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \left(\left(\frac{113}{8} + \frac{5\sqrt{505}}{8} - z_0 \right)^k + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(\frac{5}{8} (21 + \sqrt{505}) - z_0 \right)^k \right) z_0^{-k} \right)^3 \right)^{\wedge} \right. \\
& \quad \left. \left. (1/14) \right)^{30} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$(1/27((1/6(((3.32593429313)*2.911116655774*((11.5730481521/(3.39813955444+3.32593429313) *1/2.911116655774)))))-0.9270108-((\sqrt{((113+5\sqrt{505})/8)+\sqrt{((105+5\sqrt{505})/8))^3})^{1/14})^2)^{15}-18-1/\sqrt{2})^2$$

Input interpretation

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(3.32593429313 \times 2.911116655774 \left(\frac{11.5730481521}{3.39813955444 + 3.32593429313} \times \frac{1}{2.911116655774} \right) - 0.9270108 - \sqrt[14]{ \left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3 } \right)^{2 \cdot 15} - 18 - \frac{1}{\sqrt{2}} \right) \right)^2 \right)$$

Result

4096.02...

4096.02..... $\approx 4096 = 64^2$

Now, we have:

$$G_{117} = \frac{1}{2} \left(\frac{3 + \sqrt{13}}{2} \right)^{\frac{1}{4}} (2\sqrt{3} + \sqrt{13})^{\frac{1}{6}} \{ 3^{\frac{1}{4}} + \sqrt{(4 + \sqrt{3})} \},$$

$$G_{121} + \frac{1}{G_{121}} = \left(\frac{11}{2} \right)^{\frac{1}{6}} \left\{ \left(3 + \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} + \left(3 - \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} \right\}$$

$$\frac{1}{G_{121}} = \frac{1}{3\sqrt{2}} [(11 - 3\sqrt{11})^{\frac{1}{3}} \{ (3\sqrt{11} + 3\sqrt{3} - 4)^{\frac{1}{3}} + (3\sqrt{11} - 3\sqrt{3} - 4)^{\frac{1}{3}} \} - 2]$$

From:

$$G_{117} = \frac{1}{2} \left(\frac{3 + \sqrt{13}}{2} \right)^{\frac{1}{4}} (2\sqrt{3} + \sqrt{13})^{\frac{1}{6}} \{ 3^{\frac{1}{4}} + \sqrt{(4 + \sqrt{3})} \},$$

$$\frac{1}{2} \left(\frac{1}{2} (3 + \sqrt{13}) \right)^{0.25} (2\sqrt{3} + \sqrt{13})^{(1/6)} [3^{0.25} + (4 + \sqrt{3})^{0.5}]$$

Input

$$\frac{1}{2} \left(\frac{1}{2} (3 + \sqrt{13}) \right)^{0.25} \left(\sqrt[6]{2\sqrt{3} + \sqrt{13}} \left(3^{0.25} + \sqrt{4 + \sqrt{3}} \right) \right)$$

Result

3.4646439795293348074582824939380623882292908597117985414969183228

...

3.464643979529....

From:

$$\left(\frac{11}{2} \right)^{\frac{1}{6}} \left\{ \left(3 + \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} + \left(3 - \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} \right\}$$

$$\left(\frac{11}{2} \right)^{(1/6)} \left[\left(3 + \frac{1}{3\sqrt{3}} \right)^{(1/3)} + \left(3 - \frac{1}{3\sqrt{3}} \right)^{(1/3)} \right]$$

Input

$$\sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right)$$

Decimal approximation

3.8305864363226477774611570629145635545308778360411986963564438610

...

3.830586436322.....

Alternate forms

$$3^{-1/3} \sqrt[6]{\frac{11}{2}} \sqrt[3]{9 - 3^{-1/2}} + 3^{-1/3} \sqrt[6]{\frac{11}{2}} \sqrt[3]{9 + 3^{-1/2}}$$

root of $x^6 - 22x^4 + 121x^2 - 198$ near $x = 3.83059$

$$\frac{\sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{27 - \sqrt{3}} + \sqrt[3]{27 + \sqrt{3}} \right)}{3^{2/3}}$$

Minimal polynomial

$$x^6 - 22x^4 + 121x^2 - 198$$

Expanded form

$$\sqrt[6]{\frac{11}{2}} \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} + \sqrt[6]{\frac{11}{2}} \sqrt[3]{3 + \frac{1}{3\sqrt{3}}}$$

From:

$$\frac{1}{3\sqrt{2}} \left[(11 - 3\sqrt{11})^{1/3} \left\{ (3\sqrt{11} + 3\sqrt{3} - 4)^{1/3} + (3\sqrt{11} - 3\sqrt{3} - 4)^{1/3} \right\} - 2 \right]$$

$$1/(3\sqrt{2}) \left[(11 - 3\sqrt{11})^{1/3} \left[(3\sqrt{11} + 3\sqrt{3} - 4)^{1/3} + (3\sqrt{11} - 3\sqrt{3} - 4)^{1/3} \right] - 2 \right]$$

Input

$$\frac{1}{3\sqrt{2}} \left(\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2 \right)$$

Result

$$\frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{-4 - 3\sqrt{3} + 3\sqrt{11}} + \sqrt[3]{-4 + 3\sqrt{3} + 3\sqrt{11}} \right) - 2}{3\sqrt{2}}$$

Decimal approximation

0.2817853021919029973538826574520947893730103442412604309559156517

...

0.2817853021....

Alternate forms

$$\frac{1}{6} \left(\sqrt[3]{-143 + 33\sqrt{3} + 45\sqrt{11} - 9\sqrt{33}} + \sqrt[3]{-143 - 33\sqrt{3} + 45\sqrt{11} + 9\sqrt{33}} - 2 \right) \sqrt{2}$$

root of $x^{12} - 16x^{10} + 48x^8 - 68x^6 + 48x^4 - 16x^2 + 1$ near $x = 0.281785$

$$\sqrt{\text{root of } x^6 - 16x^5 + 48x^4 - 68x^3 + 48x^2 - 16x + 1 \text{ near } x = 0.079403}$$

Minimal polynomial

$$x^{12} - 16x^{10} + 48x^8 - 68x^6 + 48x^4 - 16x^2 + 1$$

Expanded forms

$$-\frac{\sqrt{2}}{3} + \frac{\sqrt[3]{-143 + 33\sqrt{3} + 45\sqrt{11} - 9\sqrt{33}}}{3\sqrt{2}} + \frac{\sqrt[3]{-143 - 33\sqrt{3} + 45\sqrt{11} + 9\sqrt{33}}}{3\sqrt{2}}$$

$$-\frac{\sqrt{2}}{3} + \frac{\sqrt[3]{(11-3\sqrt{11})(-4-3\sqrt{3}+3\sqrt{11})}}{3\sqrt{2}} + \frac{\sqrt[3]{(11-3\sqrt{11})(-4+3\sqrt{3}+3\sqrt{11})}}{3\sqrt{2}}$$

From which:

$$\frac{1}{2} \left(3.464643979529 + \left(\left(\frac{11}{2} \right)^{1/6} \left[\left(3 + \frac{1}{3\sqrt{3}} \right)^{1/3} + \left(3 - \frac{1}{3\sqrt{3}} \right)^{1/3} \right] \right) + \frac{1}{3\sqrt{2}} \left[\left(11 - 3\sqrt{11} \right)^{1/3} \left[\left(3\sqrt{11} + 3\sqrt{3} - 4 \right)^{1/3} + \left(3\sqrt{11} - 3\sqrt{3} - 4 \right)^{1/3} \right] - 2 \right] \right) - \frac{\log(34)}{\log(233)}$$

where

$$\frac{\log(34)}{\log(233)} \approx 0.64691536386$$

Input interpretation

$$\frac{1}{2} \left(3.464643979529 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{1}{3\sqrt{2}} \left(\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2 \right) \right) - \frac{\log(34)}{\log(233)}$$

$\log(x)$ is the natural logarithm

Result

3.141592495159...

3.141592495159... $\approx \pi$

Alternative representations

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) - \frac{\log(34)}{\log(233)} = -\frac{\log_e(34)}{\log_e(233)} + \frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 - \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 + \frac{1}{3\sqrt{3}}} \right) + \frac{-2 + \sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{-4 - 3\sqrt{3} + 3\sqrt{11}} + \sqrt[3]{-4 + 3\sqrt{3} + 3\sqrt{11}} \right)}{3\sqrt{2}} \right)$$

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) - \frac{\log(34)}{\log(233)} = -\frac{\log(a) \log_a(34)}{\log(a) \log_a(233)} + \frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 - \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 + \frac{1}{3\sqrt{3}}} \right) + \frac{-2 + \sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{-4 - 3\sqrt{3} + 3\sqrt{11}} + \sqrt[3]{-4 + 3\sqrt{3} + 3\sqrt{11}} \right)}{3\sqrt{2}} \right)$$

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) -$$

$$\frac{\log(34)}{\log(233)} = -\frac{\text{Li}_1(-33)}{\text{Li}_1(-232)} +$$

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 - \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 + \frac{1}{3\sqrt{3}}} \right) + \frac{-2 + \sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{-4 - 3\sqrt{3} + 3\sqrt{11}} + \sqrt[3]{-4 + 3\sqrt{3} + 3\sqrt{11}} \right)}{3\sqrt{2}} \right)$$

Series representations

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right)$$

$$\frac{\log(34)}{\log(233)} = \left(0.1666666666666667 \right.$$

$$\left. -2.0000000000000000 \log(233) - 6.0000000000000000 \right.$$

$$\exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \log(34) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} +$$

$$10.393931938587000 \exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \log(233) \sqrt{x}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + 2.763596398624306$$

$$\exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \log(233) \sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\sqrt[3]{\frac{-1 + 9 \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}} +$$

$$2.763596398624306 \exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \log(233)$$

$$\sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\sqrt[3]{\frac{1 + 9 \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}} +$$

$$1.0000000000000000 \log(233)$$

$$\sqrt[3]{11 - 3 \exp\left(i\pi \left[\frac{\arg(11-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left(-4 + \sum_{k=0}^{\infty} -\frac{1}{k!} 3 (-1)^k x^{-k} (3-x)^k \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) - \right.$$

$$\left. (11-x)^k \exp\left(i\pi \left[\frac{\arg(11-x)}{2\pi} \right]\right) \left(-\frac{1}{2}\right)_k \sqrt{x} \right)^{\wedge}$$

$$(1/3) + 1.0000000000000000 \log(233)$$

$$\sqrt[3]{11 - 3 \exp\left(i\pi \left[\frac{\arg(11-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left(-4 + \sum_{k=0}^{\infty} \frac{1}{k!} 3 (-1)^k x^{-k} (3-x)^k \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right]\right) + (11-x)^k \right.$$

$$\left. \exp\left(i\pi \left[\frac{\arg(11-x)}{2\pi} \right]\right) \left(-\frac{1}{2}\right)_k \sqrt{x} \right)^{\wedge (1/3)} \Bigg/$$

$$\left(\exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi} \right]\right) \log(233) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for
 $(x \in \mathbb{R})$
 and
 \dots

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) -$$

$$\frac{\log(34)}{\log(233)} = 1.7323219897645000 -$$

$$\frac{2i\pi \left\lfloor \frac{\arg(34-x)}{2\pi} \right\rfloor}{2i\pi \left\lfloor \frac{\arg(233-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-x)^k x^{-k}}{k}}$$

$$\frac{2i\pi \left\lfloor \frac{\arg(233-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-x)^k x^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k (34-x)^k x^{-k}}{k}} +$$

$$\frac{2i\pi \left\lfloor \frac{\arg(233-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-x)^k x^{-k}}{k}}{1} +$$

$$3 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{\frac{1}{2} \sqrt[6]{\frac{11}{2}} \sqrt[3]{3 - \frac{1}{3 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}}}{3 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} +$$

$$\frac{\frac{1}{2} \sqrt[6]{\frac{11}{2}} \sqrt[3]{3 + \frac{1}{3 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}}}{3 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} +$$

$$\left(\sqrt[3]{11 - 3 \exp\left(i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right.$$

$$\left. \left(-4 + \sum_{k=0}^{\infty} \frac{1}{k!} 3 (-1)^k x^{-k} \left((3-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \right) - \right.$$

$$\left. \left. (11-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor\right) \right) \left(-\frac{1}{2}\right)_k \sqrt{x} \right)^{\wedge(1/3)} /$$

$$\left(6 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) +$$

$$\left(\sqrt[3]{11 - 3 \exp\left(i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right.$$

$$\left. \left(-4 + \sum_{k=0}^{\infty} \frac{1}{k!} 3 (-1)^k x^{-k} \left((3-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \right) + \right.$$

$$\left. \left. (11-x)^k \exp\left(i\pi \left\lfloor \frac{\arg(11-x)}{2\pi} \right\rfloor\right) \right) \left(-\frac{1}{2}\right)_k \sqrt{x} \right)^{\wedge(1/3)} /$$

$$\left(6 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) -$$

$$\frac{\log(34)}{\log(233)} = 1.7323219897645000 -$$

$$\frac{1}{\frac{3 \exp(i\pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} (-\frac{1}{2})_k}{k!} + \frac{1}{2} \sqrt[6]{\frac{11}{2}} \sqrt[3]{3 - \frac{1}{3 \exp(i\pi \lfloor \frac{\arg(3-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} (-\frac{1}{2})_k}{k!} + \frac{1}{2} \sqrt[6]{\frac{11}{2}} \sqrt[3]{3 + \frac{1}{3 \exp(i\pi \lfloor \frac{\arg(3-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} (-\frac{1}{2})_k}{k!} + \left(\sqrt[3]{11 - 3 \exp(i\pi \lfloor \frac{\arg(11-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} (-\frac{1}{2})_k}{k!} \right. - \left. \left(-4 + \sum_{k=0}^{\infty} \frac{1}{k!} 3(-1)^k x^{-k} \left((3-x)^k \exp(i\pi \lfloor \frac{\arg(3-x)}{2\pi} \rfloor) \right) - (11-x)^k \exp(i\pi \lfloor \frac{\arg(11-x)}{2\pi} \rfloor) \right) \left(-\frac{1}{2} \right)_k \sqrt{x} \right)^{\wedge (1/3)}} /$$

$$\left(\frac{6 \exp(i\pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} (-\frac{1}{2})_k}{k!} \right) + \left(\sqrt[3]{11 - 3 \exp(i\pi \lfloor \frac{\arg(11-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (11-x)^k x^{-k} (-\frac{1}{2})_k}{k!} \right. - \left. \left(-4 + \sum_{k=0}^{\infty} \frac{1}{k!} 3(-1)^k x^{-k} \left((3-x)^k \exp(i\pi \lfloor \frac{\arg(3-x)}{2\pi} \rfloor) \right) + (11-x)^k \exp(i\pi \lfloor \frac{\arg(11-x)}{2\pi} \rfloor) \right) \left(-\frac{1}{2} \right)_k \sqrt{x} \right)^{\wedge (1/3)} /$$

$$\left(\frac{6 \exp(i\pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} (-\frac{1}{2})_k}{k!} \right) - \frac{\lfloor \frac{\arg(34-z_0)}{2\pi} \rfloor \log(\frac{1}{z_0})}{\log(z_0)} -$$

$$\frac{\log(z_0) + \lfloor \frac{\arg(233-z_0)}{2\pi} \rfloor \left(\log(\frac{1}{z_0}) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-z_0)^k z_0^{-k}}{k}}{\log(z_0)} -$$

$$\frac{\log(z_0) + \lfloor \frac{\arg(233-z_0)}{2\pi} \rfloor \left(\log(\frac{1}{z_0}) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-z_0)^k z_0^{-k}}{k}}{\lfloor \frac{\arg(34-z_0)}{2\pi} \rfloor \log(z_0)} +$$

$$\frac{\log(z_0) + \lfloor \frac{\arg(233-z_0)}{2\pi} \rfloor \left(\log(\frac{1}{z_0}) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-z_0)^k z_0^{-k}}{k}}{\sum_{k=1}^{\infty} \frac{(-1)^k (34-z_0)^k z_0^{-k}}{k}} +$$

$$\frac{\log(z_0) + \lfloor \frac{\arg(233-z_0)}{2\pi} \rfloor \left(\log(\frac{1}{z_0}) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-z_0)^k z_0^{-k}}{k}}{\log(z_0) + \lfloor \frac{\arg(233-z_0)}{2\pi} \rfloor \left(\log(\frac{1}{z_0}) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (233-z_0)^k z_0^{-k}}{k}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representations

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) -$$

$$\frac{\log(34)}{\log(233)} = \frac{1}{\sqrt{2} \int_1^{233} \frac{1}{t} dt} 0.1666666666666667$$

$$\left(-2.0000000000000000 \int_1^{233} \frac{1}{t} dt - 6.0000000000000000 \sqrt{2} \int_1^{34} \frac{1}{t} dt + 10.393931938587000 \sqrt{2} \int_1^{233} \frac{1}{t} dt + 2.763596398624306 \sqrt{2} \sqrt[3]{\frac{-1 + 9\sqrt{3}}{\sqrt{3}}} \int_1^{233} \frac{1}{t} dt + 2.763596398624306 \sqrt{2} \sqrt[3]{\frac{1 + 9\sqrt{3}}{\sqrt{3}}} \int_1^{233} \frac{1}{t} dt + 1.0000000000000000 \sqrt[3]{11 - 3\sqrt{11}} \sqrt[3]{-4 - 3\sqrt{3} + 3\sqrt{11}} \int_1^{233} \frac{1}{t} dt + 1.0000000000000000 \sqrt[3]{11 - 3\sqrt{11}} \sqrt[3]{-4 + 3\sqrt{3} + 3\sqrt{11}} \int_1^{233} \frac{1}{t} dt \right)$$

$$\frac{1}{2} \left(3.4646439795290000 + \sqrt[6]{\frac{11}{2}} \left(\sqrt[3]{3 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{3 - \frac{1}{3\sqrt{3}}} \right) + \frac{\sqrt[3]{11 - 3\sqrt{11}} \left(\sqrt[3]{3\sqrt{11} + 3\sqrt{3} - 4} + \sqrt[3]{3\sqrt{11} - 3\sqrt{3} - 4} \right) - 2}{3\sqrt{2}} \right) -$$

$$\frac{\log(34)}{\log(233)} = \left(0.1666666666666667 \right.$$

$$\left. \begin{aligned} & \left(-2.0000000000000000 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - \right. \\ & 6.0000000000000000 \sqrt{2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{33^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \\ & 10.393931938587000 \sqrt{2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \\ & 2.763596398624306 \sqrt{2} \sqrt[3]{\frac{-1+9\sqrt{3}}{\sqrt{3}}} \\ & \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + 2.763596398624306 \\ & \sqrt{2} \sqrt[3]{\frac{1+9\sqrt{3}}{\sqrt{3}}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \\ & 1.0000000000000000 \sqrt[3]{11-3\sqrt{11}} \sqrt[3]{-4-3\sqrt{3}+3\sqrt{11}} \\ & \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \\ & 1.0000000000000000 \sqrt[3]{11-3\sqrt{11}} \sqrt[3]{-4+3\sqrt{3}+3\sqrt{11}} \\ & \left. \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right) / \end{aligned}$$

$$\left(\sqrt{2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{232^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 <$$

$$\gamma < 0$$

We have:

$$g_{126} = \sqrt{\left(\frac{\sqrt{3} + \sqrt{7}}{2}\right)} (\sqrt{6} + \sqrt{7})^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{3 + \sqrt{2}}{4}\right)} + \sqrt{\left(\frac{\sqrt{2} - 1}{4}\right)} \right\}^2,$$

$$g_{138}^2 = \sqrt{\left(\frac{3\sqrt{3} + \sqrt{23}}{2}\right)} (78\sqrt{2} + 23\sqrt{23})^{\frac{1}{6}} \times \left\{ \sqrt{\left(\frac{5 + 2\sqrt{6}}{4}\right)} + \sqrt{\left(\frac{1 + 2\sqrt{6}}{4}\right)} \right\},$$

$$G_{141}^2 = (4\sqrt{3} + \sqrt{47})^{\frac{1}{4}} \left(\frac{7 + \sqrt{47}}{\sqrt{2}}\right)^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{18 + 9\sqrt{3}}{4}\right)} + \sqrt{\left(\frac{14 + 9\sqrt{3}}{4}\right)} \right\},$$

From:

$$\sqrt{\left(\frac{\sqrt{3} + \sqrt{7}}{2}\right)} (\sqrt{6} + \sqrt{7})^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{3 + \sqrt{2}}{4}\right)} + \sqrt{\left(\frac{\sqrt{2} - 1}{4}\right)} \right\}^2,$$

$$[(1/2*(\text{sqrt}3+\text{sqrt}7))]^{0.5} (\text{sqrt}6+\text{sqrt}7)^{(1/6)} [(1/4*(3+\text{sqrt}2))^{0.5}+(1/4(\text{sqrt}2 - 1))^{0.5}]^2$$

Input

$$\sqrt{\frac{1}{2}(\sqrt{3} + \sqrt{7})} \sqrt[6]{\sqrt{6} + \sqrt{7}} \left(\sqrt{\frac{1}{4}(3 + \sqrt{2})} + \sqrt{\frac{1}{4}(\sqrt{2} - 1)} \right)^2$$

Exact result

$$\sqrt{\frac{1}{2}(\sqrt{3} + \sqrt{7})} \sqrt[6]{\sqrt{6} + \sqrt{7}} \left(\frac{1}{2} \sqrt{\sqrt{2} - 1} + \frac{\sqrt{3 + \sqrt{2}}}{2} \right)^2$$

Decimal approximation

3.6548630551090639866346902876604774336644477160510791945069930158

...

3.6548630551090639....

Alternate forms

$$\frac{\sqrt{\sqrt{3} + \sqrt{7}} \sqrt[6]{\sqrt{6} + \sqrt{7}} \left(1 + \sqrt{2} + \sqrt{2\sqrt{2} - 1}\right)}{2\sqrt{2}}$$

$$\sqrt[6]{\text{root of } x^8 - 2384x^7 + 1044x^6 - 6256x^5 + 15974x^4 - 6256x^3 + 1044x^2 - 2384x + 1 \text{ near } x = 2383.56}$$

Minimal polynomial

$$x^{48} - 2384x^{42} + 1044x^{36} - 6256x^{30} + 15974x^{24} - 6256x^{18} + 1044x^{12} - 2384x^6 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt{\sqrt{3} + \sqrt{7}} \sqrt[6]{\sqrt{6} + \sqrt{7}} + \frac{\sqrt{\sqrt{3} + \sqrt{7}} \sqrt[6]{\sqrt{6} + \sqrt{7}}}{2\sqrt{2}} + \frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{2} - 1)(3 + \sqrt{2})(\sqrt{3} + \sqrt{7})} \sqrt[6]{\sqrt{6} + \sqrt{7}}$$

$$\frac{1}{4} \sqrt[6]{\sqrt{6} + \sqrt{7}} \sqrt{2\sqrt{3} + 2\sqrt{7}} + \frac{1}{4} \sqrt[6]{\sqrt{6} + \sqrt{7}} \sqrt{4\sqrt{3} + 4\sqrt{7}} + \frac{1}{4} \sqrt[6]{\sqrt{6} + \sqrt{7}} \sqrt{-2\sqrt{3} + 4\sqrt{6} - 2\sqrt{7} + 4\sqrt{14}}$$

From:

$$\sqrt{\left(\frac{3\sqrt{3} + \sqrt{23}}{2}\right)} (78\sqrt{2} + 23\sqrt{23})^{\frac{1}{6}} \times \left\{ \sqrt{\left(\frac{5 + 2\sqrt{6}}{4}\right)} + \sqrt{\left(\frac{1 + 2\sqrt{6}}{4}\right)} \right\},$$

we obtain:

$$\left[\left(\frac{1}{2}(3\sqrt{3}+\sqrt{23})\right)^{0.5} (78\sqrt{2}+23\sqrt{23})^{1/6}\right] \\ \left[\left(\frac{1}{4}(5+2\sqrt{6})\right)^{0.5} + \left(\frac{1}{4}(1+2\sqrt{6})\right)^{0.5}\right]$$

Input

$$\sqrt{\frac{1}{2}(3\sqrt{3}+\sqrt{23})} \sqrt[6]{78\sqrt{2}+23\sqrt{23}} \left(\sqrt{\frac{1}{4}(5+2\sqrt{6})} + \sqrt{\frac{1}{4}(1+2\sqrt{6})}\right)$$

Exact result

$$\sqrt{\frac{1}{2}(3\sqrt{3}+\sqrt{23})} \sqrt[6]{78\sqrt{2}+23\sqrt{23}} \left(\frac{1}{2}\sqrt{1+2\sqrt{6}} + \frac{1}{2}\sqrt{5+2\sqrt{6}}\right)$$

Decimal approximation

15.315614485200225835806318595854580375614238222494746288654476828

...

15.3156144852002258.....

Alternate forms

$$\sqrt[3]{\boxed{\text{root of } x^8 - 3584x^7 - 30724x^6 - 21248x^5 + 29446x^4 + 21248x^3 - 30724x^2 + 3584x + 1 \text{ near } x = 3592.55}}$$

$$\frac{1}{4} \sqrt{3\sqrt{3}+\sqrt{23}} \sqrt[6]{78\sqrt{2}+23\sqrt{23}} \\ \left(\sqrt{1-i\sqrt{23}} + \sqrt{2} \left(\sqrt{2} + \sqrt{3} + \sqrt{\frac{1}{2}i(\sqrt{23}+i)}\right)\right)$$

$$\frac{1}{2\sqrt{2}} \sqrt[6]{78\sqrt{2}+23\sqrt{23}} \\ \left(\sqrt{18\sqrt{2}+3\sqrt{3}+\sqrt{23}+2\sqrt{138}} + \sqrt{18\sqrt{2}+15\sqrt{3}+5\sqrt{23}+2\sqrt{138}}\right)$$

Minimal polynomial

$$x^{24} - 3584 x^{21} - 30724 x^{18} - 21248 x^{15} + 29446 x^{12} + 21248 x^9 - 30724 x^6 + 3584 x^3 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt[6]{78\sqrt{2} + 23\sqrt{23}} \left(\sqrt{9\sqrt{2} + \frac{3\sqrt{3}}{2} + \frac{\sqrt{23}}{2} + \sqrt{138}} + \sqrt{9\sqrt{2} + \frac{15\sqrt{3}}{2} + \frac{5\sqrt{23}}{2} + \sqrt{138}} \right)$$

$$\frac{\sqrt{(1+2\sqrt{6})(3\sqrt{3} + \sqrt{23})} \sqrt[6]{78\sqrt{2} + 23\sqrt{23}}}{2\sqrt{2}} + \frac{\sqrt{(5+2\sqrt{6})(3\sqrt{3} + \sqrt{23})} \sqrt[6]{78\sqrt{2} + 23\sqrt{23}}}{2\sqrt{2}}$$

From:

$$(4\sqrt{3} + \sqrt{47})^{\frac{1}{4}} \left(\frac{7 + \sqrt{47}}{\sqrt{2}} \right)^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{18 + 9\sqrt{3}}{4} \right)} + \sqrt{\left(\frac{14 + 9\sqrt{3}}{4} \right)} \right\},$$

$$[(4\sqrt{3} + \sqrt{47})]^{0.25} ((7 + \sqrt{47}) / (\sqrt{2}))^{(1/6)}$$

$$[(1/4 * (18 + 9\sqrt{3}))^{0.5} + (1/4 * (14 + 9\sqrt{3}))^{0.5}]$$

Input

$$(4\sqrt{3} + \sqrt{47})^{0.25} \sqrt[6]{\frac{7 + \sqrt{47}}{\sqrt{2}}} \left(\sqrt{\frac{1}{4}(18 + 9\sqrt{3})} + \sqrt{\frac{1}{4}(14 + 9\sqrt{3})} \right)$$

Result

15.833404207477886190304510217011509695032073020671272071483322463

...

15.833404207477....

From the sum of the last three results, we obtain, after some calculations:

$$(3.6548630551090639 + 15.3156144852002258 + 15.833404207477)^{1/3} - \left(\frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} \right)$$

where

$$\frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} \approx 0.123352534903879$$

Input interpretation

$$\sqrt[3]{3.6548630551090639 + 15.3156144852002258 + 15.833404207477} - \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)}$$

Result

3.14159265366263...

3.14159265366263..... $\approx \pi$

Alternative representations

$$\begin{aligned} & (3.65486305510906390000 + 15.31561448520022580000 + \\ & \quad 15.8334042074770000) \wedge (1/3) - \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = \\ & \sqrt[3]{34.8038817477862897} - \frac{2^{3/5} \log_e^{7/5}(2)}{3 \sqrt[5]{3} \log_e^{36/5}(3)} \end{aligned}$$

$$\begin{aligned} & (3.65486305510906390000 + 15.31561448520022580000 + \\ & \quad 15.8334042074770000) \wedge (1/3) - \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = \\ & \sqrt[3]{34.8038817477862897} - \frac{2^{3/5} (\log(a) \log_a(2))^{7/5}}{3 \sqrt[5]{3} (\log(a) \log_a(3))^{36/5}} \end{aligned}$$

$$\begin{aligned} & (3.65486305510906390000 + 15.31561448520022580000 + \\ & \quad 15.8334042074770000) \wedge (1/3) - \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = \\ & \sqrt[3]{34.8038817477862897} - \frac{2^{3/5} (2 \coth^{-1}(3))^{7/5}}{3 \sqrt[5]{3} (2 \coth^{-1}(2))^{36/5}} \end{aligned}$$

Series representations

$$\begin{aligned} & (3.65486305510906390000 + \\ & \quad 15.31561448520022580000 + 15.8334042074770000) \wedge (1/3) - \\ & \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = 3.264945188566514061 - \\ & \frac{2^{3/5} \left(2 i \pi \left[\frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right)^{7/5}}{3 \sqrt[5]{3} \left(2 i \pi \left[\frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^{36/5}} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned}
& (3.65486305510906390000 + \\
& \quad 15.31561448520022580000 + 15.8334042074770000) ^{1/3} - \\
& \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = 3.264945188566514061 - \\
& \frac{2^{3/5} \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)^{7/5}}{3 \sqrt[5]{3} \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{36/5}}
\end{aligned}$$

$$\begin{aligned}
& (3.65486305510906390000 + \\
& \quad 15.31561448520022580000 + 15.8334042074770000) ^{1/3} - \\
& \frac{2^{3/5} \log^{7/5}(2)}{3 \sqrt[5]{3} \log^{36/5}(3)} = 3.264945188566514061 - \\
& \frac{2^{3/5} \left(2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right)^{7/5}}{3 \sqrt[5]{3} \left(2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^{36/5}}
\end{aligned}$$

Now, we have:

$$\begin{aligned}
G_{145}^2 &= \sqrt{\left\{ \frac{(2 + \sqrt{5})(5 + \sqrt{29})}{2} \right\}} \left\{ \sqrt{\left(\frac{17 + \sqrt{145}}{8} \right)} + \sqrt{\left(\frac{9 + \sqrt{145}}{8} \right)} \right\}, \\
\frac{1}{G_{147}} &= 2^{-\frac{1}{12}} \left[\frac{1}{2} + \frac{1}{\sqrt{3}} \left\{ \sqrt{\left(\frac{7}{4} \right)} - (28)^{\frac{1}{6}} \right\} \right], \\
G_{153} &= \left\{ \sqrt{\left(\frac{5 + \sqrt{17}}{8} \right)} + \sqrt{\left(\frac{\sqrt{17} - 3}{8} \right)} \right\}^2 \\
&\quad \times \left\{ \sqrt{\left(\frac{37 + 9\sqrt{17}}{4} \right)} + \sqrt{\left(\frac{33 + 9\sqrt{17}}{4} \right)} \right\}^{\frac{1}{3}},
\end{aligned}$$

From:

$$\sqrt{\left\{ \frac{(2 + \sqrt{5})(5 + \sqrt{29})}{2} \right\}} \left\{ \sqrt{\left(\frac{17 + \sqrt{145}}{8} \right)} + \sqrt{\left(\frac{9 + \sqrt{145}}{8} \right)} \right\},$$

$$[(1/2*(2+sqrt5)(5+sqrt29))]^0.5 [(1/8*(17+sqrt145))^0.5+(1/8(9+sqrt145))^0.5]$$

Input

$$\sqrt{\frac{1}{2}(2 + \sqrt{5})(5 + \sqrt{29})} \left(\sqrt{\frac{1}{8}(17 + \sqrt{145})} + \sqrt{\frac{1}{8}(9 + \sqrt{145})} \right)$$

Exact result

$$\sqrt{\frac{1}{2}(2 + \sqrt{5})(5 + \sqrt{29})} \left(\frac{1}{2} \sqrt{\frac{1}{2}(9 + \sqrt{145})} + \frac{1}{2} \sqrt{\frac{1}{2}(17 + \sqrt{145})} \right)$$

Decimal approximation

16.542097553485330822915990352914694829810037886057345799143360740

...

16.542097553485.....

Alternate forms

$$\frac{1}{4} \sqrt{(2 + \sqrt{5})(5 + \sqrt{29})} \left(\sqrt{9 + \sqrt{145}} + \sqrt{17 + \sqrt{145}} \right)$$

| |
|---------------------------------------------------------------------------------------------|
| root of $x^8 - 17x^7 + 7x^6 + 12x^5 - 42x^4 + 12x^3 + 7x^2 - 17x + 1$ near $x = 16.5421$ |
|---------------------------------------------------------------------------------------------|

$$\left(\frac{\sqrt{5}}{4} + \frac{1}{4} \sqrt{9-8i} + \frac{1}{4} \sqrt{9+8i} + \frac{\sqrt{29}}{4} \right) \sqrt{\frac{1}{2} (2+\sqrt{5})(5+\sqrt{29})}$$

Minimal polynomial

$$x^8 - 17x^7 + 7x^6 + 12x^5 - 42x^4 + 12x^3 + 7x^2 - 17x + 1$$

Expanded forms

$$\frac{1}{4} \left(\sqrt{235 + 103\sqrt{5} + 43\sqrt{29} + 19\sqrt{145}} + \sqrt{315 + 143\sqrt{5} + 59\sqrt{29} + 27\sqrt{145}} \right)$$

$$\frac{1}{4} \sqrt{(2+\sqrt{5})(5+\sqrt{29})(9+\sqrt{145})} + \frac{1}{4} \sqrt{(2+\sqrt{5})(5+\sqrt{29})(17+\sqrt{145})}$$

From:

$$2^{-1/12} \left[\frac{1}{2} + \frac{1}{\sqrt{3}} \left\{ \sqrt{\left(\frac{7}{4}\right) - (28)^{1/6}} \right\} \right],$$

$$2^{(-1/12)} [1/2+1/(\text{sqrt}3)*((((((7/4)-(28)^(1/6))))^0.5))]$$

Input

$$2^{-1/12} \left(\frac{1}{2} + \frac{1}{\sqrt{3}} \sqrt{\frac{7}{4} - \sqrt[6]{28}} \right)$$

Exact result

$$\frac{\frac{1}{2} + \sqrt{\frac{1}{3} \left(\frac{7}{4} - \sqrt[3]{2} \sqrt[6]{7} \right)}}{\sqrt[12]{2}}$$

Decimal approximation

0.5188749434999687959010657336945004482607030768726858739422437444

...

0.518874943499968.....

Alternate forms

$$\frac{3 + \sqrt{3 \left(7 - 4 \sqrt[3]{2} \sqrt[6]{7} \right)}}{6 \sqrt[12]{2}}$$

$$\frac{1 + \sqrt{\frac{1}{3} \left(7 - 4 \sqrt[3]{2} \sqrt[6]{7} \right)}}{2 \sqrt[12]{2}}$$

$$\frac{1}{2 \sqrt[12]{2}} + \frac{\sqrt{\frac{1}{3} \left(7 - 4 \sqrt[3]{2} \sqrt[6]{7} \right)}}{2 \sqrt[12]{2}}$$

Minimal polynomial

$$\begin{aligned}
 & 326\,723\,350\,842\,869\,800\,400\,326\,656\,x^{144} - \\
 & 16\,341\,545\,704\,297\,282\,115\,496\,173\,568\,x^{132} - \\
 & 1\,338\,957\,944\,346\,231\,244\,113\,023\,121\,408\,x^{120} - \\
 & 157\,939\,659\,405\,791\,447\,429\,667\,759\,151\,104\,x^{108} - \\
 & 1\,329\,450\,825\,053\,077\,093\,196\,513\,802\,641\,664\,x^{96} - \\
 & 4524\,230\,361\,502\,210\,749\,330\,592\,606\,288\,896\,x^{84} + \\
 & 7\,624\,332\,260\,767\,144\,585\,977\,660\,460\,800\,x^{72} - \\
 & 4622\,475\,386\,147\,111\,255\,592\,048\,384\,x^{60} + \\
 & 1\,249\,894\,205\,071\,086\,146\,543\,856\,x^{48} - 141\,913\,350\,866\,464\,274\,016\,x^{36} + \\
 & 4030\,122\,393\,011\,272\,x^{24} - 30\,115\,950\,392\,x^{12} + 531\,441
 \end{aligned}$$

Expanded forms

$$\frac{1}{2^{12}\sqrt{2}} + \frac{\sqrt{\frac{7}{12} - \frac{1}{3}\sqrt[3]{2}\sqrt[6]{7}}}{\sqrt[12]{2}}$$

$$\frac{1}{2^{12}\sqrt{2}} + \frac{\sqrt{\frac{7}{4} - \sqrt[3]{2}\sqrt[6]{7}}}{\sqrt[12]{2}\sqrt{3}}$$

From:

$$\begin{aligned}
 & \left\{ \sqrt{\left(\frac{5 + \sqrt{17}}{8}\right)} + \sqrt{\left(\frac{\sqrt{17} - 3}{8}\right)} \right\}^2 \\
 & \quad \times \left\{ \sqrt{\left(\frac{37 + 9\sqrt{17}}{4}\right)} + \sqrt{\left(\frac{33 + 9\sqrt{17}}{4}\right)} \right\}^{\frac{1}{3}},
 \end{aligned}$$

$$\left[\left(\frac{1}{8}(5+\sqrt{17}) \right)^{0.5} + \left(\frac{1}{8}(\sqrt{17}-3) \right)^{0.5} \right]^2$$

$$\left[\left(\frac{1}{4}(37+9\sqrt{17}) \right)^{0.5} + \left(\frac{1}{4}(33+9\sqrt{17}) \right)^{0.5} \right]^{1/3}$$

Input

$$\left(\sqrt{\frac{1}{8}(5+\sqrt{17})} + \sqrt{\frac{1}{8}(\sqrt{17}-3)} \right)^2 \sqrt[3]{\sqrt{\frac{1}{4}(37+9\sqrt{17})} + \sqrt{\frac{1}{4}(33+9\sqrt{17})}}$$

Exact result

$$\left(\frac{1}{2} \sqrt{\frac{1}{2}(\sqrt{17}-3)} + \frac{1}{2} \sqrt{\frac{1}{2}(5+\sqrt{17})} \right)^2 \sqrt[3]{\frac{1}{2} \sqrt{33+9\sqrt{17}} + \frac{1}{2} \sqrt{37+9\sqrt{17}}}$$

Decimal approximation

4.2454719769464359360505946941857961435771422243258440000712880354

...

4.245471976946.....

Alternate forms

$$\frac{1}{4} \left(1 + \sqrt{17} + \sqrt{2(1+\sqrt{17})} \right)$$

root of $x^{24} - 70x^{18} - 150x^{12} - 70x^6 + 1$ near $x = 2.04009$

$$\sqrt[6]{\text{root of } x^8 - 5860x^7 + 27000x^6 - 59660x^5 + 77102x^4 - 59660x^3 + 27000x^2 - 5860x + 1 \text{ near } x = 5855.39}$$

$$\frac{\left(\sqrt{\sqrt{17}-3} + \sqrt{5+\sqrt{17}} \right)^2}{8 \sqrt[3]{\sqrt{\frac{2}{3(11+3\sqrt{17})} + \sqrt{37+9\sqrt{17}}}}}$$

Minimal polynomial

$$x^{48} - 5860x^{42} + 27000x^{36} - 59660x^{30} + 77102x^{24} - 59660x^{18} + 27000x^{12} - 5860x^6 + 1$$

Expanded forms

$$\frac{1 + \sqrt{17}}{4 \sqrt[3]{\frac{2}{\sqrt{33+9\sqrt{17}} + \sqrt{37+9\sqrt{17}}}}} + \frac{\sqrt{8 + 8\sqrt{17}}}{8 \sqrt[3]{\frac{2}{\sqrt{33+9\sqrt{17}} + \sqrt{37+9\sqrt{17}}}}}$$

$$\begin{aligned} & \frac{1}{4} \sqrt[3]{\frac{1}{2} \sqrt{33+9\sqrt{17}} + \frac{1}{2} \sqrt{37+9\sqrt{17}}} + \\ & \frac{1}{4} \sqrt{17} \sqrt[3]{\frac{1}{2} \sqrt{33+9\sqrt{17}} + \frac{1}{2} \sqrt{37+9\sqrt{17}}} + \\ & \frac{1}{4} \sqrt{(\sqrt{17} - 3)(5 + \sqrt{17})} \sqrt[3]{\frac{1}{2} \sqrt{33+9\sqrt{17}} + \frac{1}{2} \sqrt{37+9\sqrt{17}}} \end{aligned}$$

From the results of the above expressions, we obtain, after some calculations:

$$(16.542097553485+0.518874943499968+4.245471976946)^{(1/e)} + (((e^{(1/4)} \log^{(13/16)}(2))/(9 \cdot 2^{(1/8)} \cdot 3^{(3/8)} \log^{(11/16)}(3))))$$

where

$$\frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} \cdot 3^{3/8} \log^{11/16}(3)} \approx 0.0603075960251772$$

Input interpretation

$$\frac{e^{16.542097553485 + 0.518874943499968 + 4.245471976946} + \sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} \times 3^{3/8} \log^{11/16}(3)}$$

$\log(x)$ is the natural logarithm

Result

3.1415926535903...

3.1415926535903..... $\approx \pi$

Alternative representations

$$\sqrt[9]{16.5420975534850000 + 0.5188749434999680000 + 4.2454719769460000} + \frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log^{11/16}(3)} = \sqrt[9]{21.3064444739309680} + \frac{\sqrt[4]{e} \log_e^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log_e^{11/16}(3)}$$

$$\sqrt[9]{16.5420975534850000 + 0.5188749434999680000 + 4.2454719769460000} + \frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log^{11/16}(3)} = \sqrt[9]{21.3064444739309680} + \frac{\sqrt[4]{e} (\log(a) \log_a(2))^{13/16}}{9 \sqrt[8]{2} 3^{3/8} (\log(a) \log_a(3))^{11/16}}$$

$$\sqrt[9]{16.5420975534850000 + 0.5188749434999680000 + 4.2454719769460000} + \frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log^{11/16}(3)} = \sqrt[9]{21.3064444739309680} + \frac{\sqrt[4]{e} (2 \coth^{-1}(3))^{13/16}}{9 \sqrt[8]{2} 3^{3/8} (2 \coth^{-1}(2))^{11/16}}$$

Integral representations

$$\sqrt[9]{16.5420975534850000 + 0.5188749434999680000 + 4.2454719769460000} + \frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log^{11/16}(3)} = \frac{2^{7/8} \times 3^{5/8} \sqrt[4]{e} \left(\int_1^2 \frac{1}{t} dt\right)^{13/16} + 54 \sqrt[9]{21.3064444739309680} \left(\int_1^3 \frac{1}{t} dt\right)^{11/16}}{54 \left(\int_1^3 \frac{1}{t} dt\right)^{11/16}}$$

$$\begin{aligned}
& \sqrt[e]{16.5420975534850000 + 0.5188749434999680000 + 4.2454719769460000} + \\
& \frac{\sqrt[4]{e} \log^{13/16}(2)}{9 \sqrt[8]{2} 3^{3/8} \log^{11/16}(3)} = \\
& \left(54 \sqrt[e]{21.3064444739309680} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \right. \\
& \quad \left. 2^{3/4} \times 3^{5/8} \sqrt[4]{e} i\pi \left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{13/16} \right. \\
& \quad \left. \left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{5/16} \right) / \\
& \left(54 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0
\end{aligned}$$

We have:

$$\begin{aligned}
g_{154}^2 &= \sqrt{\left\{ (2\sqrt{2} + \sqrt{7}) \left(\frac{\sqrt{7} + \sqrt{11}}{2} \right) \right\}} \\
&\quad \times \left\{ \sqrt{\left(\frac{13 + 2\sqrt{22}}{4} \right)} + \sqrt{\left(\frac{9 + 2\sqrt{22}}{4} \right)} \right\}, \\
g_{158} + \frac{1}{g_{158}} &= \frac{1}{2} \{ \sqrt{(9 + \sqrt{2})} + \sqrt{(17 + 13\sqrt{2})} \},
\end{aligned}$$

From:

$$\begin{aligned}
& \sqrt{\left\{ (2\sqrt{2} + \sqrt{7}) \left(\frac{\sqrt{7} + \sqrt{11}}{2} \right) \right\}} \\
& \quad \times \left\{ \sqrt{\left(\frac{13 + 2\sqrt{22}}{4} \right)} + \sqrt{\left(\frac{9 + 2\sqrt{22}}{4} \right)} \right\},
\end{aligned}$$

we obtain:

$$\sqrt{\left(\left(\left(2\sqrt{2}+\sqrt{7}\right)\left(\frac{1}{2}\left(\sqrt{7}+\sqrt{11}\right)\right)\right)\right)^{\frac{1}{2}}\left[\left(\frac{1}{4}\left(13+2\sqrt{22}\right)\right)^{0.5}+\left(\frac{1}{4}\left(9+2\sqrt{22}\right)\right)^{0.5}\right]}$$

Input

$$\sqrt{\left(2\sqrt{2}+\sqrt{7}\right)\left(\frac{1}{2}\left(\sqrt{7}+\sqrt{11}\right)\right)\left(\sqrt{\frac{1}{4}\left(13+2\sqrt{22}\right)}+\sqrt{\frac{1}{4}\left(9+2\sqrt{22}\right)}\right)}$$

Exact result

$$\sqrt{\frac{1}{2}\left(2\sqrt{2}+\sqrt{7}\right)\left(\sqrt{7}+\sqrt{11}\right)\left(\frac{1}{2}\sqrt{9+2\sqrt{22}}+\frac{1}{2}\sqrt{13+2\sqrt{22}}\right)}$$

Decimal approximation

18.215472549990102680612133470937714740862729392434624039434624861

...

18.21547254999....

Alternate forms

$$\frac{1}{2}\sqrt{\frac{7}{2}+\sqrt{14}+\sqrt{22}+\frac{\sqrt{77}}{2}}\left(\sqrt{9+2\sqrt{22}}+\sqrt{13+2\sqrt{22}}\right)$$

| |
|---------------------------------------------------------------------------------------------|
| root of $x^8 - 18x^7 - 3x^6 - 18x^5 + 20x^4 + 18x^3 - 3x^2 + 18x + 1$ near $x = 18.2155$ |
|---------------------------------------------------------------------------------------------|

$$\frac{1}{4} \sqrt{(2\sqrt{2} + \sqrt{7})(\sqrt{7} + \sqrt{11})} \left(\sqrt{9 - i\sqrt{7}} + \sqrt{2} \left(\sqrt{2} + \sqrt{11} + \sqrt{\frac{1}{2}i(\sqrt{7} - 9i)} \right) \right)$$

$$\frac{1}{4} \left(\sqrt{302 + 80\sqrt{14} + 64\sqrt{22} + 34\sqrt{77}} + \sqrt{358 + 96\sqrt{14} + 80\sqrt{22} + 42\sqrt{77}} \right)$$

Minimal polynomial

$$x^8 - 18x^7 - 3x^6 - 18x^5 + 20x^4 + 18x^3 - 3x^2 + 18x + 1$$

Expanded forms

$$\frac{1}{2} \left(\sqrt{\frac{151}{2} + 20\sqrt{14} + 16\sqrt{22} + \frac{17\sqrt{77}}{2}} + \sqrt{\frac{179}{2} + 24\sqrt{14} + 20\sqrt{22} + \frac{21\sqrt{77}}{2}} \right)$$

$$\frac{1}{2} \sqrt{\frac{1}{2}(2\sqrt{2} + \sqrt{7})(\sqrt{7} + \sqrt{11})(9 + 2\sqrt{22})} + \frac{1}{2} \sqrt{\frac{1}{2}(2\sqrt{2} + \sqrt{7})(\sqrt{7} + \sqrt{11})(13 + 2\sqrt{22})}$$

and:

$$\frac{1}{2} \{ \sqrt{(9 + \sqrt{2})} + \sqrt{(17 + 13\sqrt{2})} \},$$

$$\frac{1}{2} [((9+\sqrt{2})^{0.5})+((17+13\sqrt{2})^{0.5})]$$

Input

$$\frac{1}{2} \left(\sqrt{9 + \sqrt{2}} + \sqrt{17 + 13 \sqrt{2}} \right)$$

Decimal approximation

4.5878082470404325582043862534461921013553428662511752760599755784

...

4.587808247....

Alternate forms

$$\sqrt{\frac{1}{2} \left(13 + 7 \sqrt{2} + \sqrt{179 + 134 \sqrt{2}} \right)}$$

$$\frac{1}{2} \left(\sqrt{\frac{17}{2} - \frac{7i}{2}} + \sqrt{\frac{17}{2} + \frac{7i}{2}} + \sqrt{9 + \sqrt{2}} \right)$$

$$\frac{1}{2} \sqrt{\frac{17}{2} - \frac{7i}{2}} + \frac{1}{2} \sqrt{\frac{17}{2} + \frac{7i}{2}} + \frac{\sqrt{9 + \sqrt{2}}}{2}$$

Minimal polynomial

$$x^8 - 26x^6 + 115x^4 - 236x^2 + 196$$

Expanded form

$$\frac{\sqrt{9 + \sqrt{2}}}{2} + \frac{1}{2} \sqrt{17 + 13 \sqrt{2}}$$

From which:

$$(18.21547254999010268+4.5878082470404325582)^{1/3}+\left(\frac{1}{13}(-e^\pi+21\pi-\log(4096)+2\log(\pi)-26\tan^{-1}(\pi))\right)$$

Input interpretation

$$\sqrt[3]{18.21547254999010268 + 4.5878082470404325582} + \frac{1}{13}(-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi))$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result

3.1415926537604090956...

(result in radians)

$$3.14159265376\dots \approx \pi$$

Alternative representations

$$\sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \frac{1}{13}(-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \sqrt[3]{22.803280797030535238200} + \frac{1}{13}(21\pi - 26\tan^{-1}(1, \pi) - \log(4096) + 2\log(\pi) - e^\pi)$$

$$\sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \frac{1}{13}(-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \sqrt[3]{22.803280797030535238200} + \frac{1}{13}(21\pi - 26\tan^{-1}(\pi) - \log(a)\log_a(4096) + 2\log(a)\log_a(\pi) - e^\pi)$$

$$\sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) =$$

$$\sqrt[3]{22.803280797030535238200} + \frac{1}{13} (21\pi - 26\tan^{-1}(\pi) - \log_e(4096) + 2\log_e(\pi) - e^\pi)$$

Series representations

$$\sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) =$$

$$2.8357358806885520288858 - 0.076923076923076923076923 e^\pi + 1.61538461538461538461538 \pi - 2.0000000000000000000000 \tan^{-1}(\pi) - 0.076923076923076923076923 \log(4095) + 0.153846153846153846153846 \log(-1 + \pi) + \sum_{k=1}^{\infty} \frac{1}{k} \left(0.07692307692307692308 \left(-\frac{1}{4095} \right)^k - 0.15384615384615384615 (-1)^k (-1 + \pi)^{-k} \right)$$

$$\sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) =$$

$$2.8357358806885520288858 - 0.076923076923076923076923 e^\pi + 1.61538461538461538461538 \pi - 0.076923076923076923076923 \log(4096) + 0.153846153846153846153846 \log(\pi) - 2.0000000000000000000000 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5} \right)^k 2^{1+2k} F_{1+2k} \left(\frac{\pi}{1 + \sqrt{1 + \frac{4\pi^2}{5}}} \right)^{1+2k}}{1 + 2k}$$

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& 2.8357358806885520288858 - 0.076923076923076923076923 e^\pi + \\
& 1.61538461538461538461538 \pi - 2.0000000000000000000000 \tan^{-1}(x) - \\
& 2.0000000000000000000000 \pi \left[\frac{\arg(i(\pi - x))}{2\pi} \right] - \\
& 0.076923076923076923076923 \log(4096) + \\
& 0.153846153846153846153846 \log(\pi) - \\
& 1.0000000000000000000000 i \sum_{k=1}^{\infty} \frac{(-(-i-x)^{-k} + (i-x)^{-k})(\pi-x)^k}{k}
\end{aligned}$$

for $(ix \in \mathbb{R} \text{ and } ix < -1)$

F_n is the n^{th} Fibonacci number

Integral representations

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& 2.8357358806885520288858 - 0.076923076923076923076923 e^\pi + \\
& 1.61538461538461538461538 \pi - 2.0000000000000000000000 \pi \\
& \int_0^1 \frac{1}{1 + \pi^2 t^2} dt - 0.076923076923076923076923 \log(4096) + \\
& 0.153846153846153846153846 \log(\pi)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& 2.8357358806885520288858 - 0.076923076923076923076923 e^\pi + \\
& 1.61538461538461538461538 \pi + \\
& \int_0^1 \left(- \frac{0.076923076923076923076923}{0.0002442002442002442002442 + 1.0000000000000000000000 t} + \right. \\
& \quad \left. (-0.1538461538461538461538 + 0.1538461538461538461538 \pi) / \right. \\
& \quad \left. (1.0000000000000000000000 + (-1.0000000000000000000000 + \right. \\
& \quad \left. \pi)t) - \frac{2.0000000000000000000000}{1 + \pi^2 t^2} \right) dt
\end{aligned}$$

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& 2.83573588068855202888578 - 0.076923076923076923076923 e^\pi + \\
& 1.61538461538461538461538\pi + \\
& \frac{i}{2\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} (1+\pi^2)^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds - \\
& 0.076923076923076923076923 \log(4096) + \frac{2\log(\pi)}{13} \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

Continued fraction representations

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& \frac{1}{13} \left(36.864566448951176375515 - e^\pi + 21\pi - \right. \\
& \left. \log(4096) + 2\log(\pi) - \frac{26\pi}{1 + \prod_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}} \right) = \\
& \frac{1}{13} \left(73.669013146537022012782 - \frac{26\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& \frac{1}{13} \left(36.864566448951176375515 - e^\pi - 5\pi - \right. \\
& \left. \log(4096) + 2\log(\pi) + \frac{26\pi^3}{3 + \sum_{k=1}^{\infty} \frac{(1+(-1)^{1+k}+k)^2 \pi^2}{3+2k}} \right) = \\
& \frac{1}{13} \left(-8.012395846797602187247 + \frac{26\pi^3}{3 + \frac{9\pi^2}{5 + \frac{4\pi^2}{7 + \frac{25\pi^2}{9 + \frac{16\pi^2}{11 + \dots}}}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[3]{18.215472549990102680000 + 4.58780824704043255820000} + \\
& \frac{1}{13} (-e^\pi + 21\pi - \log(4096) + 2\log(\pi) - 26\tan^{-1}(\pi)) = \\
& \frac{1}{13} \left(36.864566448951176375515 - e^\pi + 21\pi - \frac{26\pi}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{k^2\pi^2}{1+2k}} - \right. \\
& \left. \frac{4095}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{4095 \left| \frac{1+k}{2} \right|^2}{1+k}} + \frac{2(-1+\pi)}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{(-1+\pi) \left| \frac{1+k}{2} \right|^2}{1+k}} \right) = \\
& \frac{1}{13} \left(79.697319541557565377502 - \frac{26\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} - \right. \\
& \left. \frac{4095}{1 + \frac{4095}{2 + \frac{4095}{3 + \frac{16380}{4 + \frac{16380}{5 + \dots}}}}} + \frac{2(-1+\pi)}{1 + \frac{-1+\pi}{2 + \frac{-1+\pi}{3 + \frac{4(-1+\pi)}{4 + \frac{4(-1+\pi)}{5 + \dots}}}} \right)
\end{aligned}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

From the results of the following expressions:

$$\begin{aligned}
g_{62} &+ \frac{1}{g_{62}} = \frac{1}{2} \{ \sqrt{(1 + \sqrt{2})} + \sqrt{(9 + 5\sqrt{2})} \}, \\
G_{65}^2 &= \sqrt{ \left\{ \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3 + \sqrt{13}}{2} \right) \right\} \left\{ \sqrt{ \left(\frac{1 + \sqrt{65}}{8} \right) } + \sqrt{ \left(\frac{9 + \sqrt{65}}{8} \right) } \right\} }, \\
g_{66}^2 &= \sqrt{(\sqrt{2} + \sqrt{3})(7\sqrt{2} + 3\sqrt{11})}^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{7 + \sqrt{33}}{8} \right) } + \sqrt{ \left(\frac{\sqrt{33} - 1}{8} \right) } \right\}, \\
G_{69}^2 &= (3\sqrt{3} + \sqrt{23})^{\frac{1}{4}} \left(\frac{5 + \sqrt{23}}{4} \right)^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{6 + 3\sqrt{3}}{4} \right) } + \sqrt{ \left(\frac{2 + 3\sqrt{3}}{4} \right) } \right\}, \\
G_{77}^2 &= \left\{ \frac{1}{2} (\sqrt{7} + \sqrt{11})(8 + 3\sqrt{7}) \right\}^{\frac{1}{4}} \left\{ \sqrt{ \left(\frac{6 + \sqrt{11}}{4} \right) } + \sqrt{ \left(\frac{2 + \sqrt{11}}{4} \right) } \right\}, \\
G_{81}^3 &= \frac{(2\sqrt{3} + 2)^{\frac{1}{3}} + 1}{(2\sqrt{3} - 2)^{\frac{1}{3}} - 1}, \\
g_{90} &= \{ (2 + \sqrt{5})(\sqrt{5} + \sqrt{6}) \}^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{3 + \sqrt{6}}{4} \right) } + \sqrt{ \left(\frac{\sqrt{6} - 1}{4} \right) } \right\}, \\
g_{94} &+ \frac{1}{g_{94}} = \frac{1}{2} \{ \sqrt{(7 + \sqrt{2})} + \sqrt{(7 + 5\sqrt{2})} \}, \\
g_{98} &+ \frac{1}{g_{98}} = \frac{1}{2} \{ \sqrt{2} + \sqrt{(14 + 4\sqrt{14})} \}, \\
g_{114}^2 &= \sqrt{(\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{19})}^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{23 + 3\sqrt{57}}{8} \right) } + \sqrt{ \left(\frac{15 + 3\sqrt{57}}{8} \right) } \right\}, \\
G_{117} &= \frac{1}{2} \left(\frac{3 + \sqrt{13}}{2} \right)^{\frac{1}{4}} (2\sqrt{3} + \sqrt{13})^{\frac{1}{6}} \{ 3^{\frac{1}{3}} + \sqrt{(4 + \sqrt{3})} \}, \\
G_{121} &+ \frac{1}{G_{121}} = \left(\frac{11}{2} \right)^{\frac{1}{6}} \left\{ \left(3 + \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} + \left(3 - \frac{1}{3\sqrt{3}} \right)^{\frac{1}{3}} \right\} \\
\frac{1}{G_{121}} &= \frac{1}{3\sqrt{2}} [(11 - 3\sqrt{11})^{\frac{1}{3}} \{ (3\sqrt{11} + 3\sqrt{3} - 4)^{\frac{1}{3}} + (3\sqrt{11} - 3\sqrt{3} - 4)^{\frac{1}{3}} \} - 2] \\
g_{126} &= \sqrt{ \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) (\sqrt{6} + \sqrt{7}) }^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{3 + \sqrt{2}}{4} \right) } + \sqrt{ \left(\frac{\sqrt{2} - 1}{4} \right) } \right\}^2, \\
g_{138}^2 &= \sqrt{ \left(\frac{3\sqrt{3} + \sqrt{23}}{2} \right) (78\sqrt{2} + 23\sqrt{23}) }^{\frac{1}{6}} \times \left\{ \sqrt{ \left(\frac{5 + 2\sqrt{6}}{4} \right) } + \sqrt{ \left(\frac{1 + 2\sqrt{6}}{4} \right) } \right\}, \\
G_{141}^2 &= (4\sqrt{3} + \sqrt{47})^{\frac{1}{4}} \left(\frac{7 + \sqrt{47}}{\sqrt{2}} \right)^{\frac{1}{6}} \left\{ \sqrt{ \left(\frac{18 + 9\sqrt{3}}{4} \right) } + \sqrt{ \left(\frac{14 + 9\sqrt{3}}{4} \right) } \right\},
\end{aligned}$$

$$\begin{aligned}
G_{145}^2 &= \sqrt{\left\{ \frac{(2 + \sqrt{5})(5 + \sqrt{29})}{2} \right\}} \left\{ \sqrt{\left(\frac{17 + \sqrt{145}}{8} \right)} + \sqrt{\left(\frac{9 + \sqrt{145}}{8} \right)} \right\}, \\
\frac{1}{G_{147}} &= 2^{-\frac{1}{12}} \left[\frac{1}{2} + \frac{1}{\sqrt{3}} \left\{ \sqrt{\left(\frac{7}{4} \right)} - (28)^{\frac{1}{8}} \right\} \right], \\
G_{153} &= \left\{ \sqrt{\left(\frac{5 + \sqrt{17}}{8} \right)} + \sqrt{\left(\frac{\sqrt{17} - 3}{8} \right)} \right\}^2 \\
&\quad \times \left\{ \sqrt{\left(\frac{37 + 9\sqrt{17}}{4} \right)} + \sqrt{\left(\frac{33 + 9\sqrt{17}}{4} \right)} \right\}^{\frac{1}{3}}, \\
g_{154}^2 &= \sqrt{\left\{ (2\sqrt{2} + \sqrt{7}) \left(\frac{\sqrt{7} + \sqrt{11}}{2} \right) \right\}} \\
&\quad \times \left\{ \sqrt{\left(\frac{13 + 2\sqrt{22}}{4} \right)} + \sqrt{\left(\frac{9 + 2\sqrt{22}}{4} \right)} \right\}, \\
g_{158} + \frac{1}{g_{158}} &= \frac{1}{2} \{ \sqrt{(9 + \sqrt{2})} + \sqrt{(17 + 13\sqrt{2})} \},
\end{aligned}$$

where the g_n are highlighted in black and the G_m are highlighted in red

2.781323803920547....

5.8364372603724....

5.93160414841568....

6.2220252193329....

7.033656610253....

20.37749997725....

2.911116655774....

3.32593429313....

3.39813955444....

11.5730481521....

3.464643979529....

3.830586436322.....

0.2817853021....

3.6548630551090639....

15.3156144852002258.....

15.833404207477....

16.542097553485.....

0.518874943499968.....

4.245471976946.....

18.21547254999....

4.587808247....

FIRST TWENTY-ONE RESULTS

We obtain:

(2.781323803920547+5.93160414841568+2.911116655774+ 3.32593429313 +
3.39813955444+ 11.5730481521 + 3.6548630551090639 + 15.3156144852002258 +
18.21547254999 + 4.587808247)

Input interpretation

2.781323803920547 + 5.93160414841568 + 2.911116655774 +
3.32593429313 + 3.39813955444 + 11.5730481521 + 3.6548630551090639 +
15.3156144852002258 + 18.21547254999 + 4.587808247

Result

71.6949249450795167

71.6949249450795167

And:

(5.8364372603724 + 6.2220252193329 + 7.033656610253 + 20.37749997725 +
3.464643979529 + 3.830586436322 + 0.2817853021 + 15.833404207477
+16.542097553485 + 0.518874943499968 + 4.245471976946)

Input interpretation

5.8364372603724 + 6.2220252193329 + 7.033656610253 + 20.37749997725 +
3.464643979529 + 3.830586436322 + 0.2817853021 + 15.833404207477 +
16.542097553485 + 0.518874943499968 + 4.245471976946

Result

84.186483466567268

84.186483466567268

From these two results, we obtain:

$((84.186483466567268)/(71.6949249450795167))+2*(1/3(1/(1.0018674362$
 $)+0.9568666373+0.9991104684))$

where

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}} - \varphi} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

are Rogers-Ramanujan continued fractions

Input interpretation

$$\frac{84.186483466567268}{71.6949249450795167} + 2 \left(\frac{1}{3} \left(\frac{1}{1.0018674362} + 0.9568666373 + 0.9991104684 \right) \right)$$

Result

3.1436408834621626444919482719365045063068112499917305679005298665

...

3.1436408834.... $\approx \pi$

and also:

$$1.0018674362 + (1 / (((84.186483466567268) * (71.6949249450795167))))^{1/18}$$

Input interpretation

$$1.0018674362 + \frac{1}{\sqrt[18]{84.186483466567268 \times 71.6949249450795167}}$$

Result

1.6184047996993898980281228267159897410801242114829245654695610151

...

1.618404799699.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Observe that we can to obtain approximations to π also as follows:

$$1/11(4\ln(((84.186483466567268)*(71.6949249450795167))))$$

Input interpretation

$$\frac{1}{11} (4 \log(84.186483466567268 \times 71.6949249450795167))$$

log(x) is the natural logarithm

Result

3.16561976100504820...

3.165619761....

Alternative representations

$$\frac{\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000)}{4 \log_e(6035.74361352571792740)} = \frac{1}{11}$$

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4}{11} \log(a) \log_a(6035.74361352571792740)$$

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$- \frac{4 \operatorname{Li}_1(-6034.74361352571792740)}{11}$$

Series representations

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4 \log(6034.74361352571792740)}{11} - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.70528864936916245621k}}{k}$$

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{8}{11} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2\pi} \right] + \frac{4 \log(x)}{11} -$$

$$\frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$\frac{4 \log(z_0)}{11} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log(z_0) -$$

$$\frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}$$

Integral representations

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) = \frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt$$

$$\frac{4}{11} \log(84.1864834665672680000 \times 71.69492494507951670000) = \frac{2}{11 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$1/22(8\ln(((84.186483466567268)*(71.6949249450795167))))$$

Input interpretation

$$\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167))$$

$\log(x)$ is the natural logarithm

Result

3.16561976100504820...

[3.165619761.... result equal to previous one](#)

Alternative representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) = \frac{8 \log_e(6035.74361352571792740)}{22}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) = \frac{8}{22} \log(a) \log_a(6035.74361352571792740)$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$-\frac{8 \operatorname{Li}_1(-6034.74361352571792740)}{22}$$

Series representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4 \log(6034.74361352571792740)}{11} - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.70528864936916245621 k}}{k}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{8}{11} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2 \pi} \right] + \frac{4 \log(x)}{11} -$$

$$\frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$\frac{4 \log(z_0)}{11} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log(z_0) -$$

$$\frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}$$

Integral representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{2}{11 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

From which:

$$\frac{1}{22}(8 \ln(((84.186483466567268) * (71.6949249450795167)))) - ((2^5 * 3^5) / (e^{12} \log^3(2) \log^{19}(3)))$$

where

$$\frac{7776}{e^{12} \log^3(2) \log^{19}(3)} \approx 0.02402710726014$$

$$(7776 = 2^5 * 3^5)$$

Input interpretation

$$\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)}$$

$\log(x)$ is the natural logarithm

Result

3.14159265374490574...

$$3.141592653\dots \approx \pi$$

Alternative representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) -$$

$$\frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{8 \log_e(6035.74361352571792740)}{22} - \frac{2^5 \times 3^5}{e^{12} \log_e^3(2) \log_e^{19}(3)}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{8}{22} \log(a) \log_a(6035.74361352571792740) - \frac{2^5 \times 3^5}{e^{12} (\log(a) \log_a(2))^3 (\log(a) \log_a(3))^{19}}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{16}{22} \coth^{-1}\left(\frac{6036.74361352571792740}{6034.74361352571792740}\right) - \frac{2^5 \times 3^5}{e^{12} (2 \coth^{-1}(2))^{19} (2 \coth^{-1}(3))^3}$$

Series representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{8}{11} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2 \pi} \right] + \frac{4 \log(x)}{11} - 7776 / \left(e^{12} \left(2 i \pi \left[\frac{\arg(2 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k}}{k} \right)^3 \left(2 i \pi \left[\frac{\arg(3 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - x)^k x^{-k}}{k} \right)^{19} \right) - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\begin{aligned}
& \frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \\
& \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \\
& \frac{4 \log(z_0)}{11} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log(z_0) - \frac{7776}{\left(e^{12} \left(\log(z_0) + \left[\frac{\arg(2 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right)^3 \right.} \\
& \left. \left(\log(z_0) + \left[\frac{\arg(3 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{19} \right) - \\
& \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

$$\begin{aligned}
& \frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \\
& \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \\
& \frac{8}{11} i \pi \left[\frac{\pi - \arg\left(\frac{6035.74361352571792740}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{4 \log(z_0)}{11} - \\
& \frac{7776}{\left(e^{12} \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right)^3 \right.} \\
& \left. \left(2 i \pi \left[\frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)^{19} \right) - \\
& \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

Integral representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \frac{4 \left(-21384 + e^{12} \left(\int_1^3 \frac{1}{t} dt \right)^3 \left(\int_1^3 \frac{1}{t} dt \right)^{19} \int_1^{6035.74361352571792740} \frac{1}{t} dt \right)}{11 e^{12} \left(\int_1^2 \frac{1}{t} dt \right)^3 \left(\int_1^3 \frac{1}{t} dt \right)^{19}}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{2^5 \times 3^5}{e^{12} \log^3(2) \log^{19}(3)} = \left(2 \left(-179381993472 i^{23} \pi^{23} + e^{12} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^3 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{19} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \right) / \left(11 e^{12} i \pi \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^3 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{19} \right)$$

for $-1 < \gamma < 0$

Or:

$$1/22(8\ln(((84.186483466567268)*(71.6949249450795167))))-(24/10^3)$$

Input interpretation

$$\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{24}{10^3}$$

$\log(x)$ is the natural logarithm

Result

3.14161976100504820...

3.141619761.... $\approx \pi$

Alternative representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} =$$

$$\frac{8 \log_e(6035.74361352571792740)}{22} - \frac{24}{10^3}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} =$$

$$\frac{8}{22} \log(a) \log_a(6035.74361352571792740) - \frac{24}{10^3}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} =$$

$$- \frac{8 \operatorname{Li}_1(-6034.74361352571792740)}{22} - \frac{24}{10^3}$$

Series representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} =$$

$$- \frac{3}{125} + \frac{4 \log(6034.74361352571792740)}{11} - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.70528864936916245621k}}{k}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} =$$

$$- \frac{3}{125} + \frac{8}{11} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2 \pi} \right] + \frac{4 \log(x)}{11} -$$

$$\frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\begin{aligned} & \frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} = \\ & - \frac{3}{125} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \\ & \frac{4 \log(z_0)}{11} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right] \log(z_0) - \\ & \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k} \end{aligned}$$

Integral representations

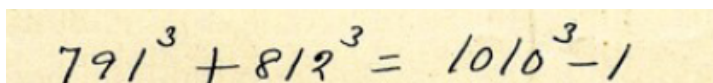
$$\begin{aligned} & \frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} = \\ & - \frac{3}{125} + \frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt \end{aligned}$$

$$\begin{aligned} & \frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{10^3} = - \frac{3}{125} + \\ & \frac{2}{11 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0 \end{aligned}$$

Or again, utilizing the Ramanujan taxicab numbers:

$$\begin{aligned} & 1/22(8\ln(((84.186483466567268)*(71.6949249450795167))))- \\ & (24/(791^3+812^3+1))^{1/3} \end{aligned}$$

where we obtain easily $(791^3+812^3+1)^{1/3}$ from the following identity:



$$791^3 + 812^3 = 1010^3 - 1$$

Input interpretation

$$\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}}$$

$\log(x)$ is the natural logarithm

Result

3.14185738476742444...

3.141857384.... $\approx \pi$

Alternative representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = \frac{8 \log_e(6035.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = \frac{8}{22} \log(a) \log_a(6035.74361352571792740) - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = \frac{8 \operatorname{Li}_1(-6034.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}}$$

Series representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = -\frac{12}{505} + \frac{4 \log(6034.74361352571792740)}{11} - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.70528864936916245621 k}}{k}$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = -\frac{12}{505} + \frac{8}{11} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2 \pi} \right] + \frac{4 \log(x)}{11} - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = -\frac{12}{505} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \frac{4 \log(z_0)}{11} + \frac{4}{11} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log(z_0) - \frac{4}{11} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}$$

Integral representations

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = -\frac{12}{505} + \frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt$$

$$\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} = -\frac{12}{505} + \frac{2}{11i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

From which:

$$\left(\frac{1}{6} \left(\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2$$

Input interpretation

$$\left(\frac{1}{6} \left(\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2$$

$\log(x)$ is the natural logarithm

Result

1728.157763100057...

1728.157763.....

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 =$$

$$-23 + \left(\frac{1}{6} \left(\frac{8 \log_e(6035.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15}$$

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 =$$

$$-23 + \left(\frac{1}{6} \left(-\frac{8 \operatorname{Li}_1(-6034.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15}$$

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 =$$

$$-23 + \left(\frac{1}{6} \left(\frac{8}{22} \log(a) \log_a(6035.74361352571792740) - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15}$$

Series representations

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \right. \right. \\ \left. \left. \frac{\frac{24}{\sqrt[3]{791^3 + 812^3 + 1}}}{2} \right)^{15} - 21 - 2 = \right. \\ \left. -23 + \frac{1}{470184984576} \left(-\frac{12}{505} + \frac{4}{11} \left(\log(6034.74361352571792740) - \right. \right. \right. \\ \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-0.000165707122628820916395)^k}{k} \right) \right) \right)^{30}$$

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \right. \right. \\ \left. \left. \frac{\frac{24}{\sqrt[3]{791^3 + 812^3 + 1}}}{2} \right)^{15} - 21 - 2 = -23 + \frac{1}{470184984576} \right. \\ \left. \left(-\frac{12}{505} + \frac{4}{11} \left(2i\pi \left\lfloor \frac{\arg(6035.74361352571792740 - x)}{2\pi} \right\rfloor + \log(x) - \right. \right. \right. \\ \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \right) \right) \right)^{30} \text{ for } x < 0$$

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \right. \right. \\ \left. \left. \frac{\frac{24}{\sqrt[3]{791^3 + 812^3 + 1}}}{2} \right)^{15} - \right. \\ \left. 21 - 2 = -23 + \frac{1}{470184984576} \left(-\frac{12}{505} + \frac{4}{11} \left(\log(z_0) + \right. \right. \right. \\ \left. \left. \left. \left\lfloor \frac{\arg(6035.74361352571792740 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \right. \\ \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k} \right) \right) \right)^{30}$$

Integral representations

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 = -23 + \frac{\left(\frac{12}{505} - \frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt \right)^{30}}{470184984576}$$

$$\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 = -23 + \frac{\left(\frac{12}{505} - \frac{2}{11i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.70528864936916245621s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{30}}{470184984576}$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{1}{22} (8 \ln(84.186483466567268) * (71.6949249450795167)) \right) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right)^2 - \frac{1}{\sqrt{2}}$$

Input interpretation

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{1}{22} (8 \log(84.186483466567268 \times 71.6949249450795167)) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right)^2 - \frac{1}{\sqrt{2}} \right)$$

Result

4096.040841316199...

$4096.0408\dots \approx 4096 = 64^2$

Alternative representations

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} = \left(\frac{1}{27} \left(-23 + \left(\frac{1}{6} \left(\frac{8 \log_e(6035.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15} \right) \right)^2 - \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} = \left(\frac{1}{27} \left(-23 + \left(\frac{1}{6} \left(-\frac{8 \operatorname{Li}_1(-6034.74361352571792740)}{22} - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15} \right) \right)^2 - \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} =$$

$$\left(\frac{1}{27} \left(-23 + \left(\frac{1}{6} \left(\frac{8}{22} \log(a) \log_a(6035.74361352571792740) - \frac{24}{\sqrt[3]{1 + 791^3 + 812^3}} \right)^2 \right)^{15} \right) \right)^2 - \frac{1}{\sqrt{2}}$$

Series representations

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 -$$

$$\frac{1}{\sqrt{2}} = \frac{1}{729} \left(23 - \frac{\left(\frac{12}{505} - \frac{4 \log(6035.74361352571792740)}{11} \right)^{30}}{470\,184\,984\,576} \right)^2 -$$

$$\frac{1}{\exp(i \pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \right. \right. \right. \right. \\ \left. \left. \left. \left. \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \\ \frac{1}{\sqrt{2}} = \frac{1}{729} \left(-23 + \frac{\left(-\frac{12}{505} + \frac{4 \log(6035.74361352571792740)}{11} \right)^{30}}{470\,184\,984\,576} \right)^2 - \\ \frac{\left(\frac{1}{z_0} \right)^{-1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(-1-\lfloor \arg(2-z_0)/(2\pi) \rfloor)}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \right. \right. \right. \right. \\ \left. \left. \left. \left. \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} = \\ \frac{1}{729} \left(-23 + \frac{1}{470\,184\,984\,576} \left(-\frac{12}{505} + \frac{4}{11} \left(\log(6034.74361352571792740) - \right. \right. \right. \right. \\ \left. \left. \left. \sum_{k=1}^{\infty} \frac{(-0.000165707122628820916395)^k}{k} \right) \right)^{30} \right)^2 - \\ \frac{\exp(i\pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{1} \text{ for} \\ (x \in \mathbb{R} \\ \text{and} \\ x < 0)$$

Integral representations

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} = \frac{1}{729} \left(23 - \frac{\left(\frac{12}{505} - \frac{4}{11} \int_1^{6035.74361352571792740} \frac{1}{t} dt \right)^{30}}{470184984576} \right)^2 - \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{27} \left(\left(\frac{1}{6} \left(\frac{8}{22} \log(84.1864834665672680000 \times 71.69492494507951670000) - \frac{24}{\sqrt[3]{791^3 + 812^3 + 1}} \right)^2 \right)^{15} - 21 - 2 \right) \right)^2 - \frac{1}{\sqrt{2}} = \frac{1}{729} \left(-23 + \frac{\left(-\frac{12}{505} + \frac{2}{11i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6034.74361352571792740^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{30}}{470184984576} \right)^2 - \frac{1}{\sqrt{2}}$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

We obtain an approximation to π , also:

$$1/36(13\ln(((84.186483466567268)*(71.6949249450795167))))$$

Input interpretation

$$\frac{1}{36} (13 \log(84.186483466567268 \times 71.6949249450795167))$$

$\log(x)$ is the natural logarithm

Result

3.14363629044251314...

$$3.14363629\dots \approx \pi$$

Alternative representations

$$\frac{\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000)}{13 \log_e(6035.74361352571792740)} = \frac{\quad}{36}$$

$$\frac{\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000)}{\frac{13}{36} \log(a) \log_a(6035.74361352571792740)} =$$

$$\frac{\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000)}{13 \operatorname{Li}_1(-6034.74361352571792740)} = \frac{\quad}{36}$$

Series representations

$$\frac{\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000)}{13 \log(6034.74361352571792740)} = \frac{13 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.70528864936916245621 k}}{k}}{36}$$

$$\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{13}{18} i \pi \left[\frac{\arg(6035.74361352571792740 - x)}{2 \pi} \right] + \frac{13 \log(x)}{36} -$$

$$\frac{13}{36} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{13}{36} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$\frac{13 \log(z_0)}{36} + \frac{13}{36} \left[\frac{\arg(6035.74361352571792740 - z_0)}{2 \pi} \right] \log(z_0) -$$

$$\frac{13}{36} \sum_{k=1}^{\infty} \frac{(-1)^k (6035.74361352571792740 - z_0)^k z_0^{-k}}{k}$$

Integral representations

$$\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{13}{36} \int_1^{6035.74361352571792740} \frac{1}{t} dt$$

$$\frac{13}{36} \log(84.1864834665672680000 \times 71.69492494507951670000) =$$

$$\frac{13}{72 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-8.70528864936916245621 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

We have these other expressions:

$$\left. \begin{aligned} G_{169} + \frac{1}{G_{169}} &= \left(\frac{13}{4}\right)^{\frac{1}{6}} \left\{ \left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} + \left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} \right\}^2 \\ \frac{1}{G_{169}} &= \frac{1}{3} \left[(\sqrt{13} - 2) + \left(\frac{13 - 3\sqrt{13}}{2}\right)^{\frac{1}{3}} \right. \\ &\quad \left. \times \left\{ \left(3\sqrt{3} - \frac{11 - \sqrt{13}}{2}\right)^{\frac{1}{3}} - \left(3\sqrt{3} + \frac{11 - \sqrt{13}}{2}\right)^{\frac{1}{3}} \right\} \right] \end{aligned} \right\},$$

From:

$$G_{169} + \frac{1}{G_{169}} = \left(\frac{13}{4}\right)^{\frac{1}{6}} \left\{ \left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} + \left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} \right\}^2$$

$$\left(\frac{13}{4}\right)^{\frac{1}{6}} \left[\left(1 + \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} + \left(1 - \frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}} \right]^2$$

Input

$$\sqrt[6]{\frac{13}{4}} \left(\sqrt[3]{1 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{1 - \frac{1}{3\sqrt{3}}} \right)^2$$

Exact result

$$\frac{\sqrt[6]{13} \left(\sqrt[3]{1 - \frac{1}{3\sqrt{3}}} + \sqrt[3]{1 + \frac{1}{3\sqrt{3}}} \right)^2}{\sqrt[3]{2}}$$

Decimal approximation

4.8277165856693115058508599034137527538403435680843835066370462266

...

4.8277165856693115....

Alternate forms

root of $x^6 - 26x^4 + 65x^2 - 52$ near $x = 4.82772$

$$\frac{\sqrt[6]{13} \left(\sqrt[3]{9 - \sqrt{3}} + \sqrt[3]{9 + \sqrt{3}} \right)^2}{3 \sqrt[3]{6}}$$

$$\frac{1}{\sqrt{\frac{3}{26 + \sqrt[3]{13(821 - 72\sqrt{3})}} + \sqrt[3]{13(821 + 72\sqrt{3})}}}}$$

Minimal polynomial

$$x^6 - 26x^4 + 65x^2 - 52$$

Expanded forms

$$\frac{2\sqrt{13}}{3} + \frac{\sqrt[6]{13} \sqrt[3]{28 - 6\sqrt{3}}}{3 \sqrt[3]{2}} + \frac{\sqrt[6]{13} \sqrt[3]{28 + 6\sqrt{3}}}{3 \sqrt[3]{2}}$$

$$\frac{\sqrt[6]{13} \left(1 - \frac{1}{3\sqrt{3}}\right)^{2/3}}{\sqrt[3]{2}} + \frac{\sqrt[6]{13} \left(1 + \frac{1}{3\sqrt{3}}\right)^{2/3}}{\sqrt[3]{2}} + 2^{2/3} \sqrt[6]{13} \sqrt[3]{\left(1 - \frac{1}{3\sqrt{3}}\right)\left(1 + \frac{1}{3\sqrt{3}}\right)}$$

and:

$$\frac{1}{G_{169}} = \frac{1}{3} \left[(\sqrt{13} - 2) + \left(\frac{13 - 3\sqrt{13}}{2} \right)^{\frac{1}{3}} \right. \\ \left. \times \left\{ \left(3\sqrt{3} - \frac{11 - \sqrt{13}}{2} \right)^{\frac{1}{3}} - \left(3\sqrt{3} + \frac{11 - \sqrt{13}}{2} \right)^{\frac{1}{3}} \right\} \right]$$

$$\frac{1}{3} \left[(\sqrt{13} - 2) + \left(\frac{13 - 3\sqrt{13}}{2} \right)^{\frac{1}{3}} \left(\left(3\sqrt{3} - \frac{11 - \sqrt{13}}{2} \right)^{\frac{1}{3}} - \left(3\sqrt{3} + \frac{11 - \sqrt{13}}{2} \right)^{\frac{1}{3}} \right) \right]$$

Input

$$\frac{1}{3} \left((\sqrt{13} - 2) + \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13})} \left(\sqrt[3]{3\sqrt{3} - \frac{1}{2}(11 - \sqrt{13})} - \sqrt[3]{3\sqrt{3} + \frac{1}{2}(11 - \sqrt{13})} \right) \right)$$

Result

$$\frac{1}{3} \left(-2 + \sqrt{13} + \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13})} \left(\sqrt[3]{3\sqrt{3} + \frac{1}{2}(\sqrt{13} - 11)} - \sqrt[3]{3\sqrt{3} + \frac{1}{2}(11 - \sqrt{13})} \right) \right)$$

Decimal approximation

0.2168804008460634642084835038906402147766632806686521327986670993

...

0.21688040084606.....

Alternate forms

$$\frac{1}{6} \left(\left(\sqrt[3]{-11 + 6\sqrt{3} + \sqrt{13}} - \sqrt[3]{11 + 6\sqrt{3} - \sqrt{13}} \right) \sqrt[3]{2(13 - 3\sqrt{13})} + 2\sqrt{13} - 4 \right)$$

root of $x^6 + 4x^5 - 2x^4 + 4x^3 + 2x^2 + 4x - 1$ near $x = 0.21688$

$$\frac{1}{6} \left(-4 + 2\sqrt{13} - \sqrt[3]{2(13 - 3\sqrt{13})(11 + 6\sqrt{3} - \sqrt{13})} + \sqrt[3]{2(13 - 3\sqrt{13})(-11 + 6\sqrt{3} + \sqrt{13})} \right)$$

Minimal polynomial

$$x^6 + 4x^5 - 2x^4 + 4x^3 + 2x^2 + 4x - 1$$

Expanded forms

$$\frac{1}{3} \left(-2 + \sqrt{13} + \sqrt[3]{\frac{1}{2}(-91 + 39\sqrt{3} + 23\sqrt{13} - 9\sqrt{39})} \right) - \frac{1}{3} \sqrt[3]{\frac{1}{2}(91 + 39\sqrt{3} - 23\sqrt{13} - 9\sqrt{39})}$$

$$-\frac{2}{3} + \frac{\sqrt{13}}{3} - \frac{1}{3} \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13}) \left(3\sqrt{3} + \frac{1}{2}(11 - \sqrt{13}) \right)} + \frac{1}{3} \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13}) \left(3\sqrt{3} + \frac{1}{2}(\sqrt{13} - 11) \right)}$$

From the two above expression, we obtain, after some easy calculations:

$$3 * (((13/4)^{(1/6)} [(1+1/(3\sqrt{3}))^{(1/3)} + (1-1/(3\sqrt{3}))^{(1/3)}]^2)) * ((1/3 [(\sqrt{13} - 2) + (1/2 * (13 - 3\sqrt{13}))^{(1/3)} * (((3\sqrt{3} - 1/2(11 - \sqrt{13}))^{(1/3)} - (3\sqrt{3} + 1/2(11 - \sqrt{13}))^{(1/3)}))]))))$$

Input

$$3 \left(\sqrt[6]{\frac{13}{4}} \left(\sqrt[3]{1 + \frac{1}{3\sqrt{3}}} + \sqrt[3]{1 - \frac{1}{3\sqrt{3}}} \right)^2 \right) \left(\frac{1}{3} \left((\sqrt{13} - 2) + \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13})} \right) \left(\sqrt[3]{3\sqrt{3} - \frac{1}{2}(11 - \sqrt{13})} - \sqrt[3]{3\sqrt{3} + \frac{1}{2}(11 - \sqrt{13})} \right) \right)$$

Exact result

$$\frac{1}{\sqrt[3]{2}} \sqrt[6]{13} \left(\sqrt[3]{1 - \frac{1}{3\sqrt{3}}} + \sqrt[3]{1 + \frac{1}{3\sqrt{3}}} \right)^2 \left(-2 + \sqrt{13} + \sqrt[3]{\frac{1}{2}(13 - 3\sqrt{13})} \left(\sqrt[3]{3\sqrt{3} + \frac{1}{2}(\sqrt{13} - 11)} - \sqrt[3]{3\sqrt{3} + \frac{1}{2}(11 - \sqrt{13})} \right) \right)$$

Decimal approximation

3.1411113248134474974049996707086011772829231957938193624736993149

...

3.141111324813447..... $\approx \pi$

Alternate forms

root of $x^6 - 78x^5 + 819x^4 - 4914x^3 + 17901x^2 - 37908x + 37908$
near $x = 3.14111$

$$-\frac{1}{18\sqrt[3]{2}}\sqrt[6]{13}\left(\sqrt[3]{27-3\sqrt{3}}+\sqrt[3]{3(9+\sqrt{3})}\right)^2$$

$$\left(4-2\sqrt{13}+\sqrt[3]{2(13-3\sqrt{13})(11+6\sqrt{3}-\sqrt{13})}-\sqrt[3]{2(13-3\sqrt{13})(-11+6\sqrt{3}+\sqrt{13})}\right)$$

$$\frac{1}{6\sqrt[3]{6}}\sqrt[6]{13}\left(\sqrt[3]{9-\sqrt{3}}+\sqrt[3]{9+\sqrt{3}}\right)^2$$

$$\left(-4+2\sqrt{13}-2^{2/3}\sqrt[3]{91+39\sqrt{3}-23\sqrt{13}-9\sqrt{39}}+2^{2/3}\sqrt[3]{-91+39\sqrt{3}+23\sqrt{13}-9\sqrt{39}}\right)$$

We have:

$$g_{198} = \sqrt{(1+\sqrt{2})(4\sqrt{2}+\sqrt{33})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{9+\sqrt{33}}{8}\right)} + \sqrt{\left(\frac{1+\sqrt{33}}{8}\right)} \right\},$$

$$G_{205} = \left(\frac{1+\sqrt{5}}{2}\right) \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^{\frac{1}{4}} \left\{ \sqrt{\left(\frac{7+\sqrt{41}}{8}\right)} + \sqrt{\left(\frac{\sqrt{41}-1}{8}\right)} \right\},$$

From:

$$g_{198} = \sqrt{(1+\sqrt{2})(4\sqrt{2}+\sqrt{33})}^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{9+\sqrt{33}}{8}\right)} + \sqrt{\left(\frac{1+\sqrt{33}}{8}\right)} \right\},$$

$$(1+\sqrt{2})^{0.5} (4\sqrt{2}+\sqrt{33})^{(1/6)} \left[(1/8*(9+\sqrt{33}))^{0.5} + (1/8(1+\sqrt{33}))^{0.5} \right]$$

Input

$$\sqrt{1+\sqrt{2}} \sqrt[6]{4\sqrt{2}+\sqrt{33}} \left(\sqrt{\frac{1}{8}(9+\sqrt{33})} + \sqrt{\frac{1}{8}(1+\sqrt{33})} \right)$$

Exact result

$$\sqrt{1+\sqrt{2}} \sqrt[6]{4\sqrt{2}+\sqrt{33}} \left(\frac{1}{2} \sqrt{\frac{1}{2}(1+\sqrt{33})} + \frac{1}{2} \sqrt{\frac{1}{2}(9+\sqrt{33})} \right)$$

Decimal approximation

5.3049225181864859298455872334358072272926307626544499803523659465

...

5.30492251818648...

Alternate forms

$$\frac{1}{2} \sqrt[12]{65+8\sqrt{66}} \sqrt{(1+\sqrt{2})(5+\sqrt{33}+\sqrt{42+10\sqrt{33}})}$$

$$\frac{1}{8} (\sqrt{2-2i} + \sqrt{2+2i}) \sqrt[6]{4\sqrt{2}+\sqrt{33}} \left(\sqrt{2(1+\sqrt{33})} + \sqrt{2(9+\sqrt{33})} \right)$$

$$\sqrt[6]{\begin{array}{l} \text{root of } x^8 - 22288x^7 - 3628x^6 + 1904x^5 + \\ 26854x^4 - 1904x^3 - 3628x^2 + 22288x + 1 \text{ near } x = 22288.2 \end{array}}$$

$$\frac{\sqrt[6]{4\sqrt{2}+\sqrt{33}} \left(\sqrt{(1+\sqrt{2})(1+\sqrt{33})} + \sqrt{(1+\sqrt{2})(9+\sqrt{33})} \right)}{2\sqrt{2}}$$

Minimal polynomial

$$x^{48} - 22288x^{42} - 3628x^{36} + 1904x^{30} + 26854x^{24} - 1904x^{18} - 3628x^{12} + 22288x^6 + 1$$

Expanded forms

$$\frac{1}{2} \sqrt[6]{4\sqrt{2} + \sqrt{33}}$$

$$\left(\sqrt{\frac{9}{\sqrt{2}} + \frac{1}{2}(9 + \sqrt{33} + \sqrt{66})} + \sqrt{\frac{1}{2} + \frac{1}{2}(\sqrt{2} + \sqrt{33} + \sqrt{66})} \right)$$

$$\frac{\sqrt{(1 + \sqrt{2})(1 + \sqrt{33})} \sqrt[6]{4\sqrt{2} + \sqrt{33}}}{2\sqrt{2}} +$$

$$\frac{\sqrt{(1 + \sqrt{2})(9 + \sqrt{33})} \sqrt[6]{4\sqrt{2} + \sqrt{33}}}{2\sqrt{2}}$$

and:

$$G_{205} = \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3\sqrt{5} + \sqrt{41}}{2} \right)^{\frac{1}{4}} \left\{ \sqrt{\left(\frac{7 + \sqrt{41}}{8} \right)} + \sqrt{\left(\frac{\sqrt{41} - 1}{8} \right)} \right\},$$

$$\left(\frac{1}{2}(1 + \sqrt{5}) \right) \left(\frac{1}{2}(3\sqrt{5} + \sqrt{41}) \right)^{0.25} \left[\left(\frac{1}{8}(7 + \sqrt{41}) \right)^{0.5} + \left(\frac{1}{8}(\sqrt{41} - 1) \right)^{0.5} \right]$$

Input

$$\left(\frac{1}{2}(1 + \sqrt{5}) \right) \left(\frac{1}{2}(3\sqrt{5} + \sqrt{41}) \right)^{0.25} \left(\sqrt{\frac{1}{8}(7 + \sqrt{41})} + \sqrt{\frac{1}{8}(\sqrt{41} - 1)} \right)$$

Result

5.4789397137741778631422933388746486067850216067301672332619100408

...

5.47893971377417.....

From the two above expressions, after some easy calculations, we obtain:

$$47 \cdot 1 / \left(\left(\left(\left(\left(\frac{1}{2}(1+\sqrt{5}) \right) \left(\frac{1}{2}(3\sqrt{5}+\sqrt{41}) \right)^{0.25} \left[\left(\frac{1}{8}(7+\sqrt{41}) \right)^{0.5} + \left(\frac{1}{8}(\sqrt{41}-1) \right)^{0.5} \right] \right) \right) \cdot \left(\left((1+\sqrt{2})^{0.5} (4\sqrt{2}+\sqrt{33})^{1/6} \right) \left[\left(\frac{1}{8}(9+\sqrt{33}) \right)^{0.5} + \left(\frac{1}{8}(1+\sqrt{33}) \right)^{0.5} \right] \right) \right) \right)$$

Input

47 ×

$$\frac{1}{\left(\left(\left(\left(\frac{1}{2}(1+\sqrt{5}) \right) \left(\frac{1}{2}(3\sqrt{5}+\sqrt{41}) \right)^{0.25} \left(\sqrt{\frac{1}{8}(7+\sqrt{41})} + \sqrt{\frac{1}{8}(\sqrt{41}-1)} \right) \right) \right) \left(\sqrt{1+\sqrt{2}} \sqrt[6]{4\sqrt{2}+\sqrt{33}} \left(\sqrt{\frac{1}{8}(9+\sqrt{33})} + \sqrt{\frac{1}{8}(1+\sqrt{33})} \right) \right) \right) \right)}$$

Result

1.6170456893611514740626547600902401324209904177855159070994338865

...

1.61704568936115.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have:

$$G_{213}^2 = (5\sqrt{3} + \sqrt{71})^{\frac{1}{4}} \left(\frac{59 + 7\sqrt{71}}{4} \right)^{\frac{1}{6}} \times \left\{ \sqrt{\left(\frac{21 + 12\sqrt{3}}{2} \right)} + \sqrt{\left(\frac{19 + 12\sqrt{3}}{2} \right)} \right\},$$

$$G_{217}^2 = \left\{ \sqrt{\left(\frac{9 + 4\sqrt{7}}{2} \right)} + \sqrt{\left(\frac{11 + 4\sqrt{7}}{2} \right)} \right\} \times \left\{ \sqrt{\left(\frac{12 + 5\sqrt{7}}{4} \right)} + \sqrt{\left(\frac{16 + 5\sqrt{7}}{4} \right)} \right\},$$

From:

$$G_{213}^2 = (5\sqrt{3} + \sqrt{71})^{\frac{1}{4}} \left(\frac{59 + 7\sqrt{71}}{4} \right)^{\frac{1}{6}} \times \left\{ \sqrt{\left(\frac{21 + 12\sqrt{3}}{2} \right)} + \sqrt{\left(\frac{19 + 12\sqrt{3}}{2} \right)} \right\},$$

$$(5\sqrt{3} + \sqrt{71})^{0.25} (1/4 * (59 + 7\sqrt{71}))^{(1/6)} [(1/2 * (21 + 12\sqrt{3}))^{0.5} + (1/2 * (19 + 12\sqrt{3}))^{0.5}]$$

Input

$$(5\sqrt{3} + \sqrt{71})^{0.25} \sqrt[6]{\frac{1}{4} (59 + 7\sqrt{71})} \left(\sqrt{\frac{1}{2} (21 + 12\sqrt{3})} + \sqrt{\frac{1}{2} (19 + 12\sqrt{3})} \right)$$

Result

32.274023643400838325366335166633123694416994026577802276726711006

...

32.2740236434008.....

and:

$$G_{217}^2 = \left\{ \sqrt{\left(\frac{9 + 4\sqrt{7}}{2} \right)} + \sqrt{\left(\frac{11 + 4\sqrt{7}}{2} \right)} \right\} \times \left\{ \sqrt{\left(\frac{12 + 5\sqrt{7}}{4} \right)} + \sqrt{\left(\frac{16 + 5\sqrt{7}}{4} \right)} \right\},$$

$$\left[\left(\frac{1}{2} (9 + 4\sqrt{7}) \right)^{0.5} + \left(\frac{1}{2} (11 + 4\sqrt{7}) \right)^{0.5} \right] \\ \left[\left(\frac{1}{4} (12 + 5\sqrt{7}) \right)^{0.5} + \left(\frac{1}{4} (16 + 5\sqrt{7}) \right)^{0.5} \right]$$

Input

$$\left(\sqrt{\frac{1}{2} (9 + 4\sqrt{7})} + \sqrt{\frac{1}{2} (11 + 4\sqrt{7})} \right) \left(\sqrt{\frac{1}{4} (12 + 5\sqrt{7})} + \sqrt{\frac{1}{4} (16 + 5\sqrt{7})} \right)$$

Exact result

$$\left(\sqrt{\frac{1}{2} (9 + 4\sqrt{7})} + \sqrt{\frac{1}{2} (11 + 4\sqrt{7})} \right) \left(\frac{1}{2} \sqrt{12 + 5\sqrt{7}} + \frac{1}{2} \sqrt{16 + 5\sqrt{7}} \right)$$

Decimal approximation

33.447338319059795840025663675972009634588228128275780414941784732

...

33.4477338319059....

Alternate forms

$$\frac{1}{4} \left(2 + \sqrt{7} + \sqrt{9 + 4\sqrt{7}} \right) \left(5 + \sqrt{7} + \sqrt{24 + 10\sqrt{7}} \right)$$

$$\frac{1}{4} \left(\sqrt{12 + 5\sqrt{7}} + \sqrt{16 + 5\sqrt{7}} \right) \left(\sqrt{18 + 8\sqrt{7}} + \sqrt{22 + 8\sqrt{7}} \right)$$

root of $x^8 - 34x^7 + 18x^6 + 16x^5 + 7x^4 + 16x^3 + 18x^2 - 34x + 1$
near $x = 33.4473$

$$\frac{\sqrt{31(8 + 3\sqrt{7})} + \sqrt{272 + 103\sqrt{7}} + \sqrt{284 + 109\sqrt{7}} + \sqrt{316 + 119\sqrt{7}}}{2\sqrt{2}}$$

Minimal polynomial

$$x^8 - 34x^7 + 18x^6 + 16x^5 + 7x^4 + 16x^3 + 18x^2 - 34x + 1$$

Expanded forms

$$\frac{1}{2} \left(\sqrt{\left(\frac{9}{2} + 2\sqrt{7}\right)(12 + 5\sqrt{7})} + \sqrt{\left(\frac{11}{2} + 2\sqrt{7}\right)(12 + 5\sqrt{7})} + \sqrt{\left(\frac{9}{2} + 2\sqrt{7}\right)(16 + 5\sqrt{7})} + \sqrt{\left(\frac{11}{2} + 2\sqrt{7}\right)(16 + 5\sqrt{7})} \right)$$

$$\frac{1}{2} \sqrt{\frac{1}{2}(9 + 4\sqrt{7})(12 + 5\sqrt{7})} + \frac{1}{2} \sqrt{\frac{1}{2}(11 + 4\sqrt{7})(12 + 5\sqrt{7})} + \frac{1}{2} \sqrt{\frac{1}{2}(9 + 4\sqrt{7})(16 + 5\sqrt{7})} + \frac{1}{2} \sqrt{\frac{1}{2}(11 + 4\sqrt{7})(16 + 5\sqrt{7})}$$

Dividing the two above expression, after some easy calculations, we obtain:

$$\left(\left(\left(\left(\frac{1}{2}(9 + 4\sqrt{7}) \right)^{0.5} + \left(\frac{1}{2}(11 + 4\sqrt{7}) \right)^{0.5} \right) \left(\frac{1}{4}(12 + 5\sqrt{7}) \right)^{0.5} + \left(\frac{1}{4}(16 + 5\sqrt{7}) \right)^{0.5} \right) \right) / \left((5\sqrt{3} + \sqrt{71})^{0.25} \left(\frac{1}{4}(59 + 7\sqrt{71}) \right)^{1/6} \left(\left(\frac{1}{2}(21 + 12\sqrt{3}) \right)^{0.5} + \left(\frac{1}{2}(19 + 12\sqrt{3}) \right)^{0.5} \right) \right) \right)^{14}$$

Input

$$\left(\left(\left(\sqrt{\frac{1}{2}(9 + 4\sqrt{7})} + \sqrt{\frac{1}{2}(11 + 4\sqrt{7})} \right) \left(\sqrt{\frac{1}{4}(12 + 5\sqrt{7})} + \sqrt{\frac{1}{4}(16 + 5\sqrt{7})} \right) \right) / \left((5\sqrt{3} + \sqrt{71})^{0.25} \sqrt[6]{\frac{1}{4}(59 + 7\sqrt{71})} \left(\sqrt{\frac{1}{2}(21 + 12\sqrt{3})} + \sqrt{\frac{1}{2}(19 + 12\sqrt{3})} \right) \right) \right)^{14}$$

Result

1.6486114189252492362932869545049375152963148797542877613812872744

...

1.648611418925.... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$ (trace of the instanton shape)

From which, after some calculations, we obtain:

$$\frac{\left(\left(\left(\left(\left(\frac{1}{2}(9+4\sqrt{7})\right)^{0.5} + \left(\frac{1}{2}(11+4\sqrt{7})\right)^{0.5}\right)\left[\left(\frac{1}{4}(12+5\sqrt{7})\right)^{0.5} + \left(\frac{1}{4}(16+5\sqrt{7})\right)^{0.5}\right]\right)\right)\right)\left(\left(\left(5\sqrt{3} + \sqrt{71}\right)^{0.25} \left(\frac{1}{4}(59+7\sqrt{71})\right)^{1/6} \left[\left(\frac{1}{2}(21+12\sqrt{3})\right)^{0.5} + \left(\frac{1}{2}(19+12\sqrt{3})\right)^{0.5}\right]\right)\right)\right)^{14}\right)^{15} - 76 - 2\Phi$$

Input

$$\left(\left(\left(\left(\sqrt{\frac{1}{2}(9+4\sqrt{7})} + \sqrt{\frac{1}{2}(11+4\sqrt{7})}\right)\left(\sqrt{\frac{1}{4}(12+5\sqrt{7})} + \sqrt{\frac{1}{4}(16+5\sqrt{7})}\right)\right)\right)\left(\left(5\sqrt{3} + \sqrt{71}\right)^{0.25} \sqrt[6]{\frac{1}{4}(59+7\sqrt{71})}\left(\sqrt{\frac{1}{2}(21+12\sqrt{3})} + \sqrt{\frac{1}{2}(19+12\sqrt{3})}\right)\right)\right)^{14}\right)^{15} - 76 - 2\Phi$$

Φ is the golden ratio conjugate

Result

1729.00...

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$\left(\frac{1}{27}\left(\left(\left(\left(\left(\left(\frac{1}{2}(9+4\sqrt{7})\right)^{0.5}+\left(\frac{1}{2}(11+4\sqrt{7})\right)^{0.5}\right)\left[\left(\frac{1}{4}(12+5\sqrt{7})\right)^{0.5}+\left(\frac{1}{4}(16+5\sqrt{7})\right)^{0.5}\right]\right)\right)\right)\right)\right)\left(\left(5\sqrt{3}+\sqrt{71}\right)^{0.25}\left(\frac{1}{4}(59+7\sqrt{71})\right)^{\frac{1}{6}}\left[\left(\frac{1}{2}(21+12\sqrt{3})\right)^{0.5}+\left(\frac{1}{2}(19+12\sqrt{3})\right)^{0.5}\right]\right)\right)^{14}\right)^{15}-76-2\Phi-1\right)^2$$

Input

$$\left(\frac{1}{27}\left(\left(\left(\left(\left(\sqrt{\frac{1}{2}(9+4\sqrt{7})}+\sqrt{\frac{1}{2}(11+4\sqrt{7})}\right)\left(\sqrt{\frac{1}{4}(12+5\sqrt{7})}+\sqrt{\frac{1}{4}(16+5\sqrt{7})}\right)\right)\right)\right)\right)\right)\left(\left(5\sqrt{3}+\sqrt{71}\right)^{0.25}\left(\frac{1}{4}(59+7\sqrt{71})\right)^{\frac{1}{6}}\left(\sqrt{\frac{1}{2}(21+12\sqrt{3})}+\sqrt{\frac{1}{2}(19+12\sqrt{3})}\right)\right)\right)^{14}\right)^{15}-76-2\Phi-1\right)^2$$

Φ is the golden ratio conjugate

Result

4096.00...

$$4096 = 64^2$$

Now, we have:

$$G_{225} = \left(\frac{1+\sqrt{5}}{4}\right) (2+\sqrt{3})^{\frac{1}{3}} \left\{\sqrt{(4+\sqrt{15})} + 15^{\frac{1}{4}}\right\},$$

$$g_{238} = \left\{\sqrt{\left(\frac{1+2\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{5+2\sqrt{2}}{4}\right)}\right\} \\ \times \left\{\sqrt{\left(\frac{1+3\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{5+3\sqrt{2}}{4}\right)}\right\},$$

From:

$$G_{225} = \left(\frac{1+\sqrt{5}}{4}\right) (2+\sqrt{3})^{\frac{1}{3}} \left\{\sqrt{(4+\sqrt{15})} + 15^{\frac{1}{4}}\right\},$$

$$\left(\frac{1}{4}(1+\sqrt{5})\right) (2+\sqrt{3})^{(1/3)} [(4+\sqrt{15})^{0.5}+15^{0.25}]$$

Input

$$\left(\frac{1}{4}(1+\sqrt{5})\right) \sqrt[3]{2+\sqrt{3}} \left(\sqrt{4+\sqrt{15}} + 15^{0.25}\right)$$

Result

5.9907020764867049706652105816600582973816363043537276451561120254

...

5.990702076....

and:

$$g_{238} = \left\{ \sqrt{\left(\frac{1+2\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{5+2\sqrt{2}}{4}\right)} \right\} \times \left\{ \sqrt{\left(\frac{1+3\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{5+3\sqrt{2}}{4}\right)} \right\},$$

$$[(1/4*(1+2\sqrt{2}))^{0.5}+(1/4(5+2\sqrt{2}))^{0.5}]$$

$$[(1/4*(1+3\sqrt{2}))^{0.5}+(1/4(5+3\sqrt{2}))^{0.5}]$$

Input

$$\left(\sqrt{\frac{1}{4}(1+2\sqrt{2})} + \sqrt{\frac{1}{4}(5+2\sqrt{2})}\right) \left(\sqrt{\frac{1}{4}(1+3\sqrt{2})} + \sqrt{\frac{1}{4}(5+3\sqrt{2})}\right)$$

Exact result

$$\left(\frac{1}{2}\sqrt{1+2\sqrt{2}} + \frac{1}{2}\sqrt{5+2\sqrt{2}}\right) \left(\frac{1}{2}\sqrt{1+3\sqrt{2}} + \frac{1}{2}\sqrt{5+3\sqrt{2}}\right)$$

Decimal approximation

6.3352883328635442071542858654823131521497473581500473659495217862

...

6.335288332....

Alternate forms

$$\frac{1}{2} \sqrt{\left(3 + 2\sqrt{2} + \sqrt{13 + 12\sqrt{2}}\right)\left(3 + 3\sqrt{2} + \sqrt{23 + 18\sqrt{2}}\right)}$$

$$\frac{1}{4} \left(\sqrt{1 + 2\sqrt{2}} + \sqrt{5 + 2\sqrt{2}}\right)\left(\sqrt{1 + 3\sqrt{2}} + \sqrt{5 + 3\sqrt{2}}\right)$$

$$\sqrt{\begin{array}{l} \text{root of } x^8 - 42x^7 + 75x^6 - 6x^5 - 52x^4 - 6x^3 + 75x^2 - 42x + 1 \\ \text{near } x = 40.1359 \end{array}}$$

$$\frac{1}{4} \left(\sqrt{17(1 + \sqrt{2})} + \sqrt{13 + 5\sqrt{2}} + \sqrt{17 + 13\sqrt{2}} + \sqrt{37 + 25\sqrt{2}}\right)$$

Minimal polynomial

$$x^{16} - 42x^{14} + 75x^{12} - 6x^{10} - 52x^8 - 6x^6 + 75x^4 - 42x^2 + 1$$

Expanded forms

$$\frac{1}{4} \left(\sqrt{(1 + 2\sqrt{2})(1 + 3\sqrt{2})} + \sqrt{(5 + 2\sqrt{2})(1 + 3\sqrt{2})} + \sqrt{(1 + 2\sqrt{2})(5 + 3\sqrt{2})} + \sqrt{(5 + 2\sqrt{2})(5 + 3\sqrt{2})}\right)$$

$$\frac{1}{4} \sqrt{(1+2\sqrt{2})(1+3\sqrt{2})} + \frac{1}{4} \sqrt{(5+2\sqrt{2})(1+3\sqrt{2})} +$$

$$\frac{1}{4} \sqrt{(1+2\sqrt{2})(5+3\sqrt{2})} + \frac{1}{4} \sqrt{(5+2\sqrt{2})(5+3\sqrt{2})}$$

From which:

$$\left(\frac{\left(\left(\left(\left(\frac{1}{4}(1+2\sqrt{2}) \right)^{0.5} + \left(\frac{1}{4}(5+2\sqrt{2}) \right)^{0.5} \right) \right) \right) \left(\left(\frac{1}{4}(1+3\sqrt{2}) \right)^{0.5} + \left(\frac{1}{4}(5+3\sqrt{2}) \right)^{0.5} \right) \right)}{\left(\frac{1}{4}(1+\sqrt{5}) \right) (2+\sqrt{3})^{1/3} \left[(4+\sqrt{15})^{0.5} + 15^{0.25} \right] \right)} \right)^9$$

Input

$$\left(\frac{\left(\sqrt{\frac{1}{4}(1+2\sqrt{2})} + \sqrt{\frac{1}{4}(5+2\sqrt{2})} \right) \left(\sqrt{\frac{1}{4}(1+3\sqrt{2})} + \sqrt{\frac{1}{4}(5+3\sqrt{2})} \right) \right)^9}{\left(\frac{1}{4}(1+\sqrt{5}) \right) \sqrt[3]{2+\sqrt{3}} \left(\sqrt{4+\sqrt{15}} + 15^{0.25} \right)}$$

Result

1.6542379056918814233976130480854520295510340941827758884008662695

...

1.654237905.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Or also:

$$\left(\frac{\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{4}(1+2\sqrt{2}) \right)^{0.5} + \left(\frac{1}{4}(5+2\sqrt{2}) \right)^{0.5} \right) \right) \right) \right) \right) \left(\left(\frac{1}{4}(1+3\sqrt{2}) \right)^{0.5} + \left(\frac{1}{4}(5+3\sqrt{2}) \right)^{0.5} \right) \right) \right) \right) \left(\left(\frac{1}{4}(1+\sqrt{5}) \right) (2+\sqrt{3})^{1/3} \left[(4+\sqrt{15})^{0.5} + 15^{0.25} \right] \right) \right)} \right)^{15}$$

Series representations

$$\left(\frac{1}{27} \left(\left(\left(\left(\sqrt{\frac{1}{4}(1+2\sqrt{2})} + \sqrt{\frac{1}{4}(5+2\sqrt{2})} \right) \left(\sqrt{\frac{1}{4}(1+3\sqrt{2})} + \sqrt{\frac{1}{4}(5+3\sqrt{2})} \right) - \frac{1}{4}(1+\sqrt{5}) \right) \sqrt[3]{2+\sqrt{3}} \left(\sqrt{4+\sqrt{15}} + 15^{0.25} \right) \right) \right) \right)^{15} - 27 \right)^2 - \frac{\phi}{2} = -\frac{\phi}{2} + \frac{1}{729} \left(-27 + \right.$$

225 051 707 283 248 404 043 264 398 919 956 893 908 530 667 320 ;
 312 401 148 007 480 414 880 605 588 527 456 387 252 533 530 ;
 387 658 761 101 443

$$\left(\left(\frac{1}{2} \sqrt{1+2\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + \frac{1}{2} \sqrt{5+2\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \left(\frac{1}{2} \sqrt{1+3\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + \frac{1}{2} \sqrt{5+3\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) - \frac{1}{4} \sqrt[3]{2+\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right) \left(\sqrt{4+\sqrt{z_0}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (15-z_0)^k z_0^{-k}}{k!} \right) \right)^{225} \right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\left(\frac{1}{27} \left(\left(\left(\left(\sqrt{\frac{1}{4}(1+2\sqrt{2})} + \sqrt{\frac{1}{4}(5+2\sqrt{2})} \right) \left(\sqrt{\frac{1}{4}(1+3\sqrt{2})} + \sqrt{\frac{1}{4}(5+3\sqrt{2})} \right) - \frac{1}{4}(1+\sqrt{5}) \right) \sqrt[3]{2+\sqrt{3}} \left(\sqrt{4+\sqrt{15}} + 15^{0.25} \right) \right) \right) \right)^{15} \right)^{15} - 27 \right)^2 - \frac{\phi}{2} = -\frac{\phi}{2} + \frac{1}{729} \left(-27 + \right.$$

225 051 707 283 248 404 043 264 398 919 956 893 908 530 667 320 `.
 312 401 148 007 480 414 880 605 588 527 456 387 252 533 530 `.
 387 658 761 101 443

$$\left(\left(\frac{1}{2} \sqrt{ \left(1 + 2 \exp \left(i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \frac{1}{2} \sqrt{ \left(5 + 2 \exp \left(i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) } \right) \left(\frac{1}{2} \sqrt{ \left(1 + 3 \exp \left(i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \frac{1}{2} \sqrt{ \left(5 + 3 \exp \left(i \pi \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) } \right) - \frac{1}{4} \left(2 + \exp \left(i \pi \left[\frac{\arg(3-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^{\wedge (1/3)} \left(1 + \exp \left(i \pi \left[\frac{\arg(5-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left(1.96799 + \sqrt{ \left(4 + \exp \left(i \pi \left[\frac{\arg(15-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (15-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right)^{225} \right)^2$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\left(\frac{1}{27} \left(\left(\left(\left(\sqrt{\frac{1}{4}(1+2\sqrt{2})} + \sqrt{\frac{1}{4}(5+2\sqrt{2})} \right) \left(\sqrt{\frac{1}{4}(1+3\sqrt{2})} + \sqrt{\frac{1}{4}(5+3\sqrt{2})} \right) - \frac{1}{4}(1+\sqrt{5}) \right) \sqrt[3]{2+\sqrt{3}} \left(\sqrt{4+\sqrt{15}} + 15^{0.25} \right) \right) \right) \right)^{15} - 27 \right)^2 - \frac{\phi}{2} = -\frac{\phi}{2} + \frac{1}{729} \left(-27 + \right.$$

225 051 707 283 248 404 043 264 398 919 956 893 908 530 667 320 ·
 312 401 148 007 480 414 880 605 588 527 456 387 252 533 530 387 ·
 658 761 101 443

$$\left(\left(\frac{1}{2} \sqrt{\left(1 + 2 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) + \frac{1}{2} \sqrt{\left(5 + 2 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)} \right) \left(\frac{1}{2} \sqrt{\left(1 + 3 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) + \frac{1}{2} \sqrt{\left(5 + 3 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)} \right) - \frac{1}{4} \left(2 + \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(3-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right)^{\wedge (1/3)} \left(1 + \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(5-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right) \left(1.96799 + \sqrt{\left(4 + \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(15-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(15-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (15-z_0)^k z_0^{-k}}{k!} \right)} \right)^{225} \right)^2$$

We have:

$$G_{265}^2 = \sqrt{\left\{ (2 + \sqrt{5}) \left(\frac{7 + \sqrt{53}}{2} \right) \right\}} \left\{ \sqrt{\left(\frac{89 + 5\sqrt{265}}{8} \right)} + \sqrt{\left(\frac{81 + 5\sqrt{265}}{8} \right)} \right\},$$

$$G_{289} = \left[\sqrt{\left\{ \frac{17 + \sqrt{17} + 17^{\frac{1}{4}}(5 + \sqrt{17})}{16} \right\}} + \sqrt{\left\{ \frac{1 + \sqrt{17} + 17^{\frac{1}{4}}(5 + \sqrt{17})}{16} \right\}} \right]^2,$$

From:

$$G_{265}^2 = \sqrt{\left\{ (2 + \sqrt{5}) \left(\frac{7 + \sqrt{53}}{2} \right) \right\}} \left\{ \sqrt{\left(\frac{89 + 5\sqrt{265}}{8} \right)} + \sqrt{\left(\frac{81 + 5\sqrt{265}}{8} \right)} \right\},$$

$$[(((2+\sqrt{5}))(1/2(7+\sqrt{53}))))^{0.5}]$$

$$[(1/8*(89+5\sqrt{265}))^{0.5}+(1/8(81+5\sqrt{265}))^{0.5}]$$

Input

$$\sqrt{(2 + \sqrt{5}) \left(\frac{1}{2} (7 + \sqrt{53}) \right)} \left(\sqrt{\frac{1}{8} (89 + 5 \sqrt{265})} + \sqrt{\frac{1}{8} (81 + 5 \sqrt{265})} \right)$$

Exact result

$$\sqrt{\frac{1}{2} (2 + \sqrt{5}) (7 + \sqrt{53})} \left(\frac{1}{2} \sqrt{\frac{1}{2} (81 + 5 \sqrt{265})} + \frac{1}{2} \sqrt{\frac{1}{2} (89 + 5 \sqrt{265})} \right)$$

Decimal approximation

50.159687983020382568668548270349931779128724519937291476731752066

...

50.159687983....

Alternate forms

$$\frac{1}{4} \sqrt{\frac{1}{2} (2 + \sqrt{5})(7 + \sqrt{53}) (5\sqrt{5} + \sqrt{53} + \sqrt{81 - 8i} + \sqrt{81 + 8i})}$$

$$\frac{1}{4} \sqrt{(2 + \sqrt{5})(7 + \sqrt{53}) \left(\sqrt{81 + 5\sqrt{265}} + \sqrt{89 + 5\sqrt{265}} \right)}$$

root of $x^8 - 47x^7 - 153x^6 - 268x^5 - 362x^4 - 268x^3 - 153x^2 - 47x + 1$
near $x = 50.1597$

Minimal polynomial

$$x^8 - 47x^7 - 153x^6 - 268x^5 - 362x^4 - 268x^3 - 153x^2 - 47x + 1$$

Expanded forms

$$\frac{1}{4} \sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(81 + 5\sqrt{265})} +$$
$$\frac{1}{4} \sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(89 + 5\sqrt{265})}$$

$$\frac{1}{2} \left(\sqrt{\frac{2459}{4} + \frac{1097\sqrt{5}}{4} + \frac{337\sqrt{53}}{4} + \frac{151\sqrt{265}}{4}} + \right.$$
$$\left. \sqrt{\frac{2571}{4} + \frac{1153\sqrt{5}}{4} + \frac{353\sqrt{53}}{4} + \frac{159\sqrt{265}}{4}} \right)$$

and:

$$G_{289} = \left[\sqrt{\left\{ \frac{17 + \sqrt{17} + 17^{\frac{1}{4}}(5 + \sqrt{17})}{16} \right\}} + \sqrt{\left\{ \frac{1 + \sqrt{17} + 17^{\frac{1}{4}}(5 + \sqrt{17})}{16} \right\}} \right]^2,$$

$$[(1/16(17+\sqrt{17}+17^{(1/4)}(5+\sqrt{17})))^{0.5}+(1/16(1+\sqrt{17}+17^{(1/4)}(5+\sqrt{17})))^{0.5}]^2$$

Input

$$\left(\sqrt{\frac{1}{16} (17 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17}))} + \sqrt{\frac{1}{16} (1 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17}))} \right)^2$$

Exact result

$$\left(\frac{1}{4} \sqrt{1 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17})} + \frac{1}{4} \sqrt{17 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17})} \right)^2$$

Decimal approximation

7.7835147315253990681077446230553615847817956976413614423585213085

...

7.78351473....

Alternate forms

$$\frac{1}{16} \left(\sqrt{1 + 5 \sqrt[4]{17} + \sqrt{17} + 17^{3/4}} + \sqrt{17 + 5 \sqrt[4]{17} + \sqrt{17} + 17^{3/4}} \right)^2$$

$$\frac{1}{16} \left(\sqrt{1 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17})} + \sqrt{17 + \sqrt{17} + \sqrt[4]{17} (5 + \sqrt{17})} \right)^2$$

root of $x^8 - 9x^7 + 11x^6 - 12x^5 + 2x^4 - 12x^3 + 11x^2 - 9x + 1$
 near $x = 7.78351$

Minimal polynomial

$$x^8 - 9x^7 + 11x^6 - 12x^5 + 2x^4 - 12x^3 + 11x^2 - 9x + 1$$

Expanded form

$$\frac{9}{8} + \frac{5\sqrt[4]{17}}{8} + \frac{\sqrt{17}}{8} + \frac{17^{3/4}}{8} + \frac{\frac{1}{8} \sqrt{(1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})) (17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17}))}}{1}$$

Dividing the two expressions, after some calculations, we obtain:

$$\frac{1/4 \left(\left(\left(\left((2 + \sqrt{5}) \left(\frac{1}{2} (7 + \sqrt{53}) \right) \right)^{0.5} \right) \right) \right)}{\left(\left(\left(\left(\frac{1}{8} (89 + 5\sqrt{265}) \right)^{0.5} + \left(\frac{1}{8} (81 + 5\sqrt{265}) \right)^{0.5} \right) \right) \right) / \left(\left(\left(\left(\frac{1}{16} (17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})) \right)^{0.5} + \left(\frac{1}{16} (1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})) \right)^{0.5} \right) \right)^2 \right)$$

Input

$$\frac{1}{4} \times \frac{\sqrt{(2 + \sqrt{5}) \left(\frac{1}{2} (7 + \sqrt{53}) \right)} \left(\sqrt{\frac{1}{8} (89 + 5\sqrt{265})} + \sqrt{\frac{1}{8} (81 + 5\sqrt{265})} \right)}{\left(\sqrt{\frac{1}{16} (17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17}))} + \sqrt{\frac{1}{16} (1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17}))} \right)^2}$$

Exact result

$$\frac{\sqrt{\frac{1}{2} (2 + \sqrt{5}) (7 + \sqrt{53})} \left(\frac{1}{2} \sqrt{\frac{1}{2} (81 + 5\sqrt{265})} + \frac{1}{2} \sqrt{\frac{1}{2} (89 + 5\sqrt{265})} \right)}{4 \left(\frac{1}{4} \sqrt{1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} + \frac{1}{4} \sqrt{17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} \right)^2}$$

Decimal approximation

1.6110873337163382646641491876274149528948393234064137086327227638

...

1.611087333716.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$\frac{\sqrt{(2 + \sqrt{5})(7 + \sqrt{53})} \left(\sqrt{81 + 5\sqrt{265}} + \sqrt{89 + 5\sqrt{265}} \right)}{\left(\sqrt{1 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} + \sqrt{17 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} \right)^2}$$

$$\left(\sqrt{\frac{1}{2}(2 + \sqrt{5})(7 + \sqrt{53})} \left(\begin{array}{l} \text{root of } x^4 - 162x^2 + 6625 \text{ near } x = 9.01094 - 0.443905i \\ + \\ \text{root of } x^4 - 162x^2 + 6625 \text{ near } x = 9.01094 + 0.443905i \\ + \\ 5\sqrt{5} + \sqrt{53} \end{array} \right) \right) / \left(\begin{array}{l} \text{root of } x^8 - 68x^6 + 1360x^4 - 9792x^2 + 17408 \text{ near } x = 6.29666 \\ + \\ \text{root of } x^8 - 4x^6 - 368x^4 - 2112x^2 - 4096 \text{ near } x = 4.86292 \end{array} \right)^2$$

$$\frac{\sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(81 + 5\sqrt{265})}}{\left(\sqrt{1 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} + \sqrt{17 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} \right)^2} + \frac{\sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(89 + 5\sqrt{265})}}{\left(\sqrt{1 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} + \sqrt{17 + 5\sqrt[4]{17}} + \sqrt{17} + 17^{3/4} \right)^2}$$

$$\left(\frac{\sqrt{2459 + 1097\sqrt{5} + 337\sqrt{53} + 151\sqrt{265}} + \sqrt{2571 + 1153\sqrt{5} + 353\sqrt{53} + 159\sqrt{265}}}{\left(\sqrt{1 + 5\sqrt[4]{17} + \sqrt{17} + 17^{3/4}} + \sqrt{17 + 5\sqrt[4]{17} + \sqrt{17} + 17^{3/4}} \right)^2} \right)^2$$

Expanded form

$$\frac{\sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(81 + 5\sqrt{265})}}{16 \left(\frac{1}{4} \sqrt{1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} + \frac{1}{4} \sqrt{17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} \right)^2} + \frac{\sqrt{(2 + \sqrt{5})(7 + \sqrt{53})(89 + 5\sqrt{265})}}{16 \left(\frac{1}{4} \sqrt{1 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} + \frac{1}{4} \sqrt{17 + \sqrt{17} + \sqrt[4]{17}(5 + \sqrt{17})} \right)^2}$$

We have:

$$G_{301}^2 = \left\{ (8 + 3\sqrt{7}) \left(\frac{23\sqrt{43} + 57\sqrt{7}}{2} \right) \right\}^{\frac{1}{4}} \times \left\{ \sqrt{\left(\frac{46 + 7\sqrt{43}}{4} \right)} + \sqrt{\left(\frac{42 + 7\sqrt{43}}{4} \right)} \right\}$$

$$g_{310} = \left(\frac{1 + \sqrt{5}}{2} \right) \sqrt{(1 + \sqrt{2})} \left\{ \sqrt{\left(\frac{7 + 2\sqrt{10}}{4} \right)} + \sqrt{\left(\frac{3 + 2\sqrt{10}}{4} \right)} \right\},$$

From:

$$G_{301}^2 = \left\{ (8 + 3\sqrt{7}) \left(\frac{23\sqrt{43} + 57\sqrt{7}}{2} \right) \right\}^{\frac{1}{4}} \times \left\{ \sqrt{\left(\frac{46 + 7\sqrt{43}}{4} \right)} + \sqrt{\left(\frac{42 + 7\sqrt{43}}{4} \right)} \right\},$$

$$\begin{aligned} & [((8+3\sqrt{7}))(1/2(23\sqrt{43}+57\sqrt{7}))]^{\wedge}0.25 \\ & [(1/4*(46+7\sqrt{43}))^{\wedge}0.5+(1/4(42+7\sqrt{43}))^{\wedge}0.5] \end{aligned}$$

Input

$$\left((8 + 3\sqrt{7}) \left(\frac{1}{2} (23\sqrt{43} + 57\sqrt{7}) \right) \right)^{0.25} \left(\sqrt{\frac{1}{4} (46 + 7\sqrt{43})} + \sqrt{\frac{1}{4} (42 + 7\sqrt{43})} \right)$$

Result

66.385325477751844759596355416246994104640619768798504535794893761

...

66.3853254777....

and:

$$g_{310} = \left(\frac{1 + \sqrt{5}}{2} \right) \sqrt{1 + \sqrt{2}} \left\{ \sqrt{\left(\frac{7 + 2\sqrt{10}}{4} \right)} + \sqrt{\left(\frac{3 + 2\sqrt{10}}{4} \right)} \right\},$$

$$(1/2*(1+\sqrt{5})) (1+\sqrt{2})^{\wedge}0.5 [(1/4*(7+2\sqrt{10}))^{\wedge}0.5+(1/4(3+2\sqrt{10}))^{\wedge}0.5]$$

Input

$$\left(\frac{1}{2} (1 + \sqrt{5}) \right) \sqrt{1 + \sqrt{2}} \left(\sqrt{\frac{1}{4} (7 + 2\sqrt{10})} + \sqrt{\frac{1}{4} (3 + 2\sqrt{10})} \right)$$

Exact result

$$\frac{1}{2} \sqrt{1+\sqrt{2}} (1+\sqrt{5}) \left(\frac{1}{2} \sqrt{3+2\sqrt{10}} + \frac{1}{2} \sqrt{7+2\sqrt{10}} \right)$$

Decimal approximation

8.4269941500362075657975840194824130333145055263384990174962280582

...

8.42699415....

Alternate forms

$$\frac{1}{4} \sqrt{1+\sqrt{2}} (1+\sqrt{5}) \left(\sqrt{2} + \sqrt{5} + \sqrt{3+2\sqrt{10}} \right)$$

$$\frac{1}{8} (\sqrt{2-2i} + \sqrt{2+2i}) (1+\sqrt{5}) \left(\sqrt{3+2\sqrt{10}} + \sqrt{7+2\sqrt{10}} \right)$$

$$\sqrt{\boxed{\begin{array}{l} \text{root of } x^8 - 70x^7 - 73x^6 + 70x^5 - 52x^4 - 70x^3 - 73x^2 + 70x + 1 \\ \text{near } x = 71.0142 \end{array}}}$$

Minimal polynomial

$$x^{16} - 70x^{14} - 73x^{12} + 70x^{10} - 52x^8 - 70x^6 - 73x^4 + 70x^2 + 1$$

Expanded forms

$$\begin{aligned} & \frac{1}{4} \sqrt{(1+\sqrt{2})(3+2\sqrt{10})} + \frac{1}{4} \sqrt{5(1+\sqrt{2})(3+2\sqrt{10})} + \\ & \frac{1}{4} \sqrt{(1+\sqrt{2})(7+2\sqrt{10})} + \frac{1}{4} \sqrt{5(1+\sqrt{2})(7+2\sqrt{10})} \end{aligned}$$

$$\frac{1}{4} \left(\sqrt{3+3\sqrt{2}+4\sqrt{5}+2\sqrt{10}} + \sqrt{7+7\sqrt{2}+4\sqrt{5}+2\sqrt{10}} + \sqrt{15+15\sqrt{2}+20\sqrt{5}+10\sqrt{10}} + \sqrt{35+35\sqrt{2}+20\sqrt{5}+10\sqrt{10}} \right)$$

From the two above expression, after some calculations, we obtain:

$$(2 \times 48) / \left(\left(\left(\left(\left((8+3\sqrt{7}) \left(\frac{1}{2}(23\sqrt{43}+57\sqrt{7}) \right) \right) \right) \right) \right)^{0.25} \right. \\ \left. \left[\left(\frac{1}{4}(46+7\sqrt{43}) \right)^{0.5} + \left(\frac{1}{4}(42+7\sqrt{43}) \right)^{0.5} \right] - \left(\left(\frac{1}{2}(1+\sqrt{5}) \right) (1+\sqrt{2})^{0.5} \right) \right. \right. \\ \left. \left. \left[\left(\frac{1}{4}(7+2\sqrt{10}) \right)^{0.5} + \left(\frac{1}{4}(3+2\sqrt{10}) \right)^{0.5} \right] \right] \right)$$

Input

$$(2 \times 48) / \left(\left(\left((8+3\sqrt{7}) \left(\frac{1}{2}(23\sqrt{43}+57\sqrt{7}) \right) \right) \right)^{0.25} \right. \\ \left. \left(\sqrt{\frac{1}{4}(46+7\sqrt{43})} + \sqrt{\frac{1}{4}(42+7\sqrt{43})} \right) - \left(\frac{1}{2}(1+\sqrt{5}) \right) \sqrt{1+\sqrt{2}} \left(\sqrt{\frac{1}{4}(7+2\sqrt{10})} + \sqrt{\frac{1}{4}(3+2\sqrt{10})} \right) \right)$$

Result

1.6563623865770762974360354869724932555996902085765333907820185402

...

1.656362386577.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Or also:

$$11^2 / \left(\left(\left(\left((8+3\sqrt{7}) \left(\frac{1}{2}(23\sqrt{43}+57\sqrt{7}) \right) \right) \right) \right)^{0.25} \right. \\ \left. \left[\left(\frac{1}{4}(46+7\sqrt{43}) \right)^{0.5} + \left(\frac{1}{4}(42+7\sqrt{43}) \right)^{0.5} \right] + \left(\left(\frac{1}{2}(1+\sqrt{5}) \right) (1+\sqrt{2})^{0.5} \right) \right. \right. \\ \left. \left. \left[\left(\frac{1}{4}(7+2\sqrt{10}) \right)^{0.5} + \left(\frac{1}{4}(3+2\sqrt{10}) \right)^{0.5} \right] \right] \right)$$

Input

$$11^2 / \left(\left((8 + 3\sqrt{7}) \left(\frac{1}{2} (23\sqrt{43} + 57\sqrt{7}) \right) \right)^{0.25} \right. \\ \left. \left(\sqrt{\frac{1}{4} (46 + 7\sqrt{43})} + \sqrt{\frac{1}{4} (42 + 7\sqrt{43})} \right) + \right. \\ \left. \left(\frac{1}{2} (1 + \sqrt{5}) \right) \sqrt{1 + \sqrt{2}} \left(\sqrt{\frac{1}{4} (7 + 2\sqrt{10})} + \sqrt{\frac{1}{4} (3 + 2\sqrt{10})} \right) \right)$$

Result

1.6173806747606331561957152438717956189094732754730469370388638980

...

1.61738067476.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have:

$$G_{333} = \frac{1}{2} (6 + \sqrt{37})^{\frac{1}{4}} (7\sqrt{3} + 2\sqrt{37})^{\frac{1}{6}} \{ \sqrt{(7 + 2\sqrt{3})} + \sqrt{(3 + 2\sqrt{3})} \},$$

$$1/2(6+\text{sqrt}37)^{0.25} (7\text{sqrt}3+2\text{sqrt}37)^{(1/6)} [(7+2\text{sqrt}3)^{0.5}+(3+2\text{sqrt}3)^{0.5}]$$

Input

$$\left(\frac{1}{2} (6 + \sqrt{37})^{0.25} \right)^6 \sqrt[6]{7\sqrt{3} + 2\sqrt{37}} \left(\sqrt{7 + 2\sqrt{3}} + \sqrt{3 + 2\sqrt{3}} \right)$$

Result

9.1651519881933109158432796709185493122029784122313483581527017164

...

9.165151988....

From which:

$$89 \frac{1}{6 \left(\left(\frac{1}{2} (6 + \sqrt{37}) \right)^{0.25} (7\sqrt{3} + 2\sqrt{37})^{1/6} \right. \\ \left. \left[(7 + 2\sqrt{3})^{0.5} + (3 + 2\sqrt{3})^{0.5} \right] \right)}$$

Input

$$89 \times \frac{1}{6 \left(\left(\frac{1}{2} (6 + \sqrt{37}) \right)^{0.25} \sqrt[6]{7\sqrt{3} + 2\sqrt{37}} \left(\sqrt{7 + 2\sqrt{3}} + \sqrt{3 + 2\sqrt{3}} \right) \right)}$$

Result

1.6184492469346782672429942886734947331771367899887667207021347574

...

1.6184492469.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have:

$$G_{441}^2 = \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) (2 + \sqrt{3})^{\frac{1}{3}} \left\{ \frac{2 + \sqrt{7} + \sqrt{(7 + 4\sqrt{7})}}{2} \right\} \left\{ \frac{\sqrt{(3 + \sqrt{7})} + (6\sqrt{7})^{\frac{1}{4}}}{\sqrt{(3 + \sqrt{7})} - (6\sqrt{7})^{\frac{1}{4}}} \right\},$$

$$G_{445} = \sqrt{(2 + \sqrt{5})} \left(\frac{21 + \sqrt{445}}{2} \right)^{\frac{1}{4}} \sqrt{\left\{ \left(\frac{13 + \sqrt{89}}{8} \right) + \sqrt{\left(\frac{5 + \sqrt{89}}{8} \right)} \right\}},$$

$$G_{465}^2 = \sqrt{\left\{ (2 + \sqrt{3}) \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3\sqrt{3} + \sqrt{31}}{2} \right) \right\} (5\sqrt{5} + 2\sqrt{31})^{\frac{1}{6}}} \\ \times \left\{ \sqrt{\left(\frac{2 + \sqrt{31}}{4} \right)} + \sqrt{\left(\frac{6 + \sqrt{31}}{4} \right)} \right\} \\ \times \left\{ \sqrt{\left(\frac{11 + 2\sqrt{31}}{2} \right)} + \sqrt{\left(\frac{13 + 2\sqrt{31}}{2} \right)} \right\},$$

From:

$$G_{441}^2 = \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) (2 + \sqrt{3})^{\frac{1}{3}} \left\{ \frac{2 + \sqrt{7} + \sqrt{(7 + 4\sqrt{7})}}{2} \right\} \left\{ \frac{\sqrt{(3 + \sqrt{7})} + (6\sqrt{7})^{\frac{1}{4}}}{\sqrt{(3 + \sqrt{7})} - (6\sqrt{7})^{\frac{1}{4}}} \right\},$$

$$\begin{aligned} & ((1/2*(\text{sqrt}3+\text{sqrt}7)) (2+\text{sqrt}3)^{(1/3)} (1/2*(2+\text{sqrt}7+(7+4\text{sqrt}7)^{0.5})) \\ & (((((3+\text{sqrt}7)^{0.5}+(6\text{sqrt}7)^{0.25}))) / (((((3+\text{sqrt}7)^{0.5}-(6\text{sqrt}7)^{0.25})))) \end{aligned}$$

Input

$$\left(\frac{1}{2} (\sqrt{3} + \sqrt{7}) \right)^{\frac{1}{3}} (2 + \sqrt{3})^{\frac{1}{3}} \left(\frac{1}{2} (2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}}) \right) \times \frac{\sqrt{3 + \sqrt{7}} + (6\sqrt{7})^{0.25}}{\sqrt{3 + \sqrt{7}} - (6\sqrt{7})^{0.25}}$$

Result

172.64087217816086422023342429510817879521053958626306652342422349

...

172.640872178....

and:

$$G_{445} = \sqrt{(2 + \sqrt{5})} \left(\frac{21 + \sqrt{445}}{2} \right)^{\frac{1}{4}} \sqrt{\left\{ \left(\frac{13 + \sqrt{89}}{8} \right) + \sqrt{\left(\frac{5 + \sqrt{89}}{8} \right)} \right\}},$$

$$((2+\text{sqrt}5)^{0.5}) (1/2*(21+\text{sqrt}445))^{0.25} \text{sqrt}[(1/8*(13+\text{sqrt}89))+(1/8(5+\text{sqrt}89))^{0.5}]$$

Input

$$\sqrt{2 + \sqrt{5}} \left(\frac{1}{2} (21 + \sqrt{445}) \right)^{0.25} \sqrt{\frac{1}{8} (13 + \sqrt{89}) + \sqrt{\frac{1}{8} (5 + \sqrt{89})}}$$

Result

8.9778697879055384610148011807619713532466986199556182457746044826

...

8.9778697879.....

From the ratio between the above two expressions, after some calculations, we obtain:

$$\left(\frac{8.9778697879 \left(\frac{1}{2}(\sqrt{3} + \sqrt{7}) \right) (2 + \sqrt{3})^{1/3} (1/2*(2 + \sqrt{7} + (7 + 4\sqrt{7})^{0.5}))}{\left(\left(\left((3 + \sqrt{7})^{0.5} + (6\sqrt{7})^{0.25} \right) \right) / \left(\left((3 + \sqrt{7})^{0.5} - (6\sqrt{7})^{0.25} \right) \right) \right)} \right)^{1/15}$$

Input interpretation

$$\left(8.9778697879 \left(\frac{1}{2} (\sqrt{3} + \sqrt{7}) \right) \sqrt[3]{2 + \sqrt{3}} \left(\frac{1}{2} (2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}}) \right) \times \frac{\sqrt{3 + \sqrt{7}} + (6\sqrt{7})^{0.25}}{\sqrt{3 + \sqrt{7}} - (6\sqrt{7})^{0.25}} \right)^{(1/15)}$$

Result

1.6318784033576116789290643572950311297075267730631510068995939352

...

1.6318784033.... result very near to the mean between $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$ and the value of golden ratio 1.61803398..., i.e. 1.63148399

From:

$$G_{465}^2 = \sqrt{\left\{ (2 + \sqrt{3}) \left(\frac{1 + \sqrt{5}}{2} \right) \left(\frac{3\sqrt{3} + \sqrt{31}}{2} \right) \right\} (5\sqrt{5} + 2\sqrt{31})^{\frac{1}{6}} \times \left\{ \sqrt{\left(\frac{2 + \sqrt{31}}{4} \right)} + \sqrt{\left(\frac{6 + \sqrt{31}}{4} \right)} \right\} \times \left\{ \sqrt{\left(\frac{11 + 2\sqrt{31}}{2} \right)} + \sqrt{\left(\frac{13 + 2\sqrt{31}}{2} \right)} \right\},$$

$$\begin{aligned} & (((2+\sqrt{3})(\frac{1}{2}*(1+\sqrt{5}))(\frac{1}{2}*(3\sqrt{3}+\sqrt{31}))))^{0.5} (5\sqrt{5}+2\sqrt{31})^{(1/6)} \\ & [(1/4*(2+\sqrt{31}))^{0.5}+(1/4(6+\sqrt{31}))^{0.5}] \\ & [(1/2*(11+2\sqrt{31}))^{0.5}+(1/2(13+2\sqrt{31}))^{0.5}] \end{aligned}$$

Input

$$\begin{aligned} & \sqrt{(2+\sqrt{3})\left(\frac{1}{2}(1+\sqrt{5})\right)\left(\frac{1}{2}(3\sqrt{3}+\sqrt{31})\right)} \sqrt[6]{5\sqrt{5}+2\sqrt{31}} \\ & \left(\sqrt{\frac{1}{4}(2+\sqrt{31})} + \sqrt{\frac{1}{4}(6+\sqrt{31})}\right) \left(\sqrt{\frac{1}{2}(11+2\sqrt{31})} + \sqrt{\frac{1}{2}(13+2\sqrt{31})}\right) \end{aligned}$$

Exact result

$$\begin{aligned} & \frac{1}{2} \sqrt{(2+\sqrt{3})(1+\sqrt{5})(3\sqrt{3}+\sqrt{31})} \sqrt[6]{5\sqrt{5}+2\sqrt{31}} \\ & \left(\frac{\sqrt{2+\sqrt{31}}}{2} + \frac{\sqrt{6+\sqrt{31}}}{2}\right) \left(\sqrt{\frac{1}{2}(11+2\sqrt{31})} + \sqrt{\frac{1}{2}(13+2\sqrt{31})}\right) \end{aligned}$$

Decimal approximation

200.10283384292805393742089378421492554827045666950208313869730977

...

200.102833842928....

Alternate forms

$$\begin{aligned} & \frac{1}{4} \sqrt[6]{5\sqrt{5}+2\sqrt{31}} \sqrt{\frac{1}{2}(1+\sqrt{5})(11+2\sqrt{31})(9+6\sqrt{3}+2\sqrt{31}+\sqrt{93})} \\ & \left(\sqrt{2+\sqrt{31}} + \sqrt{6+\sqrt{31}}\right) + \\ & \frac{1}{4} \sqrt[6]{5\sqrt{5}+2\sqrt{31}} \sqrt{\frac{1}{2}(1+\sqrt{5})(13+2\sqrt{31})(9+6\sqrt{3}+2\sqrt{31}+\sqrt{93})} \\ & \left(\sqrt{2+\sqrt{31}} + \sqrt{6+\sqrt{31}}\right) \end{aligned}$$

$$\frac{1}{4\sqrt{2}} \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} \left(\sqrt{3(1+\sqrt{5})(562 + 323\sqrt{3} + 101\sqrt{31} + 58\sqrt{93})} + \sqrt{5(1+\sqrt{5})(562 + 323\sqrt{3} + 101\sqrt{31} + 58\sqrt{93})} + \sqrt{(1+\sqrt{5})(1846 + 1055\sqrt{3} + 329\sqrt{31} + 190\sqrt{93})} + \sqrt{(1+\sqrt{5})(2578 + 1481\sqrt{3} + 463\sqrt{31} + 266\sqrt{93})} \right)$$

Expanded form

$$\begin{aligned} & \frac{1}{4} \sqrt{\frac{1}{2}(2+\sqrt{3})(1+\sqrt{5})(2+\sqrt{31})(3\sqrt{3}+\sqrt{31})(11+2\sqrt{31})} \\ & \quad \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} + \\ & \frac{1}{4} \sqrt{\frac{1}{2}(2+\sqrt{3})(1+\sqrt{5})(6+\sqrt{31})(3\sqrt{3}+\sqrt{31})(11+2\sqrt{31})} \\ & \quad \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} + \\ & \frac{1}{4} \sqrt{\frac{1}{2}(2+\sqrt{3})(1+\sqrt{5})(2+\sqrt{31})(3\sqrt{3}+\sqrt{31})(13+2\sqrt{31})} \\ & \quad \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} + \\ & \frac{1}{4} \sqrt{\frac{1}{2}(2+\sqrt{3})(1+\sqrt{5})(6+\sqrt{31})(3\sqrt{3}+\sqrt{31})(13+2\sqrt{31})} \\ & \quad \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} \end{aligned}$$

From the ratio between

$$\begin{aligned} & \frac{1}{2} \sqrt{(2+\sqrt{3})(1+\sqrt{5})(3\sqrt{3}+\sqrt{31})} \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} \\ & \left(\frac{\sqrt{2+\sqrt{31}}}{2} + \frac{\sqrt{6+\sqrt{31}}}{2} \right) \left(\sqrt{\frac{1}{2}(11+2\sqrt{31})} + \sqrt{\frac{1}{2}(13+2\sqrt{31})} \right) \end{aligned}$$

200.10283384292805393742089378421492554827045666950208313869730977

...

and:

$$\left(\frac{1}{2}(\sqrt{3} + \sqrt{7})\right)^3 \sqrt[3]{2 + \sqrt{3}} \left(\frac{1}{2}\left(2 + \sqrt{7} + \sqrt{7 + 4\sqrt{7}}\right)\right) \times \frac{\sqrt{3 + \sqrt{7}} + (6\sqrt{7})^{0.25}}{\sqrt{3 + \sqrt{7}} - (6\sqrt{7})^{0.25}}$$

172.64087217816086422023342429510817879521053958626306652342422349

...

after some easy calculations, we obtain:

$$1 + 1 / \left(\frac{1}{172.641} * \left((2 + \sqrt{3})^{1/2} * (1 + \sqrt{5})^{1/2} * (3\sqrt{3} + \sqrt{31})^{1/2} \right) \right)^{0.5} \\ (5\sqrt{5} + 2\sqrt{31})^{1/6} \left[\left(\frac{1}{4} * (2 + \sqrt{31}) \right)^{0.5} + \left(\frac{1}{4} * (6 + \sqrt{31}) \right)^{0.5} \right] \\ \left[\left(\frac{1}{2} * (11 + 2\sqrt{31}) \right)^{0.5} + \left(\frac{1}{2} * (13 + 2\sqrt{31}) \right)^{0.5} \right]^3$$

Input interpretation

1 +

$$1 / \left(\left(\frac{1}{172.641} \sqrt{(2 + \sqrt{3}) \left(\frac{1}{2} (1 + \sqrt{5}) \right) \left(\frac{1}{2} (3\sqrt{3} + \sqrt{31}) \right)} \right) \sqrt[6]{5\sqrt{5} + 2\sqrt{31}} \right. \\ \left. \left(\sqrt{\frac{1}{4}(2 + \sqrt{31})} + \sqrt{\frac{1}{4}(6 + \sqrt{31})} \right) \right. \\ \left. \left(\sqrt{\frac{1}{2}(11 + 2\sqrt{31})} + \sqrt{\frac{1}{2}(13 + 2\sqrt{31})} \right) \right)^3$$

Result

1.6422026767838305449880644938472073777291188538155727575585281099

...

$$1.64220267678\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots \text{ (trace of the instanton shape)}$$

We obtain also:

$$1 / (2\pi) \left(\frac{200.102833842928}{172.640872178} + 8.9778697879 \right)$$

Input interpretation

$$\frac{1}{2\pi} \left(\frac{200.102833842928}{172.640872178} + 8.9778697879 \right)$$

Result

1.6133440549...

1.6133440549.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{\frac{2\pi}{8.97786978790000 + \frac{200.1028338429280000}{172.6408721780000}}} = 360^\circ$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{\frac{2\pi}{8.97786978790000 + \frac{200.1028338429280000}{172.6408721780000}}} = 2i \log(-1)$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{\frac{2\pi}{8.97786978790000 + \frac{200.1028338429280000}{172.6408721780000}}} = 2 \cos^{-1}(-1)$$

Series representations

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{1.26711745765085}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{2.53423491530170}{-1.0000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{5.06846983060339}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

Integral representations

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{2.53423491530170}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{1.26711745765085}{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{\frac{200.1028338429280000}{172.6408721780000} + 8.97786978790000}{2\pi} = \frac{2.53423491530170}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

$$(4\pi)\left(\frac{200.102833842928}{172.640872178} \times \frac{1}{8.9778697879}\right)$$

Input interpretation

$$(4\pi)\left(\frac{200.102833842928}{172.640872178} \times \frac{1}{8.9778697879}\right)$$

Result

1.6223560756...

1.6223560756.... result that is a good approximation to the value of the golden ratio
1.618033988749...

Alternative representations

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = \frac{144074.0403669081600^\circ}{8.97786978790000 \times 172.6408721780000}$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = \frac{800.4113353717120000 i \log(-1)}{8.97786978790000 \times 172.6408721780000}$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = \frac{800.4113353717120000 \cos^{-1}(-1)}{8.97786978790000 \times 172.6408721780000}$$

Series representations

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = 2.06564791103375 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = -1.03282395551687 + 1.03282395551687 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} =$$

$$0.516411977758437 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50 k)}{\binom{3k}{k}}$$

Integral representations

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = 1.03282395551687 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = 2.06564791103375 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{200.1028338429280000 \times 4 \pi}{172.6408721780000 \times 8.97786978790000} = 1.03282395551687 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

We have:

$$\begin{aligned}
G_{505}^2 &= (2 + \sqrt{5}) \sqrt{\left\{ \left(\frac{1 + \sqrt{5}}{2} \right) (10 + \sqrt{101}) \right\}} \\
&\quad \times \left\{ \left(\frac{5\sqrt{5} + \sqrt{101}}{4} \right) + \sqrt{\left(\frac{105 + \sqrt{505}}{8} \right)} \right\}, \\
g_{522} &= \sqrt{\left(\frac{5 + \sqrt{29}}{2} \right)} (5\sqrt{29} + 11\sqrt{6})^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{9 + 3\sqrt{6}}{4} \right)} + \sqrt{\left(\frac{5 + 3\sqrt{6}}{4} \right)} \right\}, \\
G_{553}^2 &= \left\{ \sqrt{\left(\frac{96 + 11\sqrt{79}}{4} \right)} + \sqrt{\left(\frac{100 + 11\sqrt{79}}{4} \right)} \right\} \\
&\quad \times \left\{ \sqrt{\left(\frac{141 + 16\sqrt{79}}{2} \right)} + \sqrt{\left(\frac{143 + 16\sqrt{79}}{2} \right)} \right\}, \\
g_{630} &= (\sqrt{14} + \sqrt{15})^{\frac{1}{6}} \sqrt{\left\{ (1 + \sqrt{2}) \left(\frac{3 + \sqrt{5}}{2} \right) \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) \right\}} \\
&\quad \times \left\{ \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} + 2}{4} \right)} + \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} - 2}{4} \right)} \right\} \\
&\quad \times \left\{ \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} + 4}{8} \right)} + \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} - 4}{8} \right)} \right\}, \\
G_{765}^2 &= \left(\frac{3 + \sqrt{5}}{2} \right) (16 + \sqrt{255})^{\frac{1}{6}} \sqrt{\left\{ (4 + \sqrt{15}) \left(\frac{9 + \sqrt{85}}{2} \right) \right\}} \\
&\quad \times \left\{ \sqrt{\left(\frac{6 + \sqrt{51}}{4} \right)} + \sqrt{\left(\frac{10 + \sqrt{51}}{4} \right)} \right\} \\
&\quad \times \left\{ \sqrt{\left(\frac{18 + 3\sqrt{51}}{4} \right)} + \sqrt{\left(\frac{22 + 3\sqrt{51}}{4} \right)} \right\}, \\
G_{777}^2 &= \sqrt{\left\{ (2 + \sqrt{3})(6 + \sqrt{37}) \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) \right\}} (246\sqrt{7} + 107\sqrt{37})^{\frac{1}{6}} \\
&\quad \times \left\{ \sqrt{\left(\frac{6 + 3\sqrt{7}}{4} \right)} + \sqrt{\left(\frac{10 + 3\sqrt{7}}{4} \right)} \right\} \\
&\quad \times \left\{ \sqrt{\left(\frac{15 + 6\sqrt{7}}{2} \right)} + \sqrt{\left(\frac{17 + 6\sqrt{7}}{2} \right)} \right\},
\end{aligned}$$

From:

$$G_{505}^2 = (2 + \sqrt{5}) \sqrt{\left\{ \left(\frac{1 + \sqrt{5}}{2} \right) (10 + \sqrt{101}) \right\}} \\ \times \left\{ \left(\frac{5\sqrt{5} + \sqrt{101}}{4} \right) + \sqrt{\left(\frac{105 + \sqrt{505}}{8} \right)} \right\},$$

$$(2+\text{sqrt}5) * [(1/2*(1+\text{sqrt}5))(10+\text{sqrt}101)]^{0.5} * \\ [(1/4*(5\text{sqrt}5+\text{sqrt}101))+(1/8(105+\text{sqrt}505))^{0.5}]$$

Input

$$(2 + \sqrt{5}) \sqrt{\left(\frac{1}{2} (1 + \sqrt{5}) \right) (10 + \sqrt{101})} \left(\frac{1}{4} (5 \sqrt{5} + \sqrt{101}) + \sqrt{\frac{1}{8} (105 + \sqrt{505})} \right)$$

Exact result

$$(2 + \sqrt{5}) \sqrt{\frac{1}{2} (1 + \sqrt{5}) (10 + \sqrt{101})} \left(\frac{1}{4} (5 \sqrt{5} + \sqrt{101}) + \frac{1}{2} \sqrt{\frac{1}{2} (105 + \sqrt{505})} \right)$$

Decimal approximation

224.36895935132763918399413635761729391464432800073649303816535945

...

224.36895935....

Alternate forms

$$\sqrt{\text{root of } 256x^8 - 13134080x^7 + 12406662784x^6 + 566469885440x^5 + 8970692383216x^4 + 59000758979200x^3 + 133454526025384x^2 - 21580568998020x + 63001502001 \text{ near } x = 50341.4}$$

$$\frac{1}{4}(2 + \sqrt{5}) \sqrt{\frac{1}{2}(1 + \sqrt{5})(10 + \sqrt{101})(5\sqrt{5} + \sqrt{101})} +$$

$$\frac{1}{4}(2 + \sqrt{5}) \sqrt{(1 + \sqrt{5})(10 + \sqrt{101})(105 + \sqrt{505})}$$

$$\frac{1}{8}(2 + \sqrt{5}) \left(5 \sqrt{10(1 + \sqrt{5})(10 + \sqrt{101})} + \sqrt{202(1 + \sqrt{5})(10 + \sqrt{101})} + \right.$$

$$\left. 2 \sqrt{1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505}} \right)$$

Minimal polynomial

$$256x^{16} - 13134080x^{14} + 12406662784x^{12} +$$

$$566469885440x^{10} + 8970692383216x^8 + 59000758979200x^6 +$$

$$133454526025384x^4 - 21580568998020x^2 + 63001502001$$

Expanded forms

$$\frac{25}{4} \sqrt{\frac{1}{2}(1 + \sqrt{5})(10 + \sqrt{101})} + \frac{5}{2} \sqrt{\frac{5}{2}(1 + \sqrt{5})(10 + \sqrt{101})} +$$

$$\frac{1}{2} \sqrt{\frac{101}{2}(1 + \sqrt{5})(10 + \sqrt{101})} + \frac{1}{4} \sqrt{\frac{505}{2}(1 + \sqrt{5})(10 + \sqrt{101})} +$$

$$\frac{1}{2} \sqrt{(1 + \sqrt{5})(10 + \sqrt{101})(105 + \sqrt{505})} +$$

$$\frac{1}{4} \sqrt{5(1 + \sqrt{5})(10 + \sqrt{101})(105 + \sqrt{505})}$$

$$\frac{5}{2} \sqrt{25 + 25\sqrt{5} + \frac{5\sqrt{101}}{2} + \frac{5\sqrt{505}}{2}} +$$

$$\frac{1}{4} \sqrt{2525 + 2525\sqrt{5} + \frac{505\sqrt{101}}{2} + \frac{505\sqrt{505}}{2}} +$$

$$\frac{1}{4} \sqrt{7775 + 5755\sqrt{5} + 775\sqrt{101} + 575\sqrt{505}} +$$

$$\frac{25}{4} \sqrt{5 + 5\sqrt{5} + \frac{1}{2}(\sqrt{101} + \sqrt{505})} +$$

$$\frac{1}{2} \left(\sqrt{505 + 505\sqrt{5} + \frac{101\sqrt{101}}{2} + \frac{101\sqrt{505}}{2}} + \right.$$

$$\left. \sqrt{1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505}} \right)$$

From:

$$g_{522} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)} (5\sqrt{29} + 11\sqrt{6})^{\frac{1}{6}} \left\{ \sqrt{\left(\frac{9 + 3\sqrt{6}}{4}\right)} + \sqrt{\left(\frac{5 + 3\sqrt{6}}{4}\right)} \right\},$$

$$(1/2*(5+\sqrt{29}))^{0.5} ((5\sqrt{29}+11\sqrt{6}))^{(1/6)}$$

$$[(1/4*(9+3\sqrt{6}))^{0.5}+(1/4*(5+3\sqrt{6}))^{0.5}]$$

Input

$$\sqrt{\frac{1}{2}(5 + \sqrt{29})} \sqrt[6]{5\sqrt{29} + 11\sqrt{6}} \left(\sqrt{\frac{1}{4}(9 + 3\sqrt{6})} + \sqrt{\frac{1}{4}(5 + 3\sqrt{6})} \right)$$

Exact result

$$\sqrt{\frac{1}{2}(5 + \sqrt{29})} \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \left(\frac{1}{2} \sqrt{5 + 3\sqrt{6}} + \frac{1}{2} \sqrt{9 + 3\sqrt{6}} \right)$$

Decimal approximation

16.733627372862632001654199780484070484400153552417234409678167491

...

16.7336273728....

Alternate forms

$$\frac{1}{2} \sqrt[12]{1451 + 110\sqrt{174}} \sqrt{(5 + \sqrt{29}) \left(7 + 3\sqrt{6} + \sqrt{99 + 42\sqrt{6}}\right)}$$

$$\sqrt[6]{\text{root of } x^8 - 21955360x^7 - 352900x^6 + 19040x^5 + 382984198x^4 - 19040x^3 - 352900x^2 + 21955360x + 1 \text{ near } x = 2.19554 \times 10^7}$$

$$\frac{\sqrt{5 + \sqrt{29}} \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \left(\sqrt{3(3 + \sqrt{6})} + \sqrt{5 + 3\sqrt{6}} \right)}{2\sqrt{2}}$$

Minimal polynomial

$$x^{48} - 21955360x^{42} - 352900x^{36} + 19040x^{30} + 382984198x^{24} - 19040x^{18} - 352900x^{12} + 21955360x^6 + 1$$

Expanded forms

$$\frac{1}{4} \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \left(\sqrt{50 + 30\sqrt{6} + 10\sqrt{29} + 6\sqrt{174}} + \sqrt{90 + 30\sqrt{6} + 18\sqrt{29} + 6\sqrt{174}} \right)$$

$$\frac{\sqrt{(5+3\sqrt{6})(5+\sqrt{29})} \sqrt[6]{11\sqrt{6}+5\sqrt{29}}}{2\sqrt{2}} + \frac{\sqrt{(9+3\sqrt{6})(5+\sqrt{29})} \sqrt[6]{11\sqrt{6}+5\sqrt{29}}}{2\sqrt{2}}$$

Dividing the two above expressions, after some easy calculations, we obtain:

$$\frac{22}{\left(\left((2+\sqrt{5}) \left[\frac{1}{2}(1+\sqrt{5})(10+\sqrt{101}) \right]^{0.5} \left[\frac{1}{4}(5\sqrt{5}+\sqrt{101}) + \frac{1}{8}(105+\sqrt{505}) \right]^{0.5} \right)^{1/6} \left(\frac{1}{2}(5+\sqrt{29}) \right)^{0.5} \left((5\sqrt{29}+11\sqrt{6})^{1/6} \left[\frac{1}{4}(9+3\sqrt{6}) \right]^{0.5} + \frac{1}{4}(5+3\sqrt{6}) \right)^{0.5} \right)^{0.5}}$$

Input

$$22 / \left(\left((2+\sqrt{5}) \sqrt{\left(\frac{1}{2}(1+\sqrt{5})(10+\sqrt{101}) \right)} \left(\frac{1}{4}(5\sqrt{5}+\sqrt{101}) + \sqrt{\frac{1}{8}(105+\sqrt{505})} \right) \right) \times \frac{1}{\sqrt{\frac{1}{2}(5+\sqrt{29})} \sqrt[6]{5\sqrt{29}+11\sqrt{6}} \left(\sqrt{\frac{1}{4}(9+3\sqrt{6})} + \sqrt{\frac{1}{4}(5+3\sqrt{6})} \right)} \right)$$

Exact result

$$\frac{22 \sqrt[6]{11\sqrt{6}+5\sqrt{29}} \sqrt{\frac{5+\sqrt{29}}{(1+\sqrt{5})(10+\sqrt{101})}} \left(\frac{1}{2}\sqrt{5+3\sqrt{6}} + \frac{1}{2}\sqrt{9+3\sqrt{6}} \right)}{(2+\sqrt{5}) \left(\frac{1}{4}(5\sqrt{5}+\sqrt{101}) + \frac{1}{2}\sqrt{\frac{1}{2}(105+\sqrt{505})} \right)}$$

Decimal approximation

1.6407786677234929288130202794041134185207134487439845364524553596

...

$1.6407786677\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$ (trace of the instanton shape)

Alternate form

$$\left(44(\sqrt{5} - 2) \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \sqrt{\frac{(5 + \sqrt{29})(\sqrt{101} - 10)}{1 + \sqrt{5}}} \right. \\ \left. \left(\sqrt{3(3 + \sqrt{6})} + \sqrt{5 + 3\sqrt{6}} \right) \right) / \left(5\sqrt{5} + \sqrt{101} + \sqrt{2(105 + \sqrt{505})} \right)$$

Expanded form

$$\frac{11 \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \sqrt{\frac{(5+3\sqrt{6})(5+\sqrt{29})}{(1+\sqrt{5})(10+\sqrt{101})}}}{(2 + \sqrt{5}) \left(\frac{1}{4}(5\sqrt{5} + \sqrt{101}) + \frac{1}{2} \sqrt{\frac{1}{2}(105 + \sqrt{505})} \right)} + \\ \frac{11 \sqrt[6]{11\sqrt{6} + 5\sqrt{29}} \sqrt{\frac{(9+3\sqrt{6})(5+\sqrt{29})}{(1+\sqrt{5})(10+\sqrt{101})}}}{(2 + \sqrt{5}) \left(\frac{1}{4}(5\sqrt{5} + \sqrt{101}) + \frac{1}{2} \sqrt{\frac{1}{2}(105 + \sqrt{505})} \right)}$$

From:

$$G_{553}^2 = \left\{ \sqrt{\left(\frac{96 + 11\sqrt{79}}{4} \right)} + \sqrt{\left(\frac{100 + 11\sqrt{79}}{4} \right)} \right\} \\ \times \left\{ \sqrt{\left(\frac{141 + 16\sqrt{79}}{2} \right)} + \sqrt{\left(\frac{143 + 16\sqrt{79}}{2} \right)} \right\},$$

$$[(1/4*(96+11\sqrt{79}))^{0.5}+(1/4(100+11\sqrt{79}))^{0.5}]$$

$$[(1/2*(141+16\sqrt{79}))^{0.5}+(1/2(143+16\sqrt{79}))^{0.5}]$$

Input

$$\left(\sqrt{\frac{1}{4}(96 + 11\sqrt{79})} + \sqrt{\frac{1}{4}(100 + 11\sqrt{79})} \right)$$

$$\left(\sqrt{\frac{1}{2}(141 + 16\sqrt{79})} + \sqrt{\frac{1}{2}(143 + 16\sqrt{79})} \right)$$

Exact result

$$\left(\frac{1}{2} \sqrt{96 + 11\sqrt{79}} + \frac{1}{2} \sqrt{100 + 11\sqrt{79}} \right)$$

$$\left(\sqrt{\frac{1}{2}(141 + 16\sqrt{79})} + \sqrt{\frac{1}{2}(143 + 16\sqrt{79})} \right)$$

Decimal approximation

333.58184761212887583587576255785772649911879600771119516072400901

...

333.581847612....

Alternate forms

$$\frac{1}{4} \left(8 + \sqrt{79} + \sqrt{141 + 16\sqrt{79}} \right) \left(11 + \sqrt{79} + \sqrt{192 + 22\sqrt{79}} \right)$$

$$\frac{1}{4} \left(\sqrt{96 + 11\sqrt{79}} + \sqrt{100 + 11\sqrt{79}} \right) \left(\sqrt{282 + 32\sqrt{79}} + \sqrt{286 + 32\sqrt{79}} \right)$$

root of $x^8 - 334x^7 + 138x^6 + 496x^5 + 127x^4 + 496x^3 + 138x^2 - 334x + 1$
near $x = 333.582$

$$\frac{1}{2\sqrt{2}} \left(7\sqrt{7(80+9\sqrt{79})} + \sqrt{27632+3109\sqrt{79}} + \sqrt{28004+3151\sqrt{79}} + \sqrt{28204+3173\sqrt{79}} \right)$$

Minimal polynomial

$$x^8 - 334x^7 + 138x^6 + 496x^5 + 127x^4 + 496x^3 + 138x^2 - 334x + 1$$

Expanded forms

$$\frac{1}{2} \left(\sqrt{\left(\frac{141}{2} + 8\sqrt{79}\right)(96 + 11\sqrt{79})} + \sqrt{\left(\frac{143}{2} + 8\sqrt{79}\right)(96 + 11\sqrt{79})} + \sqrt{\left(\frac{141}{2} + 8\sqrt{79}\right)(100 + 11\sqrt{79})} + \sqrt{\left(\frac{143}{2} + 8\sqrt{79}\right)(100 + 11\sqrt{79})} \right)$$

$$\frac{1}{2} \sqrt{\frac{1}{2}(96 + 11\sqrt{79})(141 + 16\sqrt{79})} + \frac{1}{2} \sqrt{\frac{1}{2}(100 + 11\sqrt{79})(141 + 16\sqrt{79})} + \frac{1}{2} \sqrt{\frac{1}{2}(96 + 11\sqrt{79})(143 + 16\sqrt{79})} + \frac{1}{2} \sqrt{\frac{1}{2}(100 + 11\sqrt{79})(143 + 16\sqrt{79})}$$

Subtracting

$$\left(\frac{1}{2} \sqrt{96 + 11\sqrt{79}} + \frac{1}{2} \sqrt{100 + 11\sqrt{79}} \right) \left(\sqrt{\frac{1}{2}(141 + 16\sqrt{79})} + \sqrt{\frac{1}{2}(143 + 16\sqrt{79})} \right)$$

by

$$(2 + \sqrt{5}) \sqrt{\frac{1}{2}(1 + \sqrt{5})(10 + \sqrt{101})} \left(\frac{1}{4}(5\sqrt{5} + \sqrt{101}) + \frac{1}{2} \sqrt{\frac{1}{2}(105 + \sqrt{505})} \right)$$

we obtain, after some calculations:

$$1+1/\left(\left(\left(\left(\frac{1}{4}(96+11\sqrt{79})\right)^{0.5}+\left(\frac{1}{4}(100+11\sqrt{79})\right)^{0.5}\right)\right.\right. \\ \left.\left.\left(\frac{1}{2}(141+16\sqrt{79})\right)^{0.5}+\left(\frac{1}{2}(143+16\sqrt{79})\right)^{0.5}\right)\right)-\left(\left(2+\sqrt{5}\right)\right. \\ \left.\left(\frac{1}{2}(1+\sqrt{5})\right)(10+\sqrt{101})\right)^{0.5}\left[\left(\frac{1}{4}(5\sqrt{5}+\sqrt{101})\right)+\left(\frac{1}{8}(105+\sqrt{505})\right)^{0.5}\right]\right)^{1/11}$$

Input

$$1+1/\left(\left(\left(\left(\sqrt{\frac{1}{4}(96+11\sqrt{79})}+\sqrt{\frac{1}{4}(100+11\sqrt{79})}\right)\right.\right.\right. \\ \left.\left.\left(\sqrt{\frac{1}{2}(141+16\sqrt{79})}+\sqrt{\frac{1}{2}(143+16\sqrt{79})}\right)\right)-\right. \\ \left.\left(2+\sqrt{5}\right)\sqrt{\left(\frac{1}{2}(1+\sqrt{5})\right)(10+\sqrt{101})}\right. \\ \left.\left(\frac{1}{4}(5\sqrt{5}+\sqrt{101})+\sqrt{\frac{1}{8}(105+\sqrt{505})}\right)\right)^{(1/11)}$$

Exact result

$$1+1/\left(\left(\left(\left(\frac{1}{2}\sqrt{96+11\sqrt{79}}+\frac{1}{2}\sqrt{100+11\sqrt{79}}\right)\right.\right.\right. \\ \left.\left.\left(\sqrt{\frac{1}{2}(141+16\sqrt{79})}+\sqrt{\frac{1}{2}(143+16\sqrt{79})}\right)\right)-\right. \\ \left.\left(2+\sqrt{5}\right)\sqrt{\frac{1}{2}(1+\sqrt{5})(10+\sqrt{101})}\right. \\ \left.\left(\frac{1}{4}(5\sqrt{5}+\sqrt{101})+\frac{1}{2}\sqrt{\frac{1}{2}(105+\sqrt{505})}\right)\right)^{(1/11)}$$

Decimal approximation

1.6526831087008325738498636454936232383656252914784633086109704201

...

1.6526831087.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Alternate forms

$$1 + 2^{3/11} / \left(\left(14 \sqrt{14(80 + 9\sqrt{79})} + 2\sqrt{2(27632 + 3109\sqrt{79})} + \right. \right. \\ \left. \left. 2\sqrt{2(28004 + 3151\sqrt{79})} + 2\sqrt{2(28204 + 3173\sqrt{79})} - \right. \right. \\ \left. \left. 25\sqrt{2(1 + \sqrt{5})(10 + \sqrt{101})} - 10\sqrt{10(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 2\sqrt{202(1 + \sqrt{5})(10 + \sqrt{101})} - \sqrt{1010(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 4\sqrt{1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505}} - \right. \right. \\ \left. \left. 2\sqrt{5(1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505})} \right)^{(1/11)} \right)$$

$$\left(2^{3/11} + \left(14 \sqrt{14(80 + 9\sqrt{79})} + 2\sqrt{2(27632 + 3109\sqrt{79})} + \right. \right. \\ \left. \left. 2\sqrt{2(28004 + 3151\sqrt{79})} + 2\sqrt{2(28204 + 3173\sqrt{79})} - \right. \right. \\ \left. \left. 25\sqrt{2(1 + \sqrt{5})(10 + \sqrt{101})} - 10\sqrt{10(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 2\sqrt{202(1 + \sqrt{5})(10 + \sqrt{101})} - \sqrt{1010(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 4\sqrt{1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505}} - \right. \right. \\ \left. \left. 2\sqrt{5(1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505})} \right)^{(1/11)} \right) / \\ \left(\left(14 \sqrt{14(80 + 9\sqrt{79})} + 2\sqrt{2(27632 + 3109\sqrt{79})} + \right. \right. \\ \left. \left. 2\sqrt{2(28004 + 3151\sqrt{79})} + 2\sqrt{2(28204 + 3173\sqrt{79})} - \right. \right. \\ \left. \left. 25\sqrt{2(1 + \sqrt{5})(10 + \sqrt{101})} - 10\sqrt{10(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 2\sqrt{202(1 + \sqrt{5})(10 + \sqrt{101})} - \sqrt{1010(1 + \sqrt{5})(10 + \sqrt{101})} - \right. \right. \\ \left. \left. 4\sqrt{1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505}} - \right. \right. \\ \left. \left. 2\sqrt{5(1555 + 1151\sqrt{5} + 155\sqrt{101} + 115\sqrt{505})} \right)^{(1/11)} \right)$$

$$\begin{aligned}
& \left(2^{3/11} + \left(2\sqrt{2(96+11\sqrt{79})(141+16\sqrt{79})} + \right. \right. \\
& \quad 2\sqrt{2(100+11\sqrt{79})(141+16\sqrt{79})} + \\
& \quad 2\sqrt{2(96+11\sqrt{79})(143+16\sqrt{79})} + \\
& \quad 2\sqrt{2(100+11\sqrt{79})(143+16\sqrt{79})} - \\
& \quad 25\sqrt{2(1+\sqrt{5})(10+\sqrt{101})} - 10\sqrt{10(1+\sqrt{5})(10+\sqrt{101})} - \\
& \quad 2\sqrt{202(1+\sqrt{5})(10+\sqrt{101})} - \sqrt{1010(1+\sqrt{5})(10+\sqrt{101})} - \\
& \quad 4\sqrt{(1+\sqrt{5})(10+\sqrt{101})(105+\sqrt{505})} - \\
& \quad \left. \left. 2\sqrt{5(1+\sqrt{5})(10+\sqrt{101})(105+\sqrt{505})} \right)^{(1/11)} \right) / \\
& \left(\left(2\sqrt{2(96+11\sqrt{79})(141+16\sqrt{79})} + \right. \right. \\
& \quad 2\sqrt{2(100+11\sqrt{79})(141+16\sqrt{79})} + \\
& \quad 2\sqrt{2(96+11\sqrt{79})(143+16\sqrt{79})} + \\
& \quad 2\sqrt{2(100+11\sqrt{79})(143+16\sqrt{79})} - \\
& \quad 25\sqrt{2(1+\sqrt{5})(10+\sqrt{101})} - 10\sqrt{10(1+\sqrt{5})(10+\sqrt{101})} - \\
& \quad 2\sqrt{202(1+\sqrt{5})(10+\sqrt{101})} - \sqrt{1010(1+\sqrt{5})(10+\sqrt{101})} - \\
& \quad 4\sqrt{(1+\sqrt{5})(10+\sqrt{101})(105+\sqrt{505})} - \\
& \quad \left. \left. 2\sqrt{5(1+\sqrt{5})(10+\sqrt{101})(105+\sqrt{505})} \right)^{(1/11)} \right)
\end{aligned}$$

From:

$$\begin{aligned}
 g_{630} &= (\sqrt{14} + \sqrt{15})^{\frac{1}{6}} \sqrt{\left\{ (1 + \sqrt{2}) \left(\frac{3 + \sqrt{5}}{2} \right) \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) \right\}} \\
 &\quad \times \left\{ \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} + 2}{4} \right)} + \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} - 2}{4} \right)} \right\} \\
 &\quad \times \left\{ \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} + 4}{8} \right)} + \sqrt{\left(\frac{\sqrt{15} + \sqrt{7} - 4}{8} \right)} \right\},
 \end{aligned}$$

$$(\sqrt{14} + \sqrt{15})^{1/6} [(1 + \sqrt{2})(1/2(3 + \sqrt{5})) + (1/2(\sqrt{3} + \sqrt{7}))]^{0.5} [(1/4(\sqrt{15} + \sqrt{7} + 2))^{0.5} + (1/4(\sqrt{15} + \sqrt{7} - 2))^{0.5}] [(1/8(\sqrt{15} + \sqrt{7} + 4))^{0.5} + (1/8(\sqrt{15} + \sqrt{7} - 4))^{0.5}]$$

Input

$$\begin{aligned}
 &\sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{(1 + \sqrt{2}) \left(\frac{1}{2} (3 + \sqrt{5}) \right) + \frac{1}{2} (\sqrt{3} + \sqrt{7})} \\
 &\quad \left(\sqrt{\frac{1}{4} (\sqrt{15} + \sqrt{7} + 2)} + \sqrt{\frac{1}{4} (\sqrt{15} + \sqrt{7} - 2)} \right) \\
 &\quad \left(\sqrt{\frac{1}{8} (\sqrt{15} + \sqrt{7} + 4)} + \sqrt{\frac{1}{8} (\sqrt{15} + \sqrt{7} - 4)} \right)
 \end{aligned}$$

Exact result

$$\begin{aligned}
 &\sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{\frac{1}{2} (1 + \sqrt{2}) (3 + \sqrt{5}) + \frac{1}{2} (\sqrt{3} + \sqrt{7})} \\
 &\quad \left(\frac{1}{2} \sqrt{-2 + \sqrt{7} + \sqrt{15}} + \frac{1}{2} \sqrt{2 + \sqrt{7} + \sqrt{15}} \right) \\
 &\quad \left(\frac{1}{2} \sqrt{\frac{1}{2} (-4 + \sqrt{7} + \sqrt{15})} + \frac{1}{2} \sqrt{\frac{1}{2} (4 + \sqrt{7} + \sqrt{15})} \right)
 \end{aligned}$$

Decimal approximation

17.623908118731032578680892160191760490425623653614243492567598056

...

17.623908118731.....

Alternate forms

$$\frac{1}{8} \sqrt{3 + 3\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{7} + \sqrt{10}}$$
$$\sqrt[6]{\sqrt{14} + \sqrt{15}} \left(\sqrt{-2 + \sqrt{7} + \sqrt{15}} + \sqrt{2 + \sqrt{7} + \sqrt{15}} \right)$$
$$\left(\sqrt{-4 + \sqrt{7} + \sqrt{15}} + \sqrt{4 + \sqrt{7} + \sqrt{15}} \right)$$

$$\frac{1}{8} \sqrt{(3 + 3\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{7} + \sqrt{10})(-4 + \sqrt{7} + \sqrt{15})}$$
$$\sqrt[6]{\sqrt{14} + \sqrt{15}} \left(\sqrt{-2 + \sqrt{7} + \sqrt{15}} + \sqrt{2 + \sqrt{7} + \sqrt{15}} \right) +$$
$$\frac{1}{8} \sqrt{(3 + 3\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{7} + \sqrt{10})(4 + \sqrt{7} + \sqrt{15})}$$
$$\sqrt[6]{\sqrt{14} + \sqrt{15}} \left(\sqrt{-2 + \sqrt{7} + \sqrt{15}} + \sqrt{2 + \sqrt{7} + \sqrt{15}} \right)$$

Expanded form

$$\frac{1}{4} \sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{\left(\frac{1}{2}(-4 + \sqrt{7} + \sqrt{15})\right.}$$
$$\left.(-2 + \sqrt{7} + \sqrt{15})\left(\frac{1}{2}(1 + \sqrt{2})(3 + \sqrt{5}) + \frac{1}{2}(\sqrt{3} + \sqrt{7})\right)\right)} +$$
$$\frac{1}{4} \sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{\left(\frac{1}{2}(-4 + \sqrt{7} + \sqrt{15})(2 + \sqrt{7} + \sqrt{15})\right.}$$
$$\left.\left(\frac{1}{2}(1 + \sqrt{2})(3 + \sqrt{5}) + \frac{1}{2}(\sqrt{3} + \sqrt{7})\right)\right)} +$$
$$\frac{1}{4} \sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{\left(\frac{1}{2}(-2 + \sqrt{7} + \sqrt{15})(4 + \sqrt{7} + \sqrt{15})\right.}$$
$$\left.\left(\frac{1}{2}(1 + \sqrt{2})(3 + \sqrt{5}) + \frac{1}{2}(\sqrt{3} + \sqrt{7})\right)\right)} +$$
$$\frac{1}{4} \sqrt[6]{\sqrt{14} + \sqrt{15}} \sqrt{\left(\frac{1}{2}(2 + \sqrt{7} + \sqrt{15})(4 + \sqrt{7} + \sqrt{15})\right.}$$
$$\left.\left(\frac{1}{2}(1 + \sqrt{2})(3 + \sqrt{5}) + \frac{1}{2}(\sqrt{3} + \sqrt{7})\right)\right)}$$

From:

$$G_{765}^2 = \left(\frac{3 + \sqrt{5}}{2}\right) (16 + \sqrt{255})^{\frac{1}{6}} \sqrt{\left\{ (4 + \sqrt{15}) \left(\frac{9 + \sqrt{85}}{2}\right) \right\}}$$

$$\times \left\{ \sqrt{\left(\frac{6 + \sqrt{51}}{4}\right)} + \sqrt{\left(\frac{10 + \sqrt{51}}{4}\right)} \right\}$$

$$\times \left\{ \sqrt{\left(\frac{18 + 3\sqrt{51}}{4}\right)} + \sqrt{\left(\frac{22 + 3\sqrt{51}}{4}\right)} \right\},$$

$$\begin{aligned} & \left(\frac{1}{2}*(3+\text{sqrt}5)\right) (16+\text{sqrt}255)^{(1/6)} (((4+\text{sqrt}15)(\frac{1}{2}*(9+\text{sqrt}85))))^{0.5} \\ & [(1/4*(6+\text{sqrt}51))^{0.5}+(1/4(10+\text{sqrt}51))^{0.5}] \\ & [(1/4*(18+3\text{sqrt}51))^{0.5}+(1/4(22+3\text{sqrt}51))^{0.5}] \end{aligned}$$

Input

$$\begin{aligned} & \left(\frac{1}{2}(3 + \sqrt{5})\right)^6 \sqrt[6]{16 + \sqrt{255}} \sqrt{(4 + \sqrt{15}) \left(\frac{1}{2}(9 + \sqrt{85})\right)} \\ & \left(\sqrt{\frac{1}{4}(6 + \sqrt{51})} + \sqrt{\frac{1}{4}(10 + \sqrt{51})}\right) \left(\sqrt{\frac{1}{4}(18 + 3\sqrt{51})} + \sqrt{\frac{1}{4}(22 + 3\sqrt{51})}\right) \end{aligned}$$

Exact result

$$\begin{aligned} & \frac{1}{2}(3 + \sqrt{5}) \sqrt{\frac{1}{2}(4 + \sqrt{15})(9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} \\ & \left(\frac{\sqrt{6 + \sqrt{51}}}{2} + \frac{\sqrt{10 + \sqrt{51}}}{2}\right) \left(\frac{1}{2}\sqrt{18 + 3\sqrt{51}} + \frac{1}{2}\sqrt{22 + 3\sqrt{51}}\right) \end{aligned}$$

Decimal approximation

986.77267508088417580650634583062668290122626301645279729799289689

...

986.7726750808841....

Alternate forms

$$\frac{1}{16} (3 + \sqrt{5}) \sqrt{(4 + \sqrt{15})(9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} \left(\sqrt{6 + \sqrt{51}} + \sqrt{10 + \sqrt{51}} \right) \\ \left(\sqrt{18 - 3i\sqrt{15}} + \sqrt{2} \left(3\sqrt{\frac{3}{2}} + \sqrt{\frac{17}{2}} + \sqrt{\frac{3}{2}i(\sqrt{15} - 6i)} \right) \right)$$

$$\frac{1}{8} (3 + \sqrt{5}) \sqrt{\frac{3}{2}(6 + \sqrt{51})(36 + 9\sqrt{15} + 5\sqrt{51} + 4\sqrt{85})} \\ \sqrt[6]{16 + \sqrt{255}} \left(\sqrt{6 + \sqrt{51}} + \sqrt{10 + \sqrt{51}} \right) + \frac{1}{8\sqrt{2}} \\ (3 + \sqrt{5}) \sqrt{(22 + 3\sqrt{51})(36 + 9\sqrt{15} + 5\sqrt{51} + 4\sqrt{85})} \\ \sqrt[6]{16 + \sqrt{255}} \left(\sqrt{6 + \sqrt{51}} + \sqrt{10 + \sqrt{51}} \right)$$

$$\frac{1}{8\sqrt{2}} \sqrt[6]{16 + \sqrt{255}} \left(9\sqrt{2064 + 533\sqrt{15} + 289\sqrt{51} + 224\sqrt{85}} + \right. \\ 3\sqrt{5(2064 + 533\sqrt{15} + 289\sqrt{51} + 224\sqrt{85})} + \\ 5\sqrt{4092 + 1057\sqrt{15} + 573\sqrt{51} + 444\sqrt{85}} + \\ 3\sqrt{5(4092 + 1057\sqrt{15} + 573\sqrt{51} + 444\sqrt{85})} + \\ 3\sqrt{3(8076 + 2087\sqrt{15} + 1131\sqrt{51} + 876\sqrt{85})} + \\ \sqrt{15(8076 + 2087\sqrt{15} + 1131\sqrt{51} + 876\sqrt{85})} + \\ \left. 3\sqrt{26688 + 6893\sqrt{15} + 3737\sqrt{51} + 2896\sqrt{85}} + \right. \\ \left. \sqrt{5(26688 + 6893\sqrt{15} + 3737\sqrt{51} + 2896\sqrt{85})} \right)$$

Expanded form

$$\begin{aligned}
 & \frac{3}{8} \sqrt{\frac{1}{2} (4 + \sqrt{15}) (6 + \sqrt{51}) (18 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{1}{8} \sqrt{\frac{5}{2} (4 + \sqrt{15}) (6 + \sqrt{51}) (18 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{3}{8} \sqrt{\frac{1}{2} (4 + \sqrt{15}) (10 + \sqrt{51}) (18 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{1}{8} \sqrt{\frac{5}{2} (4 + \sqrt{15}) (10 + \sqrt{51}) (18 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{3}{8} \sqrt{\frac{1}{2} (4 + \sqrt{15}) (6 + \sqrt{51}) (22 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{1}{8} \sqrt{\frac{5}{2} (4 + \sqrt{15}) (6 + \sqrt{51}) (22 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{3}{8} \sqrt{\frac{1}{2} (4 + \sqrt{15}) (10 + \sqrt{51}) (22 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}} + \\
 & \frac{1}{8} \sqrt{\frac{5}{2} (4 + \sqrt{15}) (10 + \sqrt{51}) (22 + 3\sqrt{51}) (9 + \sqrt{85})} \sqrt[6]{16 + \sqrt{255}}
 \end{aligned}$$

From:

$$\begin{aligned}
 G_{777}^2 &= \sqrt{\left\{ (2 + \sqrt{3})(6 + \sqrt{37}) \left(\frac{\sqrt{3} + \sqrt{7}}{2} \right) \right\} (246\sqrt{7} + 107\sqrt{37})^{\frac{1}{6}}} \\
 &\quad \times \left\{ \sqrt{\left(\frac{6 + 3\sqrt{7}}{4} \right)} + \sqrt{\left(\frac{10 + 3\sqrt{7}}{4} \right)} \right\} \\
 &\quad \times \left\{ \sqrt{\left(\frac{15 + 6\sqrt{7}}{2} \right)} + \sqrt{\left(\frac{17 + 6\sqrt{7}}{2} \right)} \right\},
 \end{aligned}$$

$$\begin{aligned}
 & (((2+\text{sqrt}3) (6+\text{sqrt}37) (1/2*(\text{sqrt}3+\text{sqrt}7))))^0.5 (((246\text{sqrt}7+107\text{sqrt}37)))^(1/6) \\
 & [(1/4*(6+3\text{sqrt}7))^0.5+(1/4(10+3\text{sqrt}7))^0.5] \\
 & [(1/2*(15+6\text{sqrt}7))^0.5+(1/2(17+6\text{sqrt}7))^0.5]
 \end{aligned}$$

Input

$$\sqrt{(2+\sqrt{3})(6+\sqrt{37})\left(\frac{1}{2}(\sqrt{3}+\sqrt{7})\right)} \sqrt[6]{246\sqrt{7}+107\sqrt{37}} \\ \left(\sqrt{\frac{1}{4}(6+3\sqrt{7})} + \sqrt{\frac{1}{4}(10+3\sqrt{7})}\right) \left(\sqrt{\frac{1}{2}(15+6\sqrt{7})} + \sqrt{\frac{1}{2}(17+6\sqrt{7})}\right)$$

Exact result

$$\sqrt{\frac{1}{2}(2+\sqrt{3})(\sqrt{3}+\sqrt{7})(6+\sqrt{37})} \sqrt[6]{246\sqrt{7}+107\sqrt{37}} \\ \left(\frac{1}{2}\sqrt{6+3\sqrt{7}} + \frac{1}{2}\sqrt{10+3\sqrt{7}}\right) \left(\sqrt{\frac{1}{2}(15+6\sqrt{7})} + \sqrt{\frac{1}{2}(17+6\sqrt{7})}\right)$$

Decimal approximation

1044.2048545055534425664886148406305611383653989517741388463542051

...

1044.2048545055534.....

Alternate forms

$$\frac{1}{4} \sqrt{\frac{1}{2}(3+2\sqrt{3}+2\sqrt{7}+\sqrt{21})(6+\sqrt{37})} \sqrt[6]{246\sqrt{7}+107\sqrt{37}} \\ \left(\sqrt{3(2+\sqrt{7})} + \sqrt{10+3\sqrt{7}}\right) \left(\sqrt{6(5+2\sqrt{7})} + \sqrt{34+12\sqrt{7}}\right)$$

$$\frac{1}{4} \sqrt{3(5+2\sqrt{7})(3+2\sqrt{3}+2\sqrt{7}+\sqrt{21})(6+\sqrt{37})} \\ \sqrt[6]{246\sqrt{7}+107\sqrt{37}} \left(\sqrt{3(2+\sqrt{7})} + \sqrt{10+3\sqrt{7}}\right) + \\ \frac{1}{4} \sqrt{(17+6\sqrt{7})(3+2\sqrt{3}+2\sqrt{7}+\sqrt{21})(6+\sqrt{37})} \\ \sqrt[6]{246\sqrt{7}+107\sqrt{37}} \left(\sqrt{3(2+\sqrt{7})} + \sqrt{10+3\sqrt{7}}\right)$$

Expanded form

$$\begin{aligned} & \frac{1}{4} \sqrt{\frac{(2+\sqrt{3})(\sqrt{3}+\sqrt{7})(6+3\sqrt{7})(15+6\sqrt{7})(6+\sqrt{37})}{\sqrt[6]{246\sqrt{7}+107\sqrt{37}}}} \\ & \frac{1}{4} \sqrt{\frac{(2+\sqrt{3})(\sqrt{3}+\sqrt{7})(10+3\sqrt{7})(15+6\sqrt{7})(6+\sqrt{37})}{\sqrt[6]{246\sqrt{7}+107\sqrt{37}}}} \\ & \frac{1}{4} \sqrt{\frac{(2+\sqrt{3})(\sqrt{3}+\sqrt{7})(6+3\sqrt{7})(17+6\sqrt{7})(6+\sqrt{37})}{\sqrt[6]{246\sqrt{7}+107\sqrt{37}}}} \\ & \frac{1}{4} \sqrt{\frac{(2+\sqrt{3})(\sqrt{3}+\sqrt{7})(10+3\sqrt{7})(17+6\sqrt{7})(6+\sqrt{37})}{\sqrt[6]{246\sqrt{7}+107\sqrt{37}}}} \end{aligned}$$

We have:

$$\begin{aligned} G_{1225} &= \left(\frac{1+\sqrt{5}}{2} \right) (6+\sqrt{35})^{\frac{1}{4}} \left\{ \frac{7^{\frac{1}{4}} + \sqrt{(4+\sqrt{7})}}{2} \right\}^{\frac{3}{2}} \\ &\quad \times \left[\sqrt{\left\{ \frac{43+15\sqrt{7}+(8+3\sqrt{7})\sqrt{(10\sqrt{7})}}{8} \right\}} \right. \\ &\quad \left. + \sqrt{\left\{ \frac{35+15\sqrt{7}+(8+3\sqrt{7})\sqrt{(10\sqrt{7})}}{8} \right\}} \right], \\ G_{1353}^2 &= \sqrt{\left\{ (3+\sqrt{11})(5+3\sqrt{3}) \left(\frac{11+\sqrt{123}}{2} \right) \right\}} \\ &\quad \times \left(\frac{6817+321\sqrt{451}}{4} \right)^{\frac{1}{6}} \\ &\quad \times \left\{ \sqrt{\left(\frac{17+3\sqrt{33}}{8} \right)} + \sqrt{\left(\frac{25+3\sqrt{33}}{8} \right)} \right\} \\ &\quad \times \left\{ \sqrt{\left(\frac{561+99\sqrt{33}}{8} \right)} + \sqrt{\left(\frac{569+99\sqrt{33}}{8} \right)} \right\}, \end{aligned}$$

From:

$$G_{1225} = \left(\frac{1 + \sqrt{5}}{2} \right) (6 + \sqrt{35})^{\frac{1}{4}} \left\{ \frac{7^{\frac{1}{4}} + \sqrt{(4 + \sqrt{7})}}{2} \right\}^{\frac{3}{2}}$$

$$\times \left[\sqrt{\left\{ \frac{43 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{(10\sqrt{7})}}{8} \right\}} \right.$$

$$\left. + \sqrt{\left\{ \frac{35 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{(10\sqrt{7})}}{8} \right\}} \right],$$

$$\left(\frac{1}{2} * (1 + \sqrt{5}) \right) (6 + \sqrt{35})^{0.25} \left(\frac{1}{2} (7^{0.25} + \sqrt{(4 + \sqrt{7})}) \right)^{1.5}$$

$$\left[\left(\left(\frac{1}{8} * (43 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{(10\sqrt{7})}) \right) \right)^{0.5} + \left(\left(\frac{1}{8} * (35 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{(10\sqrt{7})}) \right) \right)^{0.5} \right]$$

Input

$$\left(\frac{1}{2} (1 + \sqrt{5}) \right) (6 + \sqrt{35})^{0.25} \left(\frac{1}{2} \left(7^{0.25} + \sqrt{4 + \sqrt{7}} \right) \right)^{1.5}$$

$$\left(\sqrt{\frac{1}{8} \left(43 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{10\sqrt{7}} \right)} + \right.$$

$$\left. \sqrt{\frac{1}{8} \left(35 + 15\sqrt{7} + (8 + 3\sqrt{7})\sqrt{10\sqrt{7}} \right)} \right)$$

Result

82.121716259777959592164714617421801709245946544073127264100332456

...

82.1217162597779.....

And:

$$\begin{aligned}
 G_{1353}^2 &= \sqrt{\left\{ (3 + \sqrt{11})(5 + 3\sqrt{3}) \left(\frac{11 + \sqrt{123}}{2} \right) \right\}} \\
 &\quad \times \left(\frac{6817 + 321\sqrt{451}}{4} \right)^{\frac{1}{6}} \\
 &\quad \times \left\{ \sqrt{\left(\frac{17 + 3\sqrt{33}}{8} \right)} + \sqrt{\left(\frac{25 + 3\sqrt{33}}{8} \right)} \right\} \\
 &\quad \times \left\{ \sqrt{\left(\frac{561 + 99\sqrt{33}}{8} \right)} + \sqrt{\left(\frac{569 + 99\sqrt{33}}{8} \right)} \right\},
 \end{aligned}$$

$$\begin{aligned}
 &(((3+\sqrt{11})(5+3\sqrt{3})(1/2*(11+\sqrt{123}))))^{0.5}*(1/4*(6817+321\sqrt{451}))^{(1/6)} \\
 &[(1/8*(17+3\sqrt{33}))^{0.5}+(1/8(25+3\sqrt{33}))^{0.5}] \\
 &[(1/8*(561+99\sqrt{33}))^{0.5}+(1/8(569+99\sqrt{33}))^{0.5}]
 \end{aligned}$$

Input

$$\begin{aligned}
 &\left(\sqrt{(3 + \sqrt{11})(5 + 3\sqrt{3}) \left(\frac{1}{2} (11 + \sqrt{123}) \right)} \sqrt[6]{\frac{1}{4} (6817 + 321 \sqrt{451})} \right) \\
 &\left(\sqrt{\frac{1}{8} (17 + 3 \sqrt{33})} + \sqrt{\frac{1}{8} (25 + 3 \sqrt{33})} \right) \\
 &\left(\sqrt{\frac{1}{8} (561 + 99 \sqrt{33})} + \sqrt{\frac{1}{8} (569 + 99 \sqrt{33})} \right)
 \end{aligned}$$

Exact result

$$\begin{aligned}
 &\frac{1}{2^{5/6}} \sqrt{(5 + 3 \sqrt{3})(3 + \sqrt{11})(11 + \sqrt{123})} \\
 &\sqrt[6]{6817 + 321 \sqrt{451}} \left(\frac{1}{2} \sqrt{\frac{1}{2} (17 + 3 \sqrt{33})} + \frac{1}{2} \sqrt{\frac{1}{2} (25 + 3 \sqrt{33})} \right) \\
 &\left(\frac{1}{2} \sqrt{\frac{1}{2} (561 + 99 \sqrt{33})} + \frac{1}{2} \sqrt{\frac{1}{2} (569 + 99 \sqrt{33})} \right)
 \end{aligned}$$

Decimal approximation

10756.140933072259735134541705077326995219472620706158177308491747

...

10756.140933072259....

From:

$$G_{1645}^2 = (2 + \sqrt{5}) \sqrt{\left\{ (3 + \sqrt{7}) \left(\frac{7 + \sqrt{47}}{2} \right) \right\} \left(\frac{73\sqrt{5} + 9\sqrt{329}}{2} \right)^{\frac{1}{4}}}$$

$$\times \left\{ \sqrt{\left(\frac{119 + 7\sqrt{329}}{8} \right)} + \sqrt{\left(\frac{127 + 7\sqrt{329}}{8} \right)} \right\}$$

$$\times \left\{ \sqrt{\left(\frac{743 + 41\sqrt{329}}{8} \right)} + \sqrt{\left(\frac{751 + 41\sqrt{329}}{8} \right)} \right\}.$$

$$(2+\sqrt{5})\left(\left(\left(3+\sqrt{7}\right)\left(\frac{1}{2}\left(7+\sqrt{47}\right)\right)\right)^{0.5}\left(\frac{1}{2}\left(73\sqrt{5}+9\sqrt{329}\right)\right)^{1/4}\right)$$

$$\left[\left(\frac{1}{8}\left(119+7\sqrt{329}\right)\right)^{0.5}+\left(\frac{1}{8}\left(127+7\sqrt{329}\right)\right)^{0.5}\right]$$

$$\left[\left(\frac{1}{8}\left(743+41\sqrt{329}\right)\right)^{0.5}+\left(\frac{1}{8}\left(751+41\sqrt{329}\right)\right)^{0.5}\right]$$

Input

$$\left((2 + \sqrt{5}) \sqrt{ (3 + \sqrt{7}) \left(\frac{1}{2} (7 + \sqrt{47}) \right) } \sqrt[4]{ \frac{1}{2} (73 \sqrt{5} + 9 \sqrt{329}) } \right)$$

$$\left(\sqrt{\frac{1}{8} (119 + 7 \sqrt{329})} + \sqrt{\frac{1}{8} (127 + 7 \sqrt{329})} \right)$$

$$\left(\sqrt{\frac{1}{8} (743 + 41 \sqrt{329})} + \sqrt{\frac{1}{8} (751 + 41 \sqrt{329})} \right)$$

Exact result

$$\frac{1}{2^{3/4}}(2 + \sqrt{5}) \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})} \sqrt[4]{73\sqrt{5} + 9\sqrt{329}}$$
$$\left(\frac{1}{2} \sqrt{\frac{1}{2}(119 + 7\sqrt{329})} + \frac{1}{2} \sqrt{\frac{1}{2}(127 + 7\sqrt{329})} \right)$$
$$\left(\frac{1}{2} \sqrt{\frac{1}{2}(743 + 41\sqrt{329})} + \frac{1}{2} \sqrt{\frac{1}{2}(751 + 41\sqrt{329})} \right)$$

Decimal approximation

28901.331627275895753726096134514092864156605324146787942909495805

...

28901.33162727589.....

Alternate forms

$$\frac{1}{8 \times 2^{3/4}}(2 + \sqrt{5}) \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})} \sqrt[4]{73\sqrt{5} + 9\sqrt{329}}$$
$$\left(\sqrt{7(17 + \sqrt{329})} + \sqrt{127 + 7\sqrt{329}} \right) \left(\sqrt{743 + 41\sqrt{329}} + \sqrt{751 + 41\sqrt{329}} \right)$$

$$\frac{1}{16 \times 2^{3/4}}(2 + \sqrt{5}) \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})}$$
$$\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \left(\sqrt{14(17 + \sqrt{329})} + \sqrt{254 + 14\sqrt{329}} \right)$$
$$\left(\sqrt{1486 + 82\sqrt{329}} + \sqrt{1502 + 82\sqrt{329}} \right)$$

$$\frac{1}{8 \times 2^{3/4}} (2 + \sqrt{5}) \sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(743 + 41\sqrt{329})}$$

$$\left(\sqrt{7(17 + \sqrt{329})} + \sqrt{127 + 7\sqrt{329}} \right) + \frac{1}{8 \times 2^{3/4}}$$

$$(2 + \sqrt{5}) \sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(751 + 41\sqrt{329})}$$

$$\left(\sqrt{7(17 + \sqrt{329})} + \sqrt{127 + 7\sqrt{329}} \right)$$

$$\frac{1}{4} \sqrt[4]{\frac{1}{2} (73\sqrt{5} + 9\sqrt{329})}$$

$$\left(5 \sqrt{7(25557 + 9647\sqrt{7} + 3723\sqrt{47} + 1409\sqrt{329})} + \right.$$

$$2 \sqrt{35(25557 + 9647\sqrt{7} + 3723\sqrt{47} + 1409\sqrt{329})} +$$

$$(2 + \sqrt{5}) \left(\sqrt{7(128471 + 48495\sqrt{7} + 18715\sqrt{47} + 7083\sqrt{329})} + \right.$$

$$\left. \sqrt{923587 + 348627\sqrt{7} + 134543\sqrt{47} + 50919\sqrt{329}} + \right.$$

$$\left. \left. \sqrt{928557 + 350503\sqrt{7} + 135267\sqrt{47} + 51193\sqrt{329}} \right) \right)$$

Expanded form

$$\begin{aligned}
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(119 + 7\sqrt{329})(743 + 41\sqrt{329})}}{4 \times 2^{3/4}} + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{5(3 + \sqrt{7})(7 + \sqrt{47})(119 + 7\sqrt{329})(743 + 41\sqrt{329})}}{8 \times 2^{3/4}} \\
 & + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(127 + 7\sqrt{329})(743 + 41\sqrt{329})}}{4 \times 2^{3/4}} + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{5(3 + \sqrt{7})(7 + \sqrt{47})(127 + 7\sqrt{329})(743 + 41\sqrt{329})}}{8 \times 2^{3/4}} \\
 & + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(119 + 7\sqrt{329})(751 + 41\sqrt{329})}}{4 \times 2^{3/4}} + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{5(3 + \sqrt{7})(7 + \sqrt{47})(119 + 7\sqrt{329})(751 + 41\sqrt{329})}}{8 \times 2^{3/4}} \\
 & + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{(3 + \sqrt{7})(7 + \sqrt{47})(127 + 7\sqrt{329})(751 + 41\sqrt{329})}}{4 \times 2^{3/4}} + \\
 & \frac{\sqrt[4]{73\sqrt{5} + 9\sqrt{329}} \sqrt{5(3 + \sqrt{7})(7 + \sqrt{47})(127 + 7\sqrt{329})(751 + 41\sqrt{329})}}{8 \times 2^{3/4}}
 \end{aligned}$$

From:

$$\left. \begin{aligned}
 G_{363} &= 2^{\frac{5}{12}} t, \text{ where} \\
 & 2t^3 - t^2 \{ (4 + \sqrt{33}) + \sqrt{(11 + 2\sqrt{33})} \} \\
 & - t \{ 1 + \sqrt{(11 + 2\sqrt{33})} \} - 1 = 0
 \end{aligned} \right\},$$

$$2t^3 - t^2 \left((4 + \sqrt{33}) + \sqrt{11 + 2\sqrt{33}} \right) - t \left(1 + \sqrt{11 + 2\sqrt{33}} \right) - 1 = 0$$

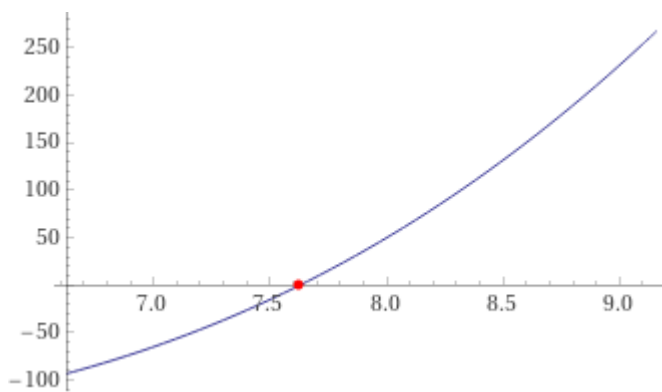
Input

$$2t^3 - t^2 \left((4 + \sqrt{33}) + \sqrt{11 + 2\sqrt{33}} \right) - t \left(1 + \sqrt{11 + 2\sqrt{33}} \right) - 1 = 0$$

Result

$$2t^3 - \left(4 + \sqrt{33} + \sqrt{11 + 2\sqrt{33}} \right) t^2 - \left(1 + \sqrt{11 + 2\sqrt{33}} \right) t - 1 = 0$$

Root plot



Alternate forms

$$t \left(- \left(-2t + \sqrt{11 + 2\sqrt{33}} + \sqrt{33} + 4 \right) t - \sqrt{11 + 2\sqrt{33}} - 1 \right) - 1 = 0$$

$$2t^3 = \left(4 + \sqrt{33} + \sqrt{11 + 2\sqrt{33}} \right) t^2 + \left(1 + \sqrt{11 + 2\sqrt{33}} \right) t + 1$$

$$t^2 \left(- \frac{\text{root of } x^4 - 44x^2 + 528 \text{ near } x = 4.74227 - 0.699375i}{\text{root of } x^4 - 44x^2 + 528 \text{ near } x = 4.74227 + 0.699375i} - 8 - 2\sqrt{33} \right) + t \left(- \frac{\text{root of } x^4 - 44x^2 + 528 \text{ near } x = 4.74227 - 0.699375i}{\text{root of } x^4 - 44x^2 + 528 \text{ near } x = 4.74227 + 0.699375i} - 2 \right) + 4t^3 = 2$$

Expanded forms

$$2t^3 - \sqrt{11 + 2\sqrt{33}} t^2 - \sqrt{33} t^2 - 4t^2 - \sqrt{11 + 2\sqrt{33}} t - t - 1 = 0$$

$$2t^3 + \left(-4 - \sqrt{33} - \sqrt{11 + 2\sqrt{33}} \right) t^2 + \left(-1 - \sqrt{11 + 2\sqrt{33}} \right) t - 1 = 0$$

Real solution

$$t \approx 7.6284$$

7.6284

Complex solutions

$$t \approx -0.19248 - 0.16880i$$

$$t \approx -0.19248 + 0.16880i$$

Polar coordinates

$$r = 0.256012 \text{ (radius), } \theta = 2.42165 \text{ (angle)}$$

0.256012

From:

$$G_{363} = 2^{\frac{5}{12}}t,$$

$$2^{(5/12)} * 0.256012$$

Input interpretation

$$2^{5/12} \times 0.256012$$

Result

0.3417350207457788378090709315657941238108708431146022663959397955

...

0.3417350207

$$2^{(5/12)} * 7.6284$$

Input interpretation

$$2^{5/12} \times 7.6284$$

Result

10.182692343550690148675517922427479548141677497990062688369244943

...

10.18269234355....

From:

$$G_{325} = \left(\frac{3 + \sqrt{13}}{2} \right)^{\frac{1}{4}} t, \text{ where } \left. \begin{aligned} &t^3 + t^2 \left(\frac{1 - \sqrt{13}}{2} \right)^2 + t \left(\frac{1 + \sqrt{13}}{2} \right)^2 + 1 \\ &= \sqrt{5} \left\{ t^3 - t^2 \left(\frac{1 + \sqrt{13}}{2} \right) + t \left(\frac{1 - \sqrt{13}}{2} \right) - 1 \right\} \end{aligned} \right\},$$

$$\text{Sqrt5} [t^3 - t^2 (1/2*(1+sqrt13)) + t(1/2*(1-sqrt13))-1]$$

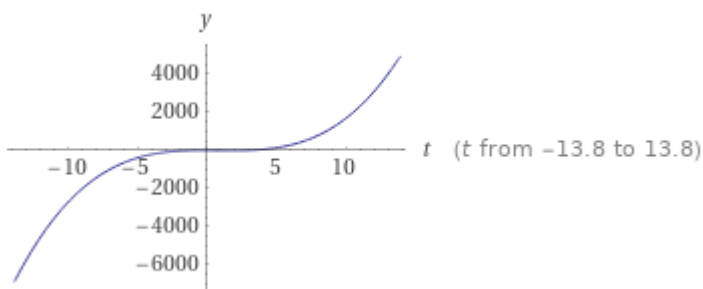
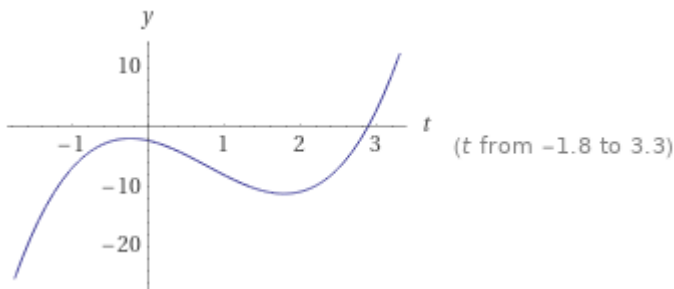
Input

$$\sqrt{5} \left(t^3 - t^2 \left(\frac{1}{2} (1 + \sqrt{13}) \right) + t \left(\frac{1}{2} (1 - \sqrt{13}) \right) - 1 \right)$$

Result

$$\sqrt{5} \left(t^3 - \frac{1}{2} (1 + \sqrt{13}) t^2 + \frac{1}{2} (1 - \sqrt{13}) t - 1 \right)$$

Plots



Alternate forms

$$\frac{1}{2} \sqrt{5} (2t^3 + (-1 - \sqrt{13}) t^2 + (1 - \sqrt{13}) t - 2)$$

$$t \left(t \left(\sqrt{5} t - \frac{\sqrt{65}}{2} - \frac{\sqrt{5}}{2} \right) - \frac{\sqrt{65}}{2} + \frac{\sqrt{5}}{2} \right) - \sqrt{5}$$

$$\sqrt{5} \left(\left(t + \frac{1}{6} (-1 - \sqrt{13}) \right)^3 + \left(\frac{1}{2} (1 - \sqrt{13}) - \frac{1}{12} (-1 - \sqrt{13})^2 \right) \left(t + \frac{1}{6} (-1 - \sqrt{13}) \right) - \frac{1}{12} (-1 - \sqrt{13}) (1 - \sqrt{13}) + \frac{1}{108} (-1 - \sqrt{13})^3 - 1 \right)$$

Expanded form

$$\sqrt{5} t^3 - \frac{\sqrt{65} t^2}{2} - \frac{\sqrt{5} t^2}{2} - \frac{\sqrt{65} t}{2} + \frac{\sqrt{5} t}{2} - \sqrt{5}$$

Real root

$$t \approx 2.8765$$

2.8765

Complex roots

$$t \approx -0.28688 - 0.51511 i$$

$$t \approx -0.28688 + 0.51511 i$$

Polar coordinates

$$r = 0.589609 \text{ (radius)}, \quad \theta = 2.07894 \text{ (angle)}$$

0.589609

Polynomial discriminant

$$\Delta = -2800$$

Properties as a real function

Domain

\mathbb{R} (all real numbers)

Range

\mathbb{R} (all real numbers)

Surjectivity

surjective onto \mathbb{R}

\mathbb{R} is the set of real numbers

Derivative

$$\frac{d}{dt} \left(\sqrt{5} \left(t^3 - \frac{1}{2} t^2 (1 + \sqrt{13}) + \frac{1}{2} t (1 - \sqrt{13}) - 1 \right) \right) = \\ -\frac{1}{2} \sqrt{5} (-6t^2 + 2(1 + \sqrt{13})t + \sqrt{13} - 1)$$

Indefinite integral

$$\int \sqrt{5} \left(-1 + \frac{1}{2} (1 - \sqrt{13}) t - \frac{1}{2} (1 + \sqrt{13}) t^2 + t^3 \right) dt = \\ \frac{1}{2} \sqrt{5} \left(\frac{t^4}{2} - \frac{1}{3} (1 + \sqrt{13}) t^3 - \frac{\sqrt{13} t^2}{2} + \frac{t^2}{2} - 2t \right) + \text{constant}$$

Local maximum

$$\max \left\{ \sqrt{5} \left(t^3 - \frac{1}{2} t^2 (1 + \sqrt{13}) + \frac{1}{2} t (1 - \sqrt{13}) - 1 \right) \right\} = \\ \frac{4}{27} \sqrt{5} \left(-16 - \sqrt{13} + \sqrt{2(1 + \sqrt{13})} + \sqrt{26(1 + \sqrt{13})} \right) \\ \text{at } t = \frac{1}{6} + \frac{\sqrt{13}}{6} - \frac{1}{3} \sqrt{2(1 + \sqrt{13})}$$

Local minimum

$$\min \left\{ \sqrt{5} \left(t^3 - \frac{1}{2} t^2 (1 + \sqrt{13}) + \frac{1}{2} t (1 - \sqrt{13}) - 1 \right) \right\} = \\ -\frac{4}{27} \sqrt{5} \left(16 + \sqrt{13} + \sqrt{2(1 + \sqrt{13})} + \sqrt{26(1 + \sqrt{13})} \right) \\ \text{at } t = \frac{1}{6} + \frac{\sqrt{13}}{6} + \frac{1}{3} \sqrt{2(1 + \sqrt{13})}$$

From:

$$G_{325} = \left(\frac{3 + \sqrt{13}}{2} \right)^{\frac{1}{4}} t,$$

$$(1/2*(3+\text{sqrt}13))^{0.25} * 2.8765$$

Input interpretation

$$\left(\frac{1}{2} (3 + \sqrt{13}) \right)^{0.25} \times 2.8765$$

Result

3.8777887510157301253271423640305832572703471601565648202950988844

...

3.87778875101573

Here all the results that we have obtained from the development of the previous expressions.

2.781323803920547....

5.8364372603724....

5.93160414841568....

6.2220252193329....

7.033656610253....

20.37749997725....

2.911116655774....

3.32593429313....

3.39813955444....

11.5730481521....

3.464643979529....

3.830586436322.....

0.2817853021....

3.6548630551090639....

15.3156144852002258.....

15.833404207477....

16.542097553485.....

0.518874943499968.....

4.245471976946.....

18.21547254999....

4.587808247....

4.8277165856693115....

0.21688040084606.....

5.30492251818648

5.47893971377417

32.2740236434008.....

33.4477338319059....

5.990702076....

6.335288332....

50.159687983....

7.78351473....

66.3853254777....

8.42699415....

9.165151988....

172.640872178....

8.9778697879.....

200.102833842928....

224.36895935....

16.7336273728....

333.581847612....

17.623908118731.....

986.7726750808841....

1044.2048545055534.....

82.1217162597779.....

10756.140933072259....

28901.33162727589.....

10.18269234355....

3.87778875101573

From the sum, we obtain:

(3.464643979529+3.830586436322+0.2817853021+3.6548630551090639
+15.3156144852002258+15.833404207477 +16.542097553485
+0.518874943499968 +4.245471976946+18.21547254999+4.587808247)

Input interpretation

3.464643979529 + 3.830586436322 + 0.2817853021 + 3.6548630551090639 +
15.3156144852002258 + 15.833404207477 + 16.542097553485 +
0.518874943499968 + 4.245471976946 + 18.21547254999 + 4.587808247

Result

86.4906227366582577

(2.781323803920547 +5.8364372603724 +5.93160414841568+6.2220252193329
+7.033656610253+20.37749997725+2.911116655774+3.32593429313+3.39813955
444+11.5730481521)

Input interpretation

2.781323803920547 + 5.8364372603724 + 5.93160414841568 +
6.2220252193329 + 7.033656610253 + 20.37749997725 +
2.911116655774 + 3.32593429313 + 3.39813955444 + 11.5730481521

Result

69.390785674988527

(4.8277165856693115+0.21688040084606+5.30492251818648 +5.47893971377417
+32.2740236434008 +33.4477338319059)

Input interpretation

4.8277165856693115 + 0.21688040084606 + 5.30492251818648 +
5.47893971377417 + 32.2740236434008 + 33.4477338319059

Result

81.5502166937827215

(5.990702076+6.335288332+50.159687983+7.78351473+66.3853254777+8.42699415+9.165151988 +172.640872178+8.9778697879+200.102833842928)

Input interpretation

5.990702076 + 6.335288332 + 50.159687983 +
7.78351473 + 66.3853254777 + 8.42699415 + 9.165151988 +
172.640872178 + 8.9778697879 + 200.102833842928

Result

535.968240545528

(224.36895935+16.7336273728+333.581847612+17.623908118731+986.7726750808841+1044.2048545055534+82.1217162597779 +10756.140933072259+28901.33162727589+10.18269234355+3.87778875101573)

Input interpretation

224.36895935 + 16.7336273728 + 333.581847612 +
17.623908118731 + 986.7726750808841 + 1044.2048545055534 +
82.1217162597779 + 10756.140933072259 +
28901.33162727589 + 10.18269234355 + 3.87778875101573

Result

42376.94062974246113

(42376.94062974246113+535.968240545528+81.5502166937827215+69.390785674988527+86.4906227366582577)

Input interpretation

42376.94062974246113 + 535.968240545528 +
81.5502166937827215 + 69.390785674988527 + 86.4906227366582577

Result

43150.3404953934186362

43150.3404953934186362 Final results of the sum

from which, we obtain:

$\text{sqrt}(6*((1/5^2(42376.9406297+535.9682405+81.5502166+69.3907856+86.4906227)+3))^1/15))$

Input interpretation

$$\sqrt{\left(6\left(\frac{1}{5^2} (42376.9406297 + 535.9682405 + 81.5502166 + 69.3907856 + 86.4906227) + 3\right)^{(1/15)}\right)}$$

Result

3.140524884790...

3.140524884790... $\approx \pi$

We obtain also:

$$((1/5^2(42376.9406297+535.9682405+81.5502166+69.3907856+86.4906227)+3))$$

Input interpretation

$$\frac{1}{5^2} (42376.9406297 + 535.9682405 + 81.5502166 + 69.3907856 + 86.4906227) + 3$$

Result

1729.013619804

1729.013619804

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$((1/5^2(42376.9406297+535.9682405+81.5502166+69.3907856+86.4906227)+3))^{1/15}$$

Input interpretation

$$\left(\frac{1}{5^2} (42376.9406297 + 535.9682405 + 81.5502166 + 69.3907856 + 86.4906227) + 3\right)^{(1/15)}$$

Result

1.64381609200...

$$1.64381609200\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots \text{ (trace of the instanton shape)}$$

$$\left(\frac{1}{27} \left(\frac{1}{5^2} (42376.9406297 + 535.9682405 + 81.5502166 + 69.3907856 + 86.4906227) + 2 \right) \right)^2$$

Input interpretation

$$\left(\frac{1}{27} \left(\frac{1}{5^2} (42376.9406297 + 535.9682405 + 81.5502166 + 69.3907856 + 86.4906227) + 2 \right) \right)^2$$

Result

4096.0645682141605775012565157750342935528120713305898491083676268

...

$$4096.0645682\dots \approx 4096 = 64^2$$

Now, we multiply the results and obtain:

$$(3.464643979529 * 3.830586436322 * 0.2817853021 * 3.6548630551090639 * 15.3156144852002258 * 15.833404207477 * 16.542097553485 * 0.518874943499968 * 4.245471976946 * 18.21547254999 * 4.587808247)$$

Input interpretation

$$3.464643979529 \times 3.830586436322 \times 0.2817853021 \times 3.6548630551090639 \times 15.3156144852002258 \times 15.833404207477 \times 16.542097553485 \times 0.518874943499968 \times 4.245471976946 \times 18.21547254999 \times 4.587808247$$

Result

$$1.00936212952519821587817530059781834993984139884351952028913\dots \times 10^7$$

(2.781323803920547 *5.8364372603724 *5.93160414841568*6.2220252193329
*7.033656610253*20.37749997725*2.911116655774*3.32593429313*3.398139554
44*11.5730481521)

Input interpretation

2.781323803920547 × 5.8364372603724 × 5.93160414841568 ×
6.2220252193329 × 7.033656610253 × 20.37749997725 ×
2.911116655774 × 3.32593429313 × 3.39813955444 × 11.5730481521

Result

3.26962246946337593106594080941002112657092362157999003002034... ×
 10^7

(4.8277165856693115*0.21688040084606*5.30492251818648 *5.47893971377417
*32.2740236434008 *33.4477338319059)

Input interpretation

4.8277165856693115 × 0.21688040084606 × 5.30492251818648 ×
5.47893971377417 × 32.2740236434008 × 33.4477338319059

Result

32851.670043110937314475841310017925468743613547972875506219857308
...

(5.990702076*6.335288332*50.159687983*7.78351473*66.3853254777*8.4269941
5*9.165151988 *172.640872178*8.9778697879*200.102833842928)

Input interpretation

5.990702076 × 6.335288332 × 50.159687983 × 7.78351473 × 66.3853254777 ×
8.42699415 × 9.165151988 × 172.640872178 × 8.9778697879 × 200.102833842928

Result

2.35629304255456518566547200179495085164046736167071218028887... ×
 10^{13}

(224.36895935*16.7336273728*333.581847612*17.623908118731*986.772675080
 8841*1044.2048545055534*82.1217162597779
 *10756.140933072259*28901.33162727589*10.18269234355*3.87778875101573)

Input interpretation

224.36895935 × 16.7336273728 × 333.581847612 ×
 17.623908118731 × 986.7726750808841 × 1044.2048545055534 ×
 82.1217162597779 × 10756.140933072259 ×
 28901.33162727589 × 10.18269234355 × 3.87778875101573

Result

2.29265888745195100525237198986843777776102699093565449676296... ×
 10²⁵

(1.0093621295251982158781753 × 10⁷ * 3.26962246946337593106 × 10⁷ *
 32851.670043110937314475 * 2.356293042554565185665472 × 10¹³ *
 2.292658887451951 × 10²⁵)

Input interpretation

1.0093621295251982158781753 × 10⁷ ×
 3.26962246946337593106 × 10⁷ × 32851.670043110937314475 ×
 2.356293042554565185665472 × 10¹³ × 2.292658887451951 × 10²⁵

Result

5.85694049658934657318543490860937198635830099097920701052578... ×
 10⁵⁷

5.856940496589346573 × 10⁵⁷ final result

From which:

$$(5.856940496589346573 \times 10^{57})^2 * (135^3 + 138^3) * (\pi^2)/49$$

where 135³ and 138³ are two Ramanujan cubes (see Ramanujan taxicab numbers)

Input interpretation

$$(5.856940496589346573 \times 10^{57})^2 (135^3 + 138^3) \times \frac{\pi^2}{49}$$

Result

3.515851670480918214... × 10¹²¹

0.3515851670480918214*10¹²²

And:

$$1/(((5.856940496589346573 \times 10^{57})^2 * (135^3+138^3) * (Pi^2)/49))$$

Input interpretation

$$\frac{1}{(5.856940496589346573 \times 10^{57})^2 (135^3 + 138^3) \times \frac{\pi^2}{49}}$$

Result

2.844261060260299050... × 10⁻¹²²
2.844261060260299050... * 10⁻¹²²

The results are equal to the observed value of ρ_Λ or Λ today that is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations. With regard the Cosmological constant, fundamental are the following results: Λ = 2.846 * 10⁻¹²² and Λ_Q = 0.3516 * 10¹²² (New Quantum Structure of the Space-Time - Norma G. SANCHEZ - arXiv:1910.13382v1 [physics.gen-ph] 28 Oct 2019)

We obtain also:

$$233 * 1 / (\ln(5.856940496589346573 \times 10^{57}) + 11)$$

Input interpretation

$$233 \times \frac{1}{\log(5.856940496589346573 \times 10^{57}) + 11}$$

log(x) is the natural logarithm

Result

1.61788727653433643777...
1.61788727653433643777.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

$$(199+29+7+2)*1/(\ln(5.856940496589346573 \times 10^{57})+11)$$

Input interpretation

$$(199 + 29 + 7 + 2) \times \frac{1}{\log(5.856940496589346573 \times 10^{57}) + 11}$$

$\log(x)$ is the natural logarithm

Result

1.64566216540187869421...

1.64566216540187869421.... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$ (trace of the instanton shape)

Mathematical connections with some sectors of String Theory

Observations

We note that, from the number 8, we obtain as follows:

$$8^2$$

$$64$$

$$8^2 \times 2 \times 8$$

$$1024$$

$$8^4 = 8^2 \times 2^6$$

True

$$8^4 = 4096$$

$$8^2 \times 2^6 = 4096$$

$$2^{13} = 2 \times 8^4$$

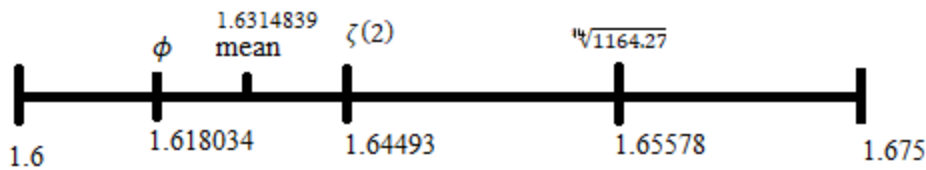
True

$$2^{13} = 8192$$

$$2 \times 8^4 = 8192$$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6, a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

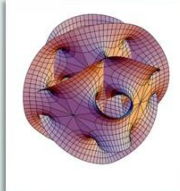
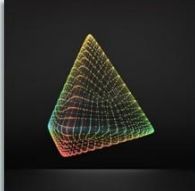
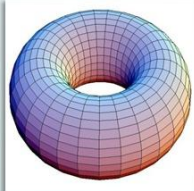
Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

We have that:

Outlook

Remarkably rich (apparently **UNIQUE**) framework

BUT :



Why a given **“shape”** of the extra dimensions ?
[**CRUCIAL**, it determines the predictions for α , ...]

A. Sagnotti – AstronomiAmo, 23.4.2020 21

From: A. Sagnotti – AstronomiAmo, 23.04.2020

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for α ”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and $\zeta(2)$, (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We obtain, in certain cases, the following connections:

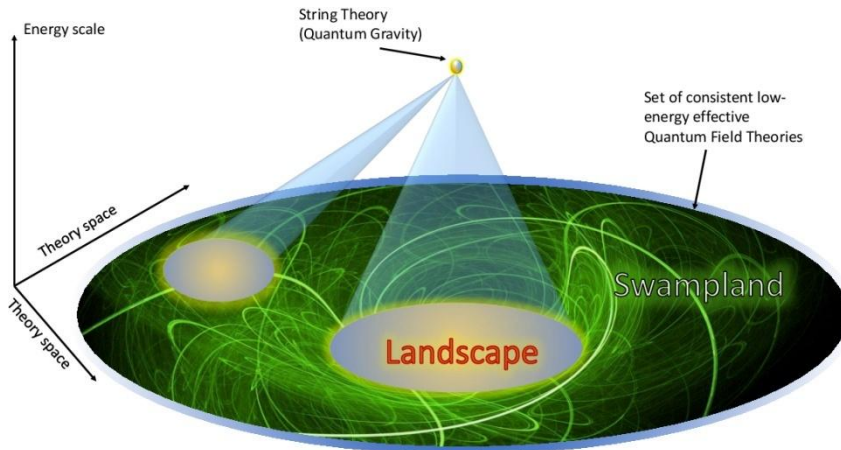
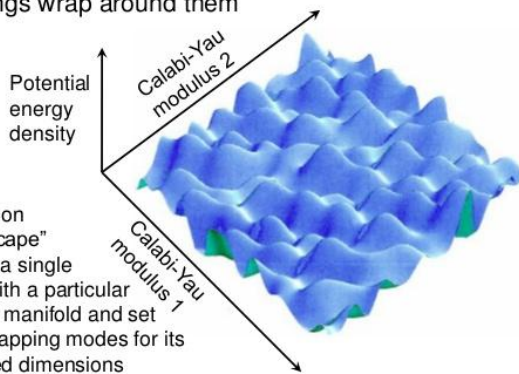


Fig. 1

The String Theory “Landscape”

- Graph axes show only 2 out of hundreds of parameters (“moduli”) that determine the exact Calabi-Yau manifolds and how strings wrap around them



- Each point on the “Landscape” represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation “bubble”

Fig. 2

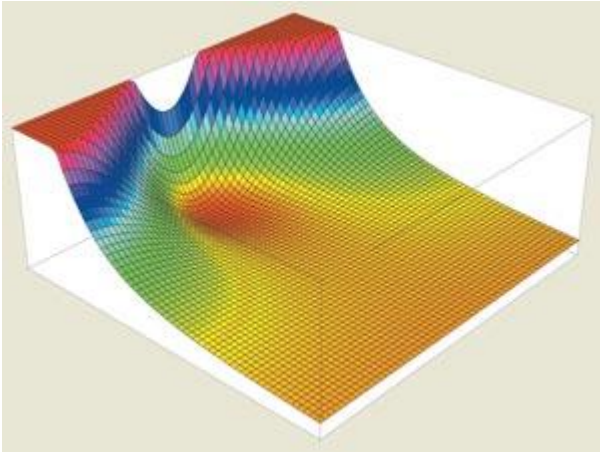


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

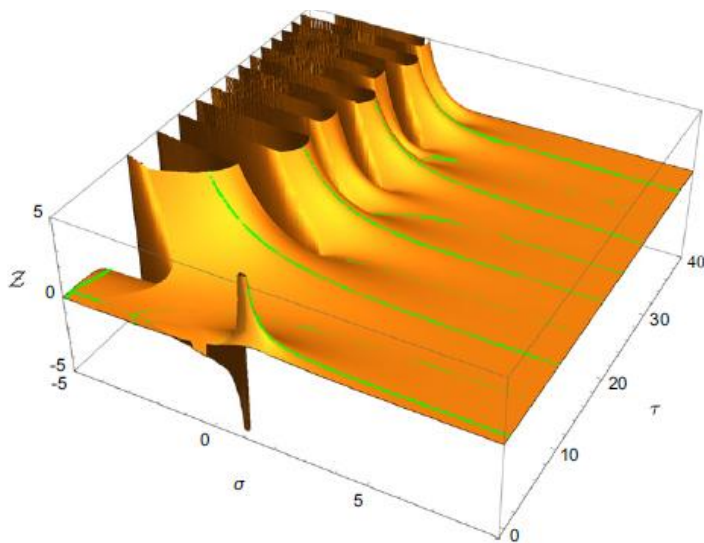


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with $k = \dots, -2, -1, 0, 1, 2, \dots$

we obtain:

$$2\pi/(\ln(2))$$

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958

...

9.06472028365....

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a) \log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 \coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[\frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t} dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

From which:

$$(2\pi/\ln(2)) \cdot (1/12 \pi \log(2))$$

Input:

$$\left(2 \times \frac{\pi}{\log(2)}\right) \left(\frac{1}{12} \pi \log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293

...

$$1.6449340668\dots = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From:

Modular equations and approximations to π - Srinivasa Ramanujan
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64 \left\{ \left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

We note that, with regard 4372, we can to obtain the following results:

$$27((4372)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi$$

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2(\sqrt{5}-1)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944....

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5(10-2\sqrt{5})}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2(5-\sqrt{5})}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2(5+\sqrt{5})}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2(\sqrt{5} - 1)}$$

Minimal polynomial

$$256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 1549835555492184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2(\sqrt{5} - 1)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2}\right) + \phi =$$

$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k}\right) / \left(2\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right)$$

$$\begin{aligned}
& 27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left(2 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left(2 \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

Or:

$$27((4096+276)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/((\sqrt{5}-1))))+\phi$$

Input

$$27 \left(\sqrt{4096 + 276} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi$$

ϕ is the golden ratio

Result

$$\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944.... as above

Alternate forms

$$\frac{1}{8} \left(-27\sqrt{5(10 - 2\sqrt{5})} + 58\sqrt{5} + 432\sqrt{1093} - 27\sqrt{2(5 - \sqrt{5})} - 374 \right)$$

$$\phi - 54 + 54\sqrt{1093} + \frac{27}{4} \left(1 + \sqrt{5} - \sqrt{2(5 + \sqrt{5})} \right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27 \left(\sqrt{10 - 2\sqrt{5}} - 2 \right)}{2(\sqrt{5} - 1)}$$

Minimal polynomial

$$\begin{aligned}
 &256x^8 + 95744x^7 - 324875080x^6 - \\
 &914210725504x^5 + 15498355554921184x^4 + \\
 &2911478392539914656x^3 - 32941144911224677091680x^2 - \\
 &3092528914069760354714456x + 26320050609744039027169013041
 \end{aligned}$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5(10-2\sqrt{5})}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)}$$

Series representations

$$\begin{aligned}
 &27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{(\sqrt{5}-1)2}\right)+\phi = \\
 &\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\right. \\
 &\quad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}- \\
 &\quad \left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)
 \end{aligned}$$

$$\begin{aligned}
& 27 \left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left(2 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 27 \left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left(2 \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

From which:

$$(27((4372)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi)^{1/15}$$

Input

$$\sqrt[15]{27 \left(\sqrt{4372} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124

...

$$1.64381856858\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

Alternate forms

$$\sqrt[15]{\phi - 54 + 54\sqrt{1093} - \frac{27(\sqrt{10 - 2\sqrt{5}} - 2)}{2(\sqrt{5} - 1)}}$$

$$\sqrt[15]{\frac{1}{166 - 108\sqrt{5} - 108\sqrt{1093} + 108\sqrt{5465} - 27\sqrt{2(5 - \sqrt{5})}}}$$

$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 1549835554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 1549835554921184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}(1 + \sqrt{5}) + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}}$$

All 15th roots of $\phi + 27(-2 + 2\sqrt{1093}) - (\sqrt{10 - 2\sqrt{5}} - 2)/(2(\sqrt{5} - 1))$

$$e^0 \sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15} \sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.50170 + 0.6686i$$

$$e^{(4i\pi)/15} \sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.0999 + 1.2216i$$

$$e^{(2i\pi)/5} \sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 0.5080 + 1.5634i$$

$$e^{(8i\pi)/15} \sqrt[15]{\phi + 27 \left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx -0.17183 + 1.63481i$$

Series representations

$$\begin{aligned} & \sqrt[15]{27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\ & \frac{1}{\sqrt[15]{2}} \left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \right. \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k} \right) / \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \right)^{(1/15)} \end{aligned}$$

$$\begin{aligned}
& \sqrt[15]{27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \right. \\
& \quad \left. \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)^{\wedge (1/15)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt[15]{27 \left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \right. \\
& \quad \left. \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)^{\wedge (1/15)}
\end{aligned}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

Integral representation

$$(1 + z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p , C , β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for $p = 5$ and $\beta_E = 1/2$:

$$e^{-6C + \phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2 , while $-6C + \phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp((-Pi*\text{sqrt}(18))$ we obtain:

Input:

$$\exp\left(-\pi \sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\pi}$$

Decimal approximation:

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

Property:

$e^{-3\sqrt{2}\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096} e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp(-\pi\sqrt{18})))))) * 1/0.000244140625$$

Input interpretation:

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785...

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \left(\frac{1}{2}\right)^k\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

From:

$$\ln(0.00666501784619)$$

Input interpretation:

$$\log(0.00666501784619)$$

Result:

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2i\pi \left[\frac{\arg(0.006665017846190000 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left[\frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for $C = 1$, we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \phi$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285, value very near to the ψ Regge slope 0.979:

| | | | | |
|--------|---|--------------|-------|-------|
| Ψ | 3 | $m_c = 1500$ | 0.979 | -0.09 |
|--------|---|--------------|-------|-------|

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 = ϕ** and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4 \sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

From

AdS Vacua from Dilaton Tadpoles and Form Fluxes - *J. Mourad and A. Sagnotti*
 - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$e^{2C} = \frac{2\xi e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right) + 5 T e^{2\phi} \right]. \quad (2.7)$$

For

$$T = \frac{16}{\pi^2}$$

$$\xi = 1$$

we obtain:

$$(2 * e^{(0.989117352243/2)}) / (1 + \text{sqrt}(((1 - 1/3 * 16 / (\text{Pi})^2 * e^{(2 * 0.989117352243)}))))$$

Input interpretation:

$$\frac{2 e^{0.989117352243/2}}{1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}}$$

Result:

0.83941881822... -
 1.4311851867... *i*

Polar coordinates:

$r = 1.65919106525$ (radius), $\theta = -59.607521917^\circ$ (angle)

1.65919106525..... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Series representations:

$$\frac{2 e^{0.9891173522430000/2}}{1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}}} = \frac{2 e^{0.4945586761215000}}{1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{2 e^{0.9891173522430000/2}}{1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}}} = \frac{2 e^{0.4945586761215000}}{1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{-\frac{1}{2}}{k}}{k!}}$$

$$\frac{2 e^{0.9891173522430000/2}}{1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}}} = \frac{2 e^{0.4945586761215000}}{1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-\frac{1}{2}}{k} \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right) + 5 T e^{2\phi} \right]$$

we obtain:

$$e^{(4 \times 0.989117352243) / (((1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{(2 \times 0.989117352243)}}))^{1/7} [42(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{(2 \times 0.989117352243)}}) + 5 \times \frac{16}{\pi^2} e^{(2 \times 0.989117352243)}]}$$

Input interpretation:

$$\frac{e^{4 \times 0.989117352243}}{\left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}\right)^7} \left(42 \left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}\right) + 5 \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}\right)$$

Result:

50.84107889... -
20.34506335... i

Polar coordinates:

r = 54.76072411 (radius), θ = -21.80979492° (angle)

54.76072411.....

Series representations:

$$\left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) e^{4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 =$$

$$\left(2 \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right. \right.$$

$$\left. \left. \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k} \right) \right) /$$

$$\left(\pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^7$$

$$\begin{aligned}
& \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) \right. \\
& \quad \left. e^{4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 = \\
& \left(2 \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \\
& \left(\pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \\
& \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) \right. \\
& \quad \left. e^{4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 = \\
& \left(2 \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \right. \right. \\
& \quad \left. \left. \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \left(\pi^2 \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right) \\
& \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

From which:

$$\begin{aligned}
& e^{(4 \times 0.989117352243)} / \left(\left(\left(1 + \sqrt{1 - \frac{16}{3 \pi^2} e^{(2 \times 0.989117352243)}} \right) + \frac{5 \times 16}{\pi^2} e^{(2 \times 0.989117352243)} \right) \right)^7 \\
& \left[42 \left(1 + \sqrt{1 - \frac{16}{3 \pi^2} e^{(2 \times 0.989117352243)}} \right) + \frac{5 \times 16}{\pi^2} e^{(2 \times 0.989117352243)} \right] \times \frac{1}{34}
\end{aligned}$$

Input interpretation:

$$\frac{e^{4 \times 0.989117352243}}{\left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}\right)^7} \left(42 \left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}\right) + 5 \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}\right) \times \frac{1}{34}$$

Result:

1.495325850... -
0.5983842161... *i*

Polar coordinates:

$r = 1.610609533$ (radius), $\theta = -21.80979492^\circ$ (angle)

[1.610609533....](#) result that is a good approximation to the value of the golden ratio
[1.618033988749...](#)

Series representations:

$$\left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) e^{4 \times 0.9891173522430000} \right) / \left(34 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) \right)^7 =$$

$$\left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right.$$

$$\left. \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k} \right) /$$

$$\left(17 \pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^7$$

$$\begin{aligned}
& \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) \right. \\
& \quad \left. e^{4 \times 0.9891173522430000} \right) / \left(\left(34 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) \right)^7 \right) = \\
& \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right. \\
& \quad \left. \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left(17 \pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \\
& \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right) \right. \\
& \quad \left. e^{4 \times 0.9891173522430000} \right) / \left(\left(34 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) \right)^7 \right) = \\
& \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \right. \\
& \quad \left. \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0\right)^k z_0^{-k}}{k!} \right) / \\
& \left(17 \pi^2 \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right)
\end{aligned}$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0)$)

Now, we have:

$$e^{2C} = \frac{2\xi e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3} e^{2\phi}}}, \quad (2.9)$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3} e^{2\phi}}\right]^7} \left[42 \left(1 + \sqrt{1 + \frac{\Lambda}{3} e^{2\phi}}\right) - 13\Lambda e^{2\phi}\right]. \quad (2.10)$$

For:

$$\xi = 1$$

$$\Lambda \simeq \frac{4\pi^2}{25}$$

$$\phi = 0.989117352243$$

From

$$e^{2C} = \frac{2\xi e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3} e^{2\phi}}},$$

we obtain:

$$\frac{2e^{-0.989117352243/2}}{\left(\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \cdot 0.989117352243}}\right)\right)^7}$$

Input interpretation:

$$\frac{2 e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \cdot 0.989117352243}}}$$

Result:

1.65430921270...

1.6543092..... We note that, the result 1.6543092... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Indeed:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5} + 2)^{1/2} \left(\frac{\sqrt{5} + 1}{2} \right)^{1/4} (\sqrt{101} + 10)^{1/4} \\ \times \left((130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} \right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} = \left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}} \right)^3,$$

which is straightforward. □

$$\sqrt[14]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}} \right)^3} = 1,65578 \dots$$

Series representations:

$$1 + \frac{1}{4(2e^{-0.9891173522430000/2})} = \\ \frac{1 + \sqrt{1 + \frac{(4\pi^2)e^{2 \times 0.9891173522430000}}{3 \times 25}}}{8} + \frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4e^{1.978234704486000} \pi^2}{75}} \\ \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k (e^{1.978234704486000} \pi^2)^{-k} \binom{\frac{1}{2}}{k}$$

$$\begin{aligned}
& 1 + \frac{1}{4(2e^{-0.9891173522430000/2})} = \\
& \frac{1}{1 + \sqrt{1 + \frac{(4\pi^2)e^{2 \times 0.9891173522430000}}{3 \times 25}}} = \\
& 1 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4e^{1.978234704486000} \pi^2}{75}} \\
& \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& 1 + \frac{1}{4(2e^{-0.9891173522430000/2})} = 1 + \frac{e^{0.4945586761215000}}{8} + \\
& \frac{1}{8} e^{0.4945586761215000} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \\
& \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

And from

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3} e^{2\phi}}\right]^7} \left[42 \left(1 + \sqrt{1 + \frac{\Lambda}{3} e^{2\phi}}\right) - 13 \Lambda e^{2\phi}\right].$$

we obtain:

$$\begin{aligned}
& e^{(-4 \times 0.989117352243)} / [1 + \sqrt{((1 + 1/3 \times (4\pi^2)/25 \times e^{(2 \times 0.989117352243))})}]^7 * \\
& [42(1 + \sqrt{((1 + 1/3 \times (4\pi^2)/25 \times e^{(2 \times 0.989117352243))})}) - \\
& 13 \times (4\pi^2)/25 \times e^{(2 \times 0.989117352243)}]
\end{aligned}$$

Input interpretation:

$$\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4 \pi^2)\right) e^{2 \times 0.989117352243}}\right)^7} \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4 \pi^2)\right) e^{2 \times 0.989117352243}}\right) - 13 \left(\frac{1}{25} (4 \pi^2)\right) e^{2 \times 0.989117352243}\right)$$

Result:

-0.034547055658...

-0.034547055658...

Series representations:

$$\left(\left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) e^{-4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 =$$

$$- \left(\left(42 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \binom{\frac{1}{2}}{k} \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \binom{\frac{1}{2}}{k} \right)^7 \right)$$

$$\left(\left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right. \\ \left. e^{-4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\ - \left(\left(42 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \right. \right. \right. \\ \left. \left. 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \left(25 e^{5.934704113458000} \right. \\ \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right)^7 \right)$$

$$\left(\left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) \\ \left. e^{-4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\ - \left(\left(42 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \right. \right. \right. \\ \left. \left. \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left(25 \right. \\ \left. e^{5.934704113458000} \right. \\ \left. \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

$$47 * 1 / (((-1 / (((((e^{-4 * 0.989117352243}) / \\ [1 + \text{sqrt}(((1 + 1/3 * (4\text{Pi}^2) / 25 * e^{(2 * 0.989117352243)})))]^7 * \\ [42(1 + \text{sqrt}(((1 + 1/3 * (4\text{Pi}^2) / 25 * e^{(2 * 0.989117352243)})))- \\ 13 * (4\text{Pi}^2) / 25 * e^{(2 * 0.989117352243)}))])))))))$$

Input interpretation:

$$47 \left(- \left(1 / 1 / \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4 \pi^2) e^{2 \times 0.989117352243} \right)} \right)^7} \right. \right. \right. \\ \left. \left. \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4 \pi^2) e^{2 \times 0.989117352243} \right)} \right) - \right. \right. \right. \\ \left. \left. \left. 13 \left(\frac{1}{25} (4 \pi^2) e^{2 \times 0.989117352243} \right) \right) \right) \right) \right)$$

Result:

1.6237116159...

1.6237116159.... result that is an approximation to the value of the golden ratio
1.618033988749...

Series representations:

$$- \left(47 / 1 / \left(e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\ \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\ \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\ \left(1974 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \right. \right. \\ \left. \left. 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{1}{k} \right) \right) / \left(25 e^{5.934704113458000} \right. \\ \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{1}{k} \right) \right)^7$$

$$\begin{aligned}
& - \left(47 / 1 / \left(e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right. \right. \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
& \left(1974 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \right. \right. \\
& \qquad \qquad \qquad \left. \left. 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
& \qquad \qquad \qquad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000} \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \left(25 e^{5.934704113458000} \right. \\
& \qquad \qquad \qquad \left. \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000} \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& - \left(47 / 1 / \left(e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right. \right. \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
& \left(1974 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \right. \right. \\
& \qquad \qquad \qquad \left. \left. \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left(25 \right. \\
& \qquad \qquad \qquad \left. \left. e^{5.934704113458000} \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right) \right)
\end{aligned}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And again:

$$32\left(\left(\frac{e^{-4 \times 0.989117352243}}{\left[1 + \sqrt{\left(1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}\right)}\right]^7} \right) \left[42 \left(1 + \sqrt{\left(1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}\right)}\right) - 13 \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}\right]\right)$$

Input interpretation:

$$32 \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}}\right)^7} \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}}\right) - 13 \left(\frac{1}{25} (4\pi^2)\right) e^{2 \times 0.989117352243}\right) \right)$$

Result:

-1.1055057810...

-1.1055057810....

We note that the result -1.1055057810.... is very near to the value of Cosmological Constant, less 10^{-52} , thence 1.1056, with minus sign

Series representations:

$$\begin{aligned}
 & \left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
 & \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
 & - \left(1344 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \right. \right. \\
 & \quad \left. \left. 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
 & \quad \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right) \right) / \left(25 e^{5.934704113458000} \right. \\
 & \quad \left. \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^7 \right)
 \end{aligned}$$

$$\begin{aligned}
& \left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
& - \left(\left(1344 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \right. \right. \right. \\
& \quad \left. \left. \left. 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \right. \\
& \quad \left. \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000} \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right) / \left(25 e^{5.934704113458000} \right. \\
& \quad \left. \left. \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000} \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
& - \left(\left(1344 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) / \left(25 \right. \\
& \quad \left. \left. \left. e^{5.934704113458000} \right. \right. \right. \\
& \quad \left. \left. \left. \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) \right)
\end{aligned}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

And:

$$- [32 \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243}} \right)^7} \right. \\ \left. \left[42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243}} \right) - 13 \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243} \right] \right)^5$$

Input interpretation:

$$- \left[32 \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243}} \right)^7} \right. \right. \\ \left. \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243}} \right) - \right. \right. \\ \left. \left. \left. \left. \left. 13 \left(\frac{1}{25} (4\pi^2) \right) e^{2 \times 0.989117352243} \right) \right) \right) \right) \right)^5$$

Result:

1.651220569...

1.651220569.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Series representations:

$$\begin{aligned}
 & - \left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
 & \qquad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
 & \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \Bigg)^5 = \\
 & \left(4385270057140224 \left(-25 + 52 e^{1.978234704486000} \pi^2 - 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
 & \qquad \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^5 / \\
 & \left(9765625 e^{19.78234704486000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
 & \qquad \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^{35}
 \end{aligned}$$

$$\begin{aligned}
& - \left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \quad \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \Big)^5 = \\
& \left(4385270057140224 \left(-25 + 52 e^{1.978234704486000} \pi^2 - 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^5 \right) / \\
& \left(9765625 e^{19.78234704486000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^{35} \right)
\end{aligned}$$

$$\begin{aligned}
& - \left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \quad \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \Big)^5 = \\
& \left(4385270057140224 \left(-25 + 52 e^{1.978234704486000} \pi^2 - \right. \right. \\
& \quad \left. \left. 25 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^5 \right) / \\
& \left(9765625 e^{19.78234704486000} \right. \\
& \quad \left. \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^{35} \right)
\end{aligned}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We obtain also:

$$-\left[32\left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)}\right)^7} \cdot \left[42\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)}\right) - 13 \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)\right]\right)^{1/2}\right]$$

Input interpretation:

$$-\left(\left(32 \frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)}\right)^7} \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)}\right) - 13 \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243}\right)\right)\right)^{1/2}\right)$$

Result:

$$-0$$

$$1.0514303501... i$$

Polar coordinates:

$$r = 1.05143035007 \text{ (radius), } \theta = -90^\circ \text{ (angle)}$$

1.05143035007

Series representations:

$$\begin{aligned}
 & - \sqrt{\left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
 & \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = -\frac{8}{5} \sqrt{21} \\
 & \sqrt{\left(\left(25 - 52 e^{1.978234704486000} \pi^2 + 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
 & \qquad \qquad \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k (e^{1.978234704486000} \pi^2)^{-k} \binom{\frac{1}{2}}{k} \right) / \left(e^{3.956469408972000} \right. \right. \\
 & \left. \left. \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k (e^{1.978234704486000} \pi^2)^{-k} \binom{\frac{1}{2}}{k} \right) \right)^7 \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& - \sqrt{\left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = -\frac{8}{5} \sqrt{21} \\
& \sqrt{\left(\left(25 - 52 e^{1.978234704486000} \pi^2 + 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
& \qquad \qquad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \\
& \left(e^{3.956469408972000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \\
& \qquad \qquad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \right) \\
& - \sqrt{\left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) / \\
& \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\
& -\frac{8}{5} \sqrt{21} \sqrt{\left(\left(25 - 52 e^{1.978234704486000} \pi^2 + \right. \right. \\
& \qquad \left. \left. 25 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \left(e^{3.956469408972000} \right. \\
& \qquad \left. \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right) \right)
\end{aligned}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$1 / -[32((((e^{(-4*0.989117352243)} / [1+\sqrt{((1+1/3*(4\pi^2)/25*e^{(2*0.989117352243)})})}]^7 * [42(1+\sqrt{((1+1/3*(4\pi^2)/25*e^{(2*0.989117352243)})})}- 13*(4\pi^2)/25*e^{(2*0.989117352243)}))])])])^{1/2}$$

Input interpretation:

$$- \left[\frac{1}{\sqrt{\left(32 \frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243} \right)} \right)^7} \right.} \right. \\ \left. \left. \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243} \right)} \right) - 13 \left(\frac{1}{25} (4\pi^2) e^{2 \times 0.989117352243} \right) \right) \right) \right) \right]$$

Result:

0.95108534763... *i*

Polar coordinates:

r = 0.95108534763 (radius), *θ* = 90° (angle)

0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

Series representations:

$$\begin{aligned} & - \left[1 / \left(\sqrt{\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right) - \right. \right. \right. \right. \\ & \qquad \qquad \qquad \left. \left. \left. \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) \right] / \\ & \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \Bigg) = \\ & - \left[5 / \left(8 \sqrt{21} \sqrt{\left(25 - 52 e^{1.978234704486000} \pi^2 + 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \right. \\ & \qquad \qquad \qquad \left. \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right) \right) \right] / \\ & \left(e^{3.956469408972000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right) \right. \\ & \qquad \qquad \qquad \left. \left. \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k} \right)^7 \right) \Bigg) \end{aligned}$$

$$\begin{aligned}
& - \left(\frac{1}{\sqrt{\left(\left(\left(\left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2)e^{2 \times 0.9891173522430000}}{3 \times 25}} \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right.} - \\
& \qquad \left. \left. \left. \left. \left. \left. \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right) \right) \right) \right) \right) / \\
& \left. \left(\left(\left(\left(\left(\left(1 + \sqrt{1 + \frac{(4\pi^2)e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \right) \right) \right) \right) \right) \right) = \\
& - \left(\frac{5}{8\sqrt{21}} \sqrt{\left(\left(\left(25 - 52 e^{1.978234704486000} \pi^2 + 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right. \right. \right. \right. \right. \\
& \qquad \left. \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right) / \\
& \left. \left(e^{3.956469408972000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right) \right. \right. \\
& \qquad \left. \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \right) \right)
\end{aligned}$$

$$1 + 1 / (((4 * ((2 * e^{(-0.989117352243/2)})) / (((1 + \sqrt{((1 + 1/3 * (4\pi^2)/25 * e^{(2 * 0.989117352243))})))))))) + (-0.034547055658)$$

Input interpretation:

$$1 + \frac{1}{4 \times \frac{2 e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} (4\pi^2) \right) e^{2 \cdot 0.989117352243}}}} - 0.034547055658$$

Result:

1.61976215705...

1.61976215705..... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$1 + \frac{1}{4 \left(2 e^{-0.9891173522430000/2} \right)} - 0.0345470556580000 = \frac{1}{1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}}} = 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75} \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \binom{\frac{1}{2}}{k}}$$

$$1 + \frac{1}{4 \left(2 e^{-0.9891173522430000/2} \right)} - 0.0345470556580000 = \frac{1}{1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}}} = 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4} \right)^k \left(e^{1.978234704486000} \pi^2 \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!}}$$

$$\begin{aligned}
& 1 + \frac{1}{4(2e^{-0.9891173522430000/2})} - 0.0345470556580000 = \\
& \frac{1}{1 + \sqrt{1 + \frac{(4\pi^2)e^{2 \times 0.9891173522430000}}{3 \times 25}}} \\
& 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} + \\
& \frac{1}{8} e^{0.4945586761215000} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4e^{1.978234704486000} \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!} \\
& \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

From

Properties of Nilpotent Supergravity

E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor-to-scalar ratio r , consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) = iM \left(\Phi + b\Phi e^{ik\Phi} \right). \quad (4.35)$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly-instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V = \frac{M^2}{3} \left(1 - a\phi e^{-\gamma\phi} \right)^2. \quad (4.36)$$

We analyzing the following equation:

$$V = \frac{M^2}{3} \left(1 - a\phi e^{-\gamma\phi} \right)^2.$$

$$\phi = \varphi - \frac{\sqrt{6}}{k},$$

$$a = \frac{b\gamma}{e} < 0, \quad \gamma = \frac{k}{\sqrt{6}} < 0.$$

We have:

$$(M^2)/3 * [1 - (b/\text{euler number} * k/\text{sqrt}6) * (\varphi - \text{sqrt}6/k) * \exp(-(k/\text{sqrt}6)(\varphi - \text{sqrt}6/k))]^2$$

i.e.

$$V = (M^2)/3 * [1 - (b/\text{euler number} * k/\text{sqrt}6) * (\varphi - \text{sqrt}6/k) * \exp(-(k/\text{sqrt}6)(\varphi - \text{sqrt}6/k))]^2$$

For $k = 2$ and $\varphi = 0.9991104684$, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

we obtain:

$$V = (M^2)/3 * [1 - (b/\text{euler number} * 2/\text{sqrt}6) * (0.9991104684 - \text{sqrt}6/2) * \exp(-(2/\text{sqrt}6)(0.9991104684 - \text{sqrt}6/2))]^2$$

Input interpretation:

$$V = \frac{M^2}{3} \left(1 - \left(\frac{b}{e} \times \frac{2}{\sqrt{6}} \right) \left(0.9991104684 - \frac{\sqrt{6}}{2} \right) \exp \left(- \frac{2}{\sqrt{6}} \left(0.9991104684 - \frac{\sqrt{6}}{2} \right) \right) \right)^2$$

Result:

$$V = \frac{1}{3} (0.0814845 b + 1)^2 M^2$$

Solutions:

$$b = \frac{225.913 \left(-0.054323 M^2 \pm 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324 (b + 12.2723)^2 M^2$$

$$V = 0.00221324 (b^2 M^2 + 24.5445 b M^2 + 150.609 M^2)$$

$$-0.00221324 b^2 M^2 - 0.054323 b M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324 b^2 M^2 + 0.054323 b M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324 (b + 12.2723)^2 M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324 b^2 M^2 + 0.054323 b M^2 + 0.333333 M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b} \left(\frac{1}{3} (0.0814845 b + 1)^2 M^2 \right) = 0.054323 (0.0814845 b + 1) M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154317775011120075}{36961748(226802245 + 18480874b)M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226802245}{18480874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154317775011120075}{2(226802245 + 18480874b)^2 M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18480874M}{226802245 + 18480874b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2(226802245 + 18480874b)^2 M}{154317775011120075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36961748(226802245 + 18480874b)M^2}{154317775011120075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845b + 1)^2 M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min \left\{ \frac{1}{3} M^2 \left(1 - \frac{(b+2) \left(0.9991104684 - \frac{\sqrt{6}}{2} \right) \exp \left(-\frac{2 \left(0.9991104684 - \frac{\sqrt{6}}{2} \right)}{\sqrt{6}} \right)}{e \sqrt{6}} \right) \right\}^2 = 0$$

for $b = -\frac{226802245}{18480874}$

$$\min \left\{ \frac{1}{3} M^2 \left(1 - \frac{(b+2) \left(0.9991104684 - \frac{\sqrt{6}}{2} \right) \exp \left(-\frac{2 \left(0.9991104684 - \frac{\sqrt{6}}{2} \right)}{\sqrt{6}} \right)}{e \sqrt{6}} \right) \right\}^2 = 0$$

for $M = 0$

From:

$$b = \frac{225.913 \left(-0.054323 M^2 \pm 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \quad (M \neq 0)$$

we obtain

$$(225.913 (-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4})) / M^2$$

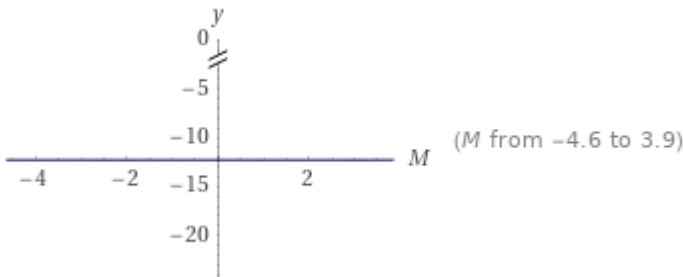
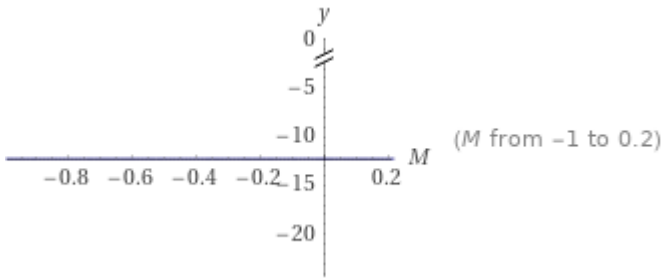
Input interpretation:

$$\frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2}$$

Result:

$$\frac{225.913 \left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2}$$

Plots:



Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value $12.1904 = \ln(196884)$

Alternate forms:

$$-\frac{12.2723 \left(M^2 - 1.21228 \times 10^{-8} \sqrt{M^4} \right)}{M^2}$$

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4} - 12.2723 M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:**Parity**

even

Series expansion at $M = 0$:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723 \right) + O(M^6)$$

(generalized Puiseux series)

Series expansion at $M = \infty$:

-12.2723

Derivative:

$$\frac{d}{dM} \left(\frac{225.913 \left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2} \right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} dM = \frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M} - 12.2723 M + \text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2\right)}{M^2}\right\} = -\frac{140119826723990341497649}{11417594849251000000000} \text{ at } M = -1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2\right)}{M^2}\right\} = -\frac{140119826723990341497649}{11417594849251000000000} \text{ at } M = -1$$

Limit:

$$\lim_{M \rightarrow \pm\infty} \frac{225.913\left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty} \left(\frac{225.913\left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} - -12.2723 \right) dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3} (0.0814845 b + 1)^2 M^2$$

we obtain:

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}))/M^2) + 1)^2 M^2$$

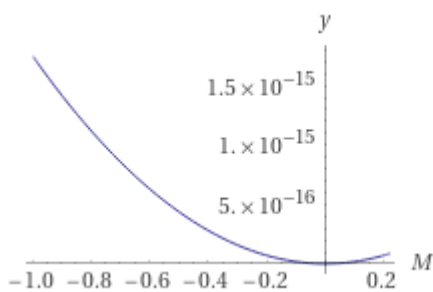
Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

Result:

0

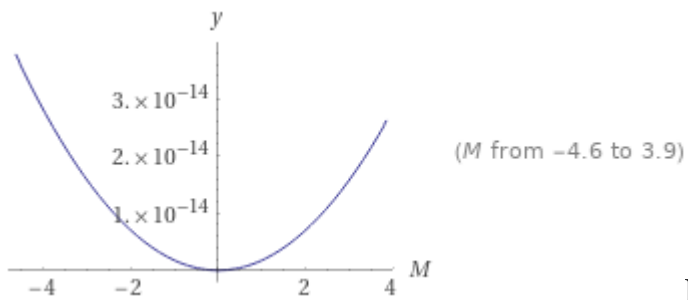
Plots: (possible mathematical connection with an open string)



(M from -1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



$$M = 2 ; M = 3$$

Root:

$$M = 0$$

Property as a function:

Parity

even

Series expansion at $M = 0$:

$$O(M^{62194})$$

(Taylor series)

Series expansion at $M = \infty$:

$$1.75541 \times 10^{-15} M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty} \left(\frac{1}{3} M^2 \left(1 + \frac{18.4084 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \right)^2 - 1.75541 \times 10^{-15} M^2 \right) dM = 0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545 \times 10^{-10} \sqrt{(-0.5)^4}))/(-0.5)^2) + 1)^2 * (-0.5^2)$$

Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 (-0.5)^2 + 6.58545 \times 10^{-10} \sqrt{(-0.5)^4} \right)}{(-0.5)^2} + 1 \right)^2$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333333333 \times 10^{-16}$$

$$-4.38851344947 * 10^{-16}$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 \cdot 3^2 + 6.58545 \times 10^{-10} \sqrt{3^4}))/3^2) + 1)^2 \cdot 3^2$$

Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 \times 3^2 + 6.58545 \times 10^{-10} \sqrt{3^4} \right)}{3^2} + 1 \right)^2 \times 3^2$$

Result:

$$1.579864841810872363256294820161116875 \times 10^{-14}$$

$$1.57986484181 \times 10^{-14}$$

For M = 2:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 \cdot 2^2 + 6.58545 \times 10^{-10} \sqrt{2^4}))/2^2) + 1)^2 \cdot 2^2$$

Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323 \times 2^2 + 6.58545 \times 10^{-10} \sqrt{2^4} \right)}{2^2} + 1 \right)^2 \times 2^2$$

We note that:

$$1/55 * (((1 / [(7.021621519 * 10^{-15} + 1.57986484181 * 10^{-14} + 7.021621519 * 10^{-17} - 4.38851344947 * 10^{-16})])^{1/7} - ((\log^{5/8}(2)) / (2 \cdot 2^{1/8} \cdot 3^{1/4} \cdot e \cdot \log^{3/2}(3))))))$$

Input interpretation:

$$\frac{1}{55} \left(\left(\frac{1}{(7.021621519 \times 10^{-15} + 1.57986484181 \times 10^{-14} + 7.021621519 \times 10^{-17} - 4.38851344947 \times 10^{-16})} \right)^{1/7} - \frac{\log^{5/8}(2)}{2 \sqrt[8]{2} \sqrt[4]{3} e \log^{3/2}(3)} \right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182...

1.6181818182... result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_P = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

$5.729475 * 10^{-35}$ Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_P = \frac{F_P}{q_P} = \sqrt{\frac{c^7}{16\pi^2 \epsilon_0 \hbar G^2}}$$

$1.820306 * 10^{61}$ V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_P^E = \mathbf{E}_P l_P^2 = \phi_P l_P = \sqrt{\frac{\hbar c}{\epsilon_0}}$$

$5.975498 * 10^{-8}$ V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\epsilon_0 G}}$$

$1.042940 * 10^{27}$ V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$$E_P * I_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$$1.042939771935 \times 10^{27}$$

$$1.042939771935 * 10^{27} \approx 1.042940 * 10^{27}$$

Or:

$$E_P * I_P^2 / I_P = (5.975498 * 10^{-8}) * 1 / (5.729475 * 10^{-35})$$

Input interpretation:

$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

1.04293988541707573556041347592929544155441816222254220500133... ×
10²⁷

$$1.042939885417 * 10^{27} \approx 1.042940 * 10^{27}$$

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

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