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Muon and electron g-2 and proton and cesium weak charges implications on dark Z_d models

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Summary. — The recent determination of the anomalous muon magnetic moment performed at Fermilab motivated us to look for an interpretation of the result in the framework of a beyond the standard model vector Z_d mediator. In this work, we derive the constraints on such model obtained from the muon and electron magnetic moment determinations and the measurements of the proton and cesium weak charge, Q_W. We first revisit the determination of the cesium Q_W from atomic parity violation experiment by exploiting recent results from other electroweak probes. From a combined fit of all the mentioned experimental results, we obtain rather precise limits on the mass and the kinetic mixing parameter of the Z_d boson, namely $m_{Z_d} = 47^{+61}_{-16}$ MeV and $\varepsilon = 2.3^{+1.1}_{-0.4} \times 10^{-3}$, when marginalizing over the $Z - Z_d$ mass mixing parameter δ.

Recently, the Muon g-2 Collaboration at Fermilab (FNAL) realeased a long awaited measurement [1] of the anomalous muon magnetic moment, referred to as $a_\mu \equiv (g_\mu - 2)/2$, with an improved precision with respect to the previous BNL measurement [2]. The combined experimental average between FNAL and BNL results $a_\mu^{\mathrm{exp}} = 116\ 592\ 061(41) \times 10^{-11}$, can be compared with the standard model (SM) prediction $a_\mu^{\mathrm{SM}} = 116\ 591\ 810(43) \times 10^{-11}$ [3], showing an intriguing 4.2σ discrepancy

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 251(59) \times 10^{-11}. \tag{1}$$

In the last years, also the electron anomalous magnetic moment experimental result [4,5] has shown a greater than 2σ discrepancy with the SM prediction [6], even if with an opposite sign with respect to the muon one. However, a new determination of the fine structure constant [7], obtained from the measurement of the recoil velocity on rubidium

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atoms, resulted into a reevaluation of the SM electron magnetic moment, bringing to a positive discrepancy of about 1.6σ. Namely $\Delta a_e = a_e^{\rm exp} - a_e^{\rm SM,\,Rb} = 0.48(30)\times 10^{-12}$ where $a_e \equiv (g_e-2)/2$. Interestingly, now the electron and muon magnetic moment discrepancies point to the same direction.

These anomalies have motivated a variety of theoretical models that predict the existence of additional particles that might contribute to the process [8-10]. In particular, they could indicate the presence of an additional sub-GeV-scale gauge boson, referred to as Z_d [11]. In the model it is assumed a $U(1)_d$ gauge symmetry whose corresponding Z_d gauge boson couples to the SM bosons via kinetic mixing, parametrized by ε, and Z-Z_d mass matrix mixing, parametrized by $\varepsilon_Z = (m_{Z_d}/m_Z)\delta$ [11], where m_{Z_d} and m_Z are the Z_d and Z masses, respectively. The parameter δ in the latter relation is usually replaced [12] by a more general parameter $\delta' \simeq \delta + \frac{m_{Z_d}}{m_Z}\varepsilon\tan\theta_W$, which incorporates higher order corrections, even if small for $m_{Z_d} \ll m_Z$. Here, θ_W is the SM predicted running of the Weinberg angle in the modified minimal subtraction ($\overline{\rm MS}$) renormalization scheme [13-15] Within this model, the new weak neutral current amplitudes at low Q^2 momentum transfer can be retrieved through the substitutions $G_F \to \rho_d G_F$, G_F being the Fermi coupling constant, and $\sin^2\theta_W(Q^2) \to \kappa_d \sin^2\theta_W(Q^2)$ [11,12], where

$$\rho_d = 1 + (\delta + \frac{m_{Z_d}}{m_Z}\,\varepsilon\,\tan\theta_W)^2 f\Big(\frac{Q^2}{m^2_{Z_d}}\Big), \quad \text{and} \tag{2}$$

$$\kappa_d = 1 - \varepsilon(\delta + \frac{m_{Z_d}}{m_Z}\,\varepsilon\,\tan\theta_W)\frac{m_Z}{m_{Z_d}}\cot\theta_W f\Big(\frac{Q^2}{m^2_{Z_d}}\Big). \tag{3}$$

The term $f(Q^2/m^2_{Z_d})$ is related to the propagator of the new boson and it may assume different forms depending on the experimental process as discussed in Refs. [16,17].

The one-loop vector contribution to the magnetic moment of the lepton $l = e,\ \mu$ which arises from this model is [11]

$$a^{Z_d}_{l,\,\rm vector} = \frac{\alpha}{2\pi}\Big(\varepsilon + \frac{m_{Z_d}}{m_Z}\delta'\frac{1-4\sin^2\theta_W}{4\sin\theta_W\cos\theta_W}\Big)^2 F_V\Big(\frac{m_{Z_d}}{m_l}\Big), \tag{4}$$

where $\sin\theta_W$ is employed at the corresponding lepton mass scale, α is the fine-structure constant, m_l the lepton mass and $F_V(x) \equiv \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2+x^2z}$. The mass mixing introduces also an axial contribution, which is although negligible, as reported in Refs. [11,18]. Considering both the vector and axial contributions, it is possible to retrieve the total Z_d induced magnetic momentum contribution $a_l^{Z_d}(\varepsilon,\delta,m_{Z_d}) = a^{Z_d}_{l,\rm vector} + a^{Z_d}_{l,\rm axial}$.

The existence of this additional Z_d boson would also introduce a new source of parity violation that could be tested by experiments sensitive to the weak charge, Q_W, of both protons and nuclei. In particular, recently the $\rm Q_{weak}$ Collaboration at JLAB [19] measured the proton weak charge, $Q_W^{p,\,\rm exp} = 0.0719(45)$, which has to be compared with the SM prediction [20,13] that, taking into account radiative corrections, is $Q_W^{p,\,\rm SM} = -2g^{ep}_{AV}(\sin^2\theta_W)\Big(1-\frac{\alpha}{2\pi}\Big) = 0.0711(2)$, where g^{ep}_{AV} is the SM electron-proton coupling, which depends on the weak mixing angle at the appropriate experimental energy scale.

Similarly, in the low-energy sector, atomic parity violation (APV) experiments provide the measurement of the weak charge of a nucleus $\mathcal{N}$ with N neutrons and Z protons, which is also very sensitive to new vector bosons. So far, the most precise measurement has been performed using cesium atoms ($N_{\mathrm{Cs}} = 78$ and $Z_{\mathrm{Cs}} = 55$), for which one can derive the following SM prediction [13] which includes radiative corrections

$$Q_W^{^{133}\mathrm{Cs,SM}} = -2[Z_{\mathrm{Cs}}(g_{AV}^{ep}(\sin^2\theta_W)+0.00005)+N_{\mathrm{Cs}}(g_{AV}^{en}+0.00006)]\left(1-\frac{\alpha}{2\pi}\right) = -73.23(1), \tag{5}$$

where g_{AV}^{en} is the SM electron-neutron coupling. The weak charge measurement depends strongly on the value of the average neutron rms radius of ^{133}Cs, $R_n(^{133}\mathrm{Cs})$ [21-23]. However, the available determinations [24,25,13,26] are based on a value for the neutron skin, that is the difference between the neutron and the proton distribution radii, extrapolated from hadronic measurements which are known to have considerable model dependencies and uncontrolled approximations [27]. Instead, we determined a new value for the cesium nuclear weak charge, whose determination is described in Ref. [18]. Namely, our new experimental value of the weak charge of ^{133}Cs is $Q_W^{^{133}\mathrm{Cs,\,exp}} = -72.94(43)$ [18]. This result can be compared to the current one presented in Ref. [13]. The uncertainty is practically the same and the central value is only marginally shifted. However, the main advantage is that it is derived from a fully electroweak determination.

Our measurements of Q_W and the proton one reported in Ref. [19] can be used to set limits on the available phase space for the Z_d model. Indeed, the presence of a Z_d mediator would change the experimental values of Q_W. More precisely, adopting the substitutions described before, the proton and the cesium weak charge expressions become [18]

$$Q_W^{p,\,Z_d} = \rho_d Q_W^{p,\,SM}(k_d \sin^2\theta_W), \tag{6}$$

$$Q_W^{^{133}\mathrm{Cs},\,Z_d} = \rho_d Q_W^{^{133}\mathrm{Cs},\,SM}(k_d \sin^2\theta_W). \tag{7}$$

In order to determine information on ε, δ and m_{Z_d}, we performed a combined fit of the anomalous magnetic moment of both muon and electron and the proton and cesium weak charge measurements from Qweak and APV respectively, with the common least-squares function $\chi^2 = \sum_i \frac{(X_i^{\mathrm{exp}} - X_i^{\mathrm{th}}(\varepsilon,\delta,m_{Z_d}))^2}{\sigma_i^2}$, where i the single measurement and σ_i are the corresponding experimental and theoretical uncertainties summed in quadrature. In order to remove the ambiguity on δ, we marginalized the result over this parameter. In fig. 1(a) we show the 1σ, 2σ, and 3σ CL contours in the plane of m_{Z_d} and ε, as well as the best fit result corresponding to a minimum $\chi^2_{\mathrm{min}} = 0.007$. For completeness, when marginalizing in turn over the other two parameters, we get the following results for m_{Z_d}, ε and δ at 1σ CL $m_{Z_d} = 47^{+61}_{-16}$ MeV, $\varepsilon = 2.3^{+1.1}_{-0.4} \times 10^{-3}$, $\delta < 2 \times 10^{-3}$. Using these best fit values(¹) and their 1σ ranges, in fig. 1(b) we show how the running of $\sin^2\vartheta_{\mathrm{W}}$ changes at low energies due to the contribution of a Z_d boson. Clearly, further measurements of $\sin^2\vartheta_{\mathrm{W}}$ in the low energy sector, as those coming from the P2 [33,35] and MOLLER [34] experiments.

In summary, in this work we studied a possible $U(1)_d$ extension of the SM which implies the presence of a sub-GeV-scale vector Z_d mediator. The existence of such additional particle would modify the experimental values of the muon and electron anomalous magnetic moments as well as the measurements of the proton and cesium weak charge, performed so far at low-energy transfer. Motivated by the recent determination of the

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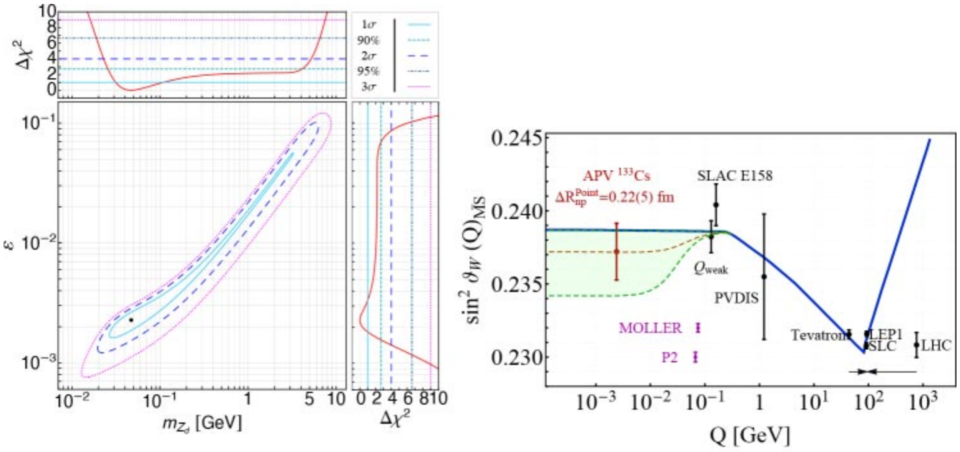
(¹) The best fit value of δ is 7.9×10^{-4}.



(a) (b)

Fig. 1. – Figure 1(a): contours at different CL of the allowed regions in the plane of m_{Z_d} and ε, together with their marginalizations, obtained from the combined fit of the $\mathrm{Q_{weak}}$, APV, a_μ and a_e experimental results. The best fit result is indicated by the black dot. Figure 1(b): running of $\sin^2\vartheta_\mathrm{W}$ with energy scale Q. The SM prediction is shown as the solid blue curve, together with experimental determinations in black [28, 24, 29, 30, 30-32, 19] and future projections in violet [33, 34] with a central value shown at an arbitrary position. The result derived in this paper for APV on cesium is shown in red. With the dashed red and green lines we indicate the best fit result and the $\pm 1\sigma$ variations, respectively, for the running of $\sin^2\vartheta_\mathrm{W}$ in the presence of a Z_d boson as described in the paper.

muon anomalous magnetic moment performed at Fermilab, we derived the constraints on such a model obtained from the aforementioned experimental measurements and by their combination. Before to do so, we revisited the determination of the cesium Q_W from the atomic parity violation experiment, which depends critically on the value of the average neutron rms radius of ^{133}Cs, by determining the latter from a practically model-independent extrapolation from other recent electroweak measurements. From a combined χ^2 fit we obtain rather precise limits on the mass and the kinetic mixing parameter of the Z_d boson, namely $m_{Z_d} = 47^{+61}_{-16}\,\mathrm{MeV}$ and $\varepsilon = 2.3^{+1.1}_{-0.4} \times 10^{-3}$, when marginalizing over the $Z - Z_d$ mass mixing parameter δ.

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