On the study of various equations concerning the Isoperimetric Theorems. Possible mathematical connections with some sectors of Number Theory and Eternal Inflation model.

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#### Abstract

In this paper, we analyze various equations concerning the Isoperimetric Theorems. We describe the new possible mathematical connections with some sectors of Number Theory and Eternal Inflation model

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#### Introduction

In 1983, it was shown that inflation could be eternal, leading to a multiverse in which space is broken up into bubbles or patches whose properties differ from patch to patch spanning all physical possibilities.

When the false vacuum decays, the lower-energy true vacuum forms through a process known as **bubble nucleation**. In this process, instanton effects cause a bubble containing the true vacuum to appear. The walls of the bubble (or domain walls) have a positive surface tension, as energy is expended as the fields roll over the potential barrier to the true vacuum.

In mathematics, a **ball** is the space bounded by a sphere. It may be a **closed ball** (including the boundary points that constitute the sphere) or an **open ball** (excluding them). (From Wikipedia)

We propose that some equations concerning the "balls", thus various sectors and theorems of Geometric Measure Theory, can be related with several parameters of some cosmological models as the "Multiverse" and the "Eternal Inflation" linked to it, which provides that space is divided into bubbles or patches whose properties differ from patch to patch and spanning all physical possibilities. From:

**Isoperimetry and Stability Properties of Balls with Respect to Nonlocal Energies** *A. Figalli, N. Fusco, F. Maggi, V. Millot, M. Morini* - Commun. Math. Phys Digital Object Identifier (DOI) 10.1007/s00220-014-2244-1

#### We have that:

In the recent paper [7], Caffarelli, Roquejoffre, and Savin have initiated the study of Plateau-type problems with respect to a family of nonlocal perimeter functionals. A regularity theory for such nonlocal minimal surfaces has been developed by several authors [4, 10, 12, 18, 35], while the relation of nonlocal perimeters with their local counterpart has been investigated in [3,8]. The isoperimetry of balls in nonlocal isoperimetric problems has been addressed in [19]. Precisely, given  $s \in (0, 1)$  and  $n \ge 2$ , one defines the *s*-perimeter of a set  $E \subset \mathbb{R}^n$  as

$$P_s(E) := \int_E \int_{E^c} \frac{dx \, dy}{|x - y|^{n+s}} \in [0, \infty].$$

As proved in [19], if  $0 < |E| < \infty$  then we have the nonlocal isoperimetric inequality

$$P_s(E) \ge \frac{P_s(B)}{|B|^{(n-s)/n}} |E|^{(n-s)/n},$$
(1.1)

where  $B_r := \{x \in \mathbb{R}^n : |x| < r\}, B := B_1$ , and |E| is the Lebesgue measure of E.

In this section we consider the family of functionals  $\text{Per}_s + \beta V_{\alpha}$  ( $\beta > 0$ ) and discuss in terms of the value of  $\beta$  the volume-constrained stability of  $\text{Per}_s + \beta V_{\alpha}$  around the unit ball *B*. Our interest in this problem lies in the fact that, as we shall prove in Sect. 8, stability is actually a necessary and sufficient condition for volume-constrained local minimality. Therefore the analysis carried on in this section will provide the basis for the proof of Theorem 1.5. We set

$$\beta_{\star}(n, s, \alpha) := \begin{cases} \frac{1-s}{\omega_{n-1}} \inf_{k \ge 2} \frac{\lambda_k^s - \lambda_1^s}{\mu_k^\alpha - \mu_1^\alpha}, & \text{if } s \in (0, 1), \\ \inf_{k \ge 2} \frac{\lambda_k^1 - \lambda_1^1}{\mu_k^\alpha - \mu_1^\alpha}, & \text{if } s = 1, \end{cases}$$
(7.1)

where, for every  $k \in \mathbb{N} \cup \{0\}$ ,

$$\lambda_k^1 = k(k+n-2), \tag{7.2}$$

$$\lambda_k^s = \frac{2^{1-s} \pi^{\frac{n-1}{2}}}{1+s} \frac{\Gamma(\frac{1-s}{2})}{\Gamma(\frac{n+s}{2})} \left( \frac{\Gamma(k+\frac{n+s}{2})}{\Gamma(k+\frac{n-2-s}{2})} - \frac{\Gamma(\frac{n+s}{2})}{\Gamma(\frac{n-2-s}{2})} \right), \quad s \in (0,1), \quad (7.3)$$

$$\mu_k^{\alpha} = \frac{2^{1+\alpha} \pi^{\frac{n-1}{2}}}{1-\alpha} \frac{\Gamma(\frac{1+\alpha}{2})}{\Gamma(\frac{n-\alpha}{2})} \left( \frac{\Gamma(k+\frac{n-\alpha}{2})}{\Gamma(k+\frac{n-2+\alpha}{2})} - \frac{\Gamma(\frac{n-\alpha}{2})}{\Gamma(\frac{n-2+\alpha}{2})} \right), \quad \alpha \in (0,1), \quad (7.4)$$

$$\mu_k^{\alpha} = 2^{\alpha} \pi^{\frac{n-1}{2}} \frac{\Gamma(\frac{\alpha-1}{2})}{\Gamma(\frac{n-\alpha}{2})} \left( \frac{\Gamma(\frac{n-\alpha}{2})}{\Gamma(\frac{n-2+\alpha}{2})} - \frac{\Gamma(k+\frac{n-\alpha}{2})}{\Gamma(k+\frac{n-2+\alpha}{2})} \right), \qquad \alpha \in (1,n), \quad (7.5)$$

$$\mu_k^1 = \frac{4\pi^{\frac{n-1}{2}}}{\Gamma(\frac{n-1}{2})} \left( \frac{\Gamma'(k+\frac{n-1}{2})}{\Gamma(k+\frac{n-1}{2})} - \frac{\Gamma'(\frac{n-1}{2})}{\Gamma(\frac{n-1}{2})} \right).$$
(7.6)

Here  $\Gamma$  denotes the Euler's Gamma function, while  $\Gamma'$  is the derivative of  $\Gamma$ , so that  $\Gamma'/\Gamma$  is the digamma function. By exploiting basic properties of the Gamma function, it is straightforward to check that  $\lambda_k^s/\mu_k^\alpha \to \infty$  as  $k \to \infty$ , so that the infimum in (7.1) is achieved, and  $\beta_* > 0$ . We shall actually prove that the infimum is always achieved at k = 2 and the formula for  $\beta_*$  considerably simplifies (see Proposition 7.4).

Now, we analyze the eqs. (7.2), (7.3), (7.4), (7.5) and (7.6)

For n = 3, k = 2,  $s = \alpha = 1/2$ , from

$$\lambda_k^1 = k(k+n-2),$$

we obtain:

2(2+3-2) = 6

From:

$$\lambda_k^s = \frac{2^{1-s} \, \pi^{\frac{n-1}{2}}}{1+s} \, \frac{\Gamma(\frac{1-s}{2})}{\Gamma(\frac{n+s}{2})} \left( \frac{\Gamma(k+\frac{n+s}{2})}{\Gamma(k+\frac{n-2-s}{2})} - \frac{\Gamma(\frac{n+s}{2})}{\Gamma(\frac{n-2-s}{2})} \right), \quad s \in (0,1),$$

we obtain:

 $(2^{(1-0.5)*Pi})/(1+1/2)* gamma(1/2*(1-1/2))/gamma(1/2*(3+1/2))*(((((gamma(2+1/2(3+1/2)))/gamma(2+1/2(3-2-1/2))))/gamma(1/2(3+1/2))/gamma(1/2(3-2-1/2)))))$ 

#### Input

$$\frac{2^{1-0.5} \pi}{1+\frac{1}{2}} \times \frac{\Gamma(\frac{1}{2} \left(1-\frac{1}{2}\right))}{\Gamma(\frac{1}{2} \left(3+\frac{1}{2}\right))} \left(\frac{\Gamma(2+\frac{1}{2} \left(3+\frac{1}{2}\right))}{\Gamma(2+\frac{1}{2} \left(3-2-\frac{1}{2}\right))} - \frac{\Gamma(\frac{1}{2} \left(3+\frac{1}{2}\right))}{\Gamma(\frac{1}{2} \left(3-2-\frac{1}{2}\right))}\right)$$

 $\Gamma(x)$  is the gamma function

#### Result

42.6517...

42.6517...

The study of this function provides the following representations:

#### **Alternative representations**

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)(2^{1-0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)}=\frac{\pi\left(-\frac{3}{4}\right)!\ 2^{0.5}\left(-\frac{\frac{3}{4}!}{\left(-\frac{3}{4}\right)!}+\frac{\frac{11}{4}!}{\frac{5}{4}!}\right)}{\frac{3\times\frac{3}{4}!}{2}}$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)(2^{1-0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)}$$

$$=\frac{\pi G\left(\frac{5}{4}\right)2^{0.5}\left(-\frac{G\left(\frac{11}{4}\right)}{\frac{G\left(\frac{7}{4}\right)G\left(\frac{5}{4}\right)}{G\left(\frac{4}{4}\right)}}+\frac{G\left(\frac{19}{4}\right)}{\frac{G\left(\frac{15}{4}\right)G\left(\frac{13}{4}\right)}{G\left(\frac{9}{4}\right)}}\right)}{\frac{3 G\left(\frac{1}{4}\right)G\left(\frac{11}{4}\right)}{2 G\left(\frac{7}{4}\right)}}$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)\left(2^{1-0.5}\pi\right)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)} = \\ \frac{\pi \ 2^{0.5} \ e^{-\log G(1/4)+\log G(5/4)} \left(-\frac{e^{-\log G(7/4)+\log G(11/4)}}{e^{-\log G(1/4)+\log G(5/4)}}+\frac{e^{-\log G(15/4)+\log G(19/4)}}{e^{-\log G(9/4)+\log G(13/4)}}\right)}{\frac{3}{2} \ e^{-\log G(7/4)+\log G(11/4)}$$

## Series representations

$$\begin{split} & \frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)\left(2^{1-0.5}\pi\right)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)} = \\ & \left(0.942809\pi\left(-\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{\left(\frac{7}{4}-z_{0}\right)^{k_{1}}\left(\frac{9}{4}-z_{0}\right)^{k_{2}}\Gamma^{(k_{1})}(z_{0})\Gamma^{(k_{2})}(z_{0})}{k_{1}!k_{2}!} + \right. \right. \\ & \left.\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{\left(\frac{1}{4}-z_{0}\right)^{k_{1}}\left(\frac{15}{4}-z_{0}\right)^{k_{2}}\Gamma^{(k_{1})}(z_{0})\Gamma^{(k_{2})}(z_{0})}{k_{1}!k_{2}!}\right)\right) / \\ & \left.\left(\left(\sum_{k=0}^{\infty}\frac{\left(\frac{7}{4}-z_{0}\right)^{k}\Gamma^{(k)}(z_{0})}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{9}{4}-z_{0}\right)^{k}\Gamma^{(k)}(z_{0})}{k!}\right)\right) \text{ for } (z_{0}\notin\mathbb{Z} \text{ or } z_{0}>0) \end{split} \right) \end{split}$$

$$\begin{split} & \left( \Gamma(\frac{1}{2} \left(1 - \frac{1}{2})\right) \left( \frac{\Gamma(2 + \frac{1}{2} \left(3 + \frac{1}{2}\right))}{\Gamma(2 + \frac{1}{2} \left(3 - 2 - \frac{1}{2}\right))} \right) (1 + \frac{1}{2}) \\ & - \left( \left( 0.942809 \, \pi \left( - \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \left( \frac{7}{4} - z_0 \right)^{k_1} \left( \frac{9}{4} - z_0 \right)^{k_2} \right) \\ & \sum_{j_1 = 0}^{k_1} \sum_{j_2 = 0}^{k_2} \left( (-1)^{j_1 + j_2} \pi^{-j_1 - j_2 + k_1 + k_2} \sin\left(\frac{1}{2} \pi \left(-j_1 + k_1 + 2 z_0\right)\right) \right) \\ & \sin\left(\frac{1}{2} \pi \left(-j_2 + k_2 + 2 z_0\right)\right) \Gamma^{(j_1)}(1 - z_0) \\ & \Gamma^{(j_2)}(1 - z_0) \right) / (j_1! \, j_2! \left(-j_1 + k_1\right)! \left(-j_2 + k_2\right)!) + \\ & \sum_{k_1 = 0}^{\infty} \sum_{k_2 = 0}^{\infty} \left(\frac{1}{4} - z_0\right)^{k_1} \left(\frac{15}{4} - z_0\right)^{k_2} \sum_{j_1 = 0}^{k_1} \sum_{j_2 = 0}^{k_2} \left( (-1)^{j_1 + j_2} \pi^{-j_1 - j_2 + k_1 + k_2} \sin\left(\frac{1}{2} \pi \left(-j_1 + k_1 + 2 z_0\right)\right) \right) \\ & \sin\left(\frac{1}{2} \pi \left(-j_2 + k_2 + 2 z_0\right)\right) \Gamma^{(j_1)}(1 - z_0) \\ & \pi^{-j_1 - j_2 + k_1 + k_2} \sin\left(\frac{1}{2} \pi \left(-j_1 + k_1 + 2 z_0\right)\right) \\ & \sin\left(\frac{1}{2} \pi \left(-j_2 + k_2 + 2 z_0\right)\right) \Gamma^{(j_1)}(1 - z_0) \\ & \Gamma^{(j_2)}(1 - z_0) \right) / (j_1! \, j_2! \left(-j_1 + k_1\right)! \left(-j_2 + k_2\right)!) \right) \\ & \left( \left( \sum_{k=0}^{\infty} \left(\frac{1}{4} - z_0\right)^k \sum_{j=0}^k \frac{\left(-1\right)^j \pi^{-j + k} \sin\left(\frac{1}{2} \pi \left(-j + k + 2 z_0\right)\right) \Gamma^{(j)}(1 - z_0)}{j! \left(-j + k\right)!} \right) \right) \\ & \sum_{k=0}^{\infty} \left( \frac{15}{4} - z_0\right)^k \sum_{j=0}^k \frac{\left(-1\right)^j \pi^{-j + k} \sin\left(\frac{1}{2} \pi \left(-j + k + 2 z_0\right)\right) \Gamma^{(j)}(1 - z_0)}{j! \left(-j + k\right)!} \right) \end{split}$$

## Integral representations

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)(2^{1-0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)}=-0.942809\pi+0.942809\exp\left(\int_{0}^{1}\frac{\sqrt[4]{x}\left(-1-x+x^{3/2}+x^{5/2}\right)}{\log(x)}\,dx\right)\pi$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)(2^{1-0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)} = \\ -0.942809\pi+0.942809\exp\left(\int_{0}^{1}\frac{\sqrt[4]{x}-x^{7/4}-x^{9/4}+x^{15/4}-\log\left(\sqrt[4]{x}\right)+\log(x^{7/4})+\log(x^{9/4})-\log(x^{15/4})}{(-1+x)\log(x)}\right) \right)$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1-\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2-\frac{1}{2}\right)\right)}\right)\right)(2^{1-0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3+\frac{1}{2}\right)\right)\left(1+\frac{1}{2}\right)} = \\ \frac{0.942809\pi\left(\int_{0}^{1}\int_{0}^{1}\log^{3/4}\left(\frac{1}{t_{1}}\right)\log^{5/4}\left(\frac{1}{t_{2}}\right)dt_{2}dt_{1}+\int_{0}^{1}\int_{0}^{1}\frac{\log^{11/4}\left(\frac{1}{t_{2}}\right)}{\log^{3/4}\left(\frac{1}{t_{1}}\right)}dt_{2}dt_{1}\right)}{\left(\int_{0}^{1}\log^{3/4}\left(\frac{1}{t_{1}}\right)dt\right)\int_{0}^{1}\log^{5/4}\left(\frac{1}{t_{1}}\right)dt}$$

For n = 3, k = 2,  $\alpha = 1/2$ 

From:

$$\mu_k^{\alpha} = \frac{2^{1+\alpha} \, \pi^{\frac{n-1}{2}}}{1-\alpha} \frac{\Gamma(\frac{1+\alpha}{2})}{\Gamma(\frac{n-\alpha}{2})} \left( \frac{\Gamma(k+\frac{n-\alpha}{2})}{\Gamma(k+\frac{n-2+\alpha}{2})} - \frac{\Gamma(\frac{n-\alpha}{2})}{\Gamma(\frac{n-2+\alpha}{2})} \right), \quad \alpha \in (0,1),$$

we obtain:

 $(2^{(1+0.5)*Pi})/(1-1/2)* gamma(1/2*(1+1/2))/gamma(1/2*(3-1/2))*(((((gamma(2+1/2(3-1/2)))/gamma(2+1/2(3-2+1/2))-gamma(1/2(3-1/2))/gamma(1/2(3-2+1/2)))))$ 

#### Input

$$\frac{2^{1+0.5} \pi}{1-\frac{1}{2}} \times \frac{\Gamma(\frac{1}{2} \left(1+\frac{1}{2}\right))}{\Gamma(\frac{1}{2} \left(3-\frac{1}{2}\right))} \left(\frac{\Gamma(2+\frac{1}{2} \left(3-\frac{1}{2}\right))}{\Gamma(2+\frac{1}{2} \left(3-2+\frac{1}{2}\right))} - \frac{\Gamma(\frac{1}{2} \left(3-\frac{1}{2}\right))}{\Gamma(\frac{1}{2} \left(3-2+\frac{1}{2}\right))}\right)$$

 $\Gamma(x)$  is the gamma function

#### Result

20.3103...

20.3103...

The study of this function provides the following representations:

#### Alternative representations

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)}=\frac{\pi\left(-\frac{1}{4}\right)!\,2^{1.5}\left(-\frac{\frac{1}{4}!}{\left(-\frac{1}{4}\right)!}+\frac{\frac{9}{4}!}{\frac{7}{4}!}\right)}{\frac{\frac{1}{4}!}{2}}$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)}$$

$$=\frac{\pi G\left(\frac{7}{4}\right)2^{1.5}\left(-\frac{G\left(\frac{9}{4}\right)}{\frac{G\left(\frac{5}{4}\right)G\left(\frac{7}{4}\right)}{G\left(\frac{3}{4}\right)}+\frac{G\left(\frac{17}{4}\right)}{\frac{G\left(\frac{13}{4}\right)G\left(\frac{15}{4}\right)}{G\left(\frac{11}{4}\right)}}\right)}{\frac{G\left(\frac{3}{4}\right)G\left(\frac{9}{4}\right)}{2G\left(\frac{5}{4}\right)}}$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)}=\frac{\pi 2^{1.5} e^{-\log G(3/4)+\log G(7/4)} \left(-\frac{e^{-\log G(5/4)+\log G(9/4)}}{e^{-\log G(3/4)+\log G(7/4)}}+\frac{e^{-\log G(13/4)+\log G(17/4)}}{e^{-\log G(11/4)+\log G(15/4)}}\right)}{\frac{1}{2} e^{-\log G(5/4)+\log G(9/4)}}$$

## Series representations

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)} = \\ \left(5.65685\pi\left(-\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{\left(\frac{5}{4}-z_{0}\right)^{k_{1}}\left(\frac{11}{4}-z_{0}\right)^{k_{2}}\Gamma^{(k_{1})}(z_{0})\Gamma^{(k_{2})}(z_{0})}{k_{1}!k_{2}!}+\right)\right)\right) \right) \\ \left(\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{\left(\frac{3}{4}-z_{0}\right)^{k_{1}}\left(\frac{13}{4}-z_{0}\right)^{k_{2}}\Gamma^{(k_{1})}(z_{0})\Gamma^{(k_{2})}(z_{0})}{k_{1}!k_{2}!}\right)}{k_{1}!k_{2}!}\right)\right) / \\ \left(\left(\sum_{k=0}^{\infty}\frac{\left(\frac{5}{4}-z_{0}\right)^{k}\Gamma^{(k)}(z_{0})}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{11}{4}-z_{0}\right)^{k}\Gamma^{(k)}(z_{0})}{k!}\right) \text{ for } (z_{0}\notin\mathbb{Z} \text{ or } z_{0}>0) \right) \right) \right)$$

## Integral representations

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)}=$$
  
-5.65685 \pi + 5.65685 \exp\left(\int\_{0}^{1}\frac{\left(-1+\sqrt{x}\right)x^{3/4}(1+x)}{\log(x)}dx\right)\pi

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)} = \\ -5.65685\pi+5.65685\exp\left(\int_{0}^{1} \frac{x^{3/4}-x^{5/4}-x^{11/4}+x^{13/4}-\log(x^{3/4})+\log(x^{5/4})+\log(x^{11/4})-\log(x^{13/4})}{(-1+x)\log(x)}\right) \right)$$

$$\frac{\left(\Gamma\left(\frac{1}{2}\left(1+\frac{1}{2}\right)\right)\left(\frac{\Gamma\left(2+\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}-\frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)}{\Gamma\left(\frac{1}{2}\left(3-2+\frac{1}{2}\right)\right)}\right)\right)(2^{1+0.5}\pi)}{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)} = \\ \frac{\Gamma\left(\frac{1}{2}\left(3-\frac{1}{2}\right)\right)\left(1-\frac{1}{2}\right)}{5.65685\pi\left(\int_{0}^{1}\int_{0}^{1}\frac{4}{\sqrt{\log\left(\frac{1}{t_{1}}\right)}}\log^{7/4}\left(\frac{1}{t_{2}}\right)dt_{2}dt_{1}+\int_{0}^{1}\int_{0}^{1}\frac{\log^{9/4}\left(\frac{1}{t_{2}}\right)}{\frac{4}{\sqrt{\log\left(\frac{1}{t_{1}}\right)}}}dt_{2}dt_{1}}\right)}{\left(\int_{0}^{1}\frac{4}{\sqrt{\log\left(\frac{1}{t_{1}}\right)}}dt\right)\int_{0}^{1}\log^{7/4}\left(\frac{1}{t_{2}}\right)dt}$$

For n = 3, k = 2,  $\alpha = 2$ 

From:

$$\mu_k^{\alpha} = 2^{\alpha} \pi^{\frac{n-1}{2}} \frac{\Gamma(\frac{\alpha-1}{2})}{\Gamma(\frac{n-\alpha}{2})} \left( \frac{\Gamma(\frac{n-\alpha}{2})}{\Gamma(\frac{n-2+\alpha}{2})} - \frac{\Gamma(k+\frac{n-\alpha}{2})}{\Gamma(k+\frac{n-2+\alpha}{2})} \right), \qquad \alpha \in (1,n),$$

we obtain:

 $(2^2*Pi)*gamma(1/2*(2-1))/gamma(1/2*(3-2))*(((((gamma(1/2(3-2)))/gamma(1/2(3-2+2))-gamma(2+1/2(3-2))/gamma(2+1/2(3-2+2)))))$ 

#### Input

$$\left(2^2 \, \pi\right) \times \frac{\Gamma\left(\frac{1}{2} \, (2-1)\right)}{\Gamma\left(\frac{1}{2} \, (3-2)\right)} \left(\frac{\Gamma\left(\frac{1}{2} \, (3-2)\right)}{\Gamma\left(\frac{1}{2} \, (3-2+2)\right)} - \frac{\Gamma\left(2+\frac{1}{2} \, (3-2)\right)}{\Gamma\left(2+\frac{1}{2} \, (3-2+2)\right)}\right)$$

 $\Gamma(x)$  is the gamma function

#### **Exact result**

 $\frac{32 \pi}{5}$ 

#### **Decimal approximation**

20.106192982974676726160917652988818458861884156000677254239645390

## 20.106192982....

The study of this function provides the following representations:

#### Property

 $\frac{32\pi}{5}$  is a transcendental number

#### **Alternative representations**

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)}-\frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^2\pi}{\Gamma\left(\frac{3-2}{2}\right)}=\frac{4\pi\left(-\frac{1}{2}\right)!\left(\frac{\left(-\frac{1}{2}\right)!}{\frac{1}{2}!}-\frac{\frac{3}{2}!}{\frac{5}{2}!}\right)}{\left(-\frac{1}{2}\right)!}$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^{2}\pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{1}{\frac{4\pi e^{-\log G(1/2) + \log G(3/2)}\left(\frac{e^{-\log G(1/2) + \log G(3/2)}}{e^{-\log G(3/2) + \log G(5/2)}} - \frac{e^{-\log G(5/2) + \log G(7/2)}}{e^{-\log G(7/2) + \log G(9/2)}}\right)}}{e^{-\log G(1/2) + \log G(3/2)}}$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)}-\frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^{2}\pi}{\Gamma\left(\frac{3-2}{2}\right)}=\frac{4\,\pi\,\Gamma\left(\frac{1}{2},\,0\right)\left(\frac{\Gamma\left(\frac{1}{2},0\right)}{\Gamma\left(\frac{3}{2},0\right)}-\frac{\Gamma\left(\frac{5}{2},0\right)}{\Gamma\left(\frac{7}{2},0\right)}\right)}{\Gamma\left(\frac{1}{2},\,0\right)}$$

## Series representations

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^2 \pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{128}{5} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{1+2 k}$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^{2}\pi}{\Gamma\left(\frac{3-2}{2}\right)}$$

$$=\frac{\Gamma\left(\frac{3-2}{2}\right)}{\sum_{k=0}^{\infty}\frac{128\left(-1\right)^{k}\left(956\times5^{-2k}-5\times239^{-2k}\right)}{5975\left(1+2k\right)}}$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^{2}\pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{32}{5}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

## Integral representations

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^2\pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{128}{5}\int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^2\pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{64}{5}\int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\left(\Gamma\left(\frac{2-1}{2}\right)\left(\frac{\Gamma\left(\frac{3-2}{2}\right)}{\Gamma\left(\frac{1}{2}\left(3-2+2\right)\right)} - \frac{\Gamma\left(2+\frac{3-2}{2}\right)}{\Gamma\left(2+\frac{1}{2}\left(3-2+2\right)\right)}\right)\right)2^2\pi}{\Gamma\left(\frac{3-2}{2}\right)} = \frac{64}{5}\int_0^\infty \frac{1}{1+t^2}\,dt$$

For 
$$n = 3$$

from:

$$\mu_k^1 = \frac{4\pi^{\frac{n-1}{2}}}{\Gamma(\frac{n-1}{2})} \left( \frac{\Gamma'(k + \frac{n-1}{2})}{\Gamma(k + \frac{n-1}{2})} - \frac{\Gamma'(\frac{n-1}{2})}{\Gamma(\frac{n-1}{2})} \right).$$

we obtain:

 $(2^2*Pi)/gamma(1/2(3-1))*(((((digamma(2+1)))/gamma(2+1/2(3-1))-digamma(1/2(3-1))/gamma(1/2(3-1)))))$ 

Input

$$\frac{2^2 \pi}{\Gamma(\frac{1}{2}(3-1))} \left( \frac{\psi(2+1)}{\Gamma(2+\frac{1}{2}(3-1))} - \frac{\psi(\frac{1}{2}(3-1))}{\Gamma(\frac{1}{2}(3-1))} \right)$$

 $\Gamma(x)$  is the gamma function

 $\psi(x)$  is the digamma function

#### **Exact result**

$$4\left(\frac{1}{2}\left(\frac{3}{2}-\gamma\right)+\gamma\right)\pi$$

 $\gamma$  is the Euler-Mascheroni constant

#### **Decimal approximation**

13.051530945552586714614199212380588690806836560870519080197129933

13.051530945...

...

The study of this function provides the following representations:

#### **Alternate forms**

 $(3+2\gamma)\pi$ 

$$2\left(\frac{3}{2}-\gamma\right)\pi + 4\gamma\pi$$

#### **Expanded form**

 $3\pi + 2\gamma\pi$ 

#### **Alternative representations**

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right) \left(2^2 \pi\right)}{\Gamma\left(\frac{3-1}{2}\right)} = \frac{4 \pi \left(-\frac{\psi(1)}{\frac{1}{1}} + \frac{\psi(3)}{2 \times \frac{1}{1}}\right)}{\frac{1}{1}}$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right) \left(2^2 \pi\right)}{\Gamma\left(\frac{3-1}{2}\right)} = \frac{4 \pi \left(-\frac{\psi(1)}{e^0} + \frac{\psi(3)}{e^{\log(2)}}\right)}{e^0}$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right)(2^2 \pi)}{\Gamma\left(\frac{3-1}{2}\right)} = \frac{4 \pi \left(-\frac{\frac{\partial 0!}{\partial 1}}{0! \times \frac{1}{1}} + \frac{\frac{\partial 2!}{\partial 3}}{2! \times 2}\right)}{\frac{1}{1}}$$

 $\psi^{(n)}(\boldsymbol{x})$  is the  $\boldsymbol{n}^{\mathrm{th}}$  derivative of the digamma function

## Series representations

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right) (2^2 \pi)}{\Gamma\left(\frac{3-1}{2}\right)} = 5 \pi + 2 \pi \sum_{k=2}^{\infty} \left(\frac{1}{k} + \log\left(\frac{-1+k}{k}\right)\right)$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right)(2^2 \pi)}{\Gamma\left(\frac{3-1}{2}\right)} = 3 \pi + 2 \pi \sum_{k=1}^{\infty} \left(\frac{1}{k} - \log\left(1 + \frac{1}{k}\right)\right)$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right)(2^2 \pi)}{\Gamma\left(\frac{3-1}{2}\right)} = \pi \left(3 + 2\sum_{k=1}^{\infty} k \sum_{j=2^k}^{-1+2^{1+k}} \frac{(-1)^j}{j}\right)$$

## **Integral representations**

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right)(2^2 \pi)}{\Gamma\left(\frac{3-1}{2}\right)} = 3 \pi - 2 \pi \int_{-\infty}^{\infty} e^{-e^t + t} t \, dt$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right) \left(2^2 \pi\right)}{\Gamma\left(\frac{3-1}{2}\right)} = -2\left(-3 + 2\int_{-\infty}^{\infty} e^{-e^t + t} t \, dt\right) \int_{0}^{\infty} \frac{1}{1+t^2} \, dt$$

$$\frac{\left(\frac{\psi(2+1)}{\Gamma\left(2+\frac{3-1}{2}\right)} - \frac{\psi\left(\frac{3-1}{2}\right)}{\Gamma\left(\frac{3-1}{2}\right)}\right) \left(2^2 \pi\right)}{\Gamma\left(\frac{3-1}{2}\right)} = 2\left(\int_0^\infty \frac{1}{1+t^2} dt\right) \left(3+2\int_0^\infty \frac{-e^{-t} + \frac{1}{1+t}}{t} dt\right)$$

From the sum of the previous results/expressions, we obtain:

 $6+42.6517+20.3103+((32\pi)/5)+[((((4(1/2(3/2-0.5772156649)+0.5772156649)\pi))))]$ 

#### **Input interpretation**

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.5772156649\right) + 0.5772156649\right)\pi$$

#### Result

102.120...

#### 102.120....

The study of this function provides the following representations:

#### Alternative representations

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 747.798^{\circ} + \frac{5760^{\circ}}{5}$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 - 4.15443 i \log(-1) - \frac{32}{5} i \log(-1)$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 4.15443\cos^{-1}(-1) + \frac{32}{5}\cos^{-1}(-1)$$

Series representations

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 42.2177\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 47.8531 + 21.1089\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 10.5544\sum_{k=0}^{\infty}\frac{2^{-k}\left(-6 + 50\,k\right)}{\binom{3\,k}{k}}$$

## **Integral representations**

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 21.1089 \int_0^\infty \frac{1}{1+t^2} dt$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 42.2177 \int_0^1 \sqrt{1 - t^2} dt$$

$$6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi = 68.962 + 21.1089 \int_0^\infty \frac{\sin(t)}{t} dt$$

From which, we obtain:

 $\begin{array}{l} 17((6+42.6517+20.3103+((32\pi)/5)+[((((4(1/2(3/2-0.5772156649)+0.5772156649)\pi))))]))-e^{*}Pi+\phi \end{array}$ 

## **Input interpretation**

$$17\left(6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.5772156649\right) + 0.5772156649\right)\pi\right) - e\pi + \phi$$

 $\phi$  is the golden ratio

#### Result

1729.11...

#### 1729.11....

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 =  $8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

#### Alternative representations

$$17\left(6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi\right) - e\pi + \phi = -180^{\circ}e + 2\cos\left(\frac{\pi}{5}\right) + 17\left(68.962 + 747.798^{\circ} + \frac{5760^{\circ}}{5}\right)$$

$$17\left(6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi\right) - e\pi + \phi = -180^{\circ}e + 17\left(68.962 + 747.798^{\circ} + \frac{5760^{\circ}}{5}\right) + 17(68.962 + 747.798^{\circ} + \frac{5760^{\circ}}{5}) + 17(68.962 + 747.798^{\circ} + \frac{5760^{\circ}}{5}) + 1260^{\circ}e^{-1} + 1260^{\circ}e^{-1$$

## Series representations

$$17\left(6+42.6517+20.3103+\frac{32\pi}{5}+4\left(\frac{1}{2}\left(\frac{3}{2}-0.577216\right)+0.577216\right)\pi\right)-e\pi+\phi=1172.35+\phi+717.701\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}-4\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}}{k_2!\left(1+2k_1\right)}$$

$$17\left(6+42.6517+20.3103+\frac{32\pi}{5}+4\left(\frac{1}{2}\left(\frac{3}{2}-0.577216\right)+0.577216\right)\pi\right)-e\pi+\phi=1172.35+\phi+717.701\sum_{k=0}^{\infty}\frac{(-1)^{k}}{1+2k}-4\sum_{k_{1}=0}^{\infty}\sum_{k_{2}=0}^{\infty}\frac{(-1)^{k_{1}}\left(-1+k_{2}\right)^{2}}{k_{2}!\left(1+2k_{1}\right)}$$

$$17\left(6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4\left(\frac{1}{2}\left(\frac{3}{2} - 0.577216\right) + 0.577216\right)\pi\right) - e\pi + \phi = 1172.35 + \phi + 179.425 \times \sum_{k=1}^{\infty} 4^{-k} \left(-1 + 3^{k}\right)\zeta(1 + k) - \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{4^{-k_{2}} \left(-1 + 3^{k_{2}}\right)\zeta(1 + k_{2})}{k_{1}!}$$

n! is the factorial function

 $\zeta(s)$  is the Riemann zeta function

## Integral representations

$$\begin{aligned} &17 \left( 6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4 \left( \frac{1}{2} \left( \frac{3}{2} - 0.577216 \right) + 0.577216 \right) \pi \right) - e\pi + \phi = \\ &1172.35 + \phi + \int_0^\infty \frac{358.851 - 2e}{1 + t^2} \, dt \end{aligned}$$

$$17\left(6+42.6517+20.3103+\frac{32\pi}{5}+4\left(\frac{1}{2}\left(\frac{3}{2}-0.577216\right)+0.577216\right)\pi\right)-e\pi+\phi=1172.35+\phi+\int_{0}^{1}(717.701-4e)\sqrt{1-t^{2}}\,dt$$

$$\begin{aligned} &17 \left( 6 + 42.6517 + 20.3103 + \frac{32\pi}{5} + 4 \left( \frac{1}{2} \left( \frac{3}{2} - 0.577216 \right) + 0.577216 \right) \pi \right) - e\pi + \phi = \\ &1172.35 + \phi + \int_0^\infty \frac{(358.851 - 2e)\sin(t)}{t} \, dt \end{aligned}$$

 $(1/27((17((6+42.6517+20.3103+((32\pi)/5)+[((((4(1/2(3/2-euler-mascheroni constant)+euler-mascheroni constant)\pi)))]))-e*Pi+\Phi)))^2-euler-mascheroni constant)$ 

#### **Input interpretation**

$$\left(\frac{1}{27}\left(17\left(6+42.6517+20.3103+\frac{32\pi}{5}+4\left(\frac{1}{2}\left(\frac{3}{2}-\gamma\right)+\gamma\right)\pi\right)-e\pi+\Phi\right)\right)^{2}-\gamma$$

 $\gamma$  is the Euler-Mascheroni constant  $\Phi$  is the golden ratio conjugate

#### Result

4095.96... 4095.96...  $\approx$  4096 = 64<sup>2</sup>

where 4096 and 64 are fundamental values indicated in the Ramanujan paper "Modular equations and Approximations to  $\pi$ "

Hence

$$\begin{array}{rcl} 64g_{22}^{24} &=& e^{\pi\sqrt{22}}-24+276e^{-\pi\sqrt{22}}-\cdots,\\ 64g_{22}^{-24} &=& 4096e^{-\pi\sqrt{22}}+\cdots, \end{array}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

 $((17((6+42.6517+20.3103+((32\pi)/5)+[((((4(1/2(3/2-euler-mascheroni constant)+euler-mascheroni constant)\pi)))]))-7))^{1/15}$ 

#### **Input interpretation**

$$\sqrt[15]{17\left(6+42.6517+20.3103+\frac{32\pi}{5}+4\left(\frac{1}{2}\left(\frac{3}{2}-\gamma\right)+\gamma\right)\pi\right)-7}$$

 $\gamma$  is the Euler-Mascheroni constant

#### Result

...

1.6438174665462117275920164888233444798961618001879383012861972638

 $1.643817466... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934...$  (trace of the instanton shape)

Now, we have that:

**Theorem 7.1.** The unit ball B is a volume-constrained stable set for  $\operatorname{Per}_s + \beta V_{\alpha}$  if and only if  $\beta \in (0, \beta_{\star}]$ .

From:

$$\mathcal{R}^{\gamma}u(x) := \frac{1}{2^{\gamma}\pi^{\frac{n-1}{2}}} \frac{\Gamma(\frac{n-1-\gamma}{2})}{\Gamma(\frac{\gamma}{2})} \int_{\partial B} \frac{u(y)}{|x-y|^{n-1-\gamma}} d\mathcal{H}_{y}^{n-1}, \quad x \in \partial B.$$
$$\gamma \in (0, n-1)$$

For n = 3 and  $\gamma = 3/2 = 1.5$ , we obtain:

 $1/(2^{1.5*Pi})*gamma(1/2(3-1-1.5))/gamma(1/2*1.5) * Integrate(((1/((x-y)^{(3-1-1.5))}u(y))))))$ 

Input

$$\frac{1}{2^{1.5}\pi} \times \frac{\Gamma\left(\frac{1}{2}\left(3-1-1.5\right)\right)}{\Gamma\left(\frac{1}{2}\times1.5\right)} \int \left(\frac{1}{(x-y)^{3-1-1.5}} u(y)\right) \mathsf{H}^2 \, dx$$

 $\Gamma(x)$  is the gamma function

**Result** 0.665936  $\operatorname{H}^{2} u(y) (x - y)^{0.5}$ 

The study of this function provides the following representations:

#### **Alternate form**

 $0.665936 \operatorname{H}^2 u(y) \sqrt{x-y}$ 

#### Series expansion of the integral at x=0

 $\begin{array}{l} 0.665936\left(-y\right)^{0.5} \mathrm{H}^{2} \, u(y) + \frac{0.332968 \, x \, \mathrm{H}^{2} \, u(y)}{\left(-y\right)^{0.5}} - \frac{0.083242 \, x^{2} \left(\mathrm{H}^{2} \, u(y)\right)}{\left(-y\right)^{1.5}} + \\ \frac{0.041621 \, x^{3} \, \mathrm{H}^{2} \, u(y)}{\left(-y\right)^{2.5}} - \frac{0.0260131 \, x^{4} \left(\mathrm{H}^{2} \, u(y)\right)}{\left(-y\right)^{3.5}} + O\left(x^{5}\right) \\ \mathrm{(Taylor \ series)} \end{array}$ 

#### Indefinite integral assuming all variables are real

 $0.443957 \,\mathrm{H}^2 \, u(y) \, (x-y)^{1.5} + \mathrm{constant}$ 

From:

$$0.665936 (-y)^{0.5} H^{2} u(y) + \frac{0.332968 x H^{2} u(y)}{(-y)^{0.5}} - \frac{0.083242 x^{2} (H^{2} u(y))}{(-y)^{1.5}} + \frac{0.041621 x^{3} H^{2} u(y)}{(-y)^{2.5}} - \frac{0.0260131 x^{4} (H^{2} u(y))}{(-y)^{3.5}} + O(x^{5})$$
(Teulor series)

#### (Taylor series)

#### we obtain:

0.665936 (-y)^0.5  $\Re^2 u(y) + (0.332968 x \, \Re^2 u(y))/(-y)^0.5 - (0.083242 \, x^2 \, (\Re^2 u(y)))/(-y)^1.5 + (0.041621 \, x^3 \, \Re^2 u(y))/(-y)^2.5 - (0.0260131 \, x^4 \, (\Re^2 u(y)))/(-y)^3.5 + O(x^5)$ 

## Input interpretation

$$\begin{array}{l} 0.665936 \sqrt{-y} \ {\mathsf H}^2 \, u(y) + \frac{0.332968 \, x \, {\mathsf H}^2 \, u(y)}{\sqrt{-y}} - \frac{0.083242 \, x^2 \left({\mathsf H}^2 \, u(y)\right)}{(-y)^{1.5}} + \\ \\ \frac{0.041621 \, x^3 \, {\mathsf H}^2 \, u(y)}{(-y)^{2.5}} - \frac{0.0260131 \, x^4 \left({\mathsf H}^2 \, u(y)\right)}{(-y)^{3.5}} + O\!\left(x^5\right) \end{array}$$

#### Result

$$O(x^{5}) - \frac{0.0260131 x^{4} H^{2} u(y)}{(-y)^{3.5}} + \frac{0.041621 x^{3} H^{2} u(y)}{(-y)^{2.5}} - \frac{0.083242 x^{2} H^{2} u(y)}{(-y)^{1.5}} + \frac{0.332968 x H^{2} u(y)}{\sqrt{-y}} + 0.665936 \sqrt{-y} H^{2} u(y)$$

The study of this function provides the following representations:

#### Alternate forms

$$\frac{1}{(-y)^{7/2}} \left( (-y)^{7/2} O(x^5) - H^2 u(y) \left( 0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4 \right) \right)$$

$$O(x^{5}) + \frac{1}{(-y)^{8}} \\ H^{2} u(y) \left(-0.0260131 x^{4} (-y)^{4.5} + 0.041621 x^{3} (-y)^{5.5} - 0.083242 x^{2} (-y)^{6.5} + 0.332968 x (-y)^{7.5} + 0.665936 (-y)^{8.5}\right)$$

$$\frac{1}{(-y)^{7.5} y} (y (-y)^{7.5} O(x^5) - 0.0260131 x^4 y (-y)^4 H^2 u(y) + 0.041621 x^3 y (-y)^5 H^2 u(y) - 0.083242 x^2 y (-y)^6 H^2 u(y) - 0.332968 x (-y)^8 H^2 u(y) + 0.665936 y (-y)^8 H^2 u(y))$$

## Alternate forms assuming x, y, and H are positive

$$\begin{split} O(x^5) + \frac{1}{(-y)^8} \mathrm{H}^2 \, u(y) \\ & \left( - \left( 1.43351 \times 10^{-17} + 0.0260131 \, i \right) x^4 \, y^{4.5} - \left( 1.01967 \times 10^{-16} + 0.041621 \, i \right) \right. \\ & x^3 \, y^{5.5} + \left( 8.16074 \times 10^{-17} - 0.083242 \, i \right) x^2 \, y^{6.5} - \\ & \left( 8.97286 \times 10^{-16} + 0.332968 \, i \right) x \, y^{7.5} + \\ & \left( -4.89757 \times 10^{-16} + 0.665936 \, i \right) y^{8.5} \right) \end{split}$$

$$\begin{split} O(x^5) + \frac{1}{y^8} & (0.665936 \, i) \, \text{H}^2 \, u(y) \\ & \left( - \left( 0.0390625 + 1.67426 \times 10^{-17} \, i \right) x^4 \, y^{4.5} - \left( 0.0625 + 1.91345 \times 10^{-17} \, i \right) \right. \\ & \left. x^3 \, y^{5.5} - \left( 0.125 + 2.29614 \times 10^{-17} \, i \right) x^2 \, y^{6.5} - 0.5 \, x \, y^{7.5} + y^{8.5} \right) \end{split}$$

$$\begin{split} O(x^5) + \frac{\left(1.11495 \times 10^{-17} - 0.0260131 \, i\right) x^4 \, \mathrm{H}^2 \, u(y)}{y^{3.5}} + \\ \frac{\left(1.27423 \times 10^{-17} - 0.041621 \, i\right) x^3 \, \mathrm{H}^2 \, u(y)}{y^{2.5}} + \\ \frac{\left(1.52908 \times 10^{-17} - 0.083242 \, i\right) x^2 \, \mathrm{H}^2 \, u(y)}{y^{1.5}} - \\ \frac{\left(0.332968 \, i\right) x \, \mathrm{H}^2 \, u(y)}{\sqrt{y}} + \left(0.665936 \, i\right) \sqrt{y} \, \mathrm{H}^2 \, u(y) \end{split}$$

## Series expansion at x=0

$$\frac{\left(O(0) + 0.665936 \left(-y\right)^{0.5} H^2 u(y)\right) + \frac{0.332968 x H^2 u(y)}{\left(-y\right)^{0.5}} - \frac{0.083242 x^2 \left(H^2 u(y)\right)}{\left(-y\right)^{1.5}} + \frac{0.041621 x^3 H^2 u(y)}{\left(-y\right)^{2.5}} - \frac{0.0260131 x^4 \left(H^2 u(y)\right)}{\left(-y\right)^{3.5}} + O\left(x^5\right)$$

(Taylor series)

## Series expansion at x=∞

$$\left( -\frac{0.0260131 \left( \mathsf{H}^2 \, u(y) \right) x^4}{(-y)^{3.5}} + \frac{0.041621 \, \mathsf{H}^2 \, u(y) \, x^3}{(-y)^{2.5}} - \frac{0.083242 \left( \mathsf{H}^2 \, u(y) \right) x^2}{(-y)^{1.5}} + \frac{0.332968 \, \mathsf{H}^2 \, u(y) \, x}{(-y)^{0.5}} + 0.665936 \, (-y)^{0.5} \, \mathsf{H}^2 \, u(y) + O\left( \left( \frac{1}{x} \right)^4 \right) \right) + O\left( x^5 \right)$$

#### Derivative

$$\begin{aligned} \frac{\partial}{\partial x} \bigg( O(x^5) - \frac{0.0260131 \, x^4 \, \text{H}^2 \, u(y)}{(-y)^{3.5}} + \frac{0.041621 \, x^3 \, \text{H}^2 \, u(y)}{(-y)^{2.5}} - \\ & \frac{0.083242 \, x^2 \, \text{H}^2 \, u(y)}{(-y)^{1.5}} + \frac{0.332968 \, x \, \text{H}^2 \, u(y)}{\sqrt{-y}} + 0.665936 \, \sqrt{-y} \, \text{H}^2 \, u(y) \bigg) = \\ & 5 \, x^4 \, O'(x^5) + \frac{1}{y^8} \text{H}^2 \, u(y) \left( -0.104052 \, x^3 \, (-y)^{4.5} + 0.124863 \, x^2 \, (-y)^{5.5} - \\ & 0.166484 \, x \, (-y)^{6.5} + 0.332968 \, (-y)^{7.5} \right) \end{aligned}$$

From the above alternate form

$$\frac{1}{(-y)^{7/2}} \left( (-y)^{7/2} O(x^5) - H^2 u(y) \left( 0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4 \right) \right)$$

we obtain:

## Input interpretation

$$\frac{1}{(-y)^{7/2}} ((-y)^{7/2} O(x^5) - H^2 u(y) (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 + y^4 \times (-0.665936)))$$

The study of this function provides the following representations:

#### **Alternate forms**

$$O(x^{5}) + \frac{1}{(-y)^{7/2}} H^{2} u(y) (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} - 0.332968 x y^{3} + 0.665936 y^{4})$$

$$\begin{array}{c} \displaystyle \frac{1}{y^4} 0.665936 \\ \displaystyle (1.50165 \ y^4 \ O(x^5) - 0.0390625 \ x^4 \ \sqrt{-y} \ \ \mathrm{H}^2 \ u(y) - 0.0625 \ x^3 \ \sqrt{-y} \ \ y \ \mathrm{H}^2 \ u(y) - \\ \displaystyle 0.125 \ x^2 \ \sqrt{-y} \ \ y^2 \ \mathrm{H}^2 \ u(y) - 0.5 \ x \ \sqrt{-y} \ \ y^3 \ \mathrm{H}^2 \ u(y) + \sqrt{-y} \ \ y^4 \ \mathrm{H}^2 \ u(y) ) \end{array}$$

## Alternate form assuming x, y, and H are positive

$$O(x^5) - \frac{1}{y^{7/2}} i \operatorname{H}^2 u(y) \left( 0.0260131 \, x^4 + 0.041621 \, x^3 \, y + 0.083242 \, x^2 \, y^2 + 0.332968 \, x \, y^3 - 0.665936 \, y^4 \right)$$

## **Expanded forms**

$$O(x^{5}) - \frac{0.0260131 x^{4} \sqrt{-y} H^{2} u(y)}{y^{4}} - \frac{0.041621 x^{3} \sqrt{-y} H^{2} u(y)}{y^{3}} - \frac{0.083242 x^{2} \sqrt{-y} H^{2} u(y)}{y^{2}} - \frac{0.332968 x \sqrt{-y} H^{2} u(y)}{y} + 0.665936 \sqrt{-y} H^{2} u(y)$$

$$O(x^{5}) - \frac{0.0260131 x^{4} H^{2} u(y)}{(-y)^{7/2}} - \frac{0.041621 x^{3} y H^{2} u(y)}{(-y)^{7/2}} - \frac{0.083242 x^{2} y^{2} H^{2} u(y)}{(-y)^{7/2}} - \frac{0.332968 x y^{3} H^{2} u(y)}{(-y)^{7/2}} + \frac{0.665936 y^{4} H^{2} u(y)}{(-y)^{7/2}}$$

## Series expansion at x=0

$$\left( \begin{array}{c} O(0) + 0.665936 \sqrt{-y} \ \mathsf{H}^2 \, u(y) \right) + \frac{0.332968 \, x \, \mathsf{H}^2 \, u(y)}{\sqrt{-y}} \\ - \frac{0.083242 \, x^2 \left( \mathsf{H}^2 \, u(y) \right)}{(-y)^{3/2}} + \frac{0.041621 \, x^3 \, \mathsf{H}^2 \, u(y)}{(-y)^{5/2}} - \frac{0.0260131 \, x^4 \left( \mathsf{H}^2 \, u(y) \right)}{(-y)^{7/2}} + O\left( x^5 \right) \right)$$

(Taylor series)

## Series expansion at x=∞

$$\left( -\frac{0.0260131 \left( \mathsf{H}^2 \, u(y) \right) x^4}{(-y)^{7/2}} + \frac{0.041621 \, \mathsf{H}^2 \, u(y) \, x^3}{(-y)^{5/2}} - \frac{0.083242 \left( \mathsf{H}^2 \, u(y) \right) x^2}{(-y)^{3/2}} + \frac{0.332968 \, \mathsf{H}^2 \, u(y) \, x}{\sqrt{-y}} + 0.665936 \, \sqrt{-y} \, \, \mathsf{H}^2 \, u(y) + O\left( \left( \frac{1}{x} \right)^4 \right) \right) + O\left( x^5 \right)$$

### Derivative

$$\begin{aligned} \frac{\partial}{\partial x} \left( \frac{1}{(-y)^{7/2}} \\ & \left( (-y)^{7/2} O(x^5) - H^2 u(y) \left( 0.0260131 \, x^4 + 0.041621 \, x^3 \, y + 0.083242 \, x^2 \, y^2 + \right. \\ & \left. 0.332968 \, x \, y^3 - 0.665936 \, y^4 \right) \right) \\ & \left. \frac{y \, H^2 \, u(y) \left( 0.104052 \, x^3 + 0.124863 \, x^2 \, y + 0.166484 \, x \, y^2 + 0.332968 \, y^3 \right)}{(-y)^{9/2}} \end{aligned}$$

Now, we have:

## $\underset{(H_2)^2}{\mathbf{Input}}$

 $H_n$  is the  $n^{\mathrm{th}}$  harmonic number

# Exact result $\frac{9}{4}$

#### **Decimal form**

2.25 2.25 From the result:

 $0.665936 \operatorname{H}^2 u(y) (x - y)^{0.5}$ 

for x = 1 and y = 2, we obtain :

 $0.665936*2.25*2(1-2)^0.5$ 

#### **Input interpretation**

 $0.665936\!\times\!2.25\!\times\!2\,\sqrt{1-2}$ 

#### Result

2.99671... i

#### **Polar coordinates**

r = 2.99671 (radius),  $\theta = 1.5708$  (angle) 2.99671

From the above alternate form:

$$\frac{1}{(-y)^{7/2}} \left( (-y)^{7/2} O(x^5) - \operatorname{H}^2 u(y) \left( 0.0260131 \, x^4 + 0.041621 \, x^3 \, y + 0.083242 \, x^2 \, y^2 + 0.332968 \, x \, y^3 - 0.665936 \, y^4 \right) \right)$$

we have, for x = 1 and y = 2:

1/((-2)^3.5) ((-2)^3.5-2.25\*2(0.0260131+0.041621\*2+0.083242\*2^2+0.332968\*2^3-0.665936\*2^4))

#### **Input interpretation**

 $\begin{array}{c} \displaystyle \frac{1}{(-2)^{3.5}} \\ \left( \left(-2\right)^{3.5} - 2.25 \times 2 \left( 0.0260131 + 0.041621 \times 2 + 0.083242 \times 2^2 + 0.332968 \times 2^3 + 2^4 \times (-0.665936) \right) \right) \end{array}$ 

#### Result

1 + 3.00260... i

Polar coordinates r = 3.16474 (radius),  $\theta = 1.24931$  (angle) 3.16474

From:

$$\frac{1}{2^{\gamma} \pi^{\frac{n-1}{2}}} \frac{\Gamma(\frac{n-1-\gamma}{2})}{\Gamma(\frac{\gamma}{2})} \int_{\partial B} \frac{d\mathcal{H}_{y}^{n-1}}{|x-y|^{n-1-\gamma}} = \mu_{0}^{*}(\gamma) \quad \text{for every } x \in \partial B.$$
(7.15)

we obtain:

 $1/(2^{1.5*Pi})*gamma(1/2(3-1-1.5))/gamma(1/2*1.5) * Integrate(((1/((x-y)^{3-1-1.5)})))))$  $\Re^2$ 

#### Input

$$\frac{1}{2^{1.5}\pi} \times \frac{\Gamma\left(\frac{1}{2}(3-1-1.5)\right)}{\Gamma\left(\frac{1}{2}\times 1.5\right)} \int \frac{1}{(x-y)^{3-1-1.5}} \, \mathrm{H}^2 \, dx$$

 $\Gamma(x)$  is the gamma function

Result

 $0.665936 \text{ H}^2 (x - y)^{0.5}$ 

The study of this function provides the following representations:

#### **Alternate form**

 $0.665936 \text{ H}^2 \sqrt{x-y}$ 

#### Series expansion of the integral at x=0

 $0.665936(-y)^{0.5} H^{2} + \frac{0.332968 x H^{2}}{(-y)^{0.5}} - \frac{0.083242 x^{2} H^{2}}{(-y)^{1.5}} + \frac{0.041621 x^{3} H^{2}}{(-y)^{2.5}} - \frac{0.0260131 x^{4} H^{2}}{(-y)^{3.5}} + O(x^{5})$ (Taylor series)

## Indefinite integral assuming all variables are real

 $0.443957 \text{ H}^2 (x - y)^{1.5} + \text{constant}$ 

From the above expression

$$0.665936(-y)^{0.5} H^{2} + \frac{0.332968 x H^{2}}{(-y)^{0.5}} - \frac{0.083242 x^{2} H^{2}}{(-y)^{1.5}} + \frac{0.041621 x^{3} H^{2}}{(-y)^{2.5}} - \frac{0.0260131 x^{4} H^{2}}{(-y)^{3.5}} + O(x^{5})$$
(Taylor series)

we obtain:

#### **Input interpretation**

$$0.665936 \sqrt{-y} H^{2} + \frac{0.332968 x H^{2}}{\sqrt{-y}} - \frac{0.083242 x^{2} H^{2}}{(-y)^{1.5}} + \frac{0.041621 x^{3} H^{2}}{(-y)^{2.5}} - \frac{0.0260131 x^{4} H^{2}}{(-y)^{3.5}} + O(x^{5})$$

#### Result

$$O(x^{5}) - \frac{0.0260131 x^{4} H^{2}}{(-y)^{3.5}} + \frac{0.041621 x^{3} H^{2}}{(-y)^{2.5}} - \frac{0.083242 x^{2} H^{2}}{(-y)^{1.5}} + \frac{0.332968 x H^{2}}{\sqrt{-y}} + 0.665936 \sqrt{-y} H^{2}$$

The study of this function provides the following representations:

#### **Alternate forms**

$$\frac{1}{(-y)^{7/2}} ((-y)^{7/2} O(x^5) - H^2 (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4))$$

$$O(x^{5}) + \frac{1}{(-y)^{8}} \\ H^{2} \left(-0.0260131 x^{4} (-y)^{4.5} + 0.041621 x^{3} (-y)^{5.5} - 0.083242 x^{2} (-y)^{6.5} + 0.332968 x (-y)^{7.5} + 0.665936 (-y)^{8.5}\right)$$

$$\frac{1}{(-y)^{7.5} y} 0.665936$$

$$(1.50165 y (-y)^{7.5} O(x^5) - 0.0390625 x^4 y (-y)^4 H^2 + 0.0625 x^3 y (-y)^5 H^2 - 0.125 x^2 y (-y)^6 H^2 - 0.5 x (-y)^8 H^2 + y (-y)^8 H^2)$$

## Alternate forms assuming x, y, and H are positive

$$O(x^{5}) + \frac{1}{(-y)^{8.5}} H^{2} ((0.332968 - 3.26204 \times 10^{-16} i) x y^{8} + 0.0260131 x^{4} y^{5} + 0.041621 x^{3} y^{6} + 0.083242 x^{2} y^{7} - 0.665936 y^{9})$$

$$O(x^{5}) + \frac{1}{y^{8}} (0.665936 i) \text{ H}^{2} (-(0.0390625 + 1.67426 \times 10^{-17} i) x^{4} y^{4.5} - (0.0625 + 1.91345 \times 10^{-17} i) x^{3} y^{5.5} - (0.125 + 2.29614 \times 10^{-17} i) x^{2} y^{6.5} - 0.5 x y^{7.5} + y^{8.5})$$

$$O(x^{5}) + \frac{(1.11495 \times 10^{-17} - 0.0260131 i) x^{4} H^{2}}{y^{3.5}} + \frac{(1.27423 \times 10^{-17} - 0.041621 i) x^{3} H^{2}}{y^{2.5}} + \frac{(1.52908 \times 10^{-17} - 0.083242 i) x^{2} H^{2}}{y^{1.5}} - \frac{(0.332968 i) x H^{2}}{\sqrt{y}} + (0.665936 i) \sqrt{y} H^{2}$$

## Series expansion at x=0

$$\left( O(0) + 0.665936 \left( -y \right)^{0.5} \text{H}^2 \right) + \frac{0.332968 \text{ x H}^2}{\left( -y \right)^{0.5}} - \frac{0.083242 \text{ x}^2 \text{H}^2}{\left( -y \right)^{1.5}} + \frac{0.041621 \text{ x}^3 \text{H}^2}{\left( -y \right)^{2.5}} - \frac{0.0260131 \text{ x}^4 \text{H}^2}{\left( -y \right)^{3.5}} + O\left( \text{x}^5 \right) \right)$$

(Taylor series)

## Series expansion at x=∞

$$\left( -\frac{0.0260131 \text{ H}^2 x^4}{(-y)^{3.5}} + \frac{0.041621 \text{ H}^2 x^3}{(-y)^{2.5}} - \frac{0.083242 \text{ H}^2 x^2}{(-y)^{1.5}} + \frac{0.332968 \text{ H}^2 x}{(-y)^{0.5}} + 0.665936 (-y)^{0.5} \text{ H}^2 + O\left(\left(\frac{1}{x}\right)^4\right)\right) + O\left(x^5\right) \right)$$

#### Derivative

$$\begin{aligned} \frac{\partial}{\partial x} \Biggl( O(x^5) &- \frac{0.0260131 \ x^4 \ \text{H}^2}{(-y)^{3.5}} + \frac{0.041621 \ x^3 \ \text{H}^2}{(-y)^{2.5}} - \\ &- \frac{0.083242 \ x^2 \ \text{H}^2}{(-y)^{1.5}} + \frac{0.332968 \ x \ \text{H}^2}{\sqrt{-y}} + 0.665936 \ \sqrt{-y} \ \text{H}^2 \Biggr) = \\ &5 \ x^4 \ O'(x^5) + \frac{1}{y^8} \ \text{H}^2 \left( -0.104052 \ x^3 \ (-y)^{4.5} + 0.124863 \ x^2 \ (-y)^{5.5} - \\ &0.166484 \ x \ (-y)^{6.5} + 0.332968 \ (-y)^{7.5} \right) \end{aligned}$$

From the previous alternate form:

$$\frac{1}{(-y)^{7/2}} ((-y)^{7/2} O(x^5) - H^2 (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4))$$

we obtain:

## Input interpretation

$$\frac{1}{(-y)^{7/2}} ((-y)^{7/2} O(x^5) - H^2 (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 + y^4 \times (-0.665936)))$$

The study of this function provides the following representations:

## Alternate forms

$$O(x^{5}) + \frac{1}{(-y)^{7/2}} H^{2} (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} - 0.332968 x y^{3} + 0.665936 y^{4})$$

$$\frac{1}{y^4} 0.665936 \left(1.50165 \ y^4 \ O(x^5) - 0.0390625 \ x^4 \ \sqrt{-y} \ H^2 - 0.0625 \ x^3 \ \sqrt{-y} \ y \ H^2 - 0.125 \ x^2 \ \sqrt{-y} \ y^2 \ H^2 - 0.5 \ x \ \sqrt{-y} \ y^3 \ H^2 + \sqrt{-y} \ y^4 \ H^2 \right)$$

## Alternate form assuming x, y, and भ are positive

$$O(x^{5}) - \frac{1}{y^{7/2}} i \operatorname{H}^{2} \left( 0.0260131 x^{4} + 0.041621 x^{3} y + 0.083242 x^{2} y^{2} + 0.332968 x y^{3} - 0.665936 y^{4} \right)$$

## **Expanded forms**

$$O(x^{5}) - \frac{0.0260131 x^{4} \sqrt{-y} H^{2}}{y^{4}} - \frac{0.041621 x^{3} \sqrt{-y} H^{2}}{y^{3}} - \frac{0.083242 x^{2} \sqrt{-y} H^{2}}{y^{2}} - \frac{0.332968 x \sqrt{-y} H^{2}}{y} + 0.665936 \sqrt{-y} H^{2}$$

$$O(x^{5}) - \frac{0.0260131 x^{4} H^{2}}{(-y)^{7/2}} - \frac{0.041621 x^{3} y H^{2}}{(-y)^{7/2}} - \frac{0.083242 x^{2} y^{2} H^{2}}{(-y)^{7/2}} - \frac{0.332968 x y^{3} H^{2}}{(-y)^{7/2}} + \frac{0.665936 y^{4} H^{2}}{(-y)^{7/2}}$$

## Series expansion at x=0

$$(O(0) + 0.665936\sqrt{-y} \ H^2) + \frac{0.332968 \ x \ H^2}{\sqrt{-y}} - \frac{0.083242 \ x^2 \ H^2}{(-y)^{3/2}} + \frac{0.041621 \ x^3 \ H^2}{(-y)^{5/2}} - \frac{0.0260131 \ x^4 \ H^2}{(-y)^{7/2}} + O(x^5)$$

(Taylor series)

#### Series expansion at x=∞

$$\left( -\frac{0.0260131 \text{ H}^2 x^4}{(-y)^{7/2}} + \frac{0.041621 \text{ H}^2 x^3}{(-y)^{5/2}} - \frac{0.083242 \text{ H}^2 x^2}{(-y)^{3/2}} + \frac{0.332968 \text{ H}^2 x}{\sqrt{-y}} + 0.665936 \sqrt{-y} \text{ H}^2 + O\left(\left(\frac{1}{x}\right)^4\right)\right) + O\left(x^5\right)$$

#### Derivative

$$\frac{\partial}{\partial x} \left( \frac{1}{(-y)^{7/2}} \left( (-y)^{7/2} O(x^5) - H^2 \left( 0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4 \right) \right) \right) = 5 x^4 O'(x^5) + \frac{H^2 \left( -0.104052 x^3 - 0.124863 x^2 y - 0.166484 x y^2 - 0.332968 y^3 \right)}{(-y)^{7/2}}$$

From the above alternate form:

$$O(x^{5}) + \frac{1}{(-y)^{7/2}} H^{2} (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} - 0.332968 x y^{3} + 0.665936 y^{4})$$

for x = 1 and y = 2, we obtain:

 $1+1/((-2)^{3.5}) \ 2.25((0.0260131-0.041621*2-0.083242*2^{2-0.0332968*2^{3}+0.665936*2^{4}))$ 

## Input interpretation

$$1 + \frac{1}{(-2)^{3.5}} \times 2.25$$

$$(0.0260131 - 0.041621 \times 2 - 0.083242 \times 2^2 + 2^3 \times (-0.332968) + 0.665936 \times 2^4)$$

#### Result

1 + 1.51165... i

#### **Polar coordinates**

r = 1.81248 (radius),  $\theta = 0.986358$  (angle) 1.81248

From the result

 $0.665936 \text{ H}^2 (x - y)^{0.5}$ 

we obtain:

0.665936\*2.25(1-2)^0.5

## Input interpretation $0.665936 \times 2.25\sqrt{1-2}$

#### Result

1.49836... *i* 

**Polar coordinates** r = 1.49836 (radius),  $\theta = 1.5708$  (angle) **1.49836** 

Dividing the two analyzed expressions, we obtain:

 $((O(x^5)+(\Re^2 u(y)(-0.0260131x^4-0.041621x^3y-0.083242x^2y^2-0.332968xy^3+0.665936y^4))/(-y)^{(7/2)}))/((O(x^5)+(\Re^2(-0.0260131x^4-0.041621x^3y-0.083242x^2y^2-0.332968xy^3+0.665936y^4))/(-y)^{(7/2)}))$ 

## Input interpretation

$$\frac{O(x^5) + \frac{H^2 u(y) \left(-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 + x y^3 \times (-0.332968) + 0.665936 y^4\right)}{(-y)^{7/2}}}{O(x^5) + \frac{H^2 \left(-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 + x y^3 \times (-0.332968) + 0.665936 y^4\right)}{(-y)^{7/2}}}$$

The study of this function provides the following representations:

## Alternate forms

$$\frac{38.4422\,(-\,y)^{7/2}\,O\!\left(x^5\right)-\mathsf{H}^2\,u(y)\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)}{38.4422\,(-\,y)^{7/2}\,O\!\left(x^5\right)-\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)}$$

$$\begin{array}{l} \left(1.50165 \; y^4 \; O\!\left(x^5\right) - 0.0390625 \; x^4 \; \sqrt{-y} \; \operatorname{H}^2 u(y) - 0.0625 \; x^3 \; \sqrt{-y} \; \; y \operatorname{H}^2 u(y) - 0.125 \; x^2 \; \sqrt{-y} \; \; y^2 \operatorname{H}^2 u(y) - 0.5 \; x \; \sqrt{-y} \; \; y^3 \operatorname{H}^2 u(y) + \sqrt{-y} \; \; y^4 \operatorname{H}^2 u(y) \right) \right) \\ \left(1.50165 \; y^4 \; O\!\left(x^5\right) - 0.0390625 \; x^4 \; \sqrt{-y} \; \operatorname{H}^2 - 0.0625 \; x^3 \; \sqrt{-y} \; \; y \operatorname{H}^2 - 0.125 \; x^2 \; \sqrt{-y} \; \; y^2 \operatorname{H}^2 - 0.5 \; x \; \sqrt{-y} \; \; y^3 \operatorname{H}^2 + \sqrt{-y} \; \; y^4 \operatorname{H}^2 \right) \end{array}$$

## Expanded forms

$$\begin{split} &- \left( (0.0260131 \ x^4 \ H^2 \ u(y)) \right/ \\ & \left( (-y)^{7/2} \left( O(x^5) - \frac{0.0260131 \ x^4 \ H^2}{(-y)^{7/2}} - \frac{0.041621 \ x^3 \ y \ H^2}{(-y)^{7/2}} - \frac{0.083242 \ x^2 \ y^2 \ H^2}{(-y)^{7/2}} - \frac{0.332968 \ x \ y^3 \ H^2}{(-y)^{7/2}} + \frac{0.665936 \ y^4 \ H^2}{(-y)^{7/2}} \right) \right) \right) - \\ & (0.041621 \ x^3 \ y \ H^2 \ u(y)) \Big/ \left( (-y)^{7/2} \left( O(x^5) - \frac{0.0260131 \ x^4 \ H^2}{(-y)^{7/2}} - \frac{0.332968 \ x \ y^3 \ H^2}{(-y)^{7/2}} + \frac{0.665936 \ y^4 \ H^2}{(-y)^{7/2}} \right) \right) - \\ & \left( 0.041621 \ x^3 \ y \ H^2 \ u(y) \right) \Big/ \left( (-y)^{7/2} \left( O(x^5) - \frac{0.0260131 \ x^4 \ H^2}{(-y)^{7/2}} - \frac{0.332968 \ x \ y^3 \ H^2}{(-y)^{7/2}} + \frac{0.665936 \ y^4 \ H^2}{(-y)^{7/2}} \right) \right) - \\ & \left( (-y)^{7/2} \left( O(x^5) - \frac{0.0260131 \ x^4 \ H^2}{(-y)^{7/2}} - \frac{0.041621 \ x^3 \ y \ H^2}{(-y)^{7/2}} - \frac{0.083242 \ x^2 \ y^2 \ H^2}{(-y)^{7/2}} - \frac{0.083242 \ x^2 \ y^2 \ H^2}{(-y)^{7/2}} - \frac{0.083242 \ x^2 \ y^2 \ H^2}{(-y)^{7/2}} - \frac{0.041621 \ x^3 \ y \ H^2}{(-y)^{7/2}} - \frac{0.041621 \ x^3 \ y \ H^2}{(-y)^{7/2}} - \frac{0.083242 \ x^2 \ y^2 \ H^2}{(-y)^{7/2}} - \frac{0.041621 \ x^3 \ y \ H^2}{(-y)^$$

$$- \left( (0.0260131 x^4 \sqrt{-y} H^2 u(y)) \right) / \left( y^4 \left( O(x^5) + \frac{1}{y^4} \sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4) \right) \right) \right) - \frac{0.041621 x^3 \sqrt{-y} H^2 u(y)}{y^3 \left( O(x^5) + \frac{\sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{y^4} \right) \right) - \frac{0.083242 x^2 \sqrt{-y} H^2 u(y)}{y^2 \left( O(x^5) + \frac{\sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{y^4} \right) \right) - \frac{0.332968 x \sqrt{-y} H^2 u(y)}{y \left( O(x^5) + \frac{\sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{y^4} \right) + \frac{0.665936 \sqrt{-y} H^2 u(y)}{O(x^5) + \frac{\sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{y^4} + \frac{O(x^5)}{O(x^5)} + \frac{\sqrt{-y} H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{y^4} \right)$$

## Alternate forms assuming x, y, and H are positive

$$\frac{\mathsf{H}^2\,u(y)\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}{\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}$$

$$\begin{split} & \left(i \, \mathsf{H}^2 \, u(y) \left(-0.0260131 \, x^4 - 0.041621 \, x^3 \, y - \\ & 0.083242 \, x^2 \, y^2 - 0.332968 \, x \, y^3 + 0.665936 \, y^4)\right) \right/ \\ & \left( y^{7/2} \left( O(x^5) + \frac{1}{y^{7/2}} i \, \mathsf{H}^2 \left(-0.0260131 \, x^4 - 0.041621 \, x^3 \, y - \\ & 0.083242 \, x^2 \, y^2 - 0.332968 \, x \, y^3 + 0.665936 \, y^4)\right) \right) \right) + \\ & \frac{O(x^5)}{O(x^5) + \frac{i \, \mathsf{H}^2 \left(-0.0260131 \, x^4 - 0.041621 \, x^3 \, y - 0.083242 \, x^2 \, y^2 - 0.332968 \, x \, y^3 + 0.665936 \, y^4)}{y^{7/2}} \end{split}$$

## Series expansion at x=0

$$\begin{aligned} & \left( \left( 0.665936 \sqrt{-y} \ \mathsf{H}^2 \, u(y) + \frac{0.332968 \ \mathsf{H}^2 \, u(y) \, x}{\sqrt{-y}} - \frac{0.083242 \left( \mathsf{H}^2 \, u(y) \right) x^2}{(-y)^{3/2}} + \right. \\ & \left. \frac{0.041621 \ \mathsf{H}^2 \, u(y) \, x^3}{(-y)^{5/2}} - \frac{0.0260131 \left( \mathsf{H}^2 \, u(y) \right) x^4}{(-y)^{7/2}} + O(x^6) \right) + O(x^5) \right) \right/ \\ & \left( \left( 0.665936 \ \sqrt{-y} \ \mathsf{H}^2 + \frac{0.332968 \ \mathsf{H}^2 \, x}{\sqrt{-y}} - \frac{0.083242 \ \mathsf{H}^2 \, x^2}{(-y)^{3/2}} + \right. \\ & \left. \frac{0.041621 \ \mathsf{H}^2 \, x^3}{(-y)^{5/2}} - \frac{0.0260131 \ \mathsf{H}^2 \, x^4}{(-y)^{7/2}} + O(x^6) \right) + O(x^5) \right) \end{aligned}$$

## Series expansion at x=∞

$$\begin{pmatrix} \left( -\frac{0.0260131 \left( H^2 u(y) \right) x^4}{(-y)^{7/2}} + \frac{0.041621 H^2 u(y) x^3}{(-y)^{5/2}} - \frac{0.083242 \left( H^2 u(y) \right) x^2}{(-y)^{3/2}} + \frac{0.332968 H^2 u(y) x}{\sqrt{-y}} + 0.665936 \sqrt{-y} H^2 u(y) + O\left( \left( \frac{1}{x} \right)^6 \right) \right) + O(x^5) \right) \\ \left( \left( -\frac{0.0260131 H^2 x^4}{(-y)^{7/2}} + \frac{0.041621 H^2 x^3}{(-y)^{5/2}} - \frac{0.083242 H^2 x^2}{(-y)^{3/2}} + \frac{0.332968 H^2 x}{\sqrt{-y}} + 0.665936 \sqrt{-y} H^2 + O\left( \left( \frac{1}{x} \right)^6 \right) \right) + O(x^5) \right) \right)$$

## Derivative

$$\frac{\partial}{\partial x} \Biggl( \frac{O(x^5) + \frac{H^2 u(y)(-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{O(x^5) + \frac{H^2 (-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4)}{(-y)^{7/2}} \Biggr) = -((3.32968 (-y)^{7/2} H^2 (x^4 O'(x^5) (u(y) (-0.0390625 x^4 - 0.0625 x^3 y - 0.125 x^2 y^2 - 0.5 x y^3 + y^4) + 0.0390625 x^4 + 0.0625 x^3 y + 0.125 x^2 y^2 - 0.5 x y^3 + y^4) + 0.0390625 x^4 + 0.0625 x^3 y + 0.0625 x^3 y + 0.125 x^2 y^2 + 0.5 x y^3 - y^4) + O(x^5) (u(y) (0.03125 x^3 + 0.0375 x^2 y + 0.05 x y^2 + 0.1 y^3) - 0.03125 x^3 - 0.0375 x^2 y - 0.05 x y^2 - 0.1 y^3))) / (y (-y)^{5/2} O(x^5) + H^2 (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4))^2)$$

From:

$$\frac{38.4422\,(-\,y)^{7/2}\,O\!\left(x^5\right)-\mathsf{H}^2\,u(y)\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)}{38.4422\,(-\,y)^{7/2}\,O\!\left(x^5\right)-\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)}$$

we obtain:

## Input interpretation

$$\frac{38.4422 \left(-y\right)^{7/2} O \!\left(x^5\right)-\mathsf{H}^2 u (y) \left(x^4+1.6 \, x^3 \, y+3.2 \, x^2 \, y^2+12.8 \, x \, y^3-25.6 \, y^4\right)}{38.4422 \left(-y\right)^{7/2} O \!\left(x^5\right)-\mathsf{H}^2 \left(x^4+1.6 \, x^3 \, y+3.2 \, x^2 \, y^2+12.8 \, x \, y^3-25.6 \, y^4\right)}$$

The study of this function provides the following representations:

### Alternate forms

$$\frac{192\,211\,(-\,y)^{7/2}\,O\bigl(x^5\bigr)-1000\,\mathsf{H}^2\,u(y)\,\bigl(5\,x^4+8\,x^3\,y+16\,x^2\,y^2+64\,x\,y^3-128\,y^4\bigr)}{192\,211\,(-\,y)^{7/2}\,O\bigl(x^5\bigr)-1000\,\mathsf{H}^2\,\bigl(5\,x^4+8\,x^3\,y+16\,x^2\,y^2+64\,x\,y^3-128\,y^4\bigr)}$$

$$\begin{array}{l} \left( 38.4422\,\sqrt{-y} \;\;y^3\;O\!\left(x^5\right) + x^4\;{\rm H}^2\;u(y) + 1.6\,x^3\;y\,{\rm H}^2\;u(y) + \\ \;\;3.2\,x^2\;y^2\;{\rm H}^2\;u(y) + 12.8\,x\;y^3\;{\rm H}^2\;u(y) - 25.6\,y^4\;{\rm H}^2\;u(y) \right) \right/ \\ \left( 38.4422\,\sqrt{-y}\;\;y^3\;O\!\left(x^5\right) + x^4\;{\rm H}^2 + 1.6\,x^3\;y\,{\rm H}^2 + \\ \;\;3.2\,x^2\;y^2\,{\rm H}^2 + 12.8\,x\;y^3\,{\rm H}^2 - 25.6\,y^4\,{\rm H}^2 \right) \end{array}$$

## Expanded forms

$$-\frac{x^4 \operatorname{H}^2 u(y)}{-38.4422 \sqrt{-y} \ y^3 O(x^5) - \operatorname{H}^2 (x^4 + 1.6 \ x^3 \ y + 3.2 \ x^2 \ y^2 + 12.8 \ x \ y^3 - 25.6 \ y^4)}{1.6 \ x^3 \ y \ H^2 u(y)} - \frac{1.6 \ x^3 \ y \ H^2 u(y)}{-38.4422 \sqrt{-y} \ y^3 O(x^5) - \operatorname{H}^2 (x^4 + 1.6 \ x^3 \ y + 3.2 \ x^2 \ y^2 + 12.8 \ x \ y^3 - 25.6 \ y^4)}{12.8 \ x \ y^3 \ H^2 u(y)} - \frac{12.8 \ x \ y^3 \ H^2 u(y)}{-38.4422 \sqrt{-y} \ y^3 O(x^5) - \operatorname{H}^2 (x^4 + 1.6 \ x^3 \ y + 3.2 \ x^2 \ y^2 + 12.8 \ x \ y^3 - 25.6 \ y^4)} + \frac{12.8 \ x \ y^3 \ H^2 u(y)}{25.6 \ y^4 \ H^2 u(y)} - \frac{12.8 \ x \ y^3 \ H^2 \ y^3 \ H^2 u(y)}{-38.4422 \ \sqrt{-y} \ y^3 \ O(x^5) - \operatorname{H}^2 (x^4 + 1.6 \ x^3 \ y + 3.2 \ x^2 \ y^2 + 12.8 \ x \ y^3 - 25.6 \ y^4)} + \frac{12.8 \ x \ y^3 \ H^2 \ y^3 \ H^2 \$$

## Alternate forms assuming x, y, and H are positive

$$\frac{\mathsf{H}^2\,u(y)\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}{\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}$$

$$-\frac{\mathsf{H}^2\,u(y)\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)}{-\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(-38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}{-\mathsf{H}^2\left(x^4+1.6\,x^3\,y+3.2\,x^2\,y^2+12.8\,x\,y^3-25.6\,y^4\right)+(-38.4422\,i)\,y^{7/2}\,O\!\left(x^5\right)}$$

## Series expansion at x=0

$$\begin{split} & \left( \left( -0.665935 \left( y^4 \text{ H}^2 u(y) \right) + 0.332967 \ y^3 \text{ H}^2 u(y) \ x + \\ & 0.0832419 \ y^2 \text{ H}^2 u(y) \ x^2 + 0.0416209 \ y \text{ H}^2 u(y) \ x^3 + \\ & 0.0260131 \text{ H}^2 u(y) \ x^4 + O(x^6) \right) - (-y)^{7/2} O(x^5) \right) / \\ & \left( \left( -0.665935 \left( y^4 \text{ H}^2 \right) + 0.332967 \ y^3 \text{ H}^2 \ x + 0.0832419 \ y^2 \text{ H}^2 \ x^2 + \\ & 0.0416209 \ y \text{ H}^2 \ x^3 + 0.0260131 \text{ H}^2 \ x^4 + O(x^6) \right) - (-y)^{7/2} O(x^5) \right) \end{split}$$

## Series expansion at x=∞

$$\begin{pmatrix} \left(0.0260131 \text{ H}^2 u(y) x^4 + 0.0416209 y \text{ H}^2 u(y) x^3 + \\ 0.0832419 y^2 \text{ H}^2 u(y) x^2 + 0.332967 y^3 \text{ H}^2 u(y) x - \\ 0.665935 (y^4 \text{ H}^2 u(y)) + O\left(\left(\frac{1}{x}\right)^6\right)\right) - (-y)^{7/2} O(x^5)\right) \\ \left(\left(0.0260131 \text{ H}^2 x^4 + 0.0416209 y \text{ H}^2 x^3 + 0.0832419 y^2 \text{ H}^2 x^2 + \\ 0.332967 y^3 \text{ H}^2 x - 0.665935 (y^4 \text{ H}^2) + O\left(\left(\frac{1}{x}\right)^6\right)\right) - (-y)^{7/2} O(x^5)\right) \right)$$

### Derivative

$$\begin{array}{l} \frac{\partial}{\partial x} \left( \left( 38.4422 \left( -y \right)^{7/2} O\left( x^5 \right) - \right. \\ \left. \left. \begin{array}{l} {\rm H}^2 \, u(y) \left( x^4 + 1.6 \, x^3 \, y + 3.2 \, x^2 \, y^2 + 12.8 \, x \, y^3 - 25.6 \, y^4 \right) \right) \right) \\ \left( 38.4422 \left( -y \right)^{7/2} O\left( x^5 \right) - \right. \\ \left. {\rm H}^2 \left( x^4 + 1.6 \, x^3 \, y + 3.2 \, x^2 \, y^2 + 12.8 \, x \, y^3 - 25.6 \, y^4 \right) \right) \right) = \\ \left( \left( -y \right)^{5/2} \, y \, {\rm H}^2 \left( x^4 \, O' \left( x^5 \right) \left( u(y) \left( - 192.211 \, x^4 - 307.538 \, x^3 \, y - 615.075 \, x^2 \, y^2 - 2460.3 \, x \, y^3 + 4920.6 \, y^4 \right) + 192.211 \, x^4 + 307.538 \, x^3 \, y + 615.075 \, x^2 \, y^2 + 2460.3 \, x \, y^3 - 4920.6 \, y^4 \right) + \\ \left. O \left( x^5 \right) \left( u(y) \left( 153.769 \, x^3 + 184.523 \, x^2 \, y - 246.03 \, x \, y^2 - 492.06 \, y^3 \right) - 153.769 \, x^3 - 184.523 \, x^2 \, y - 246.03 \, x \, y^2 - 492.06 \, y^3 \right) \right) \right) \right) \\ \left( \left( 38.4422 \left( -y \right)^{7/2} O \left( x^5 \right) - {\rm H}^2 \left( x^4 + 1.6 \, x^3 \, y + 3.2 \, x^2 \, y^2 + 12.8 \, x \, y^3 - 25.6 \, y^4 \right) \right)^2 \right) \right) \right) \right) \right) \\ \end{array} \right)$$

Subtracting the two above expressions, we obtain:

 $(\Re^2 u(y) (-0.0260131 2^4 - 0.041621 2 - 0.083242 2^2 - 0.332968 2^3 + 0.665936 2^4))/(-2)^{(7/2)} - (\Re^2 (-0.0260131 - 0.041621 2 - 0.083242 2^2 - 0.332968 2^3 + 0.665936 2^4))/(-2)^{(7/2)}$ 

#### **Input interpretation**

$$\frac{1}{(-2)^{7/2}} H^2 u(y) \left(-0.0260131 \times 2^4 - 0.041621 \times 2 - 0.083242 \times 2^2 + 2^3 \times (-0.332968) + 0.665936 \times 2^4\right) - \frac{1}{(-2)^{7/2}} H^2 \left(-0.0260131 - 0.041621 \times 2 - 0.083242 \times 2^2 + 2^3 \times (-0.332968) + 0.665936 \times 2^4\right)$$

**Result**  $(0.632756 i) \operatorname{H}^2 u(y) - (0.667244 i) \operatorname{H}^2$ 

The study of this function provides the following representations:

#### Alternate forms

 $i \operatorname{H}^2(0.632756 u(y) - 0.667244)$ 

 $\operatorname{H}^2\left((0.632756\,i)\,u(y)-0.667244\,i\right)$ 

 $(0.632756 i) \operatorname{H}^2 ((1 + 0 i) u(y) - (1.05451 + 0 i))$ 

#### Alternate form assuming y and H are real

 $0 + i (0.632756 \text{ H}^2 u(y) - 0.667244 \text{ H}^2 + 0)$ 

#### Properties as a real function Domain

Ø

#### Range

Ø

 $\boldsymbol{\phi}$  is the set with no elements

#### Series expansion at y=0

 $\begin{array}{l} ((0.632756\,i)\,u(0)-0.667244\,i)\,\mathrm{H}^2 + \\ (0.632756\,i)\,y\,\mathrm{H}^2\,u'(0)+(0.316378\,i)\,y^2\,\mathrm{H}^2\,u''(0) + \\ (0.105459\,i)\,u^{(3)}(0)\,y^3\,\mathrm{H}^2 + (0.0263648\,i)\,u^{(4)}(0)\,y^4\,\mathrm{H}^2 + O\!\left(y^5\right) \\ (\text{Taylor series}) \end{array}$ 

#### Derivative

$$\frac{\partial}{\partial y} ((0.632756 \, i) \, \mathrm{H}^2 \, u(y) - (0.667244 \, i) \, \mathrm{H}^2) = (0.632756 \, i) \, \mathrm{H}^2 \, u'(y)$$

From:

 $(0.632756 i) \operatorname{H}^2 u(y) - (0.667244 i) \operatorname{H}^2$ 

we obtain:

(0.632756 i) (2.25) (2) - (0.667244 i) (2.25)

#### **Input interpretation**

 $(0.632756\,i) \times 2.25 \times 2 - (0.667244\,i) \times 2.25$ 

#### Result

1.34610... *i* 

#### **Polar coordinates**

r = 1.3461 (radius),  $\theta = 1.5708$  (angle) 1.3461 i is the imaginary unit

Considering only the result of the second integral, we obtain also:

$$\frac{1}{(-y)^{7/2}} ((-y)^{7/2} O(x^5) - H^2 (0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4))$$

 $((O(x^5)+(9t^2(-0.0260131x^4-0.041621x^3y-0.083242x^2y^2-0.332968xy^3+0.665936y^4))/(-y)^{(7/2)}))$ 

#### **Input interpretation**

$$O(x^{5}) + \frac{1}{(-y)^{7/2}} H^{2} (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} + x y^{3} \times (-0.332968) + 0.665936 y^{4})$$

#### Result

$$O(x^5) + \frac{1}{(-y)^{7/2}} H^2 \left(-0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4\right)$$

The study of this function provides the following representations:

#### **Alternate forms**

$$\frac{1}{(-y)^{7/2}} \left( (-y)^{7/2} O(x^5) - H^2 \left( 0.0260131 x^4 + 0.041621 x^3 y + 0.083242 x^2 y^2 + 0.332968 x y^3 - 0.665936 y^4 \right) \right)$$

$$O(x^{5}) + H^{2} \left( -\frac{0.0260131 x^{4}}{(-y)^{7/2}} - \frac{0.041621 x^{3} y}{(-y)^{7/2}} - \frac{0.083242 x^{2} y^{2}}{(-y)^{7/2}} - \frac{0.332968 x y^{3}}{(-y)^{7/2}} + \frac{0.665936 y^{4}}{(-y)^{7/2}} \right)$$

$$\frac{1}{y^4} 0.665936 \left(1.50165 \ y^4 \ O(x^5) - 0.0390625 \ x^4 \ \sqrt{-y} \ H^2 - 0.0625 \ x^3 \ \sqrt{-y} \ y \ H^2 - 0.125 \ x^2 \ \sqrt{-y} \ y^2 \ H^2 - 0.5 \ x \ \sqrt{-y} \ y^3 \ H^2 + \sqrt{-y} \ y^4 \ H^2 \right)$$

## Alternate form assuming x, y, and H are positive

$$O(x^{5}) + \frac{1}{y^{7/2}}i H^{2} (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} - 0.332968 x y^{3} + 0.665936 y^{4})$$

## Expanded forms

$$O(x^{5}) - \frac{0.0260131 x^{4} \sqrt{-y} H^{2}}{y^{4}} - \frac{0.041621 x^{3} \sqrt{-y} H^{2}}{y^{3}} - \frac{0.083242 x^{2} \sqrt{-y} H^{2}}{y^{2}} - \frac{0.332968 x \sqrt{-y} H^{2}}{y} + 0.665936 \sqrt{-y} H^{2}$$

$$O(x^{5}) - \frac{0.0260131 x^{4} H^{2}}{(-y)^{7/2}} - \frac{0.041621 x^{3} y H^{2}}{(-y)^{7/2}} - \frac{0.083242 x^{2} y^{2} H^{2}}{(-y)^{7/2}} - \frac{0.332968 x y^{3} H^{2}}{(-y)^{7/2}} + \frac{0.665936 y^{4} H^{2}}{(-y)^{7/2}}$$

## Series expansion at x=0

$$\begin{split} & \left(O(0) + 0.665936 \sqrt{-y} \ \text{H}^2\right) + \frac{0.332968 \ x \ \text{H}^2}{\sqrt{-y}} - \\ & \frac{0.083242 \ x^2 \ \text{H}^2}{(-y)^{3/2}} + \frac{0.041621 \ x^3 \ \text{H}^2}{(-y)^{5/2}} - \frac{0.0260131 \ x^4 \ \text{H}^2}{(-y)^{7/2}} + O\left(x^5\right) \\ & \text{(Taylor series)} \end{split}$$

## Series expansion at x=∞

$$\left( -\frac{0.0260131 \text{ H}^2 x^4}{(-y)^{7/2}} + \frac{0.041621 \text{ H}^2 x^3}{(-y)^{5/2}} - \frac{0.083242 \text{ H}^2 x^2}{(-y)^{3/2}} + \frac{0.332968 \text{ H}^2 x}{\sqrt{-y}} + 0.665936 \sqrt{-y} \text{ H}^2 + O\left(\left(\frac{1}{x}\right)^4\right)\right) + O\left(x^5\right)$$

### Derivative

$$\frac{\partial}{\partial x} \left( O(x^5) + \frac{1}{(-y)^{7/2}} H^2 \left( -0.0260131 x^4 - 0.041621 x^3 y - 0.083242 x^2 y^2 - 0.332968 x y^3 + 0.665936 y^4 \right) \right) = 5 x^4 O'(x^5) + \frac{H^2 \left( -0.104052 x^3 - 0.124863 x^2 y - 0.166484 x y^2 - 0.332968 y^3 \right)}{(-y)^{7/2}}$$

From the previous result:

$$O(x^{5}) + \frac{1}{(-y)^{7/2}} H^{2} (-0.0260131 x^{4} - 0.041621 x^{3} y - 0.083242 x^{2} y^{2} - 0.332968 x y^{3} + 0.665936 y^{4})$$

we obtain, for x = 1 and y = 2:

## Input interpretation

$$O(1^{5}) + \frac{1}{(-2)^{7/2}} H^{2} (-0.0260131 \times 2^{4} - 0.041621 \times 2 - 0.083242 \times 2^{2} + 2^{3} \times (-0.332968) + 0.665936 \times 2^{4})$$

**Result**  $O(1) + (0.632756 i) \text{H}^2$ 

The study of this function provides the following representations:

#### **Alternate forms**

 $O(1) + (0.632756\,i)\,{\rm H}^2$ 

 $(0.632756 + 0 i) ((1.58039 + 0 i) O(1) + i H^2)$ 

 $(1 + 0 i) O(1) + (0.632756 i) \text{H}^2$ 

## Alternate form assuming भ is real

 $O(1) + i(0.632756 \text{ H}^2 + 0) + 0$ 

#### **Complex roots**

$$H = -(1.25714 i) \sqrt[4]{\text{Re}(O(1))^2} \\ \sin\left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) - \\ 1.25714 \sqrt[4]{\text{Re}(O(1))^2} \cos\left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1))\right) - \\ \left(\frac{1}{2} \tan^{-1}(1.58039 i) O(1)\right) - \\$$

$$H = (1.25714 i) \sqrt[4]{\text{Re}(O(1))^2} \sin\left(\frac{1}{2}\tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right) + (1.25714 \sqrt[4]{\text{Re}(O(1))^2} \cos\left(\frac{1}{2}\tan^{-1}(\text{Re}((1.58039 i) O(1)), \text{Im}((1.58039 i) O(1)))\right)$$

 $\operatorname{Re}(z)$  is the real part of z

 $\operatorname{Im}(z)$  is the imaginary part of z

 $\tan^{-1}(x, y)$  is the inverse tangent function

## **Polynomial discriminant**

 $\Delta_{\rm H} = (-2.53102 \, i) \, O(1)$ 

#### Property as a function Parity

even

#### Derivative

 $\frac{d}{dH} (O(1) + (0.632756 i) H^2) = (1.26551 i) H$ 

#### **Indefinite integral**

$$\int ((0.632756 i) \operatorname{H}^2 + O(1)) d\operatorname{H} = O(1) \operatorname{H} + (0.210919 i) \operatorname{H}^3 + \text{constant}$$

From:

(0.632756 i) \H^2 + O(1)

we obtain:

(0.632756 i) (2.25) + 1

#### **Input interpretation**

 $(0.632756 i) \times 2.25 + 1$ 

#### Result

1 + 1.42370... i

#### **Polar coordinates**

r = 1.73981 (radius),  $\theta = 0.958465$  (angle) 1.73981 i is the imaginary unit

From the division:

$$\frac{38.4422 (-y)^{7/2} O(x^5) - H^2 u(y) (x^4 + 1.6 x^3 y + 3.2 x^2 y^2 + 12.8 x y^3 - 25.6 y^4)}{38.4422 (-y)^{7/2} O(x^5) - H^2 (x^4 + 1.6 x^3 y + 3.2 x^2 y^2 + 12.8 x y^3 - 25.6 y^4)}$$

we obtain, for x = 1 and y = 2:

(38.4422 (-2)<sup>(7/2)</sup> - (2.25)(2) (1 + 1.6 2 + 3.2 2<sup>2</sup> + 12.8 2<sup>3</sup> - 25.6 2<sup>4</sup>))/(38.4422 (-2)<sup>(7/2)</sup> - (2.25) (1 + 1.6 2 + 3.2 2<sup>2</sup> + 12.8 2<sup>3</sup> - 25.6 2<sup>4</sup>))

#### **Input interpretation**

 $\frac{38.4422 \left(-2\right)^{7/2} - \left(2.25 \times 2\right) \left(1 + 1.6 \times 2 + 3.2 \times 2^2 + 12.8 \times 2^3 - 25.6 \times 2^4\right)}{38.4422 \left(-2\right)^{7/2} - 2.25 \left(1 + 1.6 \times 2 + 3.2 \times 2^2 + 12.8 \times 2^3 - 25.6 \times 2^4\right)}$ 

#### Result

1.69268... + 0.461385... i

#### **Polar coordinates**

r = 1.75443 (radius),  $\theta = 0.266112$  (angle) 1.75443

From:

 $(0.632756 i) \operatorname{H}^2 u(y) - (0.667244 i) \operatorname{H}^2$ 

and

$$\frac{38.4422 \left(-2\right)^{7/2}-\left(2.25 \times 2\right) \left(1+1.6 \times 2+3.2 \times 2^2+12.8 \times 2^3-25.6 \times 2^4\right)}{38.4422 \left(-2\right)^{7/2}-2.25 \left(1+1.6 \times 2+3.2 \times 2^2+12.8 \times 2^3-25.6 \times 2^4\right)}$$

we obtain, after some calculations:

1/(1/4(((38.4422 (-2)^(7/2) - (2.25)(2) (1+1.6\*2+3.2\*4+12.8\*8 - 25.6\*16))/(38.4422 (-2)^(7/2) - (2.25) (1 + 1.6\*2 + 3.2\*4 + 12.8\*8 - 25.6\*16)))+(((0.632756 i) (2.25) (2) - (0.667244 i) (2.25)))))

#### **Input interpretation**

$$1 \left/ \left( \frac{1}{4} \left( \frac{38.4422 \left(-2\right)^{7/2} - \left(2.25 \times 2\right) \left(1 + 1.6 \times 2 + 3.2 \times 4 + 12.8 \times 8 - 25.6 \times 16\right)}{38.4422 \left(-2\right)^{7/2} - 2.25 \left(1 + 1.6 \times 2 + 3.2 \times 4 + 12.8 \times 8 - 25.6 \times 16\right)} + \left( \left(0.632756 \, i\right) \times 2.25 \times 2 - \left(0.667244 \, i\right) \times 2.25 \right) \right) \right)$$

i is the imaginary unit

#### Result

1.10413... – 1.17902... i

#### **Polar coordinates**

r = 1.6153 (radius),  $\theta = -0.818188$  (angle) 1.6153 result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the two already analyzed expressions:

#### (0.632756 i) (2.25) (2) - (0.667244 i) (2.25)

#### **Input interpretation**

 $(0.632756 i) \times 2.25 \times 2 - (0.667244 i) \times 2.25$ 

#### Result

1.34610... *i* 

#### **Polar coordinates**

r = 1.3461 (radius),  $\theta = 1.5708$  (angle) 1.3461

#### And:

(0.632756 i)  $\Re^2 + O(1)$ 

(0.632756 i) (2.25) + 1

i is the imaginary unit

**Input interpretation** 

 $(0.632756\,i) \times 2.25 + 1$ 

#### Result

1 + 1.42370... i

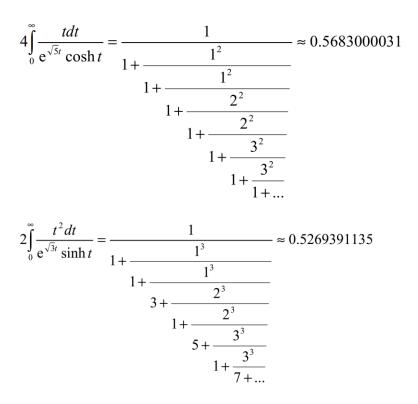
#### **Polar coordinates**

r = 1.73981 (radius),  $\theta = 0.958465$  (angle) 1.73981

after some calculations, we obtain:

 $((2((((1/(2Pi)(((0.632756 i) (2.25) + 1 + ((0.632756 i) (2.25) (2) - (0.667244 i) (2.25))))^2))))^{(1/(0.5683000031 + 0.5269391135 + 0.9568666373))))$ 

where 0.5683000031, 0.5269391135 and 0.9568666373 are the values of the following Rogers-Ramanujan continued fractions:



and

i is the imaginary unit

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}}} \approx 0.9568666373$$

#### **Input interpretation**

$$\begin{pmatrix} 2\left(\frac{1}{2\pi}\left((0.632756\,i\right) \times 2.25 + 1 + \left((0.632756\,i\right) \times 2.25 \times 2 - \left(0.667244\,i\right) \times 2.25\right)\right)^2 \\ \left(\frac{1}{0.5683000031 + 0.5269391135 + 0.9568666373}\right)$$

i is the imaginary unit

#### Result

0.604644...+ 1.52462... i

#### **Polar coordinates**

r = 1.64014 (radius),  $\theta = 1.19324$  (angle)  $1.64014 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$  ... (trace of the instanton shape)

#### From:

$$\mathscr{R}_{\alpha}u(x) := 2 \int_{\partial B} \frac{u(x) - u(y)}{|x - y|^{n - \alpha}} \, d\mathcal{H}_{y}^{n - 1}, \quad u \in C^{1}(\partial B),$$

For x = 1, y = 2, u = 8+4i, n = 3;  $\alpha = 2$ , and considering always:

## Input $(H_2)^2$

 $H_n$  is the  $n^{\mathrm{th}}$  harmonic number

# Exact result $\frac{9}{4}$

Decimal form 2.25

2.25

From

$$\mathcal{R}_{\alpha}u(x):=2\int_{\partial B}\frac{u(x)-u(y)}{|x-y|^{n-\alpha}}\,d\mathcal{H}_{y}^{n-1},\ \ u\in C^{1}(\partial B),$$

we obtain:

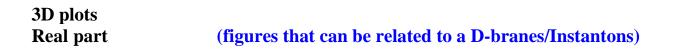
 $2* integrate(((1/(x-y)*((8+4i)x-(8+4i)y))))) \Re^{2}$ 

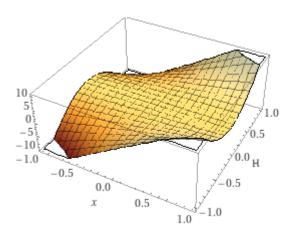
#### **Indefinite integral**

$$2\int \frac{((8+4i)x - (8+4i)y)H^2}{x - y} dx = (16+8i)xH^2 + \text{constant}$$

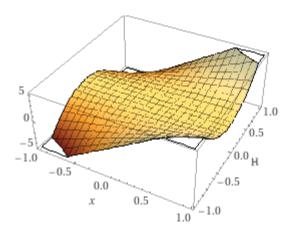
i is the imaginary unit

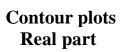
#### The study of this function provides the following representations:

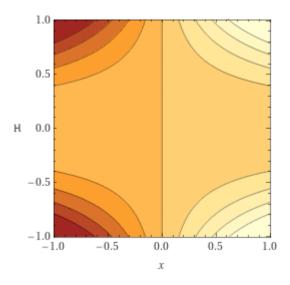




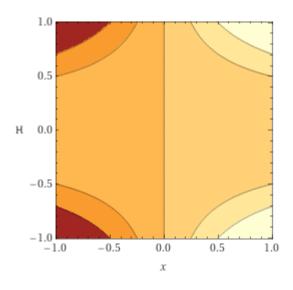
## Imaginary part







## **Imaginary part**



#### (16 + 8 i) (2.25)

#### Input

 $(16 + 8i) \times 2.25$ 

i is the imaginary unit

#### Result

36. + 18. *i* 

#### **Polar coordinates**

r = 40.2492 (radius),  $\theta = 0.463648$  (angle) 40.2492

From:

$$[u]_{\frac{1-\alpha}{2}}^2 = \iint_{\partial B \times \partial B} \frac{|u(x) - u(y)|^2}{|x - y|^{n-\alpha}} d\mathcal{H}_x^{n-1} \mathcal{H}_y^{n-1} = \int_{\partial B} u \,\mathscr{R}_\alpha u \, d\mathcal{H}^{n-1}.$$
(7.16)

If  $\alpha \in (1, n)$  then  $\gamma = \alpha - 1 \in (0, n - 1)$ , and thus we can deduce from (7.15) and (7.16) that

we obtain:

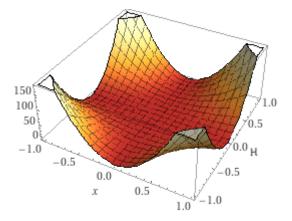
Integrate((((8+4i)\*((16 + 8i)\*x (2.25))\*(8+4i))))  $\Re^2$ 

## Indefinite integral

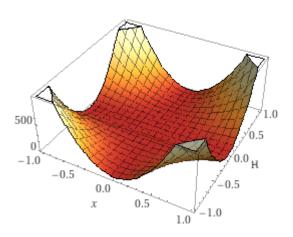
$$\int ((8+4\,i)\,((16+8\,i)\,x\,2.25)\,(8+4\,i))\,\mathsf{H}^2\,dx = (288+1584\,i)\,x^2\,\mathsf{H}^2 + \text{constant}$$

The study of this function provides the following representations:

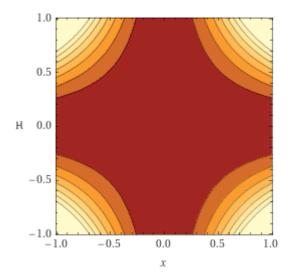
## 3D plotsReal part(figures that can be related to a D-branes/Instantons)



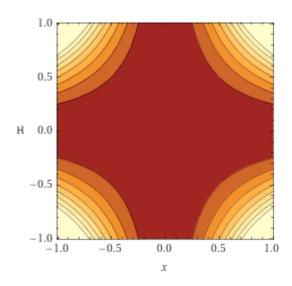
## Imaginary part



### Contour plots Real part



#### **Imaginary part**



## Alternate form assuming x and $\Re$ are real 288 $x^2 H^2 + i (1584 x^2 H^2 + 0) + 0 + constant$

(288 + 1584 i) (2.25)

#### Input

 $(288 + 1584 i) \times 2.25$ 

i is the imaginary unit

**Result**  $6.48... \times 10^2 + 3.564... \times 10^3 i$ 

#### **Polar coordinates**

r = 3622.43 (radius),  $\theta = 1.39094$  (angle) 3622.43

From the two previous expressions, after some calculations, we obtain:

(1/2((288 + 1584 i) (2.25)) - (1/2((16 + 8 i) (2.25)))i)-64i-Pi\*i

#### Input

 $\left(\frac{1}{2}\left((288+1584\,i)\times 2.25\right)-\left(\frac{1}{2}\left((16+8\,i)\times 2.25\right)\right)i\right)-64\,i-\pi\,i$ 

i is the imaginary unit

#### Result

333 + 1696.86... i

#### **Polar coordinates**

r = 1729.22 (radius),  $\theta = 1.37701$  (angle)

#### 1729.22

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 =  $8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

## **Polar forms**

 $1729.22\left(\cos(1.37701)+i\sin(1.37701)\right)$ 

 $1729.22 \, e^{1.37701 \, i}$ 

The study of this function provides the following representations:

## Alternative representations

$$\left(\frac{1}{2} \left(288 + 1584 \, i\right) 2.25 - \frac{1}{2} \, i \left((16 + 8 \, i\right) 2.25\right) \right) - i \, 64 - i \, \pi = -64 \, i - 180^{\circ} i - 1.125 \, i \left(16 + 8 \, i\right) + 1.125 \left(288 + 1584 \, i\right)$$

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
-64*i*-1.125*i*(16+8*i*)+1.125(288+1584*i*)+*i*<sup>2</sup>log(-1)

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
-64*i*-1.125*i*(16+8*i*)+1.125(288+1584*i*)-*i*cos<sup>-1</sup>(-1)

## Series representations

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
324+1700*i*-9*i*<sup>2</sup>-4*i* $\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}}{1+2\,k}$ 

$$\left(\frac{1}{2} \left(288 + 1584 \, i\right) 2.25 - \frac{1}{2} \, i \left(\left(16 + 8 \, i\right) 2.25\right)\right) - i \, 64 - i \, \pi = \\ 324 + 1702 \, i - 9 \, i^2 - 2 \, i \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi = \\ 324+1700\,i-9\,i^2-i\sum_{k=0}^{\infty}\frac{2^{-k}\left(-6+50\,k\right)}{\binom{3\,k}{k}}$$

## Integral representations

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
324+1700*i*-9*i*<sup>2</sup>-2*i* $\int_{0}^{\infty}\frac{1}{1+t^{2}}\,dt$ 

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left((16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
324+1700*i*-9*i*<sup>2</sup>-4*i* $\int_{0}^{1}\sqrt{1-t^{2}}\,dt$ 

$$\left(\frac{1}{2}\left(288+1584\,i\right)2.25-\frac{1}{2}\,i\left(\left(16+8\,i\right)2.25\right)\right)-i\,64-i\,\pi=$$
  
324+1700*i*-9*i*<sup>2</sup>-2*i* $\int_{0}^{\infty}\frac{\sin(t)}{t}\,dt$ 

From:

$$\mathscr{R}_{\alpha} = 2^{\alpha} \pi^{\frac{n-1}{2}} \frac{\Gamma(\frac{\alpha-1}{2})}{\Gamma(\frac{n-\alpha}{2})} \left( \mu_0^*(\alpha-1) \operatorname{Id} - \mathcal{R}^{\alpha-1} \right), \quad \alpha \in (1, n).$$

for:

 $\mathcal{R} = (16 + 8 i) (2.25)$ 

#### Input

 $(16+8\,i)\times2.25$ 

i is the imaginary unit

#### Result

36. + 18. *i* 

#### **Polar coordinates**

r = 40.2492 (radius),  $\theta = 0.463648$  (angle) 40.2492

#### $\mu = 0.665936 * 2.25(1-2)^{0.5}$

## **Input interpretation**

 $0.665936 \times 2.25\sqrt{1-2}$ 

#### Result

1.49836... i

## **Polar coordinates**

r = 1.49836 (radius),  $\theta = 1.5708$  (angle) 1.49836

and for: x = 1, y = 2, u = 8+4i, n = 3;  $\alpha = 2$ , we obtain:

2^2\*Pi\*gamma(1/2)/gamma(1/2) (((2\*((0.665936\*2.25(1-2)^0.5))-((((16 + 8 i) (2.25))^2)))))

#### **Input interpretation**

$$2^{2} \pi \times \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2})} \left( 2 \left( 0.665936 \times 2.25 \sqrt{1-2} \right) - \left( (16+8 i) \times 2.25 \right)^{2} \right)$$

 $\Gamma(x)$  is the gamma function

i is the imaginary unit

#### Result

– 12214.5... – 16248.4... i

#### **Polar coordinates**

r = 20327.4 (radius),  $\theta = -2.21541$  (angle)

20327.4

The study of this function provides the following representations:

#### **Polar forms**

 $20327.4(\cos(-2.21541) + i\sin(-2.21541))$ 

 $20327.4 e^{-2.21541 i}$ 

#### **Alternative representations**

$$\frac{2^2 \Gamma(\frac{1}{2}) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma(\frac{1}{2})} = \frac{\Gamma(\frac{1}{2})}{4 \pi \left(-\frac{1}{2}\right)! \left(2.99671 \sqrt{-1} - (2.25 (16+8 i))^2\right)}{\left(-\frac{1}{2}\right)!}$$

$$\frac{2^2 \Gamma(\frac{1}{2}) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma(\frac{1}{2})} = \frac{\Gamma(\frac{1}{2})}{4 \pi \Gamma(\frac{1}{2}, 0) \left(2.99671 \sqrt{-1} - (2.25 (16+8 i))^2\right)}{\Gamma(\frac{1}{2}, 0)}$$

$$\frac{2^2 \,\Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 \, i) \, 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = \frac{4 \,\pi \, e^{-\log G(1/2) + \log G(3/2)} \left(2.99671 \, \sqrt{-1} - (2.25 \, (16+8 \, i))^2\right)}{e^{-\log G(1/2) + \log G(3/2)}}$$

## Series representations

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -5184 \left((4-0.00924911 i) + 4 i + i^2\right) \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2 k}$$

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left(\left(16+8 i\right) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -2592 \left(\left(4-0.00924911 i\right) + 4 i + i^2\right) \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2 k}{k}}\right)$$

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left(\left(16+8 i\right) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -1296 \left(\left(4-0.00924911 i\right) + 4 i + i^2\right) \sum_{k=0}^{\infty} \frac{2^{-k} \left(-6+50 k\right)}{\binom{3 k}{k}}$$

#### **Integral representations**

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -2592 \left((4-0.00924911 i) + 4 i + i^2\right) \int_0^\infty \frac{1}{1+t^2} dt$$

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -5184 \left((4-0.00924911 i) + 4 i + i^2\right) \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{2^2 \Gamma\left(\frac{1}{2}\right) \pi \left(2 \times 0.665936 \left(2.25 \sqrt{1-2}\right) - \left((16+8 i) 2.25\right)^2\right)}{\Gamma\left(\frac{1}{2}\right)} = -2592 \left((4-0.00924911 i) + 4 i + i^2\right) \int_0^\infty \frac{\sin(t)}{t} dt$$

From which, we obtain, after some calculations:

 $(((2^2*Pi*gamma(1/2)/gamma(1/2))(((2*((0.665936*2.25(1-2)^{0.5}))-((((16+8i)(2.25))^{0.5})))))))^{1/20}$ 

#### **Input interpretation**

$$\sqrt[20]{2^2 \pi \times \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2})} \left(2 \left(0.665936 \times 2.25 \sqrt{1-2}\right) - \left((16+8 \, i\right) \times 2.25\right)^2\right)}$$

 $\Gamma(x)$  is the gamma function i is the imaginary unit

**Result** 1.632053... – 0.1815263... *i* 

#### **Polar coordinates**

r = 1.64212 (radius),  $\theta = -0.11077$  (angle)  $1.64212 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$  ... (trace of the instanton shape)

From:

#### AdS cycles in eternally inflating background Zhi-Guo Liu and Yun-Song Piao - arXiv:1404.5748v1 [hep-th] 23 Apr 2014

#### We have that:

We have showed that the bubble universe going through AdS cycles will fragment at certain time  $t_{Frag}$  within the 2<sup>th</sup> or 3<sup>th</sup>. We will see what is the resulting scenario.

The average square of the amplitude of field fluctuations at  $t_{Frag}$  is

$$<\delta\varphi_k^2>=\frac{1}{(2\pi)^3}\int |\delta\varphi_k|^2 d^3k$$

$$\simeq \frac{1}{(2\pi)^3}\int_{aH/e}^{aH} 3M_P^2 \left(1-\frac{\frac{H}{a}\int^t a\mathcal{R}_k\epsilon_{Mat} dt'}{\mathcal{R}_k}\right)^2 |\mathcal{R}_k|^2 d^3k$$

$$= 3M_P^2/4. \tag{26}$$

We consider:

$$M_p = \sqrt{\frac{c\hbar}{8\pi G}} = 2.4 \times 10^{18} GeV$$
$$= 4.341 \times 10^{-9} \text{ kg} = 2.435 \times 10^{18} \text{ GeV/c}^2.$$

From the result of the above integral, we obtain:

1/4(3(2.435\*10^18)^2)

#### **Input interpretation**

$$\frac{1}{4} \left( 3 \left(2.435 \times 10^{18}\right)^2 \right)$$

## Scientific notation

 $\begin{array}{c} \textbf{4.44691875} \times 10^{36} \\ \textbf{4.44691875} \ast 10^{36} \end{array}$ 

From which:

ln((((3\*(2.435\*10^18)^2))/4))+18

#### **Input interpretation**

$$\log\left(\frac{1}{4}\left(3\left(2.435 \times 10^{18}\right)^2\right)\right) + 18$$

log(x) is the natural logarithm

Result 102.385274788402969924... 102.385274.....

For:

$$H_{Kin} \sim M_P, \qquad H_{Kin} \sim H_*,$$

 $M_P = 2.435 \, * \, 10^{18} \, \text{GeV} \quad \text{and} \quad H = 2.301 \, * \, 10^{-18}$ 

From:

$$\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{k^{3/2}}{\sqrt{2\pi^2}} \left| \mathcal{R}_k \right| \sim \frac{H_{Kin}}{\sqrt{\epsilon_{Mat}} M_P},$$

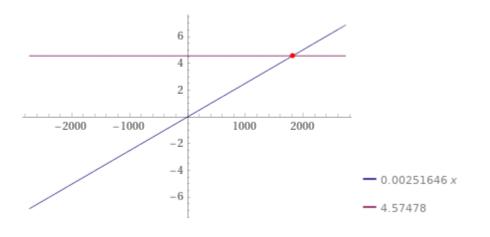
we obtain:

 $(0.050^{(1.5)})/(sqrt(2Pi^{2}))* x = (2.301*10^{-18})/(sqrt(3/2))* (2.435 \times 10^{18})$ 

#### **Input interpretation**

$$\frac{0.05^{1.5}}{\sqrt{2\,\pi^2}} x = \frac{2.301 \times 10^{-18}}{\sqrt{\frac{3}{2}}} \times 2.435 \times 10^{18}$$

#### Plot



Alternate form

0.00251646 x - 4.57478 = 0

#### Alternate form assuming x is real

0.00251646 x + 0 = 4.57478

#### Solution

 $x \approx 1817.94$ 1817.94 Thence:

(0.050^(1.5))/(sqrt(2Pi^2))\* 1817.94

# **Input interpretation**

 $\frac{0.05^{1.5}}{\sqrt{2\,\pi^2}} \times 1817.94$ 

# Result

4.5747743926615580236879738073852230386624676780275634348077544474 ... 4.574774392....

# The study of this function provides the following representations:

# **Series representations**

$$\frac{1817.94 \times 0.05^{1.5}}{\sqrt{2\,\pi^2}} = \frac{20.3252}{\sqrt{-1+2\,\pi^2}\,\sum_{k=0}^{\infty} \left(-1+2\,\pi^2\right)^{-k} \left(\frac{1}{2}\atop k\right)}$$

$$\frac{1817.94 \times 0.05^{1.5}}{\sqrt{2 \pi^2}} = \frac{20.3252}{\sqrt{-1 + 2 \pi^2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 2 \pi^2)^{-k} (-\frac{1}{2})_k}{k!}}$$

$$\frac{1817.94 \times 0.05^{1.5}}{\sqrt{2 \pi^2}} = \frac{20.3252}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 \pi^2 - z_0)^k z_0^{-k}}{k!}}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$ 

From:

$$\mathcal{R}_{k} \sim \int \frac{d\eta}{z^{2}} \sim (t_{B} - t)^{1-3n} \,.$$
  
$$\delta\phi_{k} = \sqrt{2M_{P}^{2}\epsilon} \mathcal{R}_{k} \left(1 - \frac{\frac{H}{a}\int^{t} a\mathcal{R}_{k}\epsilon \,dt'}{\mathcal{R}_{k}}\right) \simeq \sqrt{2M_{P}^{2}\epsilon} \mathcal{R}_{k}$$
  
$$\sim (t_{B} - t)^{1-3n} \,, \qquad (12)$$

Considering:

$$V_0, \lambda_1, \lambda_3 > 0, V_0 \ll 1 \text{ and } V_0 \gg \lambda_1 \phi + \frac{\lambda_3}{3!} \phi^3$$

from:

$$2\epsilon_{Mat}M_P^2 = 3M_P^2$$

we obtain:

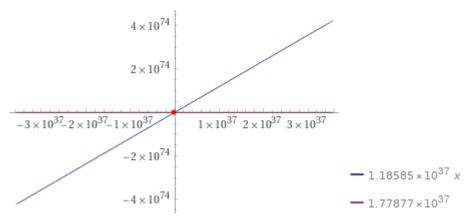
 $2*x*(2.435*10^{18})^{2} = 3(2.435*10^{18})^{2}$ 

Input interpretation  $2 x (2.435 \times 10^{18})^2 = 3 (2.435 \times 10^{18})^2$ 

# Result

 $1.18585 \times 10^{37} x = 1.77877 \times 10^{37}$ 

# Plot



Alternate form  $1.18585 \times 10^{37} x - 1.77877 \times 10^{37} = 0$ 

Alternate form assuming x is real  $1.18585 \times 10^{37} x + 0 = 1.77877 \times 10^{37}$ 

#### Solution

 $x \approx 1.5$  $x = 3/2 = \epsilon$ 

Thence:

2\*3/2\*(2.435\*10^18)^2

# **Input interpretation**

$$2\!\times\!\frac{3}{2}\,\big(2.435\!\times\!10^{18}\big)^2$$

Result 1778767500000000000000000000000000000000

# Scientific notation

 $1.7787675\!\times\!10^{37}$ 1.7787675\*10<sup>37</sup>

From:

$$\delta\phi_k = \sqrt{2M_P^2 \epsilon} \mathcal{R}_k \left( 1 - \frac{\frac{H}{a} \int^t a \mathcal{R}_k \epsilon \, dt'}{\mathcal{R}_k} \right) \simeq \sqrt{2M_P^2 \epsilon} \mathcal{R}_k$$
$$\sim (t_B - t)^{1-3n} \,, \tag{12}$$

# Input interpretation

$$\sqrt{2 \left(2.435 \times 10^{18}\right)^2 \times \frac{3}{2}} \times 1817.94$$

 $\begin{array}{l} \textbf{Result} \\ 7.66724...\times 10^{21} \\ 7.66724...*10^{21} \end{array}$ 

Furthermore, from:

$$\left(\frac{l_{Frag}}{1/H_{Frag}}\right)^3 \simeq \frac{H_{Frag}}{H_*} \sim \frac{\sqrt{\epsilon_{Mat}}M_P}{H_{Kin}} \gg 1,$$

we obtain:

# $((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18})))$

# **Input interpretation**

$$\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}$$

**Result**  $1.29607... \times 10^{36}$  $1.29607...*10^{36}$ 

# From:

$$\frac{1}{4}\left(3\left(2.435 \times 10^{18}\right)^2\right)$$

and

$$\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}$$

we obtain also:

# $\frac{1}{2}(0.9991104684+0.9568666373)+1/((((1/4(3(2.435*10^{18})^2)) / (((((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18})))))^{1/3})$

where 0.9991104684 and 0.9568666373 are the values of the following Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-$$

# **Input interpretation**

$$\frac{1}{2} \left(0.9991104684 + 0.9568666373\right) + \frac{1}{\sqrt[4]{\frac{1}{4} \left(3 \left(2.435 \times 10^{18}\right)^2\right)}{\sqrt[3]{\frac{3}{2} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}}}}\right)$$

Result

1.6410029014980904669532800942969420798336707308943552774871213258

... 1.641002901498.... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... (trace of the instanton shape) Now, we observe that, from the two expressions:

$$2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2$$

and

$$\frac{1}{4} \left( 3 \left(2.435 \times 10^{18}\right)^2 \right)$$

we obtain:

 $((2*3/2*(2.435*10^{18})^2)/(1/4(3(2.435*10^{18})^2)))$ 

# Input interpretation

 $\frac{2\!\times\!\frac{3}{2} \left(2.435\!\times\!10^{18}\right)^2}{\frac{1}{4} \left(3 \left(2.435\!\times\!10^{18}\right)^2\right)}$ 

# Result

4 4

and from which, after some easy calculations:

((2\*3/2\*(2.435\*10^18)^2)/(1/4(3(2.435\*10^18)^2)))^3

# **Input interpretation**

$$\!\!\left(\!\frac{2\!\times\!\frac{3}{2} \left(\!2.435\!\times\!10^{18}\right)^2}{\frac{1}{4} \left(\!3 \left(\!2.435\!\times\!10^{18}\right)^2\right)}\!\right)^{\!3}$$

# Result

 $64 \\ 64 = 8^2$ 

And again:

# 27((2\*3/2\*(2.435\*10^18)^2)/(1/4(3(2.435\*10^18)^2)))^3+1

# **Input interpretation**

$$27 \! \left( \! \frac{2 \! \times \! \frac{3}{2} \left( 2.435 \! \times \! 10^{18} \right)^2}{\! \frac{1}{4} \left( 3 \left( 2.435 \! \times \! 10^{18} \right)^2 \right)} \right)^{\! 3} + 1 \!$$

#### Result

1729 1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 =  $8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

 $(27((2*3/2*(2.435*10^{18})^2)/(1/4(3(2.435*10^{18})^2)))^3+1)^1/15$ 

# **Input interpretation**

$$\sqrt[15]{27 \left(\frac{2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2}{\frac{1}{4} \left(3 \left(2.435 \times 10^{18}\right)^2\right)}\right)^3 + 1}$$

**Result** 1.643815228748728131... 1.64381522874.... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$  ... (trace of the instanton shape)

# $(((2*3/2*(2.435*10^{18})^2)/(1/4(3(2.435*10^{18})^2)))^3)^2$

# **Input interpretation**

$$\left(\!\left(\frac{2\!\times\!\frac{3}{2}\left(2.435\!\times\!10^{18}\right)^2}{\frac{1}{4}\left(3\left(2.435\!\times\!10^{18}\right)^2\right)}\right)^3\right)^2$$

# Result

4096

 $4096 = 64^2$  where 4096 and 64 are fundamental values indicated in the Ramanujan paper "Modular equations and Approximations to  $\pi$ "

From:

$$2 \times \frac{3}{2} (2.435 \times 10^{18})^2$$

and

$$\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}$$

we obtain:

 $(2*3/2*(2.435*10^{18})^2) 1/(((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18})))$ 

# **Input interpretation**

$$\left( 2 \times \frac{3}{2} \left( 2.435 \times 10^{18} \right)^2 \right) \times \frac{1}{\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}}$$

# Result

13.724331811980865977622999847112253586244725947533080046050028329

...
13.72433181198....

And dividing also from:

 $\frac{0.05^{1.5}}{\sqrt{2\,\pi^2}} \times 1817.94$ 

we obtain :

 $\begin{array}{l} (2*3/2*(2.435*10^{18})^2) & 1/(((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18})) \\ & *1/((((0.050^{(1.5)})/(sqrt(2Pi^2))* 1817.94))) \end{array}$ 

# **Input interpretation**

$$\left(2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2\right) \times \frac{1}{\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}} \times \frac{1}{\frac{0.05^{1.5}}{\sqrt{2\pi^2}} \times 1817.94}$$

## Result

3.00000... 3

Multiplying by the previous expression

$$\frac{2 \!\times\! \frac{3}{2} \left(2.435 \!\times\! 10^{18}\right)^2}{\frac{1}{4} \left(3 \left(2.435 \!\times\! 10^{18}\right)^2\right)}$$

we obtain also:

 $((2*3/2*(2.435*10^{18})^{2})/(1/4(3(2.435*10^{18})^{2})))(2*3/2*(2.435*10^{18})^{2}))/(((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18}))*1/((((0.050^{(1.5)})/(sqrt(2Pi^{2}))*1817.94))))))$ 

# **Input interpretation**

$$\frac{2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2}{\frac{1}{4} \left(3 \left(2.435 \times 10^{18}\right)^2\right)} \left(2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2\right) \left(\frac{1}{\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{2.301 \times 10^{-18}}} \times \frac{1}{\frac{0.05^{1.5}}{\sqrt{2\pi^2}} \times 1817.94}\right)$$

## Result

12.0000... 12

From which:

# **Input interpretation**

$$\left( \frac{2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2}{\frac{1}{4} \left(3 \left(2.435 \times 10^{18}\right)^2\right)} \left(2 \times \frac{3}{2} \left(2.435 \times 10^{18}\right)^2\right) \\ \left(\frac{1}{\frac{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}{\sqrt{\frac{3}{2}} \times 2.435 \times 10^{18}}} \times \frac{1}{\frac{0.05^{1.5}}{\sqrt{2\pi^2}} \times 1817.94} \right) \right)^3 + 1$$

#### Result

1729.00... 1729 This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 =  $8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((((2\*3/2\*(2.435\*10^18)^2)/(1/4(3(2.435\*10^18)^2)))(2\*3/2\*(2.435\*10^18)^2)) 1/(((Sqrt(3/2)\*(2.435\*10^18)))/(2.301\*10^-18)) \*1/((((0.050^(1.5))/(sqrt(2Pi^2))\* 1817.94)))))^3+1)^1/15

# **Input interpretation**

$$\left( \left( \frac{2 \times \frac{3}{2} \left( 2.435 \times 10^{18} \right)^2}{\frac{1}{4} \left( 3 \left( 2.435 \times 10^{18} \right)^2 \right)} \left( 2 \times \frac{3}{2} \left( 2.435 \times 10^{18} \right)^2 \right) \right. \\ \left. \left( \frac{1}{\frac{\sqrt{3}}{2} \times 2.435 \times 10^{18}} \left( \frac{1}{\sqrt{\frac{3}{2} \times 2.435 \times 10^{18}}} \times \frac{1}{\sqrt{2\pi^2}} \times 1817.94} \right) \right)^3 + 1 \right)^{-1} (1/15)$$

#### Result

1.64382...

1.64382....≈ ζ(2) =  $\frac{\pi^2}{6}$  = 1.644934 ... (trace of the instanton shape)

 $\begin{array}{l} (1/27(((((2*3/2*(2.435*10^{18})^2)/(1/4(3(2.435*10^{18})^2)))(2*3/2*(2.435*10^{18})^2 \\ ) \ 1/(((Sqrt(3/2)*(2.435*10^{18})))/(2.301*10^{-18}))*1/((((0.050^{(1.5)})/(sqrt(2Pi^2))*1817.94)))))^3))^2 \end{array}$ 

# **Input interpretation**

$$\left( \frac{1}{27} \left( \frac{2 \times \frac{3}{2} \left( 2.435 \times 10^{18} \right)^2}{\frac{1}{4} \left( 3 \left( 2.435 \times 10^{18} \right)^2 \right)} \left( 2 \times \frac{3}{2} \left( 2.435 \times 10^{18} \right)^2 \right) \right. \\ \left. \left. \left( \frac{1}{\frac{\sqrt{3}}{2} \times 2.435 \times 10^{18}} \times \frac{1}{\frac{0.05^{1.5}}{\sqrt{2\pi^2}} \times 1817.94} \right) \right)^2 \right) \right)$$

#### Result

4096.02...

 $4096.02...\approx 4096 = 64^2$  where 4096 and 64 are fundamental values indicated in the Ramanujan paper "Modular equations and Approximations to  $\pi$ "

# Acknowledgments

M. Nardelli thanks **Francesco Maggi**, Professor of Mathematics at University of Texas - Austin, Department of Mathematics, for his availability and kindness towards him

# References

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