

**On the study of various equations concerning the Isoperimetric Theorems.  
Possible mathematical connections with some sectors of Number Theory, String  
Theory and some cosmological parameters.**

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**Abstract**

*In this paper, we analyze various equations concerning the Isoperimetric Theorems.  
We describe the new possible mathematical connections with some sectors of  
Number Theory, String Theory and cosmological parameters*

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## **Renato Caccioppoli**

Matematico (1904 – 1959)



## **Vesuvius landscape with gorse – Naples**



<https://www.pinterest.it/pin/95068242114589901/>

From Wikipedia:

In mathematics, a **ball** is the space bounded by a sphere. It may be a **closed ball** (including the boundary points that constitute the sphere) or an **open ball** (excluding them).

*We propose that some equations concerning the “balls”, can be related with various parameters of some cosmological models as the “Multiverse” and the “Eternal Inflation” linked to it, which provides that space is divided into bubbles or patches whose properties differ from patch to patch and spanning all physical possibilities.*

In 1983, it was shown that inflation could be eternal, leading to a multiverse in which space is broken up into bubbles or patches whose properties differ from patch to patch spanning all physical possibilities.

When the false vacuum decays, the lower-energy true vacuum forms through a process known as **bubble nucleation**. In this process, instanton effects cause a bubble containing the true vacuum to appear. The walls of the bubble (or domain walls) have a positive surface tension, as energy is expended as the fields roll over the potential barrier to the true vacuum.

From:

**Isoperimetric Theorems, Open Problems and New Results** – *Francesco Maggi* –  
ICTP, Trieste, 22 February 2017

We have:

UNLESS  $k=2 \quad 7 \leq h \leq 11$  OR  $k=3 \quad h=5$

$$C(k, h) = \frac{(k-1)^{13/8}}{(h-1)^{3/2}} \sqrt{\frac{h+k-1}{hk \omega_k \omega_h}} 2^{-12} \Rightarrow \lambda(M_{k,h} \cap B_R) \geq \frac{c}{R^2} \left(\frac{k}{h}\right)^{9/4} h^{1/4}$$

$$\frac{((3-1)^{13/8}) / ((5-1)^{3/2}) * \text{sqrt}((5+3-1)/(3*5*x*y)) * 2^{(-12)} = c/(R^2) * (3/5)^{9/4} * 5^{1/4}$$

Where  $u_k = x$  ;  $\omega_h = y$  ;  $k = 3$  and  $h = 5$

**Input**

$$\frac{\frac{(3-1)^{13/8}}{(5-1)^{3/2}} \sqrt{\frac{5+3-1}{3 \times 5 \times x \times y}}}{2^{12}} = \frac{c}{R^2} \left(\frac{3}{5}\right)^{9/4} \sqrt[4]{5}$$

**Exact result**

$$\frac{\sqrt{\frac{7}{15}} \sqrt{\frac{1}{xy}}}{8192 \times 2^{3/8}} = \frac{9 \sqrt[4]{3} c}{25 R^2}$$

**Alternate form assuming c, R, x, and y are real**

$$5 \times 2^{5/8} \sqrt[4]{3} \sqrt{35} R \sqrt{\frac{1}{xy}} = \frac{442368 c}{R}$$

### Alternate form

$$c = \frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

### Alternate form assuming c, R, x, and y are positive

$$875 \sqrt[4]{2} \sqrt{3} R^4 = 97844723712 c^2 x y$$

### Real solutions

$$c > 0, \quad R < 0, \quad x < 0, \quad y = \frac{875 R^4}{16307453952 \times 2^{3/4} \sqrt{3} c^2 x}$$

---

$$c > 0, \quad R < 0, \quad x > 0, \quad y = \frac{875 R^4}{16307453952 \times 2^{3/4} \sqrt{3} c^2 x}$$

---

$$c > 0, \quad R > 0, \quad x < 0, \quad y = \frac{875 R^4}{16307453952 \times 2^{3/4} \sqrt{3} c^2 x}$$

---

$$c > 0, \quad R > 0, \quad x > 0, \quad y = \frac{875 R^4}{16307453952 \times 2^{3/4} \sqrt{3} c^2 x}$$

### Solution for the variable y

$$y = \frac{875 R^4}{16307453952 \times 2^{3/4} \sqrt{3} c^2 x}$$

From the following alternate form:

$$c = \frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

we obtain:

$$(5 \sqrt{35} R^2 \sqrt{1/(xy)}) / (73728 2^{3/8} 3^{3/4})$$

### Input

$$\frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

### Exact result

$$\frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

### Real roots

$$R = 0, \quad x < 0, \quad y < 0$$

---

$$R = 0, \quad x > 0, \quad y > 0$$

### Properties as a function

#### Domain

$$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$$

---

#### Range

$$\{z \in \mathbb{R} : (z = 0 \text{ and } R = 0) \text{ or } (z > 0 \text{ and } R \neq 0)\}$$

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## Parity

even

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$\frac{5\sqrt{35} R^2 \sqrt{x} \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4} \sqrt{x}} + O(x^{11/2})$$

(Puiseux series)

## Series expansion at $x=\infty$

$$\frac{5\sqrt{35} R^2 \sqrt{x} \sqrt{\frac{1}{x}} \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Derivative

$$\frac{\partial}{\partial x} \left( \frac{5\sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{5\sqrt{35} R^2 y \left(\frac{1}{xy}\right)^{3/2}}{147456 \times 2^{3/8} \times 3^{3/4}}$$

## Indefinite integral

$$\int \frac{5\sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} dx = \frac{5\sqrt{35} R^2 x \sqrt{\frac{1}{xy}}}{36864 \times 2^{3/8} \times 3^{3/4}} + \text{constant}$$

## Global minimum

$$\min\left\{\frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}\right\} = 0 \text{ at } (x, y) =$$

$$\left(\left\{\begin{array}{ll} -1 & R = 0 \\ \text{indeterminate} & \text{(otherwise)} \end{array}\right\}, \left\{\begin{array}{ll} -1 & R = 0 \\ \text{indeterminate} & \text{(otherwise)} \end{array}\right\}\right)$$

## Limit

$$\lim_{x \rightarrow \pm\infty} \frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} = 0$$


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$$\lim_{y \rightarrow \pm\infty} \frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} = 0$$

## Series representations

$$\frac{5\left(\sqrt{35}R^2\sqrt{\frac{1}{xy}}\right)}{73728 \times 2^{3/8} \times 3^{3/4}} =$$

$$\frac{5R^2\sqrt{34}\sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

for  $\left|-1 + \frac{1}{xy}\right| > 1$

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$$\frac{5 \left( \sqrt{35} R^2 \sqrt{\frac{1}{xy}} \right)}{73728 \times 2^{3/8} \times 3^{3/4}} = \frac{5 R^2 \sqrt{34} \sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

for  $\left| -1 + \frac{1}{xy} \right| > 1$

---

$$\frac{5 \left( \sqrt{35} R^2 \sqrt{\frac{1}{xy}} \right)}{73728 \times 2^{3/8} \times 3^{3/4}} = \frac{5 R^2 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (35-z_0)^{k_1} \left(\frac{1}{xy} - z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

From the above derivative

$$\frac{\partial}{\partial x} \left( \frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{5 \sqrt{35} R^2 y \left(\frac{1}{xy}\right)^{3/2}}{147456 \times 2^{3/8} \times 3^{3/4}}$$

we obtain, from the result:

$$-(5 \sqrt{35} R^2 (1/(xy))^{3/2} y)/(147456 2^{3/8} 3^{3/4})$$

**Input**

$$-\frac{5 \sqrt{35} R^2 \left(\frac{1}{xy}\right)^{3/2} y}{147456 \times 2^{3/8} \times 3^{3/4}}$$

## Exact result

$$-\frac{5\sqrt{35}R^2y\left(\frac{1}{xy}\right)^{3/2}}{147456 \times 2^{3/8} \times 3^{3/4}}$$

## Real roots

$$R = 0, \quad x < 0, \quad y < 0$$

---

$$R = 0, \quad x > 0, \quad y > 0$$

## Properties as a function

### Domain

$$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$$

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### Range

$$\{z \in \mathbb{R} : \neg (R = 0 \vee z = 0)\}$$

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### Parity

odd

$e_1 \vee e_2 \vee \dots$  is the logical XOR function

$\neg \text{expr}$  is the logical NOT function

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$-\frac{5\left(\sqrt{35}R^2\sqrt{x}\sqrt{\frac{1}{xy}}\right)}{147456(2^{3/8} \times 3^{3/4})x^{3/2}} + O(x^{11/2})$$

(Puiseux series)

## Series expansion at $x=\infty$

$$-\frac{5\left(\frac{1}{x}\right)^{3/2}\left(\sqrt{35}R^2\sqrt{x}\sqrt{\frac{1}{xy}}\right)}{147456(2^{3/8}\times 3^{3/4})}+O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Derivative

$$\frac{\partial}{\partial x}\left(-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}}\right)=\frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{98304\times 2^{3/8}\times 3^{3/4}x^2}$$

## Indefinite integral

$$\int -\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}}dx=\frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{73728\times 2^{3/8}\times 3^{3/4}}+\text{constant}$$

## Limit

$$\lim_{x\rightarrow\pm\infty}-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}}=0$$

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$$\lim_{y\rightarrow\pm\infty}-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}}=0$$

## Series representations

$$-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}} = -\frac{5R^2\sqrt{\frac{1}{xy}}\sqrt{34}\sum_{k=0}^{\infty}34^{-k}\binom{\frac{1}{2}}{k}}{147456\times 2^{3/8}\times 3^{3/4}x}$$


---

$$-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}} = -\frac{5R^2\sqrt{\frac{1}{xy}}\sqrt{34}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{34}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{147456\times 2^{3/8}\times 3^{3/4}x}$$


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$$-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}} = -\frac{5R^2\sqrt{\frac{1}{xy}}\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}34^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{294912\times 2^{3/8}\times 3^{3/4}x\sqrt{\pi}}$$

From the above derivative:

$$\frac{\partial}{\partial x}\left(-\frac{5\sqrt{35}R^2\left(\frac{1}{xy}\right)^{3/2}y}{147456\times 2^{3/8}\times 3^{3/4}}\right) = \frac{5\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{98304\times 2^{3/8}\times 3^{3/4}x^2}$$

we obtain, from the result:

### Input

$$-\frac{25\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{196608\times 2^{3/8}\times 3^{3/4}x^3}$$

### Exact result

$$-\frac{25\sqrt{35}R^2\sqrt{\frac{1}{xy}}}{196608\times 2^{3/8}\times 3^{3/4}x^3}$$

## Real roots

$$R = 0, \quad x < 0, \quad y < 0$$

---

$$R = 0, \quad x > 0, \quad y > 0$$

## Properties as a function

### Domain

$$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$$

---

### Range

$$\{z \in \mathbb{R} : \neg (R = 0 \vee z = 0)\}$$

---

### Parity

odd

$e_1 \vee e_2 \vee \dots$  is the logical XOR function

$\neg \text{expr}$  is the logical NOT function

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$-\frac{25 \left( \sqrt{35} R^2 \sqrt{x} \sqrt{\frac{1}{xy}} \right)}{196608 (2^{3/8} \times 3^{3/4}) x^{7/2}} + O(x^{11/2})$$

(Puiseux series)

## Series expansion at $x=\infty$

$$-\frac{25\left(\frac{1}{x}\right)^{7/2}\left(\sqrt{35} R^2 \sqrt{x} \sqrt{\frac{1}{xy}}\right)}{196608\left(2^{3/8} \times 3^{3/4}\right)} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Derivative

$$\frac{\partial}{\partial x} \left( -\frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} \right) = \frac{175 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{393216 \times 2^{3/8} \times 3^{3/4} x^4}$$

## Indefinite integral

$$\int -\frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} dx = \frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{98304 \times 2^{3/8} \times 3^{3/4} x^2} + \text{constant}$$

## Limit

$$\lim_{x \rightarrow \pm\infty} -\frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} = 0$$

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$$\lim_{y \rightarrow \pm\infty} -\frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} = 0$$

## Series representations

$$\begin{aligned}
 & - \frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} = \\
 & - \frac{25 R^2 \sqrt{34} \sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} \\
 & \text{for } \left| -1 + \frac{1}{xy} \right| > 1
 \end{aligned}$$


---

$$\begin{aligned}
 & - \frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} = \\
 & - \frac{25 R^2 \sqrt{34} \sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} \\
 & \text{for } \left| -1 + \frac{1}{xy} \right| > 1
 \end{aligned}$$


---

$$\begin{aligned}
 & - \frac{25 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} = \\
 & - \frac{25 R^2 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (35-z_0)^{k_1} \left(\frac{1}{xy}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} \\
 & \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
 \end{aligned}$$

For:

$$\alpha s = [-\pi/2, \pi/2],$$

$$s = 32\beta \leq 1/2$$

$$|c| \geq 1/4,$$

From:

$$R = (1 - \theta)\alpha s$$

$$(1 - 1/16) \cdot \pi/6 \cdot 1/2$$

**Input**

$$\left(1 - \frac{1}{16}\right) \times \frac{\pi}{6} \times \frac{1}{2}$$

**Result**

$$\frac{5\pi}{64}$$

**Decimal approximation**

0.2454369260617025967548940143187111628279038593261801422636675462

...

$$R = 0.245436926....$$

**Property**

$\frac{5\pi}{64}$  is a transcendental number

**Alternative representations**

$$\frac{\left(1 - \frac{1}{16}\right)\pi}{2 \times 6} = \frac{90}{6} \circ \left(1 - \frac{1}{16}\right)$$

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$$\frac{\left(1 - \frac{1}{16}\right)\pi}{2 \times 6} = - \frac{i \log(-1) \left(1 - \frac{1}{16}\right)}{2 \times 6}$$

---

$$\frac{\left(1 - \frac{1}{16}\right)\pi}{2 \times 6} = \frac{1}{6} E(0) \left(1 - \frac{1}{16}\right)$$



$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$E(m)$  is the complete elliptic integral of the second kind with parameter  
 $m = k^2$

## Series representations

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \frac{5}{16} \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

---

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \sum_{k=0}^{\infty} \frac{(-1)^k (956 \times 5^{-2k} - 5 \times 239^{-2k})}{3824 (1 + 2k)}$$

---

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \frac{5}{64} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1 + 2k} + \frac{2}{1 + 4k} + \frac{1}{3 + 4k}\right)$$

## Integral representations

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \frac{5}{16} \int_0^1 \sqrt{1 - t^2} dt$$

---

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \frac{5}{32} \int_0^1 \frac{1}{\sqrt{1 - t^2}} dt$$

---

$$\frac{(1 - \frac{1}{16})\pi}{2 \times 6} = \frac{5}{32} \int_0^{\infty} \frac{1}{1 + t^2} dt$$

We have:

$$C\theta \leq 1/2$$

$$C = C(n, \lambda) > 1$$

For  $C = 8$  :  $\theta = 1/16$  ;  $R = 0.245436926$

From the previous derivative

$$\frac{\partial}{\partial x} \left( -\frac{25\sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{196608 \times 2^{3/8} \times 3^{3/4} x^3} \right) = \frac{175\sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{393216 \times 2^{3/8} \times 3^{3/4} x^4}$$

we obtain, from the result:

$$(175 \sqrt{35} ((5\pi)/64)^2 \sqrt{1/(xy)}) / (393216 2^{3/8} 3^{3/4} x^4)$$

**Input**

$$\frac{175 \sqrt{35} \left(\frac{5\pi}{64}\right)^2 \sqrt{\frac{1}{xy}}}{393216 \times 2^{3/8} \times 3^{3/4} x^4}$$

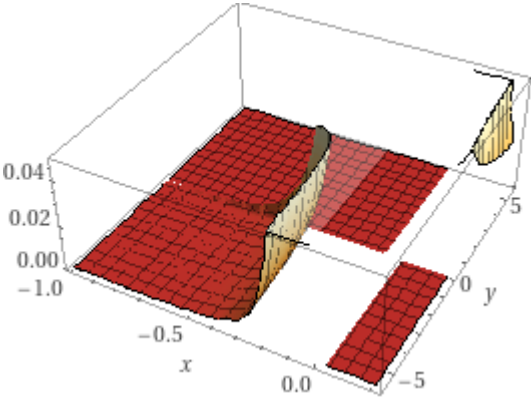
**Exact result**

$$\frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4}$$

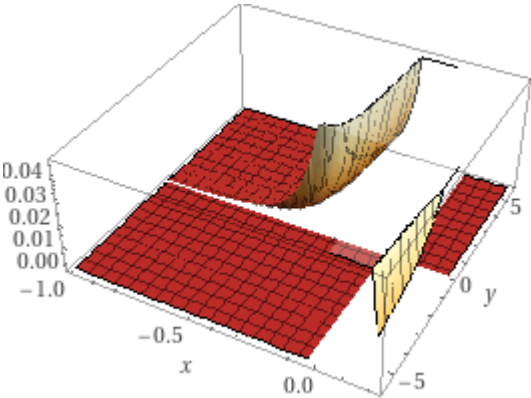
**3D plots**

**Real part**

(figures that can be related to the D-branes/Instantons)

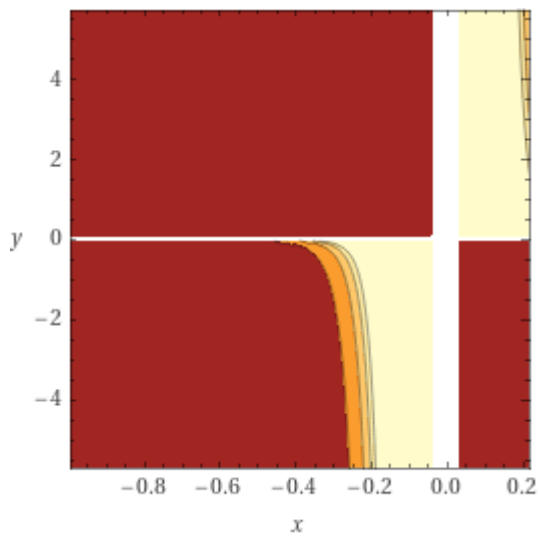


**Imaginary part**

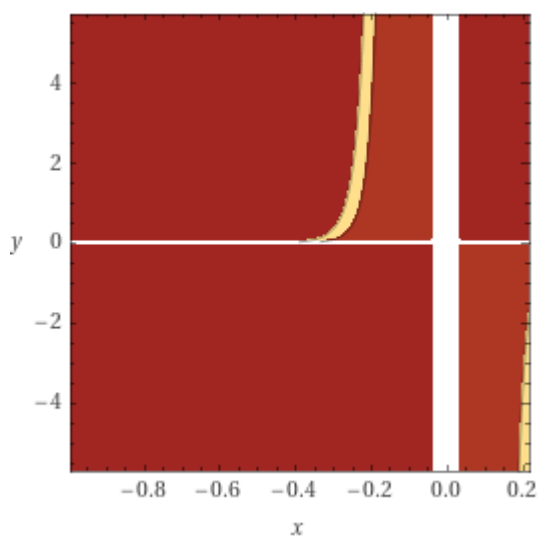


## Contour plots

### Real part



### Imaginary part



## Roots

(no roots exist)

## Properties as a function

### Domain

$$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$$

## Range

$\{z \in \mathbb{R} : z > 0\}$  (all positive real numbers)

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## Parity

even

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=\infty$

$$\frac{4375 \sqrt{35} \pi^2 \sqrt{x} \left(\frac{1}{x}\right)^{9/2} \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Partial derivatives

$$\frac{\partial}{\partial x} \left( \frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4} \right) = - \frac{4375 \sqrt[4]{3} \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1073741824 \times 2^{3/8} x^5}$$

---

$$\frac{\partial}{\partial y} \left( \frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4} \right) = - \frac{4375 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{3221225472 \times 2^{3/8} \times 3^{3/4} x^3}$$

## Indefinite integral

$$\int \frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4} dx = - \frac{625 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{805306368 \times 2^{3/8} \times 3^{3/4} x^3} + \text{constant}$$

## Limit

$$\lim_{x \rightarrow \pm\infty} \frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4} = 0$$

---

$$\lim_{y \rightarrow \pm\infty} \frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4} = 0$$

From the above result

$$\frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{1610612736 \times 2^{3/8} \times 3^{3/4} x^4}$$

For  $x = -0.4$  and  $y = -4$ , we obtain :

$$(4375 \sqrt{35} \pi^2 \sqrt{1/(-0.4 * -4)}) / (1610612736 2^{3/8} 3^{3/4} * (-0.4)^4)$$

## Input

$$\frac{4375 \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4 \times (-4)}}}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)}$$

## Result

0.00165689...

0.00165689....

## Series representations

$$\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)} = 0.0000358938 \pi^2 \sqrt{z_0}^2$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

---

$$\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)} =$$

$$0.0000358938 \pi^2 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2\pi} \right]\right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2\pi} \right]\right) \sqrt{x}^2$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (0.625 - x)^{k_1} (35 - x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

---

$$\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)} =$$

$$0.0000358938 \pi^2 \left(\frac{1}{z_0}\right)^{1/2 \lceil \arg(0.625 - z_0)/(2\pi) \rceil + 1/2 \lceil \arg(35 - z_0)/(2\pi) \rceil}$$

$$z_0^{1+1/2 \lceil \arg(0.625 - z_0)/(2\pi) \rceil + 1/2 \lceil \arg(35 - z_0)/(2\pi) \rceil}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

Inverting

$$\frac{4375 \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4 \times (-4)}}}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)}$$

we obtain:

$$1/(((4375 \text{ sqrt}(35) \pi^2 \text{ sqrt}(1/(-0.4* -4)))/(1610612736 2^{(3/8)} 3^{(3/4)} *(-0.4)^4)))$$

**Input**

$$\frac{1}{\frac{4375 \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4 \times (-4)}}}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)}}$$

**Result**

603.541...  
**603.541....**

**Series representations**

$$\frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4 \times (-4)}} \right)}{1610612736 \times 2^{3/8} (3^{3/4} (-0.4)^4)}} = \frac{27859.9}{\pi^2 \sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35 - z_0)^k z_0^{-k}}{k!}}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))



$$\frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} =$$

$$27859.9 / \left( \pi^2 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2\pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2\pi} \right] \right) \right.$$

$$\left. \sqrt{x}^{-2} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (0.625 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (35 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} = \left( 27859.9 \left( \frac{1}{z_0} \right)^{-1/2 \lfloor \arg(0.625 - z_0) / (2\pi) \rfloor - 1/2 \lfloor \arg(35 - z_0) / (2\pi) \rfloor} \right.$$

$$\left. z_0^{-1 - 1/2 \lfloor \arg(0.625 - z_0) / (2\pi) \rfloor - 1/2 \lfloor \arg(35 - z_0) / (2\pi) \rfloor} \right) /$$

$$\left( \pi^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35 - z_0)^k z_0^{-k}}{k!} \right)$$

From the previous alternate form:

$$c = \frac{5 \sqrt{35} R^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

we obtain also:

$$(5 \sqrt{35} ((5\pi)/64)^2 \sqrt{1/(xy)}) / (73728 2^{3/8} 3^{3/4})$$

**Input**

$$\frac{5 \sqrt{35} \left(\frac{5\pi}{64}\right)^2 \sqrt{\frac{1}{xy}}}{73728 \times 2^{3/8} \times 3^{3/4}}$$

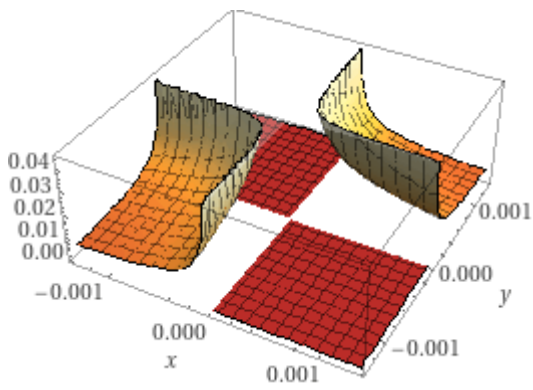
## Exact result

$$\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

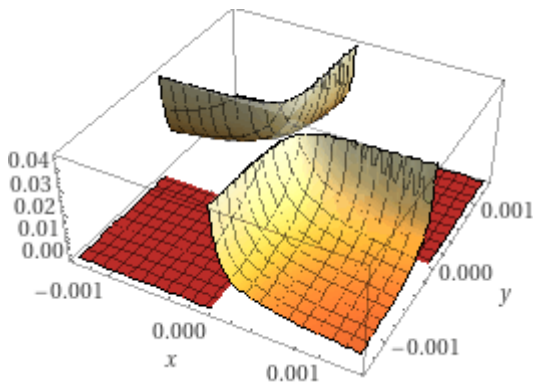
## 3D plots

### Real part

(figures that can be related to the D-branes/Instantons)

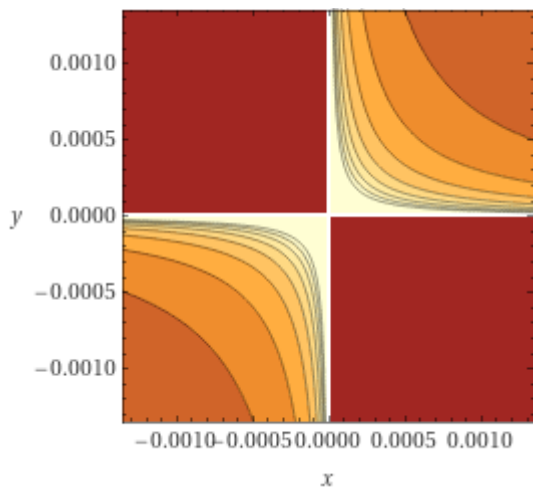


### Imaginary part

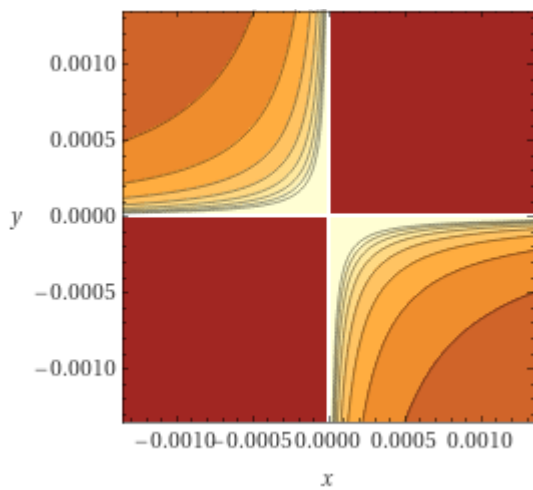


## Contour plots

### Real part



### Imaginary part



## Roots

(no roots exist)

## Properties as a function

### Domain

$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } x y > 0\}$

---

## Range

$\{z \in \mathbb{R} : z > 0\}$  (all positive real numbers)

---

## Parity

even

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$\frac{125 \sqrt{35} \pi^2 \sqrt{x} \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4} \sqrt{x}} + O(x^{11/2})$$

(Puiseux series)

## Series expansion at $x=\infty$

$$\frac{125 \sqrt{35} \pi^2 \sqrt{x} \sqrt{\frac{1}{x}} \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Partial derivatives

$$\frac{\partial}{\partial x} \left( \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{125 \sqrt{35} \pi^2 y \left(\frac{1}{xy}\right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}}$$

---

$$\frac{\partial}{\partial y} \left( \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{125 \sqrt{35} \pi^2 x \left(\frac{1}{xy}\right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}}$$

## Indefinite integral

$$\int \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} dx = \frac{125 \sqrt{35} \pi^2 x \sqrt{\frac{1}{xy}}}{150994944 \times 2^{3/8} \times 3^{3/4}} + \text{constant}$$

## Limit

$$\lim_{x \rightarrow \pm\infty} \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} = 0$$

$$\lim_{y \rightarrow \pm\infty} \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}} = 0$$

## Series representations

$$\frac{5 \left( \sqrt{35} \left( \frac{5\pi}{64} \right)^2 \sqrt{\frac{1}{xy}} \right)}{73728 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{34} \sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

for  $\left| -1 + \frac{1}{xy} \right| > 1$

$$\frac{5 \left( \sqrt{35} \left( \frac{5\pi}{64} \right)^2 \sqrt{\frac{1}{xy}} \right)}{73728 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{34} \sqrt{-1 + \frac{1}{xy}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 34^{-k_1} \left(-1 + \frac{1}{xy}\right)^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

for  $\left| -1 + \frac{1}{xy} \right| > 1$

$$\frac{5 \left( \sqrt{35} \left( \frac{5\pi}{64} \right)^2 \sqrt{\frac{1}{xy}} \right)}{73728 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (35-z_0)^{k_1} \left(\frac{1}{xy}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

From the above result

$$\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

for  $x = y = 0.001$ , we obtain:

$$(125 \sqrt{35} \pi^2 \sqrt{1/(0.001 * 0.001)}) / (301989888 2^{(3/8)} 3^{(3/4)})$$

**Input**

$$\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$

**Result**

0.00817569...

0.00817569....

## Series representations

$$\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{34} \sqrt{999999} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 34^{-k_1} e^{-13.8155 k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$


---

$$\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{34} \sqrt{999999} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{34}\right)^{k_1} (-1 \times 10^{-6})^{k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}}{301989888 \times 2^{3/8} \times 3^{3/4}}$$


---

$$\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \times 2^{3/8} \times 3^{3/4}} = \frac{\left( 125 \pi^2 \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-\frac{1}{2}+j_1} 34^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \left( \text{Res}_{s=-\frac{1}{2}+j_2} e^{-13.8155 s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \right)}{\left( 1207959552 \times 2^{3/8} \times 3^{3/4} \sqrt{\pi^2} \right)}$$

Inverting, we obtain:

$$1/\left(\left(\left(\left(125 \sqrt{35} \pi^2 \sqrt{1/(0.001 * 0.001)}\right)/\left(301989888 2^{(3/8)} 3^{(3/4)}\right)\right)\right)\right)$$

**Input**

$$\frac{1}{\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}}}{301989888 \times 2^{3/8} \times 3^{3/4}}}$$

## Result

122.314...

122.314....

## Series representations

$$\frac{1}{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)} = \frac{301\,989\,888 \cdot 2^{3/8} \times 3^{3/4}}{301\,989\,888 \times 2^{3/8} \times 3^{3/4}}$$

$$125 \pi^2 \sqrt{34} \sqrt{999\,999} \left( \sum_{k=0}^{\infty} 34^{-k} \binom{\frac{1}{2}}{k} \right) \sum_{k=0}^{\infty} e^{-13.8155k} \binom{\frac{1}{2}}{k}$$


---

$$\frac{1}{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)} = \frac{301\,989\,888 \cdot 2^{3/8} \times 3^{3/4}}{301\,989\,888 \times 2^{3/8} \times 3^{3/4}}$$

$$125 \pi^2 \sqrt{34} \sqrt{999\,999} \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{34}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1 \times 10^{-6})^k \left(-\frac{1}{2}\right)_k}{k!}$$


---

$$\frac{1}{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)} = \frac{301\,989\,888 \cdot 2^{3/8} \times 3^{3/4}}{301\,989\,888 \times 2^{3/8} \times 3^{3/4}}$$

$$125 \pi^2 \sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35-z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))



From the sum between the two previous inverted expressions, we obtain:

$$\frac{1}{\left(\frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{-0.4 \times (-4)}}}{1610612736 \cdot 2^{3/8} \cdot 3^{3/4} \cdot (-0.4)^4}\right)} + \frac{1}{\left(\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}}}{301989888 \cdot 2^{3/8} \cdot 3^{3/4}}\right)} + \pi$$

### Input

$$\frac{1}{\frac{4375 \sqrt{35} \pi^2 \sqrt{\frac{1}{-0.4 \times (-4)}}}{1610612736 \cdot 2^{3/8} \cdot 3^{3/4} \cdot (-0.4)^4}} + \frac{1}{\frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}}}{301989888 \cdot 2^{3/8} \cdot 3^{3/4}}} + \pi$$

### Result

728.996...

728.996...  $\approx$  729

### Series representations

$$\begin{aligned} & \frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{-0.4 \times (-4)}} \right)}{1610612736 \cdot 2^{3/8} \cdot 3^{3/4} \cdot (-0.4)^4}} + \frac{1}{\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \cdot 2^{3/8} \cdot 3^{3/4}}} + \pi = \\ & \left( 7.14183 \times 10^6 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} + \right. \\ & 27859.9 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} + \\ & \left. \pi^3 \sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \right. \\ & \left. (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} (1 \times 10^6 - z_0)^{k_3} z_0^{-k_1-k_2-k_3} \right) / \\ & \left( \pi^2 \sqrt{z_0}^{-2} \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} \right) \right. \\ & \left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35 - z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} \right) \end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

---


$$\begin{aligned}
& \frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} + \frac{1}{\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \cdot 2^{3/8} \cdot 3^{3/4}}} + \pi = \\
& \left( 7.14183 \times 10^6 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (0.625 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& 27859.9 \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (1 \times 10^6 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\
& \pi^3 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2 \pi} \right] \right) \\
& \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sqrt{x}^2 \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} (0.625 - x)^{k_1} (35 - x)^{k_2} \\
& \left. (1 \times 10^6 - x)^{k_3} x^{-k_1-k_2-k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \right) / \\
& \left( \pi^2 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2 \pi} \right] \right) \right. \\
& \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sqrt{x}^2 \\
& \left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k (0.625 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left( \sum_{k=0}^{\infty} \frac{(-1)^k (35 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 \times 10^6 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$


---



## Result

1729.00...

1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

## Series representations

$$\begin{aligned}
 & 10^3 + \frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} + \frac{1}{\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \cdot 2^{3/8} \times 3^{3/4}}} + \pi = \\
 & \left( 7.14183 \times 10^6 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} + \right. \\
 & \quad 27859.9 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} + \\
 & \quad 1000 \pi^2 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \\
 & \quad \left(-\frac{1}{2}\right)_{k_3} (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} (1 \times 10^6 - z_0)^{k_3} z_0^{-k_1-k_2-k_3} + \\
 & \quad \left. \pi^3 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \right. \\
 & \quad \left. (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} (1 \times 10^6 - z_0)^{k_3} z_0^{-k_1-k_2-k_3} \right) / \\
 & \left( \pi^2 \sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} \right) \right. \\
 & \quad \left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35 - z_0)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} \right)
 \end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\begin{aligned}
& 10^3 + \frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} + \frac{1}{\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \cdot 2^{3/8} \times 3^{3/4}}} + \pi = \\
& \left( 7.14183 \times 10^6 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (0.625 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& 27859.9 \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (1 \times 10^6 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\
& 1000 \pi^2 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \\
& \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2 \pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sqrt{x}^2 \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} (0.625 - x)^{k_1} (35 - x)^{k_2} \\
& (1 \times 10^6 - x)^{k_3} x^{-k_1-k_2-k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} + \\
& \pi^3 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2 \pi} \right] \right) \\
& \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sqrt{x}^2 \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} (0.625 - x)^{k_1} (35 - x)^{k_2} \\
& (1 \times 10^6 - x)^{k_3} x^{-k_1-k_2-k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \Big/ \\
& \left( \pi^2 \exp\left(i \pi \left[ \frac{\arg(0.625 - x)}{2 \pi} \right] \right) \exp\left(i \pi \left[ \frac{\arg(35 - x)}{2 \pi} \right] \right) \right. \\
& \exp\left(i \pi \left[ \frac{\arg(1 \times 10^6 - x)}{2 \pi} \right] \right) \sqrt{x}^2 \\
& \left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k (0.625 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left( \sum_{k=0}^{\infty} \frac{(-1)^k (35 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k (1 \times 10^6 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& 10^3 + \frac{1}{\frac{4375 \left( \sqrt{35} \pi^2 \sqrt{-\frac{1}{0.4(-4)}} \right)}{1610612736 \cdot 2^{3/8} (3^{3/4} (-0.4)^4)}} + \frac{1}{\frac{125 \left( \sqrt{35} \pi^2 \sqrt{\frac{1}{0.001 \times 0.001}} \right)}{301989888 \cdot 2^{3/8} \times 3^{3/4}}} + \pi = \\
& \left( \left( \frac{1}{z_0} \right)^{-1/2 [\arg(0.625 - z_0)/(2\pi)] - 1/2 [\arg(35 - z_0)/(2\pi)] - 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \right. \\
& \quad z_0^{-1 - 1/2 [\arg(0.625 - z_0)/(2\pi)] - 1/2 [\arg(35 - z_0)/(2\pi)] - 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \\
& \quad \left( 7.14183 \times 10^6 \left( \frac{1}{z_0} \right)^{1/2 [\arg(0.625 - z_0)/(2\pi)]} z_0^{1/2 [\arg(0.625 - z_0)/(2\pi)]} \right. \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} + 27859.9 \left( \frac{1}{z_0} \right)^{1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \\
& \quad z_0^{1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 1000 \pi^2 \left( \frac{1}{z_0} \right)^{1/2 [\arg(0.625 - z_0)/(2\pi)] + 1/2 [\arg(35 - z_0)/(2\pi)] + 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \\
& \quad z_0^{1 + 1/2 [\arg(0.625 - z_0)/(2\pi)] + 1/2 [\arg(35 - z_0)/(2\pi)] + 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \\
& \quad (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} (1 \times 10^6 - z_0)^{k_3} z_0^{-k_1-k_2-k_3} + \\
& \quad \left. \pi^3 \left( \frac{1}{z_0} \right)^{1/2 [\arg(0.625 - z_0)/(2\pi)] + 1/2 [\arg(35 - z_0)/(2\pi)] + 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \right. \\
& \quad z_0^{1 + 1/2 [\arg(0.625 - z_0)/(2\pi)] + 1/2 [\arg(35 - z_0)/(2\pi)] + 1/2 [\arg(1 \times 10^6 - z_0)/(2\pi)]} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (-1)^{k_1+k_2+k_3} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(-\frac{1}{2}\right)_{k_3} \\
& \quad \left. (0.625 - z_0)^{k_1} (35 - z_0)^{k_2} (1 \times 10^6 - z_0)^{k_3} z_0^{-k_1-k_2-k_3} \right) \Bigg) / \\
& \left( \pi^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.625 - z_0)^k z_0^{-k}}{k!} \right) \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (35 - z_0)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1 \times 10^6 - z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

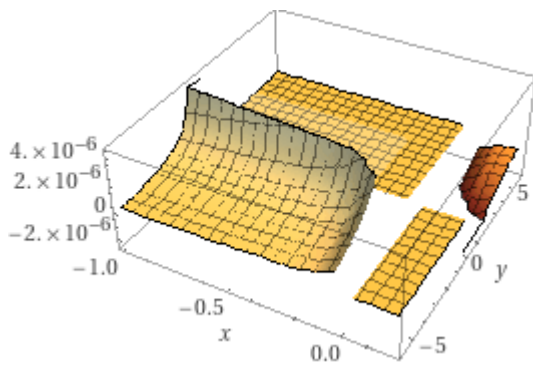


## Exact result

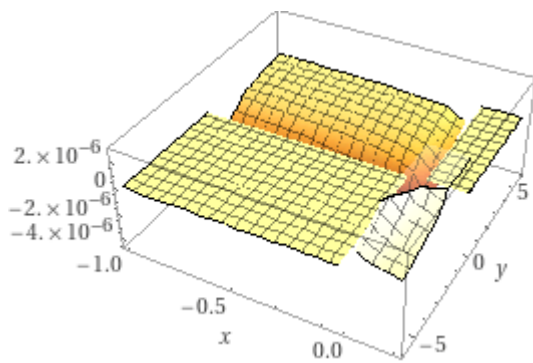
$$-\frac{125 \sqrt{35} \pi^2 x \left(\frac{1}{xy}\right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}}$$

## 3D plots

**Real part** (figures that can be related to the D-branes/Instantons)



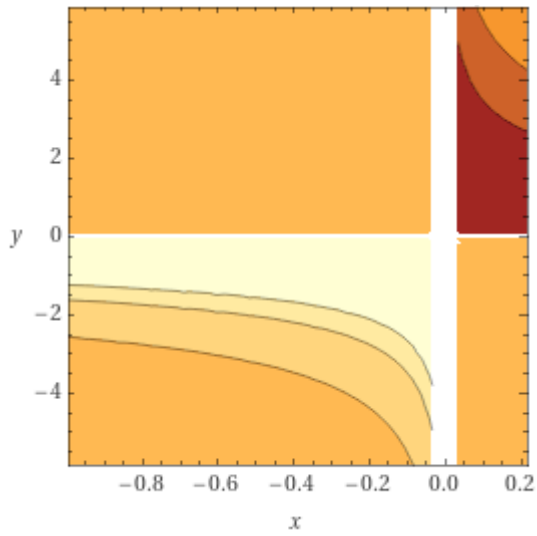
## Imaginary part



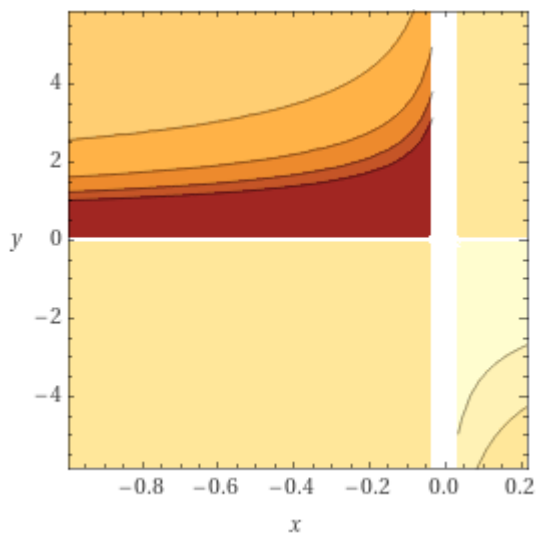


## Contour plots

### Real part



### Imaginary part



## Roots

(no roots exist)

## Properties as a function

### Domain

$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$

## Range

$$\{z \in \mathbb{R} : z \neq 0\}$$

---

## Parity

odd

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$-\frac{125 \left( \sqrt{35} \pi^2 x^{3/2} \left( \frac{1}{xy} \right)^{3/2} \right)}{603979776 \left( 2^{3/8} \times 3^{3/4} \right) \sqrt{x}} + O(x^{11/2})$$

(Puiseux series)

## Partial derivatives

$$\frac{\partial}{\partial x} \left( -\frac{125 \sqrt{35} \pi^2 x \left( \frac{1}{xy} \right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}} \right) = \frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}}$$

---

$$\frac{\partial}{\partial y} \left( -\frac{125 \sqrt{35} \pi^2 x \left( \frac{1}{xy} \right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}} \right) = \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{402653184 \times 2^{3/8} \times 3^{3/4} y^2}$$

## Indefinite integral

$$\int -\frac{125 \sqrt{35} \pi^2 x \left( \frac{1}{xy} \right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}} dx = -\frac{125 \sqrt{35} \pi^2 x^2 \left( \frac{1}{xy} \right)^{3/2}}{301989888 \times 2^{3/8} \times 3^{3/4}} + \text{constant}$$

And from:

$$\frac{\partial}{\partial x} \left( -\frac{125 \sqrt{35} \pi^2 x \left(\frac{1}{xy}\right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}} \right) = \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}}$$

we obtain:

$$(125 \sqrt{35} \pi^2 (1/(xy))^{3/2}) / (1207959552 2^{3/8} 3^{3/4})$$

**Input**

$$\frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}}$$

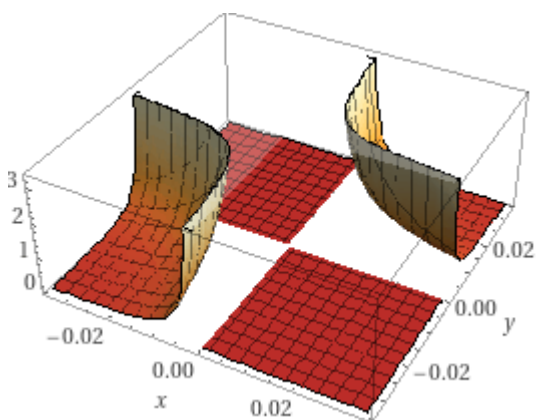
**Exact result**

$$\frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}}$$

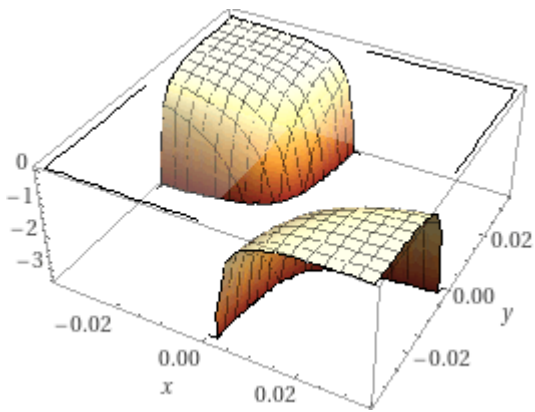
**3D plots**

**Real part**

(figures that can be related to the D-branes/Instantons)

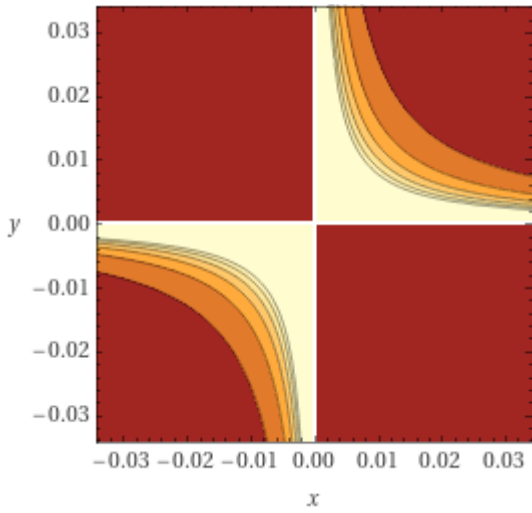


### Imaginary part

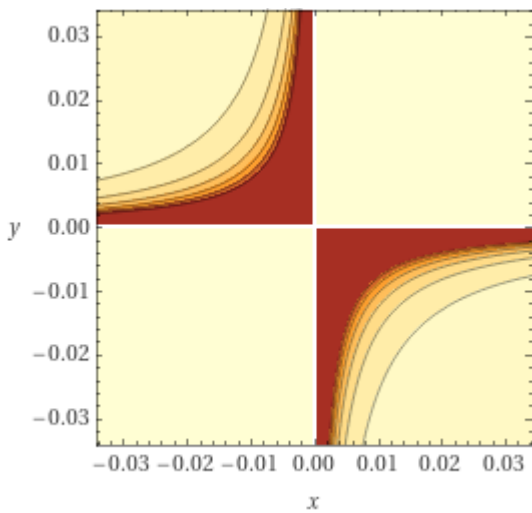


### Contour plots

#### Real part



#### Imaginary part



## Roots

(no roots exist)

## Properties as a function

### Domain

$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$

---

### Range

$\{z \in \mathbb{R} : z > 0\}$  (all positive real numbers)

---

### Parity

even

$\mathbb{R}$  is the set of real numbers

## Series expansion at $x=0$

$$\frac{125 \sqrt{35} \pi^2 x^{3/2} \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4} x^{3/2}} + O(x^{11/2})$$

(Puiseux series)

## Series expansion at $x=\infty$

$$\frac{125 \sqrt{35} \pi^2 x^{3/2} \left(\frac{1}{x}\right)^{3/2} \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} + O\left(\left(\frac{1}{x}\right)^{11/2}\right)$$

(Puiseux series)

## Partial derivatives

$$\frac{\partial}{\partial x} \left( \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{125 \sqrt{35} \pi^2 y \left(\frac{1}{xy}\right)^{5/2}}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

---

$$\frac{\partial}{\partial y} \left( \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{125 \sqrt{35} \pi^2 x \left(\frac{1}{xy}\right)^{5/2}}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

## Indefinite integral

$$\int \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} dx = - \frac{125 \sqrt{35} \pi^2 x \left(\frac{1}{xy}\right)^{3/2}}{603979776 \times 2^{3/8} \times 3^{3/4}} + \text{constant}$$

## Limit

$$\lim_{x \rightarrow \pm\infty} \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} = 0$$

---

$$\lim_{y \rightarrow \pm\infty} \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} = 0$$

## Series representations

$$\frac{125 \left(\sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{3/2}\right)}{1207959552 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sqrt{34} \sum_{k=0}^{\infty} 34^{-k} \binom{\frac{1}{2}}{k}}{1207959552 \times 2^{3/8} \times 3^{3/4} x y}$$

---


$$\frac{125 \left( \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2} \right)}{1207959552 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sqrt{34} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{34}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{1207959552 \times 2^{3/8} \times 3^{3/4} xy}$$


---

$$\frac{125 \left( \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2} \right)}{1207959552 \times 2^{3/8} \times 3^{3/4}} = \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 34^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2415919104 \times 2^{3/8} \times 3^{3/4} xy \sqrt{\pi}}$$

And again, from:

$$\frac{\partial}{\partial x} \left( \frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2}}{1207959552 \times 2^{3/8} \times 3^{3/4}} \right) = - \frac{125 \sqrt{35} \pi^2 y \left( \frac{1}{xy} \right)^{5/2}}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

$$-(125 \sqrt{35} \pi^2 (1/(xy))^{5/2} y)/(805306368 2^{3/8} 3^{3/4})$$

### Input

$$-\frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{5/2} y}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

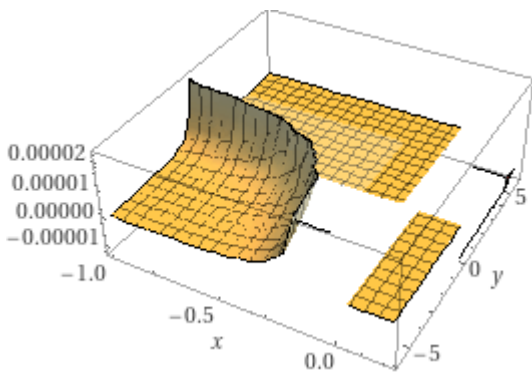
### Exact result

$$-\frac{125 \sqrt{35} \pi^2 y \left( \frac{1}{xy} \right)^{5/2}}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

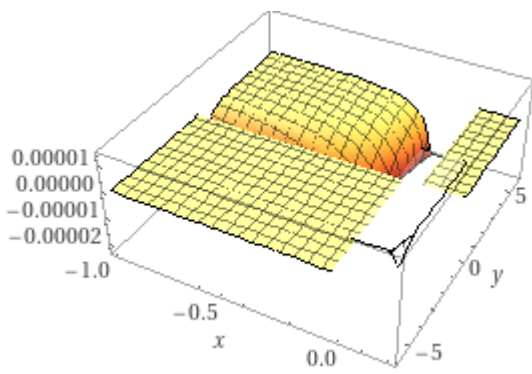
### 3D plots

#### Real part

(figures that can be related to the D-branes/Instantons)

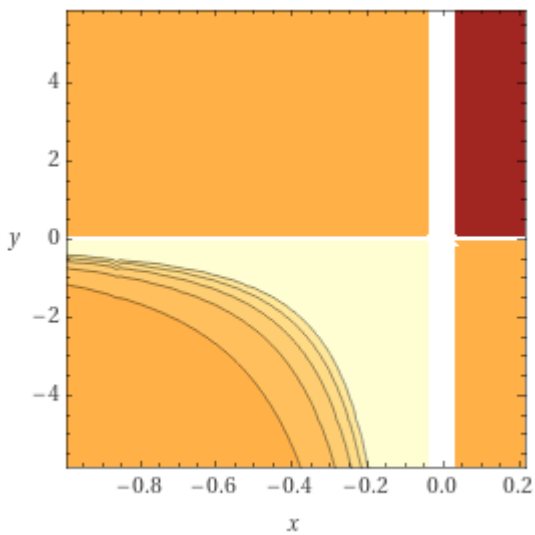


#### Imaginary part



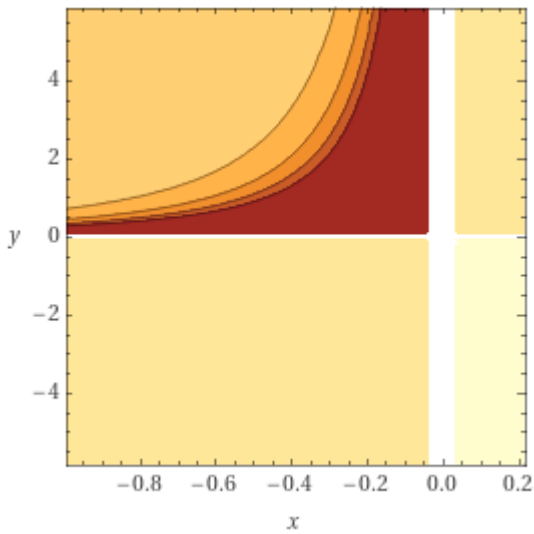
### Contour plots

#### Real part





## Imaginary part



## Alternate form assuming x and y are positive

$$-\frac{125 \sqrt{35} \pi^2}{805306368 \times 2^{3/8} \times 3^{3/4} x^{5/2} y^{3/2}}$$

## Roots

(no roots exist)

## Properties as a function

### Domain

$$\{(x, y) \in \mathbb{R}^2 : x \neq 0 \text{ and } y \neq 0 \text{ and } xy > 0\}$$

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### Range

$$\{z \in \mathbb{R} : z \neq 0\}$$

---

### Parity

odd

### Series expansion at $x=0$

$$-\frac{125 \left( \sqrt{35} \pi^2 x^{3/2} \left( \frac{1}{xy} \right)^{3/2} \right)}{805\,306\,368 \left( 2^{3/8} \times 3^{3/4} \right) x^{5/2}} + O(x^{11/2})$$

(Puiseux series)

### Series expansion at $x=\infty$

$$-\frac{125 \left( \frac{1}{x} \right)^{5/2} \left( \sqrt{35} \pi^2 x^{3/2} \left( \frac{1}{xy} \right)^{3/2} \right)}{805\,306\,368 \left( 2^{3/8} \times 3^{3/4} \right)} + O\left( \left( \frac{1}{x} \right)^{11/2} \right)$$

(Puiseux series)

### Partial derivatives

$$\frac{\partial}{\partial x} \left( -\frac{125 \sqrt{35} \pi^2 y \left( \frac{1}{xy} \right)^{5/2}}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} \right) = \frac{625 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2}}{1\,610\,612\,736 \times 2^{3/8} \times 3^{3/4} x^2}$$

$$\frac{\partial}{\partial y} \left( -\frac{125 \sqrt{35} \pi^2 y \left( \frac{1}{xy} \right)^{5/2}}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} \right) = \frac{125 \sqrt{35} \pi^2 \sqrt{\frac{1}{xy}}}{536\,870\,912 \times 2^{3/8} \times 3^{3/4} x^2 y^2}$$

### Indefinite integral

$$\int -\frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} dx = \frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{3/2}}{1\,207\,959\,552 \times 2^{3/8} \times 3^{3/4}} + \text{constant}$$

### Limit

$$\lim_{x \rightarrow \pm\infty} -\frac{125 \sqrt{35} \pi^2 \left( \frac{1}{xy} \right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} = 0$$

$$\lim_{y \rightarrow \pm\infty} - \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} = 0$$

### Series representations

$$- \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} = - \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sqrt{34} \sum_{k=0}^{\infty} 34^{-k} \binom{\frac{1}{2}}{k}}{805\,306\,368 \times 2^{3/8} \times 3^{3/4} x^2 y}$$


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$$- \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} = - \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sqrt{34} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{34}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{805\,306\,368 \times 2^{3/8} \times 3^{3/4} x^2 y}$$


---

$$- \frac{125 \sqrt{35} \pi^2 \left(\frac{1}{xy}\right)^{5/2} y}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}} = - \frac{125 \pi^2 \sqrt{\frac{1}{xy}} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 34^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{1\,610\,612\,736 \times 2^{3/8} \times 3^{3/4} x^2 y \sqrt{\pi}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

From:

$$- \frac{125 \sqrt{35} \pi^2 y \left(\frac{1}{xy}\right)^{5/2}}{805\,306\,368 \times 2^{3/8} \times 3^{3/4}}$$

For  $x = -0.8$  and  $y = -3$ , we obtain:

$$-(125 \sqrt{35} \pi^2 (1/(-0.8 * -3))^{5/2} * (-3)) / (805306368 2^{3/8} 3^{3/4})$$

### Input

$$-\frac{125 \sqrt{35} \pi^2 \left( \left( -\frac{1}{0.8(-3)} \right)^{5/2} \times (-3) \right)}{805306368 \times 2^{3/8} \times 3^{3/4}}$$

### Result

$$1.03074... \times 10^{-6}$$

$$1.03074... * 10^{-6}$$

### Series representations

$$-\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \times 2^{3/8} \times 3^{3/4}} = 1.76529 \times 10^{-8} \pi^2 \sqrt{34} \sum_{k=0}^{\infty} 34^{-k} \binom{\frac{1}{2}}{k}$$

$$-\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \times 2^{3/8} \times 3^{3/4}} = 1.76529 \times 10^{-8} \pi^2 \sqrt{34} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{34} \right)^k \left( -\frac{1}{2} \right)_k}{k!}$$

$$-\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \times 2^{3/8} \times 3^{3/4}} = \frac{8.82643 \times 10^{-9} \pi^2 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 34^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}$$

From which:

$$1 / \left( \left( -125 \sqrt{35} \pi^2 (1/(-0.8 * -3))^{5/2} * (-3) \right) / (805306368 2^{3/8} 3^{3/4}) \right)^{20} * \left( \frac{1}{2} (5 e^\pi + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)) \right)$$

## Input

$$\frac{1}{\left(-\frac{125\sqrt{35}\pi^2\left(-\frac{1}{0.8(-3)}\right)^{5/2}\times(-3)}{805306368\times 2^{3/8}\times 3^{3/4}}\right)^{20}} \left(\frac{1}{2}(5e^\pi + \pi + \log(16) + 3\log(\pi) + 3\tan^{-1}(\pi))\right)$$

$\log(x)$  is the natural logarithm

$\tan^{-1}(x)$  is the inverse tangent function

## Result

$$3.51599... \times 10^{121}$$

(result in radians)

$$0.351599... * 10^{122} \approx \Lambda_Q$$

The observed value of  $\rho_\Lambda$  or  $\Lambda$  today is precisely the classical dual of its quantum precursor values  $\rho_Q$ ,  $\Lambda_Q$  in the quantum very early precursor vacuum  $U_Q$  as determined by our dual equations. With regard the Cosmological constant, fundamental are the following results:  $\Lambda = 2.846 * 10^{-122}$  and  $\Lambda_Q = 0.3516 * 10^{122}$  (New Quantum Structure of the Space-Time - Norma G. SANCHEZ - arXiv:1910.13382v1 [physics.gen-ph] 28 Oct 2019)

## Alternative representations

$$\frac{5e^\pi + \pi + \log(16) + 3\log(\pi) + 3\tan^{-1}(\pi)}{2\left(-\frac{125\left(-\frac{1}{0.8(-3)}\right)^{5/2}(-3)\sqrt{35}\pi^2}{805306368\times 2^{3/8}\times 3^{3/4}}\right)^{20}} = \frac{\pi + 3\tan^{-1}(1, \pi) + \log(16) + 3\log(\pi) + 5e^\pi}{2\left(\frac{375\pi^2\left(\frac{1}{2.4}\right)^{5/2}\sqrt{35}}{805306368\times 2^{3/8}\times 3^{3/4}}\right)^{20}}$$

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\frac{\pi + 3 \tan^{-1}(\pi) + \log_e(16) + 3 \log_e(\pi) + 5 e^{\pi}}{2 \left( \frac{375 \pi^2 \left( \frac{1}{2.4} \right)^{5/2} \sqrt{35}}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}}$$


---

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\frac{\pi + 3 \tan^{-1}(\pi) + \log(a) \log_a(16) + 3 \log(a) \log_a(\pi) + 5 e^{\pi}}{2 \left( \frac{375 \pi^2 \left( \frac{1}{2.4} \right)^{5/2} \sqrt{35}}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}}$$

## Series representations

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\left( 2.89496 \times 10^{155} e^{\pi} + 5.78993 \times 10^{154} \pi + 1.73698 \times 10^{155} \tan^{-1}(x) - \right.$$

$$1.73698 \times 10^{155} \pi \left[ \frac{\arg(i(-\pi + x))}{2\pi} \right] + 5.78993 \times 10^{154} \log(15) +$$

$$1.73698 \times 10^{155} \log(-1 + \pi) + 5.78993 \times 10^{154} \sum_{k=1}^{\infty} \frac{-2 \left( -\frac{1}{15} \right)^k - 6(-1)^k (-1 + \pi)^{-k} + 3i \left( -(-i - x)^{-k} + (i - x)^{-k} \right) (\pi - x)^k}{2k}$$

$$\left. \right) /$$

$$\left( \pi^{40} \exp^{20} \left( i \pi \left[ \frac{\arg(35 - x)}{2\pi} \right] \right) \sqrt{x}^{20} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (35 - x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right)^{20} \right)$$

for  $(i x \in \mathbb{R} \text{ and } i x > 1 \text{ and } x \in \mathbb{R} \text{ and } x < 0)$

---

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\left( \begin{aligned} & 2.89496 \times 10^{155} e^{\pi} + 5.78993 \times 10^{154} \pi + \\ & 5.78993 \times 10^{154} \log(15) + 1.73698 \times 10^{155} \log(-1 + \pi) - \\ & 5.78993 \times 10^{154} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{15}\right)^k}{k} - 1.73698 \times 10^{155} \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \pi)^{-k}}{k} + \\ & 1.73698 \times 10^{155} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 2^{1+2k} F_{1+2k} \left( \frac{\pi}{1 + \sqrt{1 + \frac{4\pi^2}{5}}} \right)^{1+2k}}{1 + 2k} \end{aligned} \right) /$$

$$\left( \pi^{40} \exp^{20} \left( i \pi \left[ \frac{\arg(35 - x)}{2\pi} \right] \right) \sqrt{x}^{20} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (35 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^{20} \right)$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& \frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805 306 368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} = \\
& \left( \left( \frac{1}{z_0} \right)^{-10 \lfloor \arg(35-z_0)/(2\pi) \rfloor} z_0^{-10-10 \lfloor \arg(35-z_0)/(2\pi) \rfloor} \right. \\
& \quad \left( 2.89496 \times 10^{155} e^{\pi} + 5.78993 \times 10^{154} \pi + 1.73698 \times 10^{155} \tan^{-1}(x) - \right. \\
& \quad 1.73698 \times 10^{155} \pi \left[ \frac{\arg(i(-\pi+x))}{2\pi} \right] + 5.78993 \times 10^{154} \log(15) + \\
& \quad 1.73698 \times 10^{155} \log(-1+\pi) + 5.78993 \times 10^{154} \sum_{k=1}^{\infty} \frac{1}{2k} \left( -2 \left( -\frac{1}{15} \right)^k - \right. \\
& \quad \left. \left. 6(-1)^k (-1+\pi)^{-k} + 3i(-(-i-x)^{-k} + (i-x)^{-k}) (\pi-x)^k \right) \right) \left. \right) / \\
& \left( \pi^{40} \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (35-z_0)^k z_0^{-k}}{k!} \right)^{20} \right) \text{ for } (ix \in \mathbb{R} \text{ and } ix > 1)
\end{aligned}$$

## Integral representations

$$\begin{aligned}
& \frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805 306 368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} = \\
& \frac{5.78993 \times 10^{154} \left( 5 e^{\pi} + \pi + 3\pi \int_0^1 \frac{1}{1+\pi^2 t^2} dt + \log(16) + 3 \log(\pi) \right)}{\pi^{40} \sqrt{35}^{20}}
\end{aligned}$$

$$\begin{aligned}
& \frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805 306 368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} = \\
& \int_1^{16} \frac{\frac{1.73698 \times 10^{155} (-1+\pi)}{16+\pi(-1+t)-t} + \frac{5.78993 \times 10^{154}}{t} + \frac{2.60547 \times 10^{156} \pi}{225+\pi^2(1-2t+t^2)}}{\pi^{40} \sqrt{35}^{20}} dt + \\
& \frac{2.89496 \times 10^{155} e^{\pi}}{\pi^{40} \sqrt{35}^{20}} + \frac{5.78993 \times 10^{154}}{\pi^{39} \sqrt{35}^{20}}
\end{aligned}$$



$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} = \frac{2.89496 \times 10^{155} e^{\pi}}{\pi^{40} \sqrt{35}^{20}} + \frac{5.78993 \times 10^{154}}{\pi^{39} \sqrt{35}^{20}} -$$

$$\frac{4.34245 \times 10^{154} i}{\pi^{81/2} \sqrt{35}^{20}} \int_{-i\infty+\gamma}^{i\infty+\gamma} (1 + \pi^2)^{-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s) \Gamma(s)^2 ds +$$

$$\frac{5.78993 \times 10^{154} \log(16)}{\pi^{40} \sqrt{35}^{20}} + \frac{1.73698 \times 10^{155} \log(\pi)}{\pi^{40} \sqrt{35}^{20}} \text{ for } 0 < \gamma < \frac{1}{2}$$

$\Gamma(x)$  is the gamma function

### Continued fraction representations

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\frac{5.78993 \times 10^{154} \left( 5 e^{\pi} + \pi + \frac{3\pi}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}} + \frac{15}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{15 \left| \frac{1+k}{2} \right|^2}{1+k}} + \frac{3(-1+\pi)}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{(-1+\pi) \left| \frac{1+k}{2} \right|^2}{1+k}} \right)}{\pi^{40} \sqrt{35}^{20}} =$$

$$2.72896 \times 10^{119} \left( 5 e^{\pi} + \pi + \frac{3\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} + \right.$$

$$\left. \frac{15}{1 + \frac{15}{2 + \frac{15}{3 + \frac{60}{4 + \frac{60}{5 + \dots}}}}} + \frac{3(-1+\pi)}{1 + \frac{-1+\pi}{2 + \frac{-1+\pi}{3 + \frac{4(-1+\pi)}{4 + \frac{4(-1+\pi)}{5 + \dots}}}}} \right)$$

---


$$\begin{aligned}
& \frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} = \\
& \frac{1}{\pi^{40} \sqrt{35}^{20}} 5.78993 \times 10^{154} \left( 5 e^{\pi} + \pi + 3 \left( \pi - \frac{\pi^3}{3 + \sum_{k=1}^{\infty} \frac{(1+(-1)^{1+k}+k)^2 \pi^2}{3+2k}} \right) + \right. \\
& \left. \frac{15}{1 + \sum_{k=1}^{\infty} \frac{15 \left| \frac{1+k}{2} \right|^2}{1+k}} + \frac{3(-1+\pi)}{1 + \sum_{k=1}^{\infty} \frac{(-1+\pi) \left| \frac{1+k}{2} \right|^2}{1+k}} \right) = \\
& 2.72896 \times 10^{119} \left( 5 e^{\pi} + \pi + 3 \left( \pi - \frac{\pi^3}{3 + \frac{9\pi^2}{5 + \frac{4\pi^2}{7 + \frac{25\pi^2}{9 + \frac{16\pi^2}{11 + \dots}}}}} \right) + \right. \\
& \left. \frac{15}{1 + \frac{15}{2 + \frac{15}{3 + \frac{60}{4 + \frac{60}{5 + \dots}}}}} + \frac{3(-1+\pi)}{1 + \frac{-1+\pi}{2 + \frac{-1+\pi}{3 + \frac{4(-1+\pi)}{4 + \frac{4(-1+\pi)}{5 + \dots}}}} \right)
\end{aligned}$$


---

$$\frac{5 e^{\pi} + \pi + \log(16) + 3 \log(\pi) + 3 \tan^{-1}(\pi)}{2 \left( -\frac{125 \left( -\frac{1}{0.8(-3)} \right)^{5/2} (-3) \sqrt{35} \pi^2}{805306368 \cdot 2^{3/8} \cdot 3^{3/4}} \right)^{20}} =$$

$$\frac{5.78993 \times 10^{154} \left( 5 e^{\pi} + \pi + \frac{3\pi}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{k^2 \pi^2}{1+2k}} + \frac{15}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{15 \left| \frac{1+k}{2} \right|^2}{1+k}} + \frac{3(-1+\pi)}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{(-1+\pi) \left| \frac{1+k}{2} \right|^2}{1+k}} \right)}{\pi^{40} \left( 1 + 4 \left( \mathop{\text{K}}_{k=1}^{\infty} \frac{17}{\frac{8}{\frac{1}{2}}} \right) \right)^{20}} =$$

$$7.52798 \times 10^{134} \left( 5 e^{\pi} + \pi + \frac{3\pi}{1 + \frac{\pi^2}{3 + \frac{4\pi^2}{5 + \frac{9\pi^2}{7 + \frac{16\pi^2}{9 + \dots}}}}} + \frac{15}{1 + \frac{15}{2 + \frac{15}{3 + \frac{60}{4 + \frac{60}{5 + \dots}}}}} + \frac{3(-1+\pi)}{1 + \frac{-1+\pi}{2 + \frac{-1+\pi}{3 + \frac{4(-1+\pi)}{4 + \frac{4(-1+\pi)}{5 + \dots}}}}} \right)$$

$$\left( 1 + 4 \left( \frac{17}{8 \left( \frac{1}{2} + \frac{17}{8 \left( \frac{1}{2} + \frac{17}{8 \left( \frac{1}{2} + \frac{17}{8 \left( \frac{1}{2} + \frac{17}{8 \left( \frac{1}{2} + \dots \right)} \right)} \right)} \right)} \right)} \right) \right)^{20}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$  is a continued fraction

From:

**SHARP STABILITY INEQUALITIES FOR THE PLATEAU PROBLEM - G.**  
*De Philippis & F. Maggi - j. differential geometry 96 (2014) 399-456*

We have that:

$$R > 0.$$

$$\varepsilon < \frac{R}{2\sqrt{h-1}},$$

$$(4.10) \quad c = \frac{\sqrt{3}}{16},$$

From:

$$\begin{aligned} & \int_{B_R^h \setminus B_{\varepsilon\sqrt{h-1}}^h} \left( \frac{|y|}{\sqrt{h-1}} + \varepsilon \right)^k - \left( \frac{|y|}{\sqrt{h-1}} - \varepsilon \right)^k dy \\ &= \frac{h k \omega_h \varepsilon}{(h-1)^{(k-1)/2}} \int_{\varepsilon\sqrt{h-1}}^R r^{h+k-2} dr \leq \omega_h \frac{h k}{m-1} \frac{R^{m-1}}{(h-1)^{(k-1)/2}} \varepsilon, \end{aligned}$$

$$x^* (h^k)/(m-1) * R^{(m-1)} / ((h-1)^{(k-1)/2}) * y$$

**Input**

$$x \times \frac{h k}{m-1} \times \frac{R^{m-1}}{(h-1)^{(k-1)/2}} y$$

## Result

$$\frac{h k x y (h-1)^{(1-k)/2} R^{m-1}}{m-1}$$

## Alternate form

$$\frac{h k x y (h-1)^{1/2-k/2} R^{m-1}}{m-1}$$

## Roots

$$h = 1, \quad \operatorname{Re}(m) > \frac{1}{2} (\operatorname{Re}(k) + 1), \quad R = 0$$

---

$$h - 1 \neq 0, \quad \operatorname{Re}(m) > 1, \quad R = 0$$

---

$$m - 1 \neq 0, \quad R \neq 0, \quad h = 0$$

---

$$R \neq 0, \quad h = 1, \quad \operatorname{Re}(k) < 1$$

---

$$h - 1 \neq 0, \quad m - 1 \neq 0, \quad R \neq 0, \quad k = 0$$

$\operatorname{Re}(z)$  is the real part of  $z$

## Derivative

$$\frac{\partial}{\partial x} \left( \frac{x (h k) R^{m-1} y}{(m-1) (h-1)^{(k-1)/2}} \right) = \frac{h k y (h-1)^{(1-k)/2} R^{m-1}}{m-1}$$

## Indefinite integral

$$\int \frac{(-1+h)^{(1-k)/2} h k R^{-1+m} x y}{-1+m} dx = \frac{h k x^2 y (h-1)^{(1-k)/2} R^{m-1}}{2(m-1)} + \text{constant}$$

## Limit

$$\lim_{m \rightarrow -\infty} \frac{(-1+h)^{(1-k)/2} h k R^{-1+m} x y}{-1+m} = 0 \text{ for } ((h-1)^{(1-k)/2}, h, k, x, y) \in \mathbb{R}^5 \wedge \log(R) > 0$$

$\log(x)$  is the natural logarithm

$e_1 \wedge e_2 \wedge \dots$  is the logical AND function

$\mathbb{R}$  is the set of real numbers

## Series representations

$$\frac{(-1+h)^{(1-k)/2} h k R^{-1+m} x y}{-1+m} = \sum_{n=1}^{\infty} \frac{h^n (-1)^{3/2-k/2+n} \left( k R^{-1+m} x y \binom{\frac{1}{2} - \frac{k}{2}}{-1+n} \right)}{-1+m}$$

$$\frac{(-1+h)^{(1-k)/2} h k R^{-1+m} x y}{-1+m} = \sum_{n=0}^{\infty} \frac{(-1+R)^n (-1+h)^{1/2-k/2} \left( h k x y \binom{-1+m}{n} \right)}{-1+m}$$

$$\frac{(-1+h)^{(1-k)/2} h k R^{-1+m} x y}{-1+m} = \sum_{n=1}^{\infty} \frac{k^n 2^{1-n} \left( \sqrt{-1+h} h R^{-1+m} x y (-\log(-1+h))^{-1+n} \right)}{(-1+m)(-1+n)!}$$

For:  $k = 3$ ;  $h = 5$  ;  $m = 8$

From:

$$\frac{h k x y (h - 1)^{(1-k)/2} R^{m-1}}{m - 1}$$

$$(3 \times 5 x y (5 - 1)^{(1-3)/2} 2^{8-1}) / (8 - 1)$$

**Input**

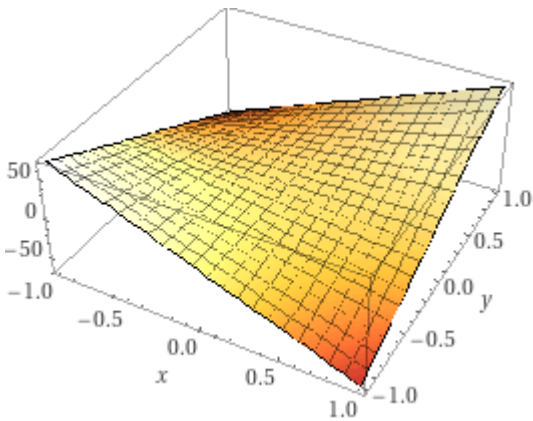
$$\frac{(3 \times 5) x y (5 - 1)^{(1-3)/2} \times 2^{8-1}}{8 - 1}$$

**Result**

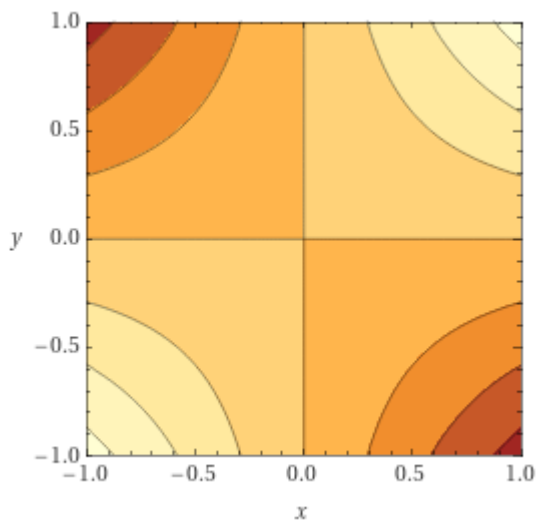
$$\frac{480 x y}{7}$$

**3D plot**

(figure that can be related to a D-brane/Instanton)



## Contour plot



## Geometric figure

hyperbolic paraboloid

## Properties as a function

### Domain

$\mathbb{R}^2$

---

### Range

$\mathbb{R}$  (all real numbers)

---

### Parity

even

$\mathbb{R}$  is the set of real numbers



## Partial derivatives

$$\frac{\partial}{\partial x} \left( \frac{480 x y}{7} \right) = \frac{480 y}{7}$$

---

$$\frac{\partial}{\partial y} \left( \frac{480 x y}{7} \right) = \frac{480 x}{7}$$

## Indefinite integral

$$\int \frac{480 x y}{7} dx = \frac{240 x^2 y}{7} + \text{constant}$$

## Definite integral over a disk of radius R

$$\iint_{x^2+y^2 < R^2} \frac{480 x y}{7} dx dy = 0$$

## Definite integral over a square of edge length 2 L

$$\int_{-L}^L \int_{-L}^L \frac{480 x y}{7} dy dx = 0$$

For  $x = y = 0.5$  :

$$(480 \cdot 0.5 \cdot 0.5) / 7$$

## Input

$$\frac{1}{7} (480 \times 0.5 \times 0.5)$$

## Result

17.142857142857142857142857142857142857142857142857142857142857142

...

## Repeating decimal

17.142857 (period 6)

17.142857

We have:

where, recall,  $m = k + h$ . We thus find

$$\left| \{p < \varepsilon\} \cap H_R \right| \leq \omega_h \omega_k (h-1)^{h/2} (k-1)^{k/2} \left( 2^k \varepsilon^{m-1} + \frac{hk}{m-1} \frac{R^{m-1}}{(h-1)^{(m-1)/2}} \right) \varepsilon,$$

since  $hk/(m-1) > 1$  and  $\varepsilon < R/(2\sqrt{h-1})$

From:

$$2^k \varepsilon^{m-1} \leq \frac{2^k}{2^{m-1}} \frac{R^{m-1}}{(h-1)^{(m-1)/2}} \leq \frac{hk}{m-1} \frac{R^{m-1}}{(h-1)^{(m-1)/2}}$$

for:  $k = 3$ ;  $h = 5$ ;  $m = 8$

$$(5 \cdot 3) / (8-1) * (2^{8-1}) / (((5-1)^{(7/2)}))$$

## Input

$$\frac{5 \times 3}{8-1} \times \frac{2^{8-1}}{(5-1)^{7/2}}$$

## Exact result

$$\frac{15}{7}$$

## Decimal approximation

2.1428571428571428571428571428571428571428571428571428571428

...

2.142857142....

We have:

$$(4.19) \quad \alpha \leq \frac{2R\delta}{c\varepsilon} + \frac{\gamma\varepsilon}{R}, \quad \text{whenever} \quad \varepsilon < \frac{R}{2\sqrt{h-1}}.$$

$$\varepsilon_0 = \sqrt{\frac{2\delta}{c\gamma}} R.$$

If  $\varepsilon_0 < R/(2\sqrt{h-1})$ , then, by (4.19),

$$(4.20) \quad \alpha \leq \varphi(\varepsilon_0) = \frac{2\gamma\varepsilon_0}{R} = \sqrt{\frac{8\gamma}{c}} \sqrt{\delta}.$$

Otherwise,  $1/(2\sqrt{h-1}) < \sqrt{2\delta/c\gamma}$ . Hence, by  $\delta < \omega_k\omega_h$ , and setting  $\gamma_0 = \gamma/\omega_k\omega_h$ ,

$$(4.21) \quad \begin{aligned} \alpha &\leq \varphi\left(\frac{R}{2\sqrt{h-1}}\right) = \frac{4\sqrt{h-1}}{c} \delta + \frac{\gamma}{R} \frac{R}{2\sqrt{h-1}} \\ &\leq \frac{4\sqrt{h-1}}{c} \delta + \gamma \sqrt{\frac{2\delta}{c\gamma}} \leq \sqrt{\omega_k\omega_h} \left( \frac{4\sqrt{h-1}}{c} + \sqrt{\frac{2\gamma_0}{c}} \right) \sqrt{\delta}. \end{aligned}$$

Combining (4.11), (4.20), and (4.21), we thus find

$$\alpha \leq \sqrt{\omega_k\omega_h} \max \left\{ 1, \frac{8\sqrt{h-1}}{c}, \sqrt{\frac{8\gamma_0}{c}} \right\} \sqrt{\delta}.$$

If  $(k, h) \neq (4, 4)$ , then, by (4.9),

$$\begin{aligned} \frac{8\sqrt{h-1}}{c} &= 2^{12} \left( \frac{h-1}{k-1} \right)^{3/2} \frac{1}{(k-1)^{1/4}}, \\ \sqrt{\frac{8\gamma_0}{c}} &= \sqrt{2^{13} \left( \frac{h-1}{k-1} \right)^{(5-k)/2} \frac{hk}{m-1} \frac{1}{(k-1)^{1/4}}}. \end{aligned}$$

Since  $hk \geq m-1$  and  $(5-k)/4 \leq 3/2$ , we have

$$\max \left\{ \frac{8\sqrt{h-1}}{c}, \sqrt{\frac{8\gamma_0}{c}} \right\} \leq \frac{2^{12}}{(k-1)^{1/8}} \sqrt{\frac{hk}{m-1}} \left( \frac{h-1}{k-1} \right)^{3/2},$$

For:  $k = 3$ ;  $h = 5$ ;  $m = 8$

From:

$$\sqrt{\frac{8\gamma_0}{c}} = \sqrt{2^{13} \left(\frac{h-1}{k-1}\right)^{(5-k)/2} \frac{hk}{m-1} \frac{1}{(k-1)^{1/4}}}$$

$$\text{Sqrt}(2^{13}((5-1)/(3-1))*(5*3)/(8-1)*1/(3-1)^{0.25})$$

**Input**

$$\sqrt{2^{13} \times \frac{5-1}{3-1} \times \frac{5 \times 3}{8-1} \times \frac{1}{(3-1)^{0.25}}}$$

**Result**

171.82162803024739706430781886821173633081273778862129400898403838

...

171.82162803....

From the algebraic sum of the three above expressions, after some calculations, we obtain:

$$12 * ( - ( ( ( (480 * 0.5 * 0.5) / 7) + ( (5 * 3) / (8 - 1) * (2^{8-1}) / ( (5-1)^{7/2} ) ) ) - ( \text{Sqrt}(2^{13} * ((5-1)/(3-1)) * (5*3)/(8-1) * 1/(3-1)^{0.25}) ) - 8) - (2\pi)$$

**Input**

$$12 \left( - \left( \frac{1}{7} (480 \times 0.5 \times 0.5) + \frac{5 \times 3}{8 - 1} \times \frac{2^{8-1}}{(5 - 1)^{7/2}} - \sqrt{2^{13} \times \frac{5 - 1}{3 - 1} \times \frac{5 \times 3}{8 - 1} \times \frac{1}{(3 - 1)^{0.25}}} \right) - 8 \right) - 2\pi$$

**Result**

1728.15...

1728.15....

This result is very near to the mass of candidate glueball  $\mathbf{f_0(1710)}$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

### Series representations

$$12 \left( - \left( \frac{480 (0.5 \times 0.5)}{7} + \frac{2^{8-1} (5 \times 3)}{(5-1)^{7/2} (8-1)} - \sqrt{\frac{(2^{13} (5-1) (5 \times 3))}{((3-1) (3-1)^{0.25}) (8-1)}} \right) - 8 \right) -$$

$$2\pi = -327.429 - 2\pi + 12 \sqrt{29521.7} \sum_{k=0}^{\infty} e^{-10.2929k} \binom{\frac{1}{2}}{k}$$


---

$$12 \left( - \left( \frac{480 (0.5 \times 0.5)}{7} + \frac{2^{8-1} (5 \times 3)}{(5-1)^{7/2} (8-1)} - \sqrt{\frac{(2^{13} (5-1) (5 \times 3))}{((3-1) (3-1)^{0.25}) (8-1)}} \right) - 8 \right) -$$

$$2\pi = -327.429 - 2\pi + 12 \sqrt{29521.7} \sum_{k=0}^{\infty} \frac{(-0.0000338734)^k \binom{-\frac{1}{2}}{k}}{k!}$$


---

$$12 \left( - \left( \frac{480 (0.5 \times 0.5)}{7} + \frac{2^{8-1} (5 \times 3)}{(5-1)^{7/2} (8-1)} - \sqrt{\frac{(2^{13} (5-1) (5 \times 3))}{((3-1) (3-1)^{0.25}) (8-1)}} \right) - 8 \right) -$$

$$2\pi = -327.429 - 2\pi + \frac{6 \cdot \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-10.2929s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{\sqrt{\pi}}$$

$$\left(\frac{1}{27}(12*((-(((480*0.5*0.5)/7) + ((5*3)/(8-1) * (2^{8-1})/(((5-1)^{7/2})))))) - (\text{Sqrt}(2^{13}((5-1)/(3-1))*(5*3)/(8-1)*1/(3-1)^{0.25}))))-8)-(2\text{Pi}))^2-\Phi$$

**Input**

$$\left(\frac{1}{27}\left(12\left(-\left(\frac{1}{7}(480 \times 0.5 \times 0.5) + \frac{5 \times 3}{8-1} \times \frac{2^{8-1}}{(5-1)^{7/2}} - \sqrt{2^{13} \times \frac{5-1}{3-1} \times \frac{5 \times 3}{8-1} \times \frac{1}{(3-1)^{0.25}}}\right) - 8\right) - 2\pi\right)\right)^2 - \Phi$$

Φ is the golden ratio conjugate

**Result**

4096.08...

4096.08.... ≈ 4096 = 64<sup>2</sup>

$$(12*((-(((480*0.5*0.5)/7) + ((5*3)/(8-1) * (2^{8-1})/(((5-1)^{7/2})))))) - (\text{Sqrt}(2^{13}((5-1)/(3-1))*(5*3)/(8-1)*1/(3-1)^{0.25}))))-8)-(2\text{Pi})^1/15$$

**Input**

$$\left(12\left(-\left(\frac{1}{7}(480 \times 0.5 \times 0.5) + \frac{5 \times 3}{8-1} \times \frac{2^{8-1}}{(5-1)^{7/2}} - \sqrt{2^{13} \times \frac{5-1}{3-1} \times \frac{5 \times 3}{8-1} \times \frac{1}{(3-1)^{0.25}}}\right) - 8\right) - 2\pi\right)^{(1/15)}$$

**Result**

1.6437612007880039882093866319653704859325035222763049160285557157

...

1.643761200788.... ≈ ζ(2) =  $\frac{\pi^2}{6}$  = 1.644934 ... (trace of the instanton shape)

We have:

If  $k = h = 4$ , then  $c$  satisfies (4.10), and

$$\frac{8\sqrt{h-1}}{c} = \frac{8\sqrt{3}16}{\sqrt{3}} = 128,$$

$$\sqrt{\frac{8\gamma_0}{c}} = \sqrt{\frac{2^8}{7} \sqrt{3} \frac{16}{\sqrt{3}}} = \sqrt{\frac{2^{12}}{7}} < 64.$$

From:

$$\frac{8\sqrt{h-1}}{c} = \frac{8\sqrt{3}16}{\sqrt{3}} = 128,$$

$$(8\sqrt{3} \cdot 16) / (\sqrt{3})$$

**Input**

$$\frac{8\sqrt{3} \times 16}{\sqrt{3}}$$

**Result**

128

128

From which:

$$27 \cdot \frac{1}{2} \cdot \left( \frac{8\sqrt{3} \cdot 16}{\sqrt{3}} \right) + 1$$

**Input**

$$27 \times \frac{1}{2} \times \frac{8\sqrt{3} \times 16}{\sqrt{3}} + 1$$

### Exact result

1729

1729

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$((27*1/2*(((8\sqrt{3}*16)/(\sqrt{3}))+1)))^{1/15}$$

### Input

$$\sqrt[15]{27 \times \frac{1}{2} \times \frac{8\sqrt{3} \times 16}{\sqrt{3}} + 1}$$

### Result

$$\sqrt[15]{1729}$$

### Decimal approximation

1.6438152287487281305800880313247695143292831436999401726452126788

...

$1.6438152287\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  (trace of the instanton shape)

$$(1/2*(((8\sqrt{3}*16)/(\sqrt{3}))))^2$$

### Input

$$\left(\frac{1}{2} \times \frac{8\sqrt{3} \times 16}{\sqrt{3}}\right)^2$$



## Exact result

4096

$$4096 = 64^2$$

From:

**STABILITY INEQUALITIES FOR LAWSON CONES** - *Zhenhua Liu* -  
arXiv:1711.06927v6 [math.DG] 22 Aug 2018

We have that

$$\operatorname{div} g = \frac{\frac{1}{16}(u-v)v^{1/4}(27u^2 - 123uv + 98v^2)}{\left(\frac{1}{16}\sqrt{v}(9u^2 - 34uv + 49v^2)\right)^{3/2}}.$$

$$\left(\frac{1}{16}(u-v)v^{0.25}(27u^2-123uv+98v^2)\right)/\left(\left(\frac{1}{16}\sqrt{v}(9u^2-34uv+49v^2)\right)\right)^{3/2}$$

## Input

$$\frac{\frac{1}{16}(u-v)v^{0.25}(27u^2-123uv+98v^2)}{\left(\frac{1}{16}\sqrt{v}(9u^2-34uv+49v^2)\right)^{3/2}}$$

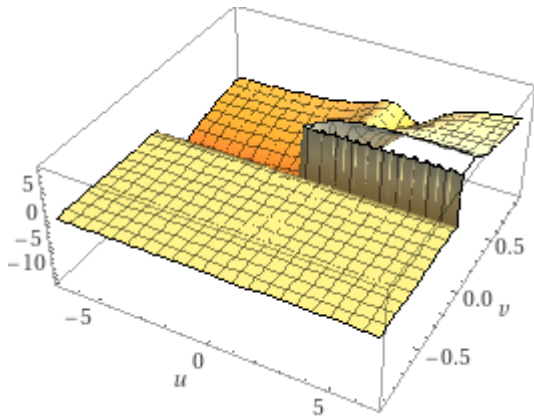
## Result

$$\frac{4v^{0.25}(u-v)(27u^2-123uv+98v^2)}{(\sqrt{v}(9u^2-34uv+49v^2))^{3/2}}$$

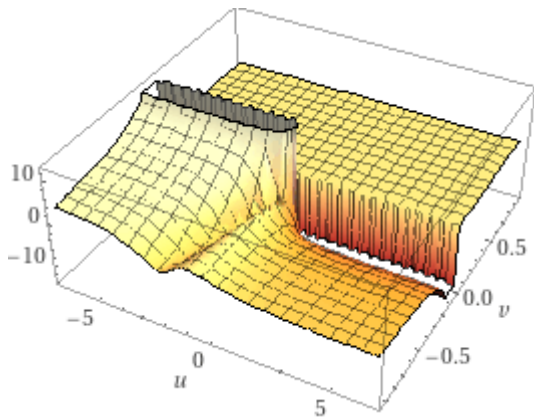
### 3D plots

Real part

(figures that can be related to the D-branes/Instantons)

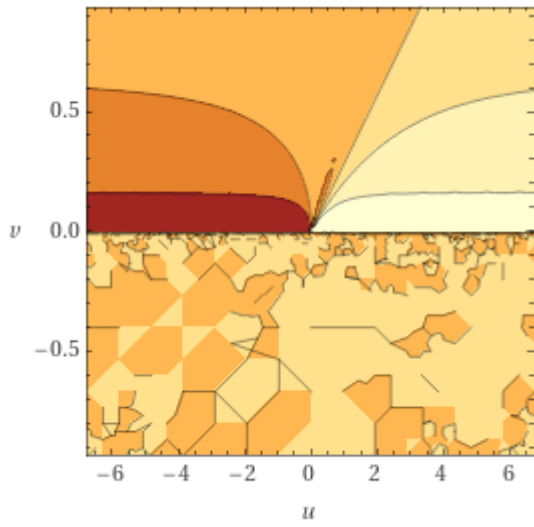


Imaginary part

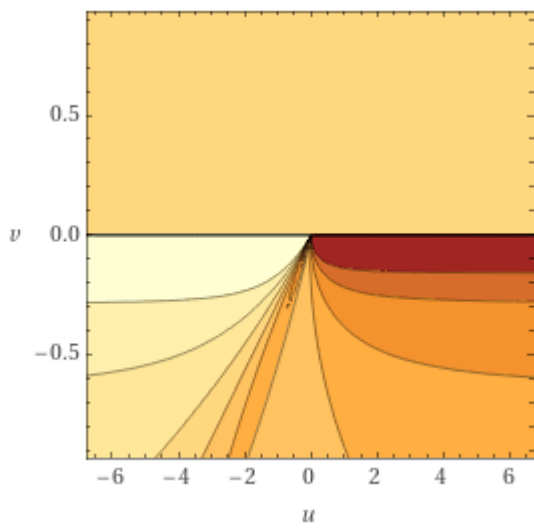


## Contour plots

### Real part



### Imaginary part



## Expanded forms

$$\begin{aligned}
 & - \frac{392 v^{2.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^2} + \\
 & \frac{884 u v^{1.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^2} - \\
 & \frac{600 u^2 v^{0.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^2} + \frac{108 u^3 \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{v^{0.75} (9u^2 - 34uv + 49v^2)^2}
 \end{aligned}$$


---

$$\begin{aligned}
 & - \frac{392 v^{3.25}}{(9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2})^{3/2}} + \frac{884 u v^{2.25}}{(9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2})^{3/2}} - \\
 & \frac{600 u^2 v^{1.25}}{(9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2})^{3/2}} + \frac{108 u^3 v^{0.25}}{(9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2})^{3/2}}
 \end{aligned}$$

## Alternate forms assuming u and v are positive

$$\frac{4(27u^3 - 150u^2v + 221uv^2 - 98v^3)}{v^{0.5}(9u^2 - 34uv + 49v^2)^{3/2}}$$


---

$$\begin{aligned}
 & - \frac{392 v^{2.5}}{(9u^2 - 34uv + 49v^2)^{3/2}} + \frac{884 u v^{1.5}}{(9u^2 - 34uv + 49v^2)^{3/2}} - \\
 & \frac{600 u^2 v^{0.5}}{(9u^2 - 34uv + 49v^2)^{3/2}} + \frac{108 u^3}{v^{0.5} (9u^2 - 34uv + 49v^2)^{3/2}}
 \end{aligned}$$

## Real roots

$$u > 0, \quad v \approx -0.0153061 \left( 22.4722 \sqrt{u^2 - 41u} \right)$$


---

$$u > 0, \quad v \approx 0.0153061 \left( 41 u + 22.4722 \sqrt{u^2} \right)$$


---

$$u > 0, \quad v = u$$

### Roots for the variable u

$$u = v$$


---

$$u \approx 1.02932 v$$


---

$$u \approx 3.52623 v$$

### Series expansion at u=0

$$-\frac{8 \sqrt{v^{5/2}}}{7 v^{1.75}} + \frac{68 u \sqrt{v^{5/2}}}{49 v^{2.75}} + \frac{3636 u^2 \sqrt{v^{5/2}}}{16 807 v^{3.75}} - \frac{146 788 u^3 \sqrt{v^{5/2}}}{823 543 v^{4.75}} - \frac{1 030 676 u^4 \sqrt{v^{5/2}}}{5 764 801 v^{5.75}} + O(u^5)$$

(Taylor series)

### Series expansion at u=∞

$$\frac{4 u}{v^{0.25} \sqrt{u^2} \sqrt{v}} + \frac{4 u v^{0.75}}{9 u \sqrt{u^2} \sqrt{v}} - \frac{508 (u v^{1.75})}{27 u^2 \sqrt{u^2} \sqrt{v}} - \frac{57 532 (u v^{2.75})}{729 u^3 \sqrt{u^2} \sqrt{v}} + O\left(\left(\frac{1}{u}\right)^4\right)$$

(generalized Puiseux series)

## Derivative

$$\frac{\partial}{\partial u} \left( \frac{(u-v) v^{0.25} (27u^2 - 123uv + 98v^2)}{16 \left( \frac{1}{16} \sqrt{v} (9u^2 - 34uv + 49v^2) \right)^{3/2}} \right) = \frac{4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297uv^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}$$

## Indefinite integral

$$\int \frac{4(u-v) v^{0.25} (27u^2 - 123uv + 98v^2)}{(\sqrt{v} (9u^2 - 34uv + 49v^2))^{3/2}} du = \left( 0.444444 v \sqrt{9u^2 - 34uv + 49v^2} \log \left( 3 \sqrt{9u^2 - 34uv + 49v^2} + 9u - 17v \right) + 12u^2 - 54.6667uv + 130.667v^2 \right) / \left( \sqrt[4]{v} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)} \right) + \text{constant}$$

(assuming a complex-valued logarithm)

From:

$$\frac{\partial}{\partial u} \left( \frac{(u-v) v^{0.25} (27u^2 - 123uv + 98v^2)}{16 \left( \frac{1}{16} \sqrt{v} (9u^2 - 34uv + 49v^2) \right)^{3/2}} \right) = \frac{4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297uv^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}$$

$(4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297uv^{3.75} + 5831v^{4.75})) / (v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)})$

Input

$$\frac{4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297uv^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}$$

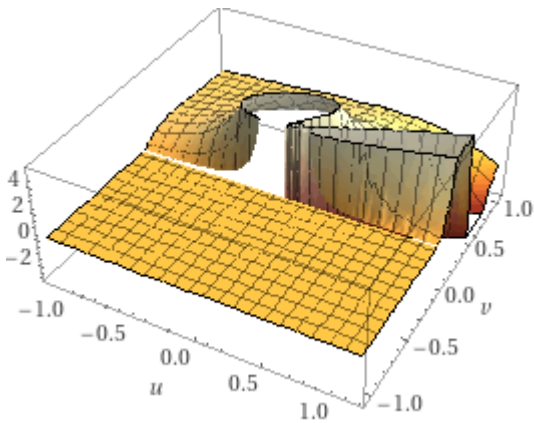
## Result

$$\frac{4(-27u^3v^{1.75} + 2541u^2v^{2.75} - 8297uv^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}}$$

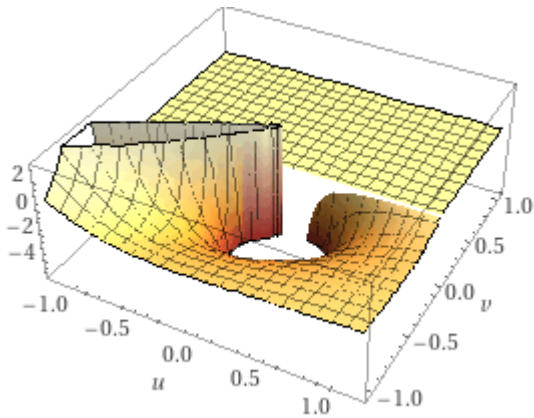
## 3D plots

Real part

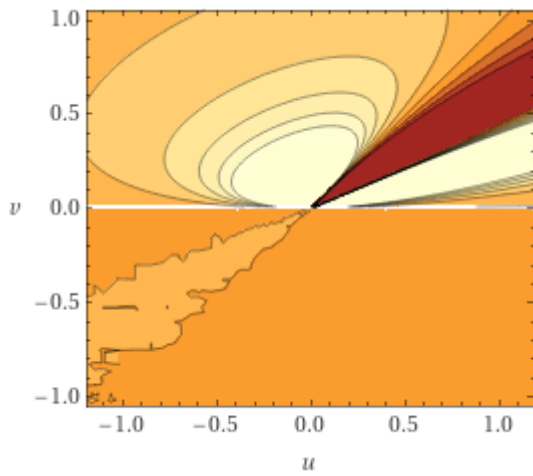
(figures that can be related to the D-branes/Instantons)



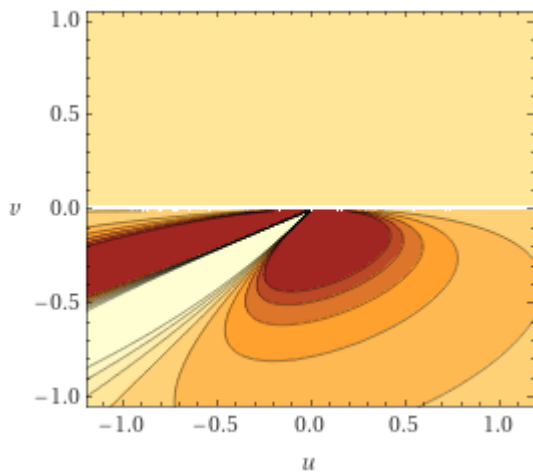
Imaginary part



**Contour plots**  
**Real part**



**Imaginary part**



**Alternate form assuming u and v are real**

$$\frac{4(-27u^3v^{0.5} + 2541u^2v^{1.5} - 8297uv^{2.5} + 5831v^{3.5})}{(9u^2 - 34uv + 49v^2)^{5/2}}$$

**Alternate forms**

$$-\frac{4v^{3/4}(27u^3 - 2541u^2v + 8297uv^2 - 5831v^3)}{(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}}$$



---


$$-\left(\left(4\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}\right.\right. \\ \left.\left.(27u^3v^{0.25}-2541u^2v^{1.25}+8297uv^{2.25}-5831v^{3.25})\right)\right) / \\ \left.(9u^2-34uv+49v^2)^3\right)$$

### Alternate form assuming u and v are positive

$$\frac{23324v^{3.5}}{(9u^2-34uv+49v^2)^{5/2}} - \frac{33188uv^{2.5}}{(9u^2-34uv+49v^2)^{5/2}} + \\ \frac{10164u^2v^{1.5}}{(9u^2-34uv+49v^2)^{5/2}} - \frac{108u^3v^{0.5}}{(9u^2-34uv+49v^2)^{5/2}}$$

### Expanded forms

$$(23324v^{3.75}) / \left(\sqrt{9u^2\sqrt{v}-34uv^{3/2}+49v^{5/2}}\right. \\ \left.(81u^4-612u^3v+1156u^2v^2+98v^2(9u^2-34uv)+2401v^4)\right) - \\ (33188uv^{2.75}) / \left(\sqrt{9u^2\sqrt{v}-34uv^{3/2}+49v^{5/2}}\right. \\ \left.(81u^4-612u^3v+1156u^2v^2+98v^2(9u^2-34uv)+2401v^4)\right) + \\ (10164u^2v^{1.75}) / \left(\sqrt{9u^2\sqrt{v}-34uv^{3/2}+49v^{5/2}}\right. \\ \left.(81u^4-612u^3v+1156u^2v^2+98v^2(9u^2-34uv)+2401v^4)\right) - \\ (108u^3v^{0.75}) / \left(\sqrt{9u^2\sqrt{v}-34uv^{3/2}+49v^{5/2}}\right. \\ \left.(81u^4-612u^3v+1156u^2v^2+98v^2(9u^2-34uv)+2401v^4)\right)$$


---

$$\frac{23324 v^{3.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^3} -$$

$$\frac{33188 u v^{2.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^3} +$$

$$\frac{10164 u^2 v^{1.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^3} -$$

$$\frac{108 u^3 v^{0.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^3}$$

## Real roots

$$u < 0, \quad v = 0$$


---

$$u > 0, \quad v = 0$$


---

$$u > 0, \quad v \approx (0.0110191 - 3.33067 \times 10^{-16} i) u$$


---

$$u > 0, \quad v \approx (0.426404 + 5.55112 \times 10^{-16} i) u$$


---

$$u > 0, \quad v \approx (0.985489 - 2.22045 \times 10^{-16} i) u$$

## Roots for the variable u

$$u \approx 1.01472 v$$


---

$$u \approx 2.34519 v$$

---


$$u \approx 90.7512 v$$

### Series expansion at $u=0$

$$\frac{68 \sqrt{v^{5/2}}}{49 v^{2.75}} + \frac{7272 u \sqrt{v^{5/2}}}{16807 v^{3.75}} - \frac{440364 u^2 \sqrt{v^{5/2}}}{823543 v^{4.75}} - \frac{4122704 u^3 \sqrt{v^{5/2}}}{5764801 v^{5.75}} - \frac{127291020 u^4 \sqrt{v^{5/2}}}{282475249 v^{6.75}} + O(u^5)$$

(Taylor series)

### Series expansion at $u=\infty$

$$-\frac{4(u v^{0.75})}{9 u^2 \sqrt{u^2 \sqrt{v}}} + \frac{1016 u v^{1.75}}{27 u^3 \sqrt{u^2 \sqrt{v}}} + \frac{57532 u v^{2.75}}{243 u^4 \sqrt{u^2 \sqrt{v}}} + \frac{5059408 u v^{3.75}}{6561 u^5 \sqrt{u^2 \sqrt{v}}} + O\left(\left(\frac{1}{u}\right)^6\right)$$

(generalized Puiseux series)

### Derivative

$$\frac{\partial}{\partial u} \left( \frac{4(-27 u^3 v^{1.75} + 2541 u^2 v^{2.75} - 8297 u v^{3.75} + 5831 v^{4.75})}{v(9 u^2 - 34 u v + 49 v^2)^2 \sqrt{\sqrt{v}(9 u^2 - 34 u v + 49 v^2)}} \right) = \frac{24(81 u^4 v^{1.75} - 11358 u^3 v^{2.75} + 56320 u^2 v^{3.75} - 72754 u v^{4.75} + 14847 v^{5.75})}{v(9 u^2 - 34 u v + 49 v^2)^3 \sqrt{\sqrt{v}(9 u^2 - 34 u v + 49 v^2)}}$$

## Indefinite integral

$$\int \frac{4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297u v^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}} du =$$

$$-\left( \left( 0.163265 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)} \right. \right.$$

$$\left. \left. (-0.27551u^3 + 1.53061u^2 v - 2.2551u v^2 + v^3) \right) \right) /$$

$$(v^{3/4} (0.183673u^2 - 0.693878uv + v^2)^2) + \text{constant}$$

From:

$$\frac{\partial}{\partial u} \left( \frac{4(-27u^3 v^{1.75} + 2541u^2 v^{2.75} - 8297u v^{3.75} + 5831v^{4.75})}{v(9u^2 - 34uv + 49v^2)^2 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}} \right) =$$

$$\frac{24(81u^4 v^{1.75} - 11358u^3 v^{2.75} + 56320u^2 v^{3.75} - 72754u v^{4.75} + 14847v^{5.75})}{v(9u^2 - 34uv + 49v^2)^3 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}}$$

$$(24(81u^4 v^{1.75} - 11358u^3 v^{2.75} + 56320u^2 v^{3.75} - 72754u v^{4.75} + 14847v^{5.75})) / (v(9u^2 - 34uv + 49v^2)^3 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)})$$

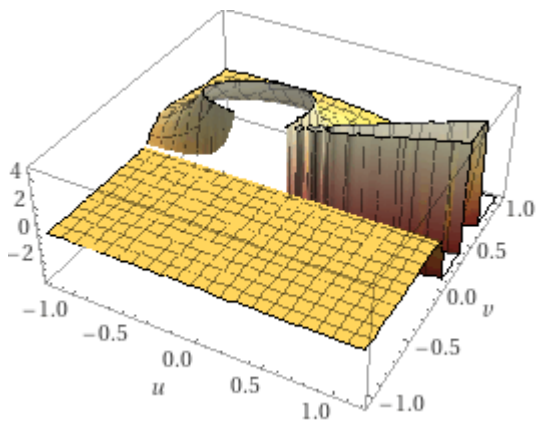
## Input

$$\frac{24(81u^4 v^{1.75} - 11358u^3 v^{2.75} + 56320u^2 v^{3.75} - 72754u v^{4.75} + 14847v^{5.75})}{v(9u^2 - 34uv + 49v^2)^3 \sqrt{\sqrt{v}(9u^2 - 34uv + 49v^2)}}$$

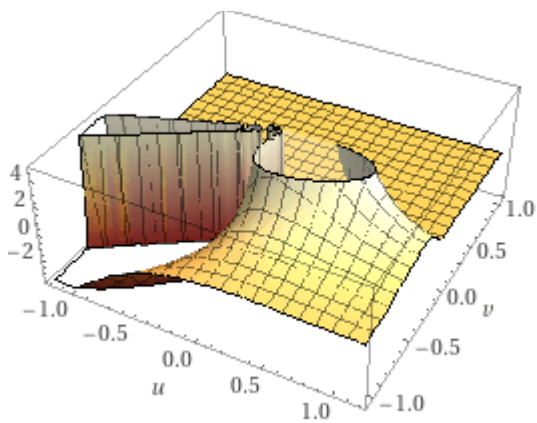
### 3D plots

#### Real part

(figures that can be related to the D-branes/Instantons)

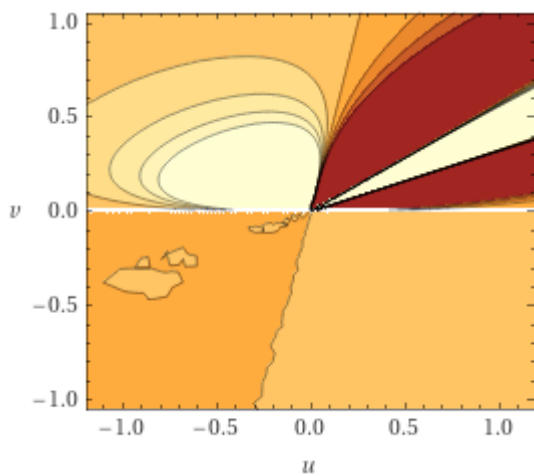


#### Imaginary part

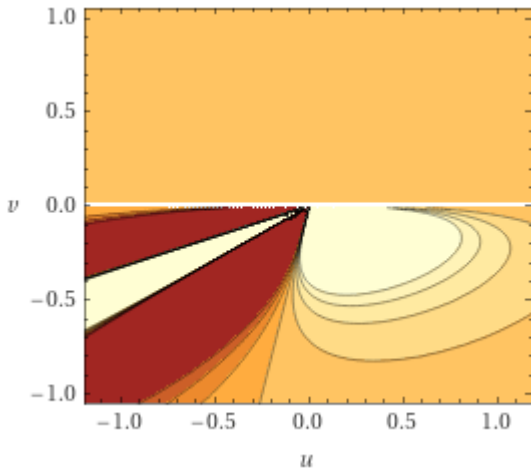


### Contour plots

#### Real part



## Imaginary part



## Alternate form assuming u and v are real

$$\frac{24 (81 u^4 v^{0.5} - 11358 u^3 v^{1.5} + 56320 u^2 v^{2.5} - 72754 u v^{3.5} + 14847 v^{4.5})}{(9 u^2 - 34 u v + 49 v^2)^{7/2}}$$

## Alternate forms

$$\frac{24 v^{3/4} (81 u^4 - 11358 u^3 v + 56320 u^2 v^2 - 72754 u v^3 + 14847 v^4)}{(9 u^2 - 34 u v + 49 v^2)^3 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)}}$$

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$$\left( 24 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} (81 u^4 v^{0.25} - 11358 u^3 v^{1.25} + 56320 u^2 v^{2.25} - 72754 u v^{3.25} + 14847 v^{4.25}) \right) / (9 u^2 - 34 u v + 49 v^2)^4$$

## Alternate form assuming u and v are positive

$$\frac{356328 v^{4.5}}{(9 u^2 - 34 u v + 49 v^2)^{7/2}} - \frac{1746096 u v^{3.5}}{(9 u^2 - 34 u v + 49 v^2)^{7/2}} + \frac{1351680 u^2 v^{2.5}}{(9 u^2 - 34 u v + 49 v^2)^{7/2}} + \frac{1944 u^4 v^{0.5}}{(9 u^2 - 34 u v + 49 v^2)^{7/2}} - \frac{272592 u^3 v^{1.5}}{(9 u^2 - 34 u v + 49 v^2)^{7/2}}$$

## Expanded forms

$$\begin{aligned}
 & (356328 v^{4.75}) / \left( \sqrt{9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2}} \right. \\
 & \quad \left. (729u^6 - 8262u^5 v + 31212u^4 v^2 - 39304u^3 v^3 + 7203v^4(9u^2 - 34uv) + \right. \\
 & \quad \left. 147v^2(81u^4 - 612u^3 v + 1156u^2 v^2) + 117649v^6) \right) - \\
 & (1746096 u v^{3.75}) / \left( \sqrt{9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2}} \right. \\
 & \quad \left. (729u^6 - 8262u^5 v + 31212u^4 v^2 - 39304u^3 v^3 + 7203v^4(9u^2 - 34uv) + \right. \\
 & \quad \left. 147v^2(81u^4 - 612u^3 v + 1156u^2 v^2) + 117649v^6) \right) + \\
 & (1351680 u^2 v^{2.75}) / \left( \sqrt{9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2}} \right. \\
 & \quad \left. (729u^6 - 8262u^5 v + 31212u^4 v^2 - 39304u^3 v^3 + 7203v^4(9u^2 - 34uv) + \right. \\
 & \quad \left. 147v^2(81u^4 - 612u^3 v + 1156u^2 v^2) + 117649v^6) \right) - \\
 & (272592 u^3 v^{1.75}) / \left( \sqrt{9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2}} \right. \\
 & \quad \left. (729u^6 - 8262u^5 v + 31212u^4 v^2 - 39304u^3 v^3 + 7203v^4(9u^2 - 34uv) + \right. \\
 & \quad \left. 147v^2(81u^4 - 612u^3 v + 1156u^2 v^2) + 117649v^6) \right) + \\
 & (1944 u^4 v^{0.75}) / \left( \sqrt{9u^2 \sqrt{v} - 34u v^{3/2} + 49v^{5/2}} \right. \\
 & \quad \left. (729u^6 - 8262u^5 v + 31212u^4 v^2 - 39304u^3 v^3 + 7203v^4(9u^2 - 34uv) + \right. \\
 & \quad \left. 147v^2(81u^4 - 612u^3 v + 1156u^2 v^2) + 117649v^6) \right)
 \end{aligned}$$


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$$\begin{aligned}
& \frac{356328 v^{4.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^4} - \\
& \frac{1746096 u v^{3.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^4} + \\
& \frac{1351680 u^2 v^{2.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^4} + \\
& \frac{1944 u^4 v^{0.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^4} - \\
& \frac{272592 u^3 v^{1.25} \sqrt{\sqrt{v} (9u^2 - 34uv + 49v^2)}}{(9u^2 - 34uv + 49v^2)^4}
\end{aligned}$$

## Real roots

$$u < 0, \quad v = 0$$


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$$u > 0, \quad v = 0$$


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$$u > 0, \quad v \approx (0.00740052 + 0i) u$$


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$$u > 0, \quad v \approx (0.323448 + 0i) u$$


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$$u > 0, \quad v \approx (0.569862 + 0i) u$$

## Roots for the variable u

$$u \approx 0.250029 v$$



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$$u \approx 1.75481 v$$

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$$u \approx 3.09169 v$$

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$$u \approx 135.126 v$$

### Series expansion at $u=0$

$$\frac{7272 \sqrt{v^{5/2}}}{16807 v^{3.75}} - \frac{880728 u \sqrt{v^{5/2}}}{823543 v^{4.75}} - \frac{12368112 u^2 \sqrt{v^{5/2}}}{5764801 v^{5.75}} - \frac{509164080 u^3 \sqrt{v^{5/2}}}{282475249 v^{6.75}} - \frac{10979584920 u^4 \sqrt{v^{5/2}}}{13841287201 v^{7.75}} + O(u^5)$$

(Taylor series)

### Series expansion at $u=\infty$

$$\frac{8 u v^{0.75}}{9 u^3 \sqrt{u^2 \sqrt{v}}} - \frac{1016 (u v^{1.75})}{9 u^4 \sqrt{u^2 \sqrt{v}}} - \frac{230128 (u v^{2.75})}{243 u^5 \sqrt{u^2 \sqrt{v}}} - \frac{25297040 (u v^{3.75})}{6561 u^6 \sqrt{u^2 \sqrt{v}}} + O\left(\left(\frac{1}{u}\right)^7\right)$$

(generalized Puiseux series)

## Derivative

$$\frac{\partial}{\partial u} \left( \frac{24 (81 u^4 v^{1.75} - 11358 u^3 v^{2.75} + 56320 u^2 v^{3.75} - 72754 u v^{4.75} + 14847 v^{5.75})}{\left( v (9 u^2 - 34 u v + 49 v^2)^3 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right)} \right) =$$

$$- \left( \frac{24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75})}{\left( v (9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right)} \right)$$

## Indefinite integral

$$\int \frac{24 (81 u^4 v^{1.75} - 11358 u^3 v^{2.75} + 56320 u^2 v^{3.75} - 72754 u v^{4.75} + 14847 v^{5.75})}{v (9 u^2 - 34 u v + 49 v^2)^3 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)}} du =$$

$$\left( 0.198251 \sqrt[4]{v} \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right.$$

$$\left. (-0.00463042 u^3 + 0.435774 u^2 v - 1.42291 u v^2 + v^3) \right) /$$

$$(0.183673 u^2 - 0.693878 u v + v^2)^3 + \text{constant}$$

From:

$$\frac{\partial}{\partial u} \left( \frac{24 (81 u^4 v^{1.75} - 11358 u^3 v^{2.75} + 56320 u^2 v^{3.75} - 72754 u v^{4.75} + 14847 v^{5.75})}{\left( v (9 u^2 - 34 u v + 49 v^2)^3 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right)} \right) =$$

$$- \left( \frac{24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75})}{\left( v (9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right)} \right)$$

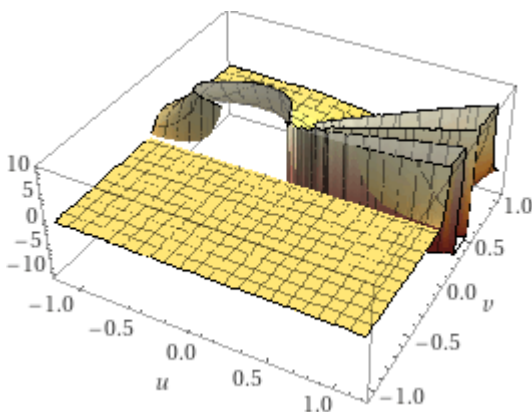
$$-(24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75}))/((v (9 u^2 - 34 u v + 49 v^2))^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)})$$

**Input**

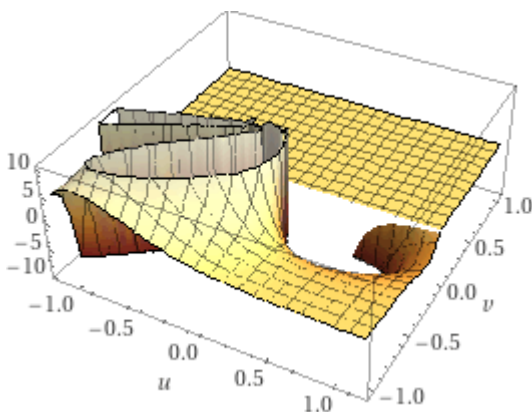
$$-\left( \left( 24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75}) \right) / \left( v (9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right) \right)$$

**3D plots**

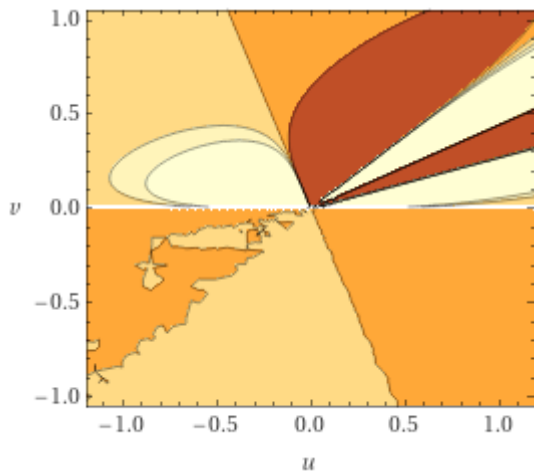
**Real part** (figures that can be related to the D-branes/Instantons)



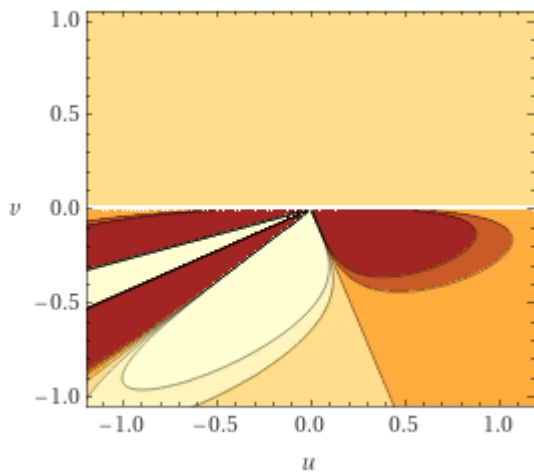
**Imaginary part**



**Contour plots**  
**Real part**



**Imaginary part**



**Alternate form assuming u and v are real**

$$-\left(\frac{24 \left(2187 u^5 v^{0.5} - 407511 u^4 v^{1.5} + 2711610 u^3 v^{2.5} - 5131410 u^2 v^{3.5} + 1600091 u v^{4.5} + 1798153 v^{5.5}\right)}{\left(9 u^2 - 34 u v + 49 v^2\right)^{9/2}}\right)$$

## Alternate forms

$$-\left(24 v^{3/4} (2187 u^5 - 407511 u^4 v + 2711610 u^3 v^2 - 5131410 u^2 v^3 + 1600091 u v^4 + 1798153 v^5)\right) / \left((9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)}\right)$$


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$$-\left(\left(24 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} (2187 u^5 v^{0.25} - 407511 u^4 v^{1.25} + 2711610 u^3 v^{2.25} - 5131410 u^2 v^{3.25} + 1600091 u v^{4.25} + 1798153 v^{5.25})\right) / (9 u^2 - 34 u v + 49 v^2)^5\right)$$

## Alternate form assuming u and v are positive

$$-\frac{43155672 v^{5.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}} - \frac{38402184 u v^{4.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}} + \frac{123153840 u^2 v^{3.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}} - \frac{52488 u^5 v^{0.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}} + \frac{9780264 u^4 v^{1.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}} - \frac{65078640 u^3 v^{2.5}}{(9 u^2 - 34 u v + 49 v^2)^{9/2}}$$

## Expanded forms

$$\begin{aligned}
& -\left((43\,155\,672\,v^{5.75})/\right. \\
& \quad \left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\,(6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - \right. \\
& \quad \quad 1\,414\,944\,v^3\,u^5 + 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6 \\
& \quad \quad (9\,u^2 - 34\,u\,v) + 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right)) - \\
& (38\,402\,184\,u\,v^{4.75})/\left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\right. \\
& \quad (6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - 1\,414\,944\,v^3\,u^5 + \\
& \quad \quad 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6(9\,u^2 - 34\,u\,v) + \\
& \quad \quad 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right)) + \\
& (123\,153\,840\,u^2\,v^{3.75})/\left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\right. \\
& \quad (6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - 1\,414\,944\,v^3\,u^5 + \\
& \quad \quad 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6(9\,u^2 - 34\,u\,v) + \\
& \quad \quad 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right)) - \\
& (65\,078\,640\,u^3\,v^{2.75})/\left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\right. \\
& \quad (6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - 1\,414\,944\,v^3\,u^5 + \\
& \quad \quad 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6(9\,u^2 - 34\,u\,v) + \\
& \quad \quad 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right)) + \\
& (9\,780\,264\,u^4\,v^{1.75})/\left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\right. \\
& \quad (6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - 1\,414\,944\,v^3\,u^5 + \\
& \quad \quad 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6(9\,u^2 - 34\,u\,v) + \\
& \quad \quad 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right)) - \\
& (52\,488\,u^5\,v^{0.75})/\left(\sqrt{49\,v^{5/2} - 34\,u\,v^{3/2} + 9\,u^2\,\sqrt{v}}\right. \\
& \quad (6561\,u^8 - 99\,144\,v\,u^7 + 561\,816\,v^2\,u^6 - 1\,414\,944\,v^3\,u^5 + \\
& \quad \quad 1\,336\,336\,v^4\,u^4 + 5\,764\,801\,v^8 + 470\,596\,v^6(9\,u^2 - 34\,u\,v) + \\
& \quad \quad 14\,406\,v^4(81\,u^4 - 612\,v\,u^3 + 1156\,v^2\,u^2) + \\
& \quad \quad \left. 196\,v^2(729\,u^6 - 8262\,v\,u^5 + 31\,212\,v^2\,u^4 - 39\,304\,v^3\,u^3)\right))
\end{aligned}$$

$$\begin{aligned}
& - \frac{43\,155\,672\,v^{5.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5} - \\
& \frac{38\,402\,184\,uv^{4.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5} + \\
& \frac{123\,153\,840\,u^2v^{3.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5} - \\
& \frac{52\,488\,u^5v^{0.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5} + \\
& \frac{9\,780\,264\,u^4v^{1.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5} - \\
& \frac{65\,078\,640\,u^3v^{2.25}\sqrt{\sqrt{v}(9u^2-34uv+49v^2)}}{(9u^2-34uv+49v^2)^5}
\end{aligned}$$

### Series expansion at u=0

$$\begin{aligned}
& - \frac{880\,728\sqrt{v^{5/2}}}{823\,543v^{4.75}} - \frac{24\,736\,224u\sqrt{v^{5/2}}}{5\,764\,801v^{5.75}} - \frac{1\,527\,492\,240u^2\sqrt{v^{5/2}}}{282\,475\,249v^{6.75}} - \\
& \frac{43\,918\,339\,680u^3\sqrt{v^{5/2}}}{13\,841\,287\,201v^{7.75}} + \frac{29\,088\,785\,160u^4\sqrt{v^{5/2}}}{678\,223\,072\,849v^{8.75}} + O(u^5)
\end{aligned}$$

(Taylor series)

### Series expansion at u=∞

$$\begin{aligned}
& - \frac{8(uv^{0.75})}{3u^4\sqrt{u^2\sqrt{v}}} + \frac{4064uv^{1.75}}{9u^5\sqrt{u^2\sqrt{v}}} + \\
& \frac{1\,150\,640uv^{2.75}}{243u^6\sqrt{u^2\sqrt{v}}} + \frac{50\,594\,080uv^{3.75}}{2187u^7\sqrt{u^2\sqrt{v}}} + O\left(\left(\frac{1}{u}\right)^8\right)
\end{aligned}$$

(generalized Puiseux series)

## Derivative

$$\frac{\partial}{\partial u} \left( - \left( (24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75})) \right) / \left( v (9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right) \right) =$$

$$(96 (19683 u^6 v^{1.75} - 4575204 u^5 v^{2.75} + 38204703 u^4 v^{3.75} - 95424696 u^3 v^{4.75} + 38192433 u^2 v^{5.75} + 114529436 u v^{6.75} - 88380467 v^{7.75})) / \left( v (9 u^2 - 34 u v + 49 v^2)^5 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right)$$

## Indefinite integral

$$\int - \left( (24 (2187 u^5 v^{1.75} - 407511 u^4 v^{2.75} + 2711610 u^3 v^{3.75} - 5131410 u^2 v^{4.75} + 1600091 u v^{5.75} + 1798153 v^{6.75})) \right) / \left( v (9 u^2 - 34 u v + 49 v^2)^4 \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} \right) du =$$

$$\left( 0.061811 \sqrt[4]{v} \sqrt{\sqrt{v} (9 u^2 - 34 u v + 49 v^2)} (0.00545565 u^4 - 0.765003 u^3 v + 3.79336 u^2 v^2 - 4.90025 u v^3 + v^4) \right) / (0.183673 u^2 - 0.693878 u v + v^2)^4 + \text{constant}$$

From:

$$- \left( (24 (2187 u^5 v^{0.5} - 407511 u^4 v^{1.5} + 2711610 u^3 v^{2.5} - 5131410 u^2 v^{3.5} + 1600091 u v^{4.5} + 1798153 v^{5.5})) \right) / (9 u^2 - 34 u v + 49 v^2)^{9/2}$$

$$-(24 (2187 u^5 v^{0.5} - 407511 u^4 v^{1.5} + 2711610 u^3 v^{2.5} - 5131410 u^2 v^{3.5} + 1600091 u v^{4.5} + 1798153 v^{5.5})) / (9 u^2 - 34 u v + 49 v^2)^{9/2}$$

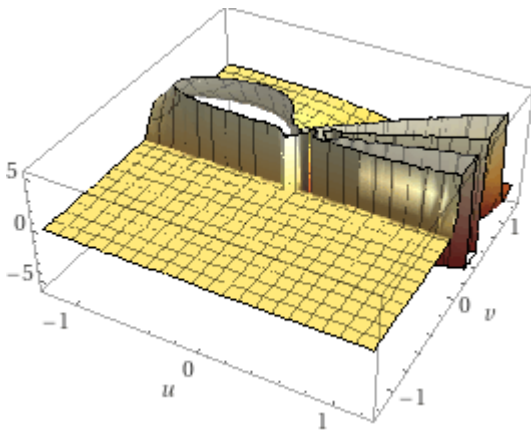


## Input

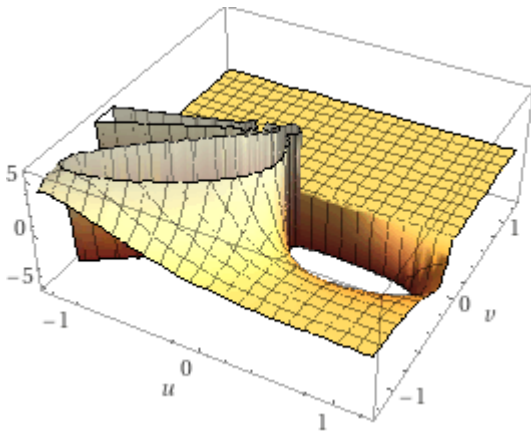
$$-\left(\frac{24(2187u^5\sqrt{v} - 407511u^4v^{1.5} + 2711610u^3v^{2.5} - 5131410u^2v^{3.5} + 1600091uv^{4.5} + 1798153v^{5.5})}{(9u^2 - 34uv + 49v^2)^{9/2}}\right)$$

## 3D plots

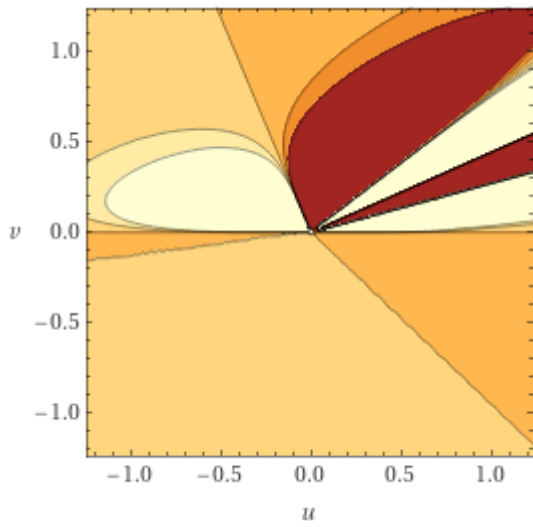
**Real part** (figures that can be related to the D-branes/Instantons)



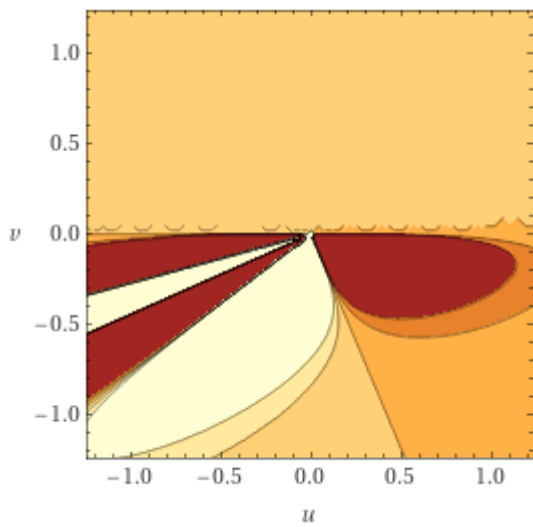
## Imaginary part



**Contour plots**  
**Real part**



**Imaginary part**



**Alternate form**

$$-\left(\frac{24\sqrt{v}\left(2187u^5 - 407511u^4v + 2711610u^3v^2 - 5131410u^2v^3 + 1600091uv^4 + 1798153v^5\right)}{\left(9u^2 - 34uv + 49v^2\right)^{9/2}}\right)$$

## Expanded form

$$\begin{aligned}
 & - \frac{43\,155\,672\,v^{5.5}}{(9u^2 - 34uv + 49v^2)^{9/2}} - \frac{38\,402\,184\,u\,v^{4.5}}{(9u^2 - 34uv + 49v^2)^{9/2}} + \frac{123\,153\,840\,u^2\,v^{3.5}}{(9u^2 - 34uv + 49v^2)^{9/2}} - \\
 & \frac{52\,488\,u^5\,\sqrt{v}}{(9u^2 - 34uv + 49v^2)^{9/2}} + \frac{9\,780\,264\,u^4\,v^{1.5}}{(9u^2 - 34uv + 49v^2)^{9/2}} - \frac{65\,078\,640\,u^3\,v^{2.5}}{(9u^2 - 34uv + 49v^2)^{9/2}}
 \end{aligned}$$

## Roots

$$2187u^5 \neq 0, \quad v = 0$$


---

$$366.25u^2 \neq 0, \quad v \approx -2.37541u$$


---

$$8.8121u^2 \neq 0, \quad v \approx 0.00557107u$$


---

$$3.38637u^2 \neq 0, \quad v \approx 0.270764u$$

$$v = 0.270764u$$


---

$$3.55412u^2 \neq 0, \quad v \approx 0.442991u$$

## Roots for the variable u

$$\begin{aligned}
 u \approx \text{Root}[ & 2187.0000000000000 \#1^5 - 407511.0000000000 \#1^4 v + \\
 & 2.711610000000000 \times 10^6 \#1^3 v^2 - 5.131410000000000 \times 10^6 \#1^2 v^3 + \\
 & 1.600091000000000 \times 10^6 \#1 v^4 + 1.798153000000000 \times 10^6 v^5 \&, 1]
 \end{aligned}$$


---

$$u \approx \text{Root}[2187.00000000000 \#1^5 - 407511.000000000 \#1^4 v + 2.71161000000000 \times 10^6 \#1^3 v^2 - 5.13141000000000 \times 10^6 \#1^2 v^3 + 1.60009100000000 \times 10^6 \#1 v^4 + 1.79815300000000 \times 10^6 v^5 \&, 2]$$


---

$$u \approx \text{Root}[2187.00000000000 \#1^5 - 407511.000000000 \#1^4 v + 2.71161000000000 \times 10^6 \#1^3 v^2 - 5.13141000000000 \times 10^6 \#1^2 v^3 + 1.60009100000000 \times 10^6 \#1 v^4 + 1.79815300000000 \times 10^6 v^5 \&, 3]$$


---

$$u \approx \text{Root}[2187.00000000000 \#1^5 - 407511.000000000 \#1^4 v + 2.71161000000000 \times 10^6 \#1^3 v^2 - 5.13141000000000 \times 10^6 \#1^2 v^3 + 1.60009100000000 \times 10^6 \#1 v^4 + 1.79815300000000 \times 10^6 v^5 \&, 4]$$


---

$$u \approx \text{Root}[2187.00000000000 \#1^5 - 407511.000000000 \#1^4 v + 2.71161000000000 \times 10^6 \#1^3 v^2 - 5.13141000000000 \times 10^6 \#1^2 v^3 + 1.60009100000000 \times 10^6 \#1 v^4 + 1.79815300000000 \times 10^6 v^5 \&, 5]$$

### Series expansion at u=0

$$-\frac{880728 \sqrt{v^2}}{823543 v^{4.5}} - \frac{24736224 u \sqrt{v^2}}{5764801 v^{5.5}} - \frac{1527492240 u^2 \sqrt{v^2}}{282475249 v^{6.5}} - \frac{43918339680 u^3 \sqrt{v^2}}{13841287201 v^{7.5}} + \frac{29088785160 u^4 \sqrt{v^2}}{678223072849 v^{8.5}} + O(u^5)$$

(Taylor series)

### Series expansion at u=∞

$$-\frac{8 \sqrt{v}}{3 u^4} + \frac{8(559 v^{1.5} - 51 v^{3/2})}{9 u^5} + \frac{8(156151 v^{2.5} - 12321 v^{5/2})}{243 u^6} + \frac{8(7006997 v^{3.5} - 682737 v^{7/2})}{2187 u^7} + O\left(\left(\frac{1}{u}\right)^8\right)$$

(Laurent series)

## Derivative

$$\frac{\partial}{\partial u} \left( - \left( (24 (2187 u^5 \sqrt{v} - 407511 u^4 v^{1.5} + 2711610 u^3 v^{2.5} - 5131410 u^2 v^{3.5} + 1600091 u v^{4.5} + 1798153 v^{5.5})) / (9 u^2 - 34 u v + 49 v^2)^{9/2} \right) \right) =$$

$$(24 (78732 u^6 \sqrt{v} - 2187 u^5 (8385 v^{1.5} - 17 v^{3/2}) + 2187 u^4 (70121 v^{2.5} - 245 v^{5/2}) - 381698784 u^3 v^{3.5} + 152769732 u^2 v^{4.5} + 458117744 u v^{5.5} - 353521868 v^{6.5})) / (9 u^2 - 34 u v + 49 v^2)^{11/2}$$

## Indefinite integral

$$\int - \left( (24 (2187 u^5 \sqrt{v} - 407511 u^4 v^{1.5} + 2711610 u^3 v^{2.5} - 5131410 u^2 v^{3.5} + 1600091 u v^{4.5} + 1798153 v^{5.5})) / (9 u^2 - 34 u v + 49 v^2)^{9/2} \right) du =$$

$$- \frac{24 \sqrt{v} (-81 u^4 + 11358 u^3 v - 56320 u^2 v^2 + 72754 u v^3 - 14847 v^4)}{(9 u^2 - 34 u v + 49 v^2)^{7/2}} +$$

constant

From:

$$- \left( (24 \sqrt{v} (2187 u^5 - 407511 u^4 v + 2711610 u^3 v^2 - 5131410 u^2 v^3 + 1600091 u v^4 + 1798153 v^5)) / (9 u^2 - 34 u v + 49 v^2)^{9/2} \right)$$

$$\begin{cases} u^2 + uv + v^2 = -p \\ u = -v \left( \frac{1}{2} \pm \frac{i\sqrt{3}}{2} \right) \\ v = -u \left( \frac{1}{2} \pm \frac{i\sqrt{3}}{2} \right) \end{cases}$$

For  $u = -v(1/2+(i*\sqrt{3})/2)$  ;  $v = -u(1/2+(i*\sqrt{3})/2)$

$$\begin{aligned}
& -(24 \sqrt{-u(1/2+(i*\sqrt{3})/2)}) (2187 (-v(1/2+(i*\sqrt{3})/2))^5 - 407511 (- \\
& v(1/2+(i*\sqrt{3})/2))^4 *(-u(1/2+(i*\sqrt{3})/2))+ 2711610 (-v(1/2+(i*\sqrt{3})/2))^3 (- \\
& u(1/2+(i*\sqrt{3})/2))^2 - 5131410 (-v(1/2+(i*\sqrt{3})/2))^2 (-u(1/2+(i*\sqrt{3})/2))^3 + \\
& 1600091 (-v(1/2+(i*\sqrt{3})/2)) (-u(1/2+(i*\sqrt{3})/2))^4 + 1798153 (- \\
& u(1/2+(i*\sqrt{3})/2))^5)/(9 (-v(1/2+(i*\sqrt{3})/2))^2 - 34 (-v(1/2+(i*\sqrt{3})/2))(- \\
& u(1/2+(i*\sqrt{3})/2)) + 49 (-u(1/2+(i*\sqrt{3})/2))^2)^{(9/2)}
\end{aligned}$$

Dividing the above long expression:

$$-(24 \sqrt{-u(1/2+(i*\sqrt{3})/2)})$$

**Input**

$$-\left(24 \sqrt{-u\left(\frac{1}{2} + \frac{1}{2}(i\sqrt{3})\right)}\right)$$

**Exact result**

$$-24 \sqrt{-u\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)}$$

**Alternate form**

$$-24 \sqrt{-u\left(\frac{1}{2}(\sqrt{3}i + 1)\right)}$$

For  $u = -1$  :

$$-24 \sqrt{\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)}$$

**Input**

$$-24 \sqrt{\frac{1}{2}(\sqrt{3}i) + \frac{1}{2}}$$

$i$  is the imaginary unit



$$407511 u((\sqrt{3} i)/2 + 1/2) v((\sqrt{3} i)/2 + 1/2)^4 - 2711610 u((\sqrt{3} i)/2 + 1/2)^2 v((\sqrt{3} i)/2 + 1/2)^3 - 2187 v((\sqrt{3} i)/2 + 1/2)^5$$

### Input

$$407511 u\left(\frac{1}{2}(\sqrt{3} i) + \frac{1}{2}\right) v\left(\frac{1}{2}(\sqrt{3} i) + \frac{1}{2}\right)^4 - 2711610 u\left(\frac{1}{2}(\sqrt{3} i) + \frac{1}{2}\right)^2 v\left(\frac{1}{2}(\sqrt{3} i) + \frac{1}{2}\right)^3 - 2187 v\left(\frac{1}{2}(\sqrt{3} i) + \frac{1}{2}\right)^5$$

### Exact result

$$407511 u\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right) v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^4 - 2711610 u\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^2 v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^3 - 2187 v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^5$$

### Alternate forms

$$-27 v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^3 \left(-15093 u\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right) v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right) + 100430 u\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^2 + 81 v\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^2\right)$$

$$-27 v\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^3 \left(-15093 u\left(\frac{1}{2}(\sqrt{3} i + 1)\right) v\left(\frac{1}{2}(\sqrt{3} i + 1)\right) + 100430 u\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^2 + 81 v\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^2\right)$$

$$407511 u\left(\frac{1}{2}(\sqrt{3} i + 1)\right) v\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^4 - 2711610 u\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^2 v\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^3 - 2187 v\left(\frac{1}{2}(\sqrt{3} i + 1)\right)^5$$



$$407511 * -((\sqrt{3} i)/2 + 1/2) ((\sqrt{3} i)/2 + 1/2)^4 - 2711610 * -((\sqrt{3} i)/2 + 1/2)^2 ((\sqrt{3} i)/2 + 1/2)^3 - 2187 ((\sqrt{3} i)/2 + 1/2)^5$$

### Input

$$407511 \times (-1) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^4 - \left( 2711610 \times (-1) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^2 \right) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^3 - 2187 \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^5$$

$i$  is the imaginary unit

### Result

$$2301912 \left( \frac{1}{2} + \frac{i \sqrt{3}}{2} \right)^5$$

### Decimal approximation

$$1.150956 \times 10^6 -$$

$$1.9935142692762447342491755314342328359670233637045804461445... \times 10^6 i$$

### Polar coordinates

$$r = 2301912 \text{ (radius), } \theta = -\frac{\pi}{3} \text{ (angle)}$$

2301912

$$-5131410 (-v(1/2+(i*\sqrt{3})/2))^2 (-u(1/2+(i*\sqrt{3})/2))^3 + 1600091 (-v(1/2+(i*\sqrt{3})/2)) (-u(1/2+(i*\sqrt{3})/2))^4 + 1798153 (-u(1/2+(i*\sqrt{3})/2))^5$$

### Input

$$-5131410 \left( -v \left( \frac{1}{2} + \frac{1}{2} (i \sqrt{3}) \right) \right)^2 \left( -u \left( \frac{1}{2} + \frac{1}{2} (i \sqrt{3}) \right) \right)^3 + 1600091 \left( -v \left( \frac{1}{2} + \frac{1}{2} (i \sqrt{3}) \right) \right) \left( -u \left( \frac{1}{2} + \frac{1}{2} (i \sqrt{3}) \right) \right)^4 + 1798153 \left( -u \left( \frac{1}{2} + \frac{1}{2} (i \sqrt{3}) \right) \right)^5$$

## Exact result

$$-1600091 u \left( \frac{\sqrt{3} i}{2} + \frac{1}{2} \right)^4 v \left( \frac{\sqrt{3} i}{2} + \frac{1}{2} \right) + \\ 5131410 u \left( \frac{\sqrt{3} i}{2} + \frac{1}{2} \right)^3 v \left( \frac{\sqrt{3} i}{2} + \frac{1}{2} \right)^2 - 1798153 u \left( \frac{\sqrt{3} i}{2} + \frac{1}{2} \right)^5$$

For  $u = -1$  ;  $v = 1$  :

$$-1600091 * -\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^4 \left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right) + 5131410 * -\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^3 \left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^2 - 1798153 * -\left(\frac{\sqrt{3} i}{2} + \frac{1}{2}\right)^5$$

## Input

$$-1600091 \times (-1) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^4 \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right) + \\ 5131410 \times (-1) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^3 \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^2 - \\ 1798153 \times (-1) \left( \frac{1}{2} (\sqrt{3} i) + \frac{1}{2} \right)^5$$

$i$  is the imaginary unit

## Result

$$-1733166 \left( \frac{1}{2} + \frac{i\sqrt{3}}{2} \right)^5$$

## Decimal approximation

$$-866583 + \\ 1.50096578497546039165689503296118339336239700526276107580... \times 10^6 i$$

## Polar coordinates

$$r = 1733166 \text{ (radius), } \theta = \frac{2\pi}{3} \text{ (angle)}$$

1733166

$$(9 (-v(1/2+(i*\sqrt{3})/2))^2 - 34 (-v(1/2+(i*\sqrt{3})/2))(-u(1/2+(i*\sqrt{3})/2)) + 49 (-u(1/2+(i*\sqrt{3})/2))^2)^{9/2}$$

### Input

$$\left(9\left(-v\left(\frac{1}{2} + \frac{1}{2}(i\sqrt{3})\right)\right)\right)^2 - 34\left(-v\left(\frac{1}{2} + \frac{1}{2}(i\sqrt{3})\right)\right)\left(-u\left(\frac{1}{2} + \frac{1}{2}(i\sqrt{3})\right)\right) + 49\left(-u\left(\frac{1}{2} + \frac{1}{2}(i\sqrt{3})\right)\right)^2\right)^{9/2}$$

### Exact result

$$\left(-34u\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)v\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right) + 49u\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)^2 + 9v\left(\frac{\sqrt{3}i}{2} + \frac{1}{2}\right)^2\right)^{9/2}$$

$$(-34 * -((\sqrt{3} i)/2 + 1/2) ((\sqrt{3} i)/2 + 1/2) + 49 * -((\sqrt{3} i)/2 + 1/2)^2 + 9 ((\sqrt{3} i)/2 + 1/2)^2)^{9/2}$$

### Input

$$\left(-34 \times (-1) \left(\frac{1}{2}(\sqrt{3}i) + \frac{1}{2}\right) \left(\frac{1}{2}(\sqrt{3}i) + \frac{1}{2}\right) + 49 \times (-1) \left(\frac{1}{2}(\sqrt{3}i) + \frac{1}{2}\right)^2 + 9 \left(\frac{1}{2}(\sqrt{3}i) + \frac{1}{2}\right)^2\right)^{9/2}$$

*i* is the imaginary unit

### Result

$$1296\sqrt{6} \left(-\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\right)^{9/2}$$

### Decimal approximation

3174.538706646998815263680160818835243987867934931044486448769567...

*i*

### Polar coordinates

$r = 1296\sqrt{6}$  (radius),  $\theta = 1.5708$  (angle)

1296√6

## Polar forms

$$1296 \sqrt{6} (\cos(1.5708) + i \sin(1.5708))$$

---

## Approximate form

$$1296 \sqrt{6} e^{1.5708 i}$$

## Alternate forms

$$1296 \sqrt{3} i \sqrt{2}$$

---

$$\frac{81}{16} \sqrt{\frac{3}{2}} (\sqrt{3} + -i)^9$$

---

$$81 \sqrt{3} (1 - i \sqrt{3})^{9/2}$$

## Expanded forms

$$1296 i \sqrt{6}$$

---

$$1944 i \sqrt{-2 \left( \frac{1}{2} + \frac{i \sqrt{3}}{2} \right)^2} - 648 \sqrt{-6 \left( \frac{1}{2} + \frac{i \sqrt{3}}{2} \right)^2}$$

$$(24(2301912-1733166))/(1296*\text{sqrt}6)$$

## Input

$$\frac{24 (2301912 - 1733166)}{1296 \sqrt{6}}$$

## Result

$$\frac{31\,597\sqrt{6}}{18}$$

## Decimal approximation

4299.8070779288932427077547171378916839971134747949336693382103915

...

4299.807077928....

## Alternate form

$$\frac{31\,597\sqrt{6}}{18}$$

From:

$$\operatorname{div} g = \frac{\frac{1}{32}u^{1/4}(u-v)(49u^2 - 72uv + 27v^2)}{\left(\frac{1}{32}\sqrt{u}(49u^2 - 10uv + 9v^2)\right)^{3/2}}.$$

For  $u = 1$  ;  $v = -1$  :

$$\left(\frac{1}{32}u^{0.25}(u-v)(49u^2-72uv+27v^2)\right)/\left(\left(\frac{1}{32}\sqrt{u}(49u^2-10uv+9v^2)\right)\right)^{3/2}$$

## Input

$$\frac{\frac{1}{32}u^{0.25}(u-v)(49u^2-72uv+27v^2)}{\left(\frac{1}{32}\sqrt{u}(49u^2-10uv+9v^2)\right)^{3/2}}$$

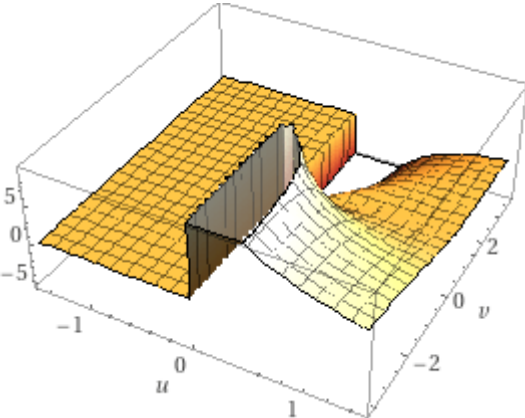
## Result

$$\frac{4\sqrt{2}u^{0.25}(u-v)(49u^2-72uv+27v^2)}{(\sqrt{u}(49u^2-10uv+9v^2))^{3/2}}$$

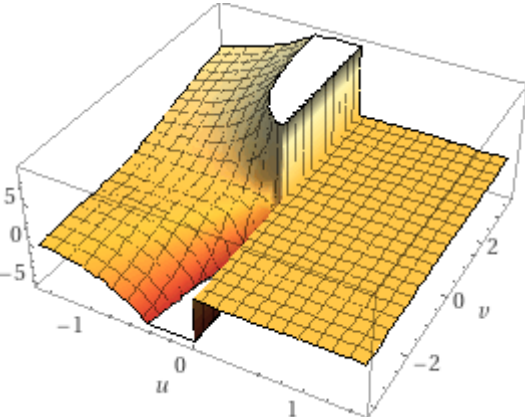
**3D plots**

**Real part**

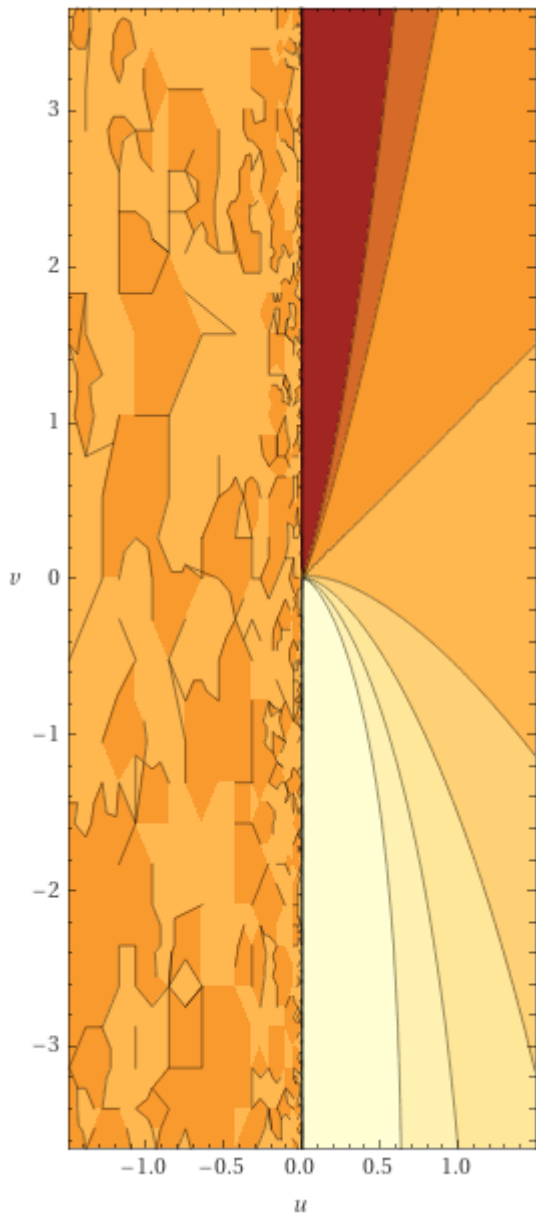
(figures that can be related to the D-branes/Instantons)



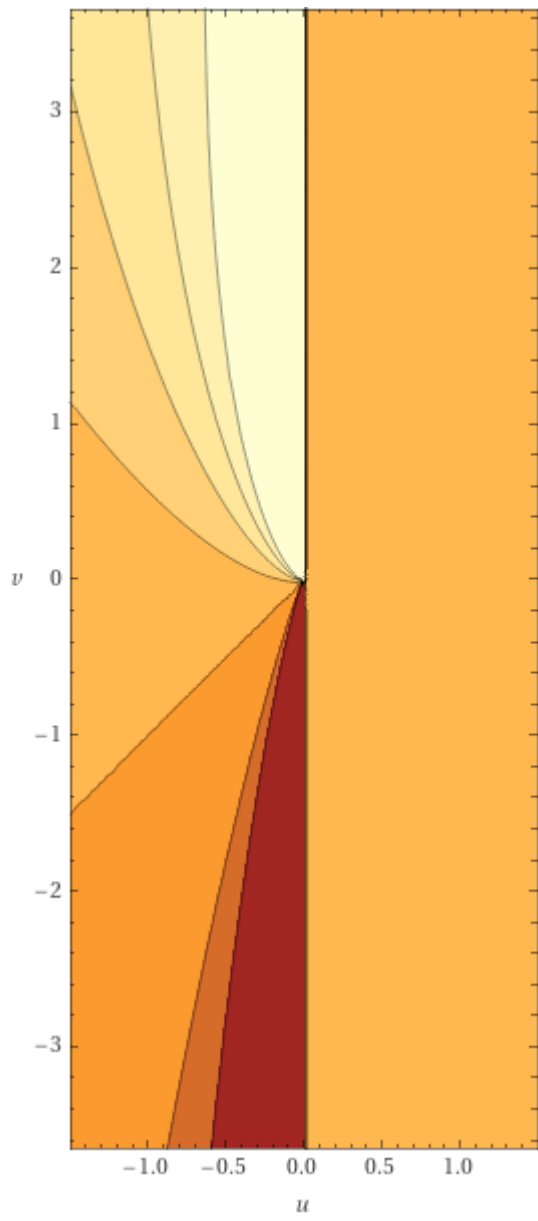
**Imaginary part**



**Contour plots**  
**Real part**



### Imaginary part





## Expanded forms

$$\frac{196 \sqrt{2} u^{2.25} \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(49 u^2 - 10 u v + 9 v^2)^2} -$$

$$\frac{484 \sqrt{2} u^{1.25} v \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(49 u^2 - 10 u v + 9 v^2)^2} -$$

$$\frac{108 \sqrt{2} v^3 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{u^{0.75} (49 u^2 - 10 u v + 9 v^2)^2} +$$

$$\frac{396 \sqrt{2} u^{0.25} v^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(49 u^2 - 10 u v + 9 v^2)^2}$$


---

$$\frac{196 \sqrt{2} u^{3.25}}{(49 u^{5/2} - 10 u^{3/2} v + 9 \sqrt{u} v^2)^{3/2}} - \frac{484 \sqrt{2} u^{2.25} v}{(49 u^{5/2} - 10 u^{3/2} v + 9 \sqrt{u} v^2)^{3/2}} +$$

$$\frac{396 \sqrt{2} u^{1.25} v^2}{(49 u^{5/2} - 10 u^{3/2} v + 9 \sqrt{u} v^2)^{3/2}} - \frac{108 \sqrt{2} u^{0.25} v^3}{(49 u^{5/2} - 10 u^{3/2} v + 9 \sqrt{u} v^2)^{3/2}}$$

## Alternate forms assuming u and v are positive

$$\frac{4 \sqrt{2} (49 u^3 - 121 u^2 v + 99 u v^2 - 27 v^3)}{u^{0.5} (49 u^2 - 10 u v + 9 v^2)^{3/2}}$$


---

$$\frac{196 \sqrt{2} u^{2.5}}{(49 u^2 - 10 u v + 9 v^2)^{3/2}} - \frac{484 \sqrt{2} u^{1.5} v}{(49 u^2 - 10 u v + 9 v^2)^{3/2}} +$$

$$\frac{396 \sqrt{2} u^{0.5} v^2}{(49 u^2 - 10 u v + 9 v^2)^{3/2}} - \frac{108 \sqrt{2} v^3}{u^{0.5} (49 u^2 - 10 u v + 9 v^2)^{3/2}}$$

## Real root

$$u > 0, \quad v = u$$

## Roots for the variable u

$$u \approx (0.734694 - 0.106044 i) v$$


---

$$u \approx (0.734694 + 0.106044 i) v$$


---

$$u = v$$

## Series expansion at u=0

$$u^{0.25} \left( -\frac{4(\sqrt{2} \sqrt[4]{u} v)}{\sqrt{\sqrt{u} v^2} u^{3/4}} + \frac{8\sqrt{2} \sqrt[4]{u} \sqrt[4]{u}}{\sqrt{\sqrt{u} v^2}} + \frac{808\sqrt{2} u^{3/4} v u^{5/4}}{27(\sqrt{u} v^2)^{3/2}} - \frac{21656(\sqrt{2} u^{3/4}) u^{9/4}}{729(\sqrt{u} v^2)^{3/2}} + O(u^{13/4}) \right)$$

## Series expansion at u=∞

$$u^{0.25} \left( \frac{4}{7} \sqrt{2} \left(\frac{1}{u}\right)^{3/4} - \frac{424}{343} (\sqrt{2} v) \left(\frac{1}{u}\right)^{7/4} + \frac{1464 \sqrt{2} v^2 \left(\frac{1}{u}\right)^{11/4}}{2401} + O\left(\left(\frac{1}{u}\right)^{13/4}\right) \right)$$

## Derivative

$$\frac{\partial}{\partial u} \left( \frac{u^{0.25} (u - v) (49u^2 - 72uv + 27v^2)}{32 \left( \frac{1}{32} \sqrt{u} (49u^2 - 10uv + 9v^2) \right)^{3/2}} \right) =$$

$$\frac{(-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5)}{\left( u^{2.25} (u^2 - 0.204082 uv + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49u^2 - 10uv + 9v^2)} \right)}$$

## Indefinite integral assuming all variables are real

$$\begin{aligned}
 & \int \frac{u^{0.25} (u - v) (49 u^2 - 72 u v + 27 v^2)}{32 \left(\frac{1}{32} \sqrt{u} (49 u^2 - 10 u v + 9 v^2)\right)^{3/2}} du = \\
 & \left( 4 \sqrt{2} \sqrt{u - 0.416246 \sqrt{-v^2} - 0.102041 v} \right. \\
 & \quad \sqrt{u + 0.416246 \sqrt{-v^2} - 0.102041 v} \\
 & \quad \sqrt{\frac{49 u + 20.3961 \sqrt{-v^2} - 5 v}{20.3961 \sqrt{-v^2} - 5 v}} \sqrt{\frac{-49 u + 20.3961 \sqrt{-v^2} + 5 v}{20.3961 \sqrt{-v^2} + 5 v}} \\
 & \quad \left. \left( 42327.6 u^{1.5} \left( 2.40242 u + \sqrt{-v^2} - 0.245145 v \right) \right. \right. \\
 & \quad \quad \left. \left( 2.40242 u - \sqrt{-v^2} - 0.245145 v \right)^2 \right. \\
 & \quad \quad \left. F_1 \left( 1.5; 1.5, 1.5; 2.5; \frac{2.40242 u}{0.245145 v - \sqrt{-v^2}}, \frac{2.40242 u}{0.245145 v + \sqrt{-v^2}} \right) \right) - \\
 & \quad 12467.4 u^{0.5} v \left( 2.40242 u + \sqrt{-v^2} - 0.245145 v \right) \\
 & \quad \left( 2.40242 u - \sqrt{-v^2} - 0.245145 v \right)^2 \\
 & \quad \left. F_1 \left( 0.5; 1.5, 1.5; 1.5; \frac{2.40242 u}{0.245145 v - \sqrt{-v^2}}, \frac{2.40242 u}{0.245145 v + \sqrt{-v^2}} \right) \right) - \\
 & \quad 5996.45 u^{1.5} (5.44444 u^2 - 1.11111 u v + v^2) \\
 & \quad \left( -2.40242 u + \sqrt{-v^2} + 0.245145 v \right) F_1 \left( 1.5; 0.5, 0.5; \right. \\
 & \quad \quad \left. 2.5; -\frac{2.40242 u}{\sqrt{-v^2} - 0.245145 v}, \frac{2.40242 u}{0.245145 v + \sqrt{-v^2}} \right) + \\
 & \quad 40751.4 u^{0.5} v (5.44444 u^2 - 1.11111 u v + v^2) \\
 & \quad \left( -2.40242 u + \sqrt{-v^2} + 0.245145 v \right) F_1 \left( 0.5; 0.5, 0.5; \right. \\
 & \quad \quad \left. 1.5; -\frac{2.40242 u}{\sqrt{-v^2} - 0.245145 v}, \frac{2.40242 u}{0.245145 v + \sqrt{-v^2}} \right) \Bigg) / \\
 & \left( (49 u^2 - 10 u v + 9 v^2)^{3/2} (49 u - 20.3961 \sqrt{-v^2} - 5 v)^{3/2} \right. \\
 & \quad \left. \sqrt{49 u + 20.3961 \sqrt{-v^2} - 5 v} \right) + \text{constant}
 \end{aligned}$$

$F_1(a; b_1, b_2; c; x, y)$

is the Appell hypergeometric function of two variables

From:

$$\frac{\partial}{\partial u} \left( \frac{u^{0.25} (u - v) (49u^2 - 72uv + 27v^2)}{32 \left( \frac{1}{32} \sqrt{u} (49u^2 - 10uv + 9v^2) \right)^{3/2}} \right) =$$

$$\frac{(-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5)}{\left( u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49u^2 - 10uv + 9v^2)} \right)}$$

$$(-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5) / (u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)})$$

### Input interpretation

$$\frac{(-2.82843 u^6 + 19.799 u^5 v + u^4 v^2 \times (-25.9754) + 9.39354 u^3 v^3 + u^2 v^4 \times (-0.222646) + 0.286259 u v^5)}{\left( u^{2.25} (u^2 + u v \times (-0.204082) + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49u^2 - 10uv + 9v^2)} \right)}$$

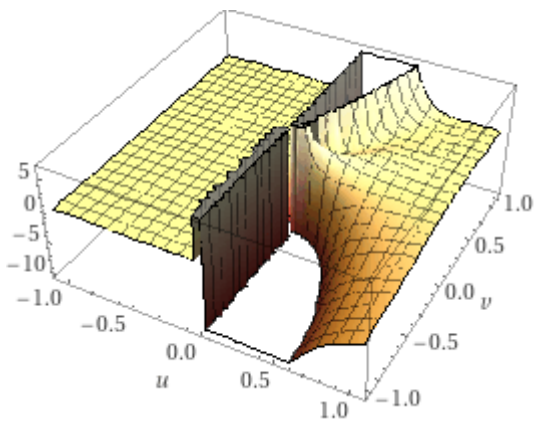
### Result

$$\frac{(-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5)}{\left( u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49u^2 - 10uv + 9v^2)} \right)}$$

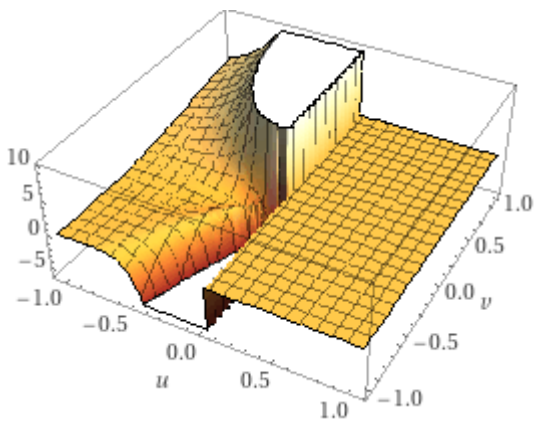
### 3D plots

Real part

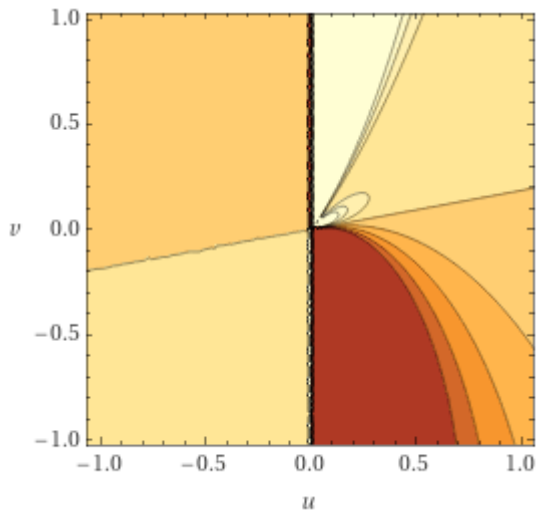
(figures that can be related to the D-branes/Instantons)



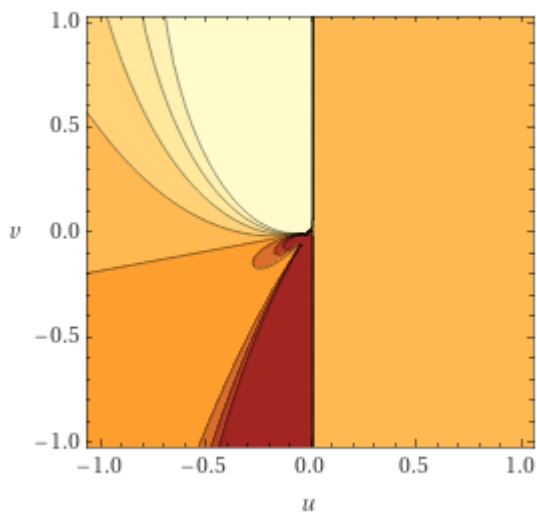
Imaginary part



**Contour plots**  
**Real part**



**Imaginary part**



**Alternate form assuming u and v are real**

$$\frac{(-2.82843 u^5 + 19.799 u^4 v - 25.9754 u^3 v^2 + 9.39354 u^2 v^3 - 0.222646 u v^4 + 0.286259 v^5)}{(u^{1.5} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2})}$$

## Alternate forms

$$-\left( (2.82843 \times 10^6 u^5 - 1.9799 \times 10^7 u^4 v + 2.59754 \times 10^7 u^3 v^2 - 9.39354 \times 10^6 u^2 v^3 + 222646. u v^4 - 286259. v^5) / \left( 1000000 u^{5/4} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)} \right) \right)$$


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$$-\left( \left( 0.0577231 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)} (-7. u^{9/2} v + 9.18368 u^{7/2} v^2 - 3.32111 u^{5/2} v^3 + 0.0787172 u^{3/2} v^4 + u^{11/2} - 0.101208 \sqrt{u} v^5) \right) / \left( u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 (u^2 - 0.204082 u v + 0.183673 v^2) \right) \right)$$

## Expanded forms

$$\begin{aligned}
 & -\left( (2.82843 u^6) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) \right) + \\
 & (19.799 v u^5) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) - \right. \\
 & (25.9754 v^2 u^4) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) + \right. \\
 & (9.39354 v^3 u^3) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) - \right. \\
 & (0.222646 v^4 u^2) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) + \right. \\
 & (0.286259 v^5 u) / \left( \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{6.25} - \right. \\
 & \quad 0.408164 v \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{5.25} + \\
 & \quad 0.408995 v^2 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{4.25} - \\
 & \quad 0.0749687 v^3 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{3.25} + \\
 & \quad \left. \left. 0.0337358 v^4 \sqrt{49 u^{5/2} - 10 v u^{3/2} + 9 v^2 \sqrt{u}} u^{2.25} \right) \right)
 \end{aligned}$$



$$\begin{aligned}
& - \frac{2.82843 u^{3.25} \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)} + \\
& \frac{19.799 u^{2.25} v \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)} + \\
& \frac{0.286259 v^5 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{u^{1.75} (u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)} - \\
& \frac{25.9754 u^{1.25} v^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)} - \\
& \frac{0.222646 v^4 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{u^{0.75} (u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)} + \\
& \frac{9.39354 u^{0.25} v^3 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)}}{(u^2 - 0.204082 u v + 0.183673 v^2)^2 (49 u^2 - 10 u v + 9 v^2)}
\end{aligned}$$

### Alternate forms assuming u and v are positive

$$\begin{aligned}
& (-2.82843 u^{5.5} + 19.799 u^{4.5} v - 25.9754 u^{3.5} v^2 + \\
& \quad 9.39354 u^{2.5} v^3 - 0.222646 u^{1.5} v^4 + 0.286259 u^{0.5} v^5) / \\
& \left( u^2 (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2} \right)
\end{aligned}$$


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$$\begin{aligned}
& - \frac{2.82843 u^{3.5}}{(u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2}} + \\
& \frac{19.799 u^{2.5} v}{25.9754 u^{1.5} v^2} - \\
& \frac{(u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2}}{0.286259 v^5} + \\
& \frac{u^{1.5} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2}}{0.222646 v^4} - \\
& \frac{u^{0.5} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2}}{9.39354 u^{0.5} v^3} + \\
& \frac{(u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{49 u^2 - 10 u v + 9 v^2}}{9.39354 u^{0.5} v^3}
\end{aligned}$$

## Derivative

$$\begin{aligned}
& \frac{\partial}{\partial u} \left( (-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + \right. \\
& \quad \left. 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5) / \right. \\
& \quad \left. \left( u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)} \right) \right) = \\
& (4.24265 u^{12.25} - 49.7861 u^{11.25} v + 99.8586 u^{10.25} v^2 - 65.6488 u^{9.25} v^3 + \\
& \quad 22.1199 u^{8.25} v^4 - 7.77408 u^{7.25} v^5 + 0.100311 u^{6.25} v^6 - \\
& \quad 0.339076 u^{5.25} v^7 + 0.0627719 u^{4.25} v^8 - 0.0144858 u^{3.25} v^9) / \\
& \left( u^{5.5} (u^2 - 0.204082 u v + 0.183673 v^2)^3 (u^2 - 0.204082 u v + 0.183673 v^2) \right. \\
& \quad \left. \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)} \right)
\end{aligned}$$

From:

$$\begin{aligned}
& (-2.82843 u^6 + 19.799 u^5 v - 25.9754 u^4 v^2 + \\
& \quad 9.39354 u^3 v^3 - 0.222646 u^2 v^4 + 0.286259 u v^5) / \\
& \left( u^{2.25} (u^2 - 0.204082 u v + 0.183673 v^2)^2 \sqrt{\sqrt{u} (49 u^2 - 10 u v + 9 v^2)} \right)
\end{aligned}$$

For  $u = 1$ ;  $v = -1$  :

$$(-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259)/(1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)})$$

**Input interpretation**

$$\frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}}$$

**Result**

-3.683960519003057812417731694680342623222913216296901275783962164  
 ...  
 -3.683960519....

From which:

$$1 + 1/(-(-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259)/(1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}))^{1/3}$$

**Input interpretation**

$$1 + \frac{1}{\sqrt[3]{-\frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}}}}$$

**Result**

1.6474829612126284868494150019848050642711573942237242773174072704  
 ...  
 1.6474829612....  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  (trace of the instanton shape)

$$(24(2301912-1733166))/(1296*\sqrt{6}) - (((-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259)/(1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)})))-233+21+5-\pi/6$$

### Input interpretation

$$\frac{24 (2301912 - 1733166)}{1296 \sqrt{6} - 2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259} - \frac{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}}{233 + 21 + 5 - \frac{\pi}{6}}$$

### Result

4095.9674...

$$4095.9674\dots \approx 4096 = 64^2$$

### Series representations

$$\frac{24 (2301912 - 1733166)}{1296 \sqrt{6} - 2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259} - \frac{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}}{233 + 21 + 5 - \frac{\pi}{6}} = -207 - \frac{\pi}{6} + \frac{31597}{3 \sqrt{5} \sum_{k=0}^{\infty} \frac{(-\frac{1}{5})^k (-\frac{1}{2})_k}{k!}} + \frac{30.3787}{\sqrt{-1 + 68 \sqrt{1}} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (-1+68\sqrt{1})^{-k}}{k!}}$$


---

$$\begin{aligned}
& \frac{24(2301912 - 1733166)}{1296\sqrt{6}} - \\
& \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25}(1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1}(49 + 10 + 9)}} - \\
& 233 + 21 + 5 - \frac{\pi}{6} = -207 - \frac{\pi}{6} + \frac{31597}{3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!}} + \\
& \frac{30.3787}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (68\sqrt{1}-z_0)^k z_0^{-k}}{k!}} \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$\begin{aligned}
& \frac{24(2301912 - 1733166)}{1296\sqrt{6}} - \\
& \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25}(1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1}(49 + 10 + 9)}} - \\
& 233 + 21 + 5 - \frac{\pi}{6} = -207 - \frac{\pi}{6} + \frac{63194\sqrt{\pi}}{3 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 5^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)} + \\
& \frac{60.7574\sqrt{\pi}}{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) (-1 + 68\sqrt{1})^{-s}}
\end{aligned}$$

$$27 \left( \left( \frac{24(2301912-1733166)}{1296*\text{sqrt}(6)} - \left( \left( \left( -2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259 \right) / \left( 1^{2.25} (1 + 0.204082 + 0.183673)^2 \text{sqrt}(\text{sqrt}(1) (49 + 10 + 9)) \right) \right) - 233 + 21 + 5 - \text{Pi}/6 \right) \right)^{1/2}$$

### Input interpretation

$$27 \sqrt{\left( \frac{24(2301912 - 1733166)}{1296\sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25}(1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1}(49 + 10 + 9)}} - 233 + 21 + 5 - \frac{\pi}{6} \right)}$$

## Result

1727.99313...

$1727.99313\dots \approx 1728$

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

## Series representations

$$\begin{aligned}
 & 27 \sqrt{\left( \frac{24(2301912 - 1733166)}{1296\sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25}(1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1}(49 + 10 + 9)}} - \right. \\
 & \left. 233 + 21 + 5 - \frac{\pi}{6} \right)} = \\
 & 27 \sqrt{\left( -207 - \frac{\pi}{6} + \frac{31597}{3\sqrt{5} \sum_{k=0}^{\infty} 5^{-k} \binom{\frac{1}{2}}{k}} + \frac{30.3787}{\sqrt{-1 + 68\sqrt{1}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 + 68\sqrt{1})^{-k}} \right)}
 \end{aligned}$$


---

$$\begin{aligned}
& 27 \sqrt{\left( \frac{24 (2301912 - 1733166)}{1296 \sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}} - \right. \\
& \left. 233 + 21 + 5 - \frac{\pi}{6} \right) = \\
& 27 \sqrt{\left( -207 - \frac{\pi}{6} + \frac{31597}{3 \sqrt{5} \sum_{k=0}^{\infty} \frac{(-\frac{1}{5})^k (-\frac{1}{2})_k}{k!}} + \frac{30.3787}{\sqrt{-1 + 68 \sqrt{1}} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (-1+68 \sqrt{1})^{-k}}{k!}} \right)}
\end{aligned}$$

$$\begin{aligned}
& 27 \sqrt{\left( \frac{24 (2301912 - 1733166)}{1296 \sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}} - \right. \\
& \left. 233 + 21 + 5 - \frac{\pi}{6} \right) = \\
& 27 \sqrt{-207 - \frac{\pi}{6} + \frac{\frac{31597}{\sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (6-20)^k z_0^{-k}}{k!}} + \frac{30.3787}{\sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (68 \sqrt{1} - 20)^k z_0^{-k}}{k!}}}{\sqrt{z_0}}}
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\left( \left( \left( \left( \left( \frac{24(2301912 - 1733166)}{1296 \sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}} - 233 + 21 + 5 - \frac{\pi}{6} \right)^{1/2} \right)^{1/15} \right) \right) \right)$$

**Input interpretation**

$$\left( 27 \sqrt{\left( \frac{24 (2301912 - 1733166)}{1296 \sqrt{6}} - \frac{-2.82843 - 19.799 - 25.9754 - 9.39354 - 0.222646 - 0.286259}{1^{2.25} (1 + 0.204082 + 0.183673)^2 \sqrt{\sqrt{1} (49 + 10 + 9)}} - 233 + 21 + 5 - \frac{\pi}{6} \right) \right)^{1/15}$$

**Result**

1.643751394...

1.643751394....  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  (trace of the instanton shape)



## Observations

We note that, from the number 8, we obtain as follows:

$$8^2$$

$$64$$

$$8^2 \times 2 \times 8$$

$$1024$$

$$8^4 = 8^2 \times 2^6$$

True

$$8^4 = 4096$$

$$8^2 \times 2^6 = 4096$$

$$2^{13} = 2 \times 8^4$$

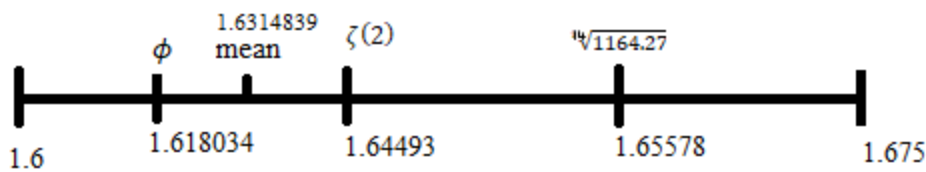
True

$$2^{13} = 8192$$

$$2 \times 8^4 = 8192$$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact  $8^2 = 64$ ,  $8^3 = 512$ ,  $8^4 = 4096$ . We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6, a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

## “Golden” Range



Finally we note how  $8^2 = 64$ , multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to  $\zeta(2)$  that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

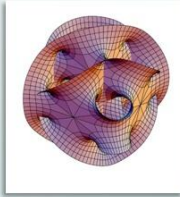
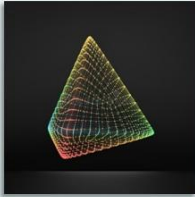
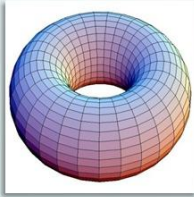
Furthermore for all the results very near to 1728 or 1729, adding  $64 = 8^2$ , one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

## Appendix

**Outlook**

Remarkably rich (apparently **UNIQUE**) framework

**BUT :**



Why a given **“shape” of the extra dimensions** ?  
[**CRUCIAL**, it determines the predictions for  $\alpha$ , ...]

A. Sagnotti – AstronomiAmo, 23.4.2020 21

From: A. Sagnotti – AstronomiAmo, 23.04.2020

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for  $\alpha$ ”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and  $\zeta(2)$ , (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally  $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

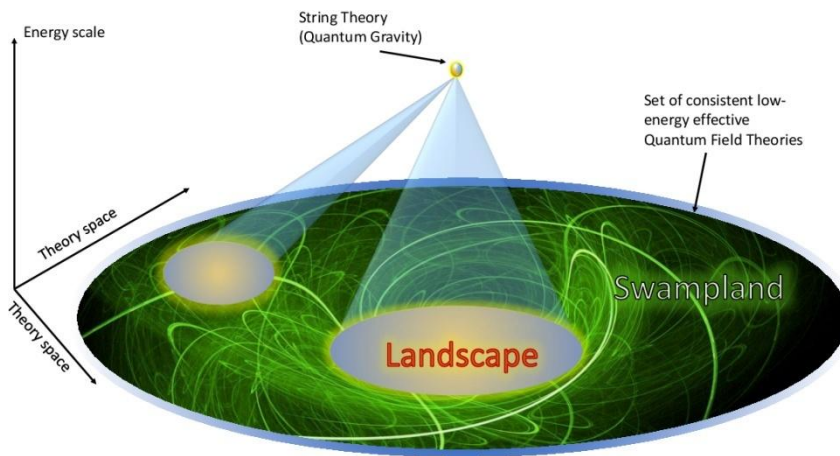
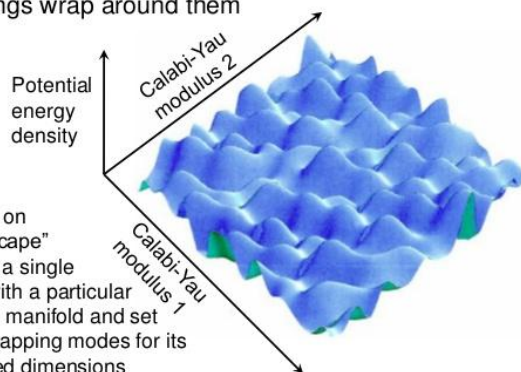


Fig. 1

### The String Theory “Landscape”

- Graph axes show only 2 out of hundreds of parameters (“moduli”) that determine the exact Calabi-Yau manifolds and how strings wrap around them



- Each point on the “Landscape” represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation “bubble”

Fig. 2

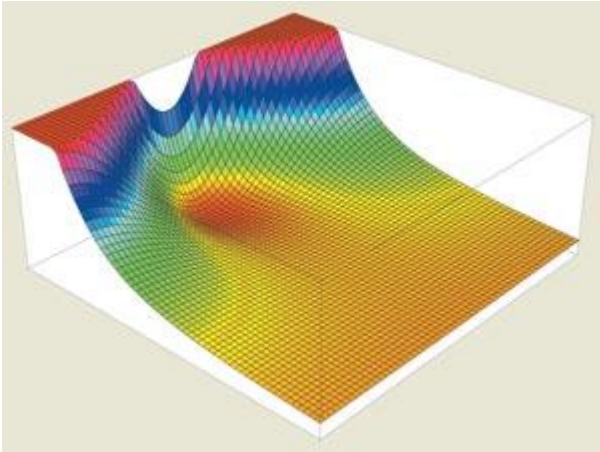
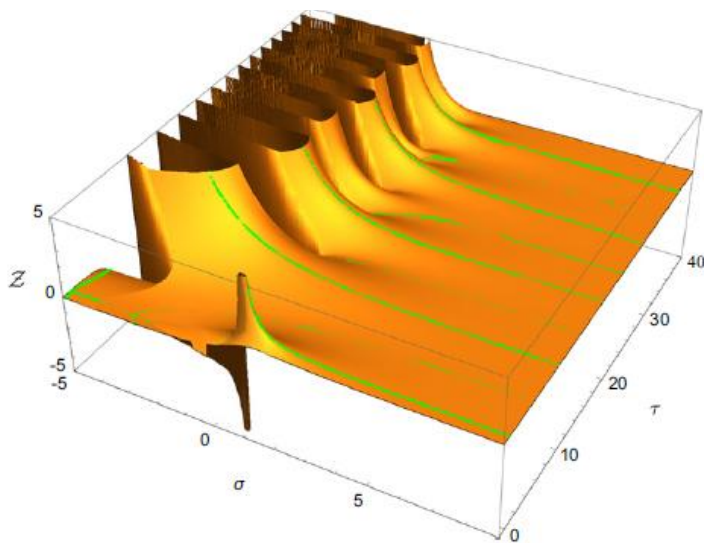


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.



**Figure 2.** Lines in the complex plane where the Riemann zeta function  $\zeta$  is real (green) depicted on a relief representing the positive absolute value of  $\zeta$  for arguments  $s \equiv \sigma + i\tau$  where the real part of  $\zeta$  is positive, and the negative absolute value of  $\zeta$  where the real part of  $\zeta$  is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of  $2\pi$  run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with  $k = \dots, -2, -1, 0, 1, 2, \dots$

we obtain:

$$2\pi/(\ln(2))$$

**Input:**

$$2 \times \frac{\pi}{\log(2)}$$

**Exact result:**

$$\frac{2\pi}{\log(2)}$$

**Decimal approximation:**

9.0647202836543876192553658914333336203437229354475911683720330958

...

9.06472028365....

**Alternative representations:**

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a) \log_a(2)}$$

---


$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 \coth^{-1}(3)}$$

### Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

---


$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

---


$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

### Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t} dt}$$

---


$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

From which:

$$(2\pi/(\ln(2))) * (1/12 \pi \log(2))$$

**Input:**

$$\left(2 \times \frac{\pi}{\log(2)}\right) \left(\frac{1}{12} \pi \log(2)\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

1.6449340668482264364724151666460251892189499012067984377355582293

...

$$1.6449340668\dots = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$



From:

**Modular equations and approximations to  $\pi$  - Srinivasa Ramanujan**  
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64\left\{\left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

We note that, with regard 4372, we can to obtain the following results:

$$27((4372)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi$$

### Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

$\phi$  is the golden ratio

### Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2(\sqrt{5}-1)}\right)$$

### Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944....

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

### Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5(10-2\sqrt{5})}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2(5-\sqrt{5})}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2(5+\sqrt{5})}\right)$$

---


$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2(\sqrt{5} - 1)}$$

### Minimal polynomial

$$256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 1549835555492184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041$$

### Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}$$

---


$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2(\sqrt{5} - 1)}$$

### Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2}\right) + \phi =$$

$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k}\right) / \left(2\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right)$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

Or:

$$27((4096+276)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi$$

## Input

$$27 \left( \sqrt{4096 + 276} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi$$

$\phi$  is the golden ratio

## Result

$$\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)$$

## Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944.... as above

## Alternate forms

$$\frac{1}{8} \left( -27 \sqrt{5(10 - 2\sqrt{5})} + 58\sqrt{5} + 432\sqrt{1093} - 27 \sqrt{2(5 - \sqrt{5})} - 374 \right)$$

---

$$\phi - 54 + 54\sqrt{1093} + \frac{27}{4} \left( 1 + \sqrt{5} - \sqrt{2(5 + \sqrt{5})} \right)$$

---

$$\phi - 54 + 54\sqrt{1093} - \frac{27 \left( \sqrt{10 - 2\sqrt{5}} - 2 \right)}{2(\sqrt{5} - 1)}$$

## Minimal polynomial

$$\begin{aligned}
 &256x^8 + 95744x^7 - 324875080x^6 - \\
 &914210725504x^5 + 15498355554921184x^4 + \\
 &2911478392539914656x^3 - 32941144911224677091680x^2 - \\
 &3092528914069760354714456x + 26320050609744039027169013041
 \end{aligned}$$

## Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5(10-2\sqrt{5})}$$


---

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)}$$

## Series representations

$$\begin{aligned}
 &27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{(\sqrt{5}-1)2}\right)+\phi= \\
 &\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\right. \\
 &\quad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}- \\
 &\quad \left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)
 \end{aligned}$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

From which:

$$(27((4372)^{1/2}-2-1/2((\sqrt{(10-2\sqrt{5})-2})/(\sqrt{5}-1))))+\phi)^{1/15}$$

### Input

$$\sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi}$$

$\phi$  is the golden ratio

### Exact result

$$\sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

### Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124

...

$$1.64381856858\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

### Alternate forms

$$\sqrt[15]{\phi - 54 + 54\sqrt{1093} - \frac{27(\sqrt{10 - 2\sqrt{5}} - 2)}{2(\sqrt{5} - 1)}}$$

---


$$\sqrt[15]{\frac{1}{\frac{2(\sqrt{5} - 1)}{166 - 108\sqrt{5} - 108\sqrt{1093} + 108\sqrt{5465} - 27\sqrt{2(5 - \sqrt{5})}}}}$$


---



$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 154983555492184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

### Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 154983555492184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

### Expanded forms

$$\sqrt[15]{\frac{1}{2}(1 + \sqrt{5}) + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}}$$

All 15th roots of  $\phi + 27(-2 + 2\sqrt{1093}) - (\sqrt{10 - 2\sqrt{5}} - 2)/(2(\sqrt{5} - 1))$

$$e^0 \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.50170 + 0.6686i$$

$$e^{(4i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.0999 + 1.2216i$$


---

$$e^{(2i\pi)/5} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 0.5080 + 1.5634i$$


---

$$e^{(8i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx -0.17183 + 1.63481i$$


---

### Series representations

$$\begin{aligned} & \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\ & \frac{1}{\sqrt[15]{2}} \left( \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \right. \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k} \right) / \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \right)^{(1/15)} \end{aligned}$$


---

$$\begin{aligned}
& \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \right. \\
& \quad \left. \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)^{(1/15)}
\end{aligned}$$


---

$$\begin{aligned}
& \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \right. \\
& \quad \left. \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)^{(1/15)}
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

## Integral representation

$$(1 + z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From:

## An Update on Brane Supersymmetry Breaking

*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left( p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left( 7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$  instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning  $p$ ,  $C$ ,  $\beta_E$  and  $\phi$  correspond to the exponents of  $e$  (i.e. of exp). Thence we obtain for  $p = 5$  and  $\beta_E = 1/2$ :

$$e^{-6C + \phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to  $64^2$ , while  $-6C + \phi$  is equal to  $-\pi\sqrt{18}$ . From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp((-Pi*\text{sqrt}(18))$  we obtain:

**Input:**

$$\exp\left(-\pi \sqrt{18}\right)$$

**Exact result:**

$$e^{-3\sqrt{2}\pi}$$

**Decimal approximation:**

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

**Property:**

$e^{-3\sqrt{2}\pi}$  is a transcendental number

**Series representations:**

$$e^{-\pi \sqrt{18}} = e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

**Input interpretation:**

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

**Result:**

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp((-Pi*\sqrt{18})))))) * 1/0.000244140625$$

**Input interpretation:**

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

**Result:**

0.00666501785...

0.00666501785...

**Series representations:**

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

From:

$$\ln(0.00666501784619)$$

**Input interpretation:**

$$\log(0.00666501784619)$$

**Result:**

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$



**Alternative representations:**

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

**Series representations:**

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left[ \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\log(0.006665017846190000) = \left[ \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

**Integral representation:**

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \phi$$

Note that the values of  $n_s$  (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

Also performing the 512<sup>th</sup> root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

**Input interpretation:**

$$\sqrt[512]{\frac{1}{139.57}}$$

**Result:**

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 =  $\phi$**  and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}} - \phi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

From

**Properties of Nilpotent Supergravity**

*E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015*

We have that:

Cosmological inflation with a tiny tensor-to-scalar ratio  $r$ , consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) = i M \left( \Phi + b \Phi e^{ik\Phi} \right) . \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly-instanton inflation of [33]. One can verify that  $\chi = 0$  solves the field equations, and that the potential along the  $\chi = 0$  trajectory is now

$$V = \frac{M^2}{3} \left( 1 - a \phi e^{-\gamma\phi} \right)^2 . \tag{4.36}$$

We analyzing the following equation:

$$V = \frac{M^2}{3} \left( 1 - a \phi e^{-\gamma\phi} \right)^2 .$$

$$\phi = \varphi - \frac{\sqrt{6}}{k},$$

$$a = \frac{b\gamma}{e} < 0, \quad \gamma = \frac{k}{\sqrt{6}} < 0.$$

We have:

$$(M^2)/3 * [1 - (b/\text{euler number} * k/\text{sqrt}6) * (\varphi - \text{sqrt}6/k) * \exp(-(k/\text{sqrt}6)(\varphi - \text{sqrt}6/k))]^2$$

i.e.

$$V = (M^2)/3 * [1 - (b/\text{euler number} * k/\text{sqrt}6) * (\varphi - \text{sqrt}6/k) * \exp(-(k/\text{sqrt}6)(\varphi - \text{sqrt}6/k))]^2$$

For  $k = 2$  and  $\varphi = 0.9991104684$ , that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

we obtain:

$$V = (M^2)/3 * [1 - (b/\text{euler number} * 2/\text{sqrt}6) * (0.9991104684 - \text{sqrt}6/2) * \exp(-(2/\text{sqrt}6)(0.9991104684 - \text{sqrt}6/2))]^2$$

**Input interpretation:**

$$V = \frac{M^2}{3} \left( 1 - \left( \frac{b}{e} \times \frac{2}{\sqrt{6}} \right) \left( 0.9991104684 - \frac{\sqrt{6}}{2} \right) \exp \left( - \frac{2}{\sqrt{6}} \left( 0.9991104684 - \frac{\sqrt{6}}{2} \right) \right) \right)^2$$

**Result:**

$$V = \frac{1}{3} (0.0814845 b + 1)^2 M^2$$

**Solutions:**

$$b = \frac{225.913 \left( -0.054323 M^2 \pm 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \quad (M \neq 0)$$

**Alternate forms:**

$$V = 0.00221324 (b + 12.2723)^2 M^2$$


---

$$V = 0.00221324 (b^2 M^2 + 24.5445 b M^2 + 150.609 M^2)$$


---

$$-0.00221324 b^2 M^2 - 0.054323 b M^2 - \frac{M^2}{3} + V = 0$$

**Expanded form:**

$$V = 0.00221324 b^2 M^2 + 0.054323 b M^2 + \frac{M^2}{3}$$

**Alternate form assuming b, M, and V are positive:**

$$V = 0.00221324 (b + 12.2723)^2 M^2$$

**Alternate form assuming b, M, and V are real:**

$$V = 0.00221324 b^2 M^2 + 0.054323 b M^2 + 0.333333 M^2 + 0$$

## Derivative:

$$\frac{\partial}{\partial b} \left( \frac{1}{3} (0.0814845 b + 1)^2 M^2 \right) = 0.054323 (0.0814845 b + 1) M^2$$

## Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154317775011120075}{36961748(226802245 + 18480874 b) M^2}$$

---

$$\frac{\partial b(M, V)}{\partial M} = - \frac{\frac{226802245}{18480874} + b}{M}$$

---

$$\frac{\partial M(b, V)}{\partial V} = \frac{154317775011120075}{2(226802245 + 18480874 b)^2 M}$$

---

$$\frac{\partial M(b, V)}{\partial b} = - \frac{18480874 M}{226802245 + 18480874 b}$$

---

$$\frac{\partial V(b, M)}{\partial M} = \frac{2(226802245 + 18480874 b)^2 M}{154317775011120075}$$

---

$$\frac{\partial V(b, M)}{\partial b} = \frac{36961748(226802245 + 18480874 b) M^2}{154317775011120075}$$

**Global minimum:**

$$\min\left\{\frac{1}{3} (0.0814845 b + 1)^2 M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

**Global minima:**

$$\min\left\{\frac{1}{3} M^2 \left(1 - \frac{(b+2) \left(0.9991104684 - \frac{\sqrt{6}}{2}\right) \exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\} = 0$$

for  $b = -\frac{226802245}{18480874}$

$$\min\left\{\frac{1}{3} M^2 \left(1 - \frac{(b+2) \left(0.9991104684 - \frac{\sqrt{6}}{2}\right) \exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\} = 0$$

for  $M = 0$

From:

$$b = \frac{225.913 \left(-0.054323 M^2 \pm 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

we obtain

$$(225.913 (-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}))/M^2$$

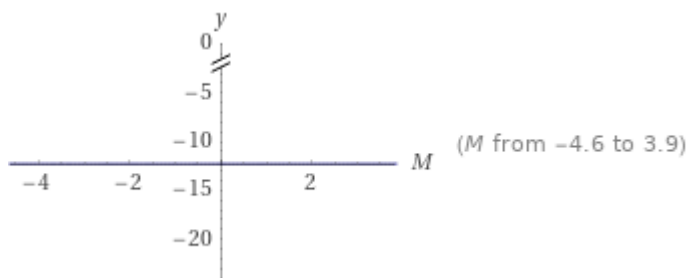
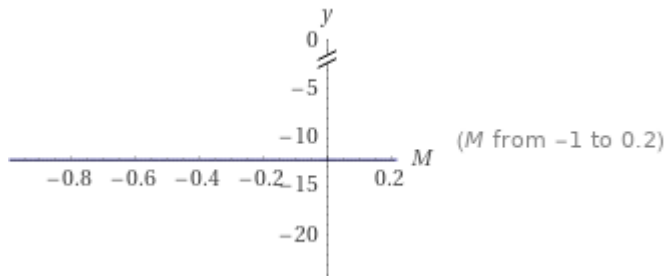
**Input interpretation:**

$$\frac{225.913 \left(-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4}\right)}{M^2}$$

**Result:**

$$\frac{225.913 \left( 6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2}$$

**Plots:**



**Alternate form assuming M is real:**

-12.2723

-12.2723 result very near to the black hole entropy value  $12.1904 = \ln(196884)$

**Alternate forms:**

$$-\frac{12.2723 \left( M^2 - 1.21228 \times 10^{-8} \sqrt{M^4} \right)}{M^2}$$



$$\frac{1.48774 \times 10^{-7} \sqrt{M^4} - 12.2723 M^2}{M^2}$$

**Expanded form:**

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

**Property as a function:**

**Parity**

even

**Series expansion at  $M = 0$ :**

$$\left( \frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723 \right) + O(M^6)$$

(generalized Puiseux series)

**Series expansion at  $M = \infty$ :**

$-12.2723$

**Derivative:**

$$\frac{d}{dM} \left( \frac{225.913 \left( 6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2} \right) = \frac{3.55271 \times 10^{-15}}{M}$$

**Indefinite integral:**

$$\int \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} dM = \frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M} - 12.2723 M + \text{constant}$$

**Global maximum:**

$$\max \left\{ \frac{225.913 \left( 6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2} \right\} = -\frac{140119826723990341497649}{1141759484925100000000} \text{ at } M = -1$$

**Global minimum:**

$$\min \left\{ \frac{225.913 \left( 6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323 M^2 \right)}{M^2} \right\} = -\frac{140119826723990341497649}{1141759484925100000000} \text{ at } M = -1$$

**Limit:**

$$\lim_{M \rightarrow \pm\infty} \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} = -12.2723$$

**Definite integral after subtraction of diverging parts:**

$$\int_0^\infty \left( \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} - -12.2723 \right) dM = 0$$

From b that is equal to

$$\frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2}$$

From:

$$V = \frac{1}{3} (0.0814845 b + 1)^2 M^2$$

we obtain:

$$\frac{1}{3} (0.0814845 \left( \frac{225.913 (-0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4})}{M^2} \right) + 1)^2 M^2$$

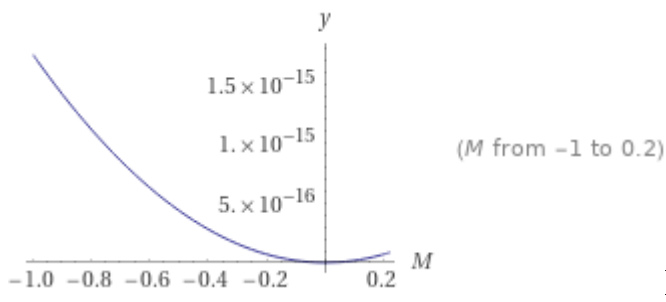
**Input interpretation:**

$$\frac{1}{3} \left( 0.0814845 \times \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

**Result:**

0

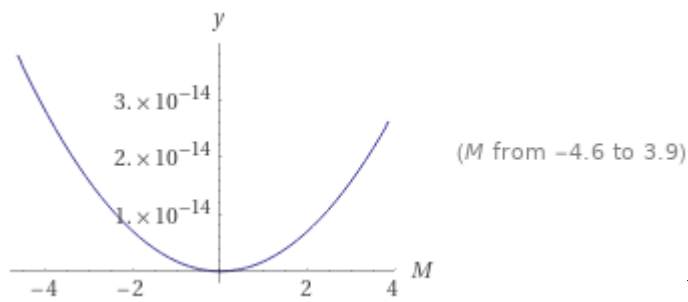
**Plots: (possible mathematical connection with an open string)**



$$M = -0.5; M = 0.2$$

---

**(possible mathematical connection with an open string)**



$$M = 2 ; M = 3$$

**Root:**

$$M = 0$$

**Property as a function:**

**Parity**

even

**Series expansion at  $M = 0$ :**

$$O(M^{62194})$$

(Taylor series)

**Series expansion at  $M = \infty$ :**

$$1.75541 \times 10^{-15} M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

**Definite integral after subtraction of diverging parts:**

$$\int_0^\infty \left( \frac{1}{3} M^2 \left( 1 + \frac{18.4084 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \right)^2 - 1.75541 \times 10^{-15} M^2 \right) dM = 0$$

For  $M = -0.5$ , we obtain:

$$\frac{1}{3} \left( 0.0814845 \times \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545 \times 10^{-10} \sqrt{(-0.5)^4}))/(-0.5)^2) + 1)^2 * (-0.5^2)$$



For M = 3:

$$\frac{1}{3} \left( 0.0814845 \times \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 \cdot 3^2 + 6.58545 \times 10^{-10} \sqrt{3^4}))/3^2) + 1)^2 \cdot 3^2$$

**Input interpretation:**

$$\frac{1}{3} \left( 0.0814845 \times \frac{225.913 \left( -0.054323 \times 3^2 + 6.58545 \times 10^{-10} \sqrt{3^4} \right)}{3^2} + 1 \right)^2 \times 3^2$$

**Result:**

$$1.579864841810872363256294820161116875 \times 10^{-14}$$

$$1.57986484181 \times 10^{-14}$$

For M = 2:

$$\frac{1}{3} \left( 0.0814845 \times \frac{225.913 \left( -0.054323 M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

$$\frac{1}{3} (0.0814845 ((225.913 (-0.054323 \cdot 2^2 + 6.58545 \times 10^{-10} \sqrt{2^4}))/2^2) + 1)^2 \cdot 2^2$$





$$\phi_P^E = \mathbf{E}_P l_P^2 = \phi_P l_P = \sqrt{\frac{\hbar c}{\epsilon_0}}$$

We note that:

$$\frac{1}{55} * \left( \left( \left( \frac{1}{\left( 7.021621519 * 10^{-15} + 1.57986484181 * 10^{-14} + 7.021621519 * 10^{-17} - 4.38851344947 * 10^{-16} \right)} \right)^{1/7} - \left( \frac{\log^{5/8}(2)}{2 \sqrt[8]{2} \sqrt[4]{3} e \log^{3/2}(3)} \right) \right) \right)$$

**Input interpretation:**

$$\frac{1}{55} \left( \left( \frac{1}{\left( 7.021621519 \times 10^{-15} + 1.57986484181 \times 10^{-14} + 7.021621519 \times 10^{-17} - 4.38851344947 \times 10^{-16} \right)} \right)^{1/7} - \frac{\log^{5/8}(2)}{2 \sqrt[8]{2} \sqrt[4]{3} e \log^{3/2}(3)} \right)$$

log(x) is the natural logarithm

**Result:**

1.6181818182...

1.6181818182... result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_P = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

$5.729475 * 10^{-35}$  Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_P = \frac{F_P}{q_P} = \sqrt{\frac{c^7}{16\pi^2 \epsilon_0 \hbar G^2}}$$

$1.820306 * 10^{61}$  V\*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_P^E = \mathbf{E}_P l_P^2 = \phi_P l_P = \sqrt{\frac{\hbar c}{\epsilon_0}}$$

$5.975498 * 10^{-8}$  V\*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\epsilon_0 G}}$$

$1.042940 * 10^{27}$  V Lorentz-Heaviside value

## Relationship between Planck's Electric Flux and Planck's Electric Potential

$$E_P * I_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$$

**Input interpretation:**

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

**Result:**

1 042 939 771 935 000 000 000 000 000

**Scientific notation:**

$$1.042939771935 \times 10^{27}$$

$$1.042939771935 * 10^{27} \approx 1.042940 * 10^{27}$$

Or:

$$E_P * I_P^2 / I_P = (5.975498 * 10^{-8}) * 1 / (5.729475 * 10^{-35})$$

**Input interpretation:**

$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

**Result:**

1.04293988541707573556041347592929544155441816222254220500133... ×  
10<sup>27</sup>

$$1.042939885417 * 10^{27} \approx 1.042940 * 10^{27}$$

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## References

**Isoperimetric Theorems, Open Problems and New Results** – *Francesco Maggi* –  
ICTP, Trieste, 22 February 2017

**SHARP STABILITY INEQUALITIES FOR THE PLATEAU PROBLEM** - *G. De Philippis & F. Maggi* - *j. differential geometry* 96 (2014) 399-456

**STABILITY INEQUALITIES FOR LAWSON CONES** - *Zhenhua Liu* -  
arXiv:1711.06927v6 [math.DG] 22 Aug 2018

**Modular equations and approximations to  $\pi$**  - *Srinivasa Ramanujan*  
*Quarterly Journal of Mathematics*, XLV, 1914, 350 – 372

**An Update on Brane Supersymmetry Breaking**  
*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

**Properties of Nilpotent Supergravity**  
*E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14  
Sep 2015