

**On the study of various Isoperimetric and Variational Problems. Possible mathematical connections with several parameters of Number Theory and sectors of String Theory.**

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### **Abstract**

*In this paper, we analyze various Isoperimetric and variational problems. We describe the possible mathematical connections obtained with several parameters of Number Theory and sectors of String Theory.*

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## **Renato Caccioppoli**

Mathematician (1904 – 1959)



## **Vesuvius landscape with gorse – Naples**



<https://www.pinterest.it/pin/95068242114589901/>

From:

**Droplet Minimizers of an Isoperimetric Problem with Long-Range Interactions - MARCO CICALESE, EMANUELE SPADARO – 2013**

We have:

than the nonlocal one. In order to identify the correct regime, we show here the different contributions to the energy of a single ball. As shown in (2.15), given a ball  $B_{r_m}(p) \subset \Omega$  of radius  $r_m$  centered at  $p$  and with average mass  $m$ , i.e.,  $m|\Omega| = \omega_n r_m^n$  (here  $|\Omega|$  stands for the  $n$ -dimensional volume of  $\Omega$ ), it holds that

$$F_{\gamma,m}(\chi_{B_{r_m}(p)}) = \begin{cases} 2\pi r_m + \gamma \left( \frac{\pi}{2} r_m^4 \log r_m + \left( \pi^2 g_{r_m}(p) - \frac{3\pi}{8} \right) r_m^4 \right) & \text{if } n = 2, \\ n\omega_n r_m^{n-1} + \gamma \left( \frac{2\omega_n}{4-n^2} r_m^{n+2} + \omega_n^2 g_{r_m}(p) r_m^{2n} \right) & \text{if } n \geq 3, \end{cases}$$

where  $g_{r_m}(p)$  is uniformly bounded for  $p$  in a compact subset of  $\Omega$ ; see Sec-

We consider  $n = 11$  ;  $r_m = 2.81794 \cdot 10^{-15}$

$|\Omega| = n$ -dimensional volume. We consider the following formula:

$$V_n(R) = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2} + 1\right)} R^n,$$

Thence:

$$(\pi^{5.5} \cdot 2.81794 \cdot 10^{-15}) / (\Gamma(11/2 + 1))$$

where  $R = 2.81794 \cdot 10^{-15}$  is the Electron radius

**Input interpretation**

$$\frac{\pi^{5.5} \times 2.81794 \times 10^{-15}}{\Gamma\left(\frac{11}{2} + 1\right)}$$

$\Gamma(x)$  is the gamma function

**Result**

$$5.30929... \times 10^{-15}$$

$$5.30929... \cdot 10^{-15}$$

Now, from:

$\omega_n r_m^n = m |\Omega|$  ; where  $m = 9.1093837015 \times 10^{-31}$  (Electron mass), we obtain:

$$9.1093837015 \times 10^{-31} * (\pi^{5.5} * 2.81794 * 10^{-15}) / (\text{gamma}(11/2+1))$$

**Input interpretation**

$$9.1093837015 \times 10^{-31} \times \frac{\pi^{5.5} \times 2.81794 \times 10^{-15}}{\Gamma\left(\frac{11}{2} + 1\right)}$$

$\Gamma(x)$  is the gamma function

**Result**

$$4.83644... \times 10^{-45}$$

$$\omega_n r_m^n = 4.83644... * 10^{-45}$$

We have:

$$\delta_0, r_0 > 0 \quad \gamma r_m^3 < \delta_0 \text{ if } n \geq 3.$$

$$r_m = 2.81794 * 10^{-15}$$

$$\delta_0 = 16 ;$$

From:

$$\gamma (2.81794 * 10^{-15})^3 = 8 ; \gamma = 3.5751540638 \times 10^{44}$$

Indeed:

$$x * (2.81794 * 10^{-15})^3 = 8$$

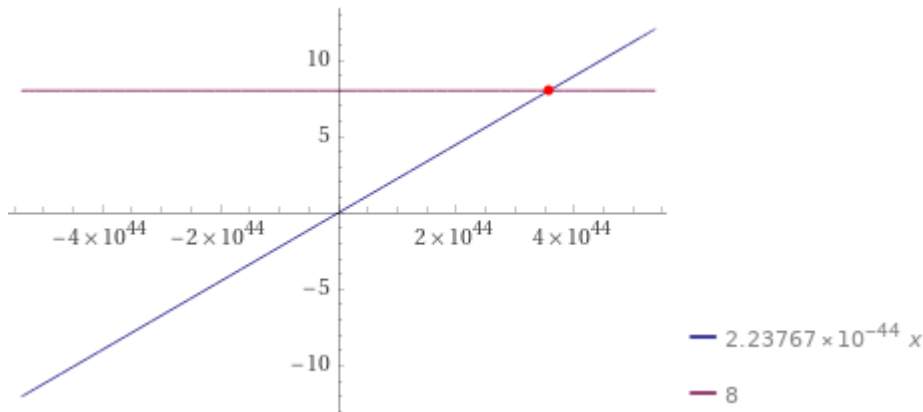
**Input interpretation**

$$x (2.81794 \times 10^{-15})^3 = 8$$

**Result**

$$2.23767 \times 10^{-44} x = 8$$

## Plot



## Alternate form

$$2.23767 \times 10^{-44} x - 8 = 0$$

## Alternate form assuming x is real

$$2.23767 \times 10^{-44} x + 0 = 8$$

## Solution

$$x = 357515406389472979487245268499604469507424256$$

$3.5751540638 \times 10^{44}$  that is equal to  $\gamma$

For:

$$\omega_n r_m^{n-1} =$$

$$((9.1093837015 \times 10^{-31} * (\pi^5 * 2.81794 * 10^{-15}) / (\Gamma(\frac{10}{2} + 1))))$$

## Input interpretation

$$9.1093837015 \times 10^{-31} \times \frac{\pi^5 \times 2.81794 \times 10^{-15}}{\Gamma(\frac{10}{2} + 1)}$$

$\Gamma(x)$  is the gamma function

## Result

$$6.54619... \times 10^{-45}$$

$$\omega_n r_m^{n-1} = 6.54619 \times 10^{-45}$$

And for:

$$\omega_n r_m^{n+2} =$$

$$((9.1093837015 \times 10^{-31} * (\text{Pi}^{6.5} * 2.81794 * 10^{-15}) / (\text{gamma}(13/2+1))))$$

### Input interpretation

$$9.1093837015 \times 10^{-31} \times \frac{\pi^{6.5} \times 2.81794 \times 10^{-15}}{\Gamma\left(\frac{13}{2} + 1\right)}$$

$\Gamma(x)$  is the gamma function

### Result

$$2.33756... \times 10^{-45}$$

$$\omega_n r_m^{n+2} = 2.33756 \times 10^{-45}$$

From:

$$n \omega_n r_m^{n-1} + \gamma\left(\frac{2\omega_n}{4-n^2} r_m^{n+2} + \omega_n^2 g_{r_m}(p) r_m^{2n}\right) \quad \text{if } n \geq 3,$$

we obtain:

$$11 * 6.54619 * 10^{-45} + 3.5751540638 \times 10^{44} (((1/(4-11^2)) * 2(2.33756 * 10^{-45})) + (4.83644 * 10^{-45})^2 * a * b))$$

### Input interpretation

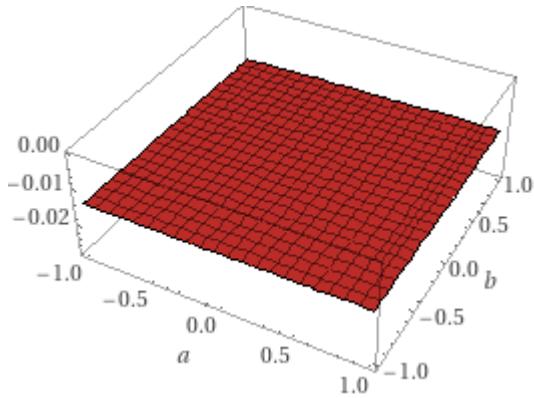
$$11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4-11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 a b \right)$$

### Result

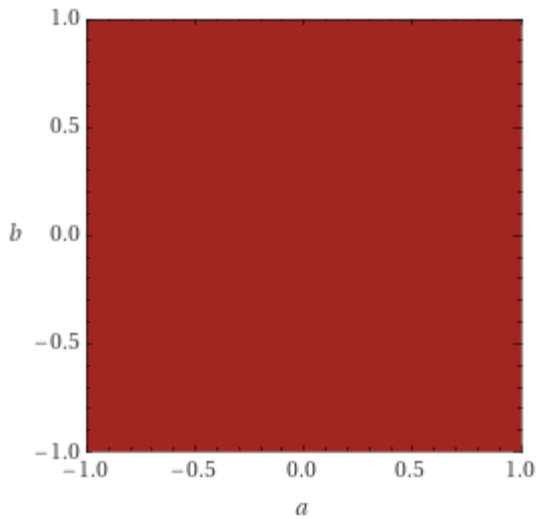
$$3.5751540638 \times 10^{44} (2.33912 \times 10^{-89} a b - 3.99583 \times 10^{-47}) + 7.20081 \times 10^{-44}$$

### 3D plot

(figure that can be related to a D-brane/Instanton)



### Contour plot



### Geometric figure

line

### Alternate form

$$2.06121 \times 10^{-139} (4.05718 \times 10^{94} a b - 6.93074 \times 10^{136})$$

### Expanded form

$$8.3627 \times 10^{-45} a b - 0.0142857$$

## Root

$$a \neq 0, \quad b \approx \frac{1.70827 \times 10^{42}}{a}$$

## Property as a function

### Parity

even

## Root for the variable b

$$b = \frac{1\,708\,265\,194\,215\,972\,392\,228\,875\,639\,649\,382\,466\,322\,432}{a}$$

## Derivative

$$\frac{\partial}{\partial a} (357\,515\,406\,379\,999\,999\,999\,999\,952\,327\,684\,080\,533\,504 \\ (2.33912 \times 10^{-89} a b - 3.99583 \times 10^{-47}) + \\ 7.20081 \times 10^{-44}) = 8.3627 \times 10^{-45} b$$

## Indefinite integral

$$\int (7.20081 \times 10^{-44} + \\ 3.5751540638 \times 10^{44} (-3.99583 \times 10^{-47} + 2.33912 \times 10^{-89} a b)) da = \\ 4.18135 \times 10^{-45} a^2 b - 0.0142857 a + \text{constant}$$

## Limit

$$\lim_{a \rightarrow \pm\infty} (7.20081 \times 10^{-44} + 3.5751540638 \times 10^{44} \\ (-3.99583 \times 10^{-47} + 2.33912 \times 10^{-89} a b)) = -0.0142857$$



$$\lim_{b \rightarrow \pm\infty} (7.20081 \times 10^{-44} + 3.5751540638 \times 10^{44} (-3.99583 \times 10^{-47} + 2.33912 \times 10^{-89} a b)) = -0.0142857$$

### Definite integral over a disk of radius R

$$\iint_{a^2+b^2 < R^2} (3.5751540638 \times 10^{44} (2.33912 \times 10^{-89} a b - 3.99583 \times 10^{-47}) + 7.20081 \times 10^{-44}) da db = 0 - 0.0448799 R^2$$

### Definite integral over a square of edge length 2 L

$$\int_{-L}^L \int_{-L}^L (7.20081 \times 10^{-44} + 3.5751540638 \times 10^{44} (-3.99583 \times 10^{-47} + 2.33912 \times 10^{-89} a b)) db da = 0 - 0.0571428 L^2$$

From:

$$11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 a b \right)$$

for a = b = 0.5 (a, b = p, g), we obtain:

$$11 * 6.54619 * 10^{-45} + 3.5751540638 \times 10^{44} (((1/(4-11^2)) * 2 * (2.33756 * 10^{-45})) + (4.83644 * 10^{-45})^2 * 0.5 * 0.5))$$

### Input interpretation

$$11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 \times 0.5 \times 0.5 \right)$$

### Result

-0.014285704501497996581196581196581196581196507097816904634595982

...

-0.0142857045.....

Inverting and changing the sign, we obtain:

$$-1/((11 * 6.54619 * 10^{-45} + 3.5751540638 \times 10^{44}(((1/(4-11^2))*2(2.33756 * 10^{-45}))+( 4.83644 * 10^{-45})^2 * 0.5 * 0.5))))))$$

**Input interpretation**

$$-\left(1 / \left(11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left(\frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 \times 0.5 \times 0.5\right)\right)\right)$$

**Result**

70.000047942692652469344415564649626382933444888948358354085253889

...

70.000047942692....

From which:

$$24 * ((-1/((11 * 6.54619 * 10^{-45} + 3.5751540638 \times 10^{44}(((1/(4-11^2))*2(2.33756 * 10^{-45}))+( 4.83644 * 10^{-45})^2 * 0.5 * 0.5)))))) + 2) + 1$$

**Input interpretation**

$$24 \left( - \left( 1 / \left( 11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 \times 0.5 \times 0.5 \right) \right) \right) + 2 \right) + 1$$

**Result**

1729.0011506246236592642659735515910331904026773347606004980460933

...

1729.0011506246....

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$(24 * ((-1 / ((11 * 6.54619 * 10^{-45} + 3.5751540638 * 10^{44} * (((1 / (4 - 11^2)) * 2 * (2.33756 * 10^{-45})) + (4.83644 * 10^{-45})^2 * 0.5 * 0.5)))))) + 2) + 1)^{1/15}$$

**Input interpretation**

$$\left( 24 \left( - \left( 1 / \left( 11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 \times 0.5 \times 0.5 \right) \right) \right) + 2 \right) + 1 \right)^{1/15}$$

**Result**

1.6438153016777310820779163280106621402276621737888968068869402952

...

$$1.64381530167\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots \text{ (trace of the instanton shape)}$$

$$(1/27 * (24 * ((-1 / ((11 * 6.54619 * 10^{-45} + 3.5751540638 * 10^{44} * (((1 / (4 - 11^2)) * 2 * (2.33756 * 10^{-45})) + (4.83644 * 10^{-45})^2 * 0.5 * 0.5)))))) + 2) + 1)^2$$

**Input interpretation**

$$\left( \frac{1}{27} \left( 24 \left( - \left( 1 / \left( 11 \times 6.54619 \times 10^{-45} + 3.5751540638 \times 10^{44} \left( \frac{1}{4 - 11^2} \times 2 \times 2.33756 \times 10^{-45} + (4.83644 \times 10^{-45})^2 \times 0.5 \times 0.5 \right) \right) \right) + 2 \right) \right) \right)^2$$

**Result**

4096.0054548148467811273994898747868575215754847986560284549030122

...

4096.005454814...  $\approx 4096 = 64^2$

We have the following Theorem:

**THEOREM 4.1.** *There exists  $\delta_0 > 0$  such that the following holds: Assume  $r_m < 1$  and*

$$\gamma r_m^3 |\log r_m| < \delta_0 \text{ if } n = 2 \text{ or } \gamma r_m^3 < \delta_0 \text{ if } n \geq 3.$$

*Then, every minimizer  $E_m \subset \mathbb{T}^n$  of  $F_{\gamma,m}$  is, up to a translation, a convex set such that*

$$\partial E_m = \{(1 + \psi_m(x))r_m x : x \in \mathbb{S}^{n-1}\}$$

*for some  $\psi_m : \mathbb{S}^{n-1} \rightarrow \mathbb{R}$  with*

$$(4.2) \quad \|\psi_m\|_{C^1} \lesssim \gamma r_m^{n+3},$$

*and its energy has the following asymptotic expansion:*

$$(4.3) \quad F_{\gamma,m}(\chi E_m) = \begin{cases} 2\pi r_m + \frac{\pi\gamma}{2} r_m^4 \log r_m + \gamma(-\frac{1}{8} + \pi^2 h(0)) r_m^4 + O(\gamma r_m^6) & \text{if } n = 2, \\ n\omega_n r_m^{n-1} + \frac{2\gamma\omega_n}{4-n^2} r_m^{n+2} + \gamma\omega_n^2 r_m^{2n} h(0) + O(\gamma r_m^{2n+2}) & \text{if } n \geq 3, \end{cases}$$

*where  $h$  is the Robin function associated to  $G$ .*

From (4.3) , we consider

$$n\omega_n r_m^{n-1} + \frac{2\gamma\omega_n}{4-n^2} r_m^{n+2} + \gamma\omega_n^2 r_m^{2n} h(0) + O(\gamma r_m^{2n+2})$$

and obtain, for  $h(0) = 1/r^{n-2}$ :

$$11*6.54619*10^{-45} + (1/(4-11^2))*(2.33756*10^{-45})*2*3.5751540638 \times 10^{44} + 3.5751540638 \times 10^{44} * ((4.83644*10^{-45})^2) * (1/(2.81794*10^{-15})^9) + (3.5751540638 \times 10^{44} * (2.81794*10^{-15})^{24})$$

### Input interpretation

$$11 \times 6.54619 \times 10^{-45} + \frac{1}{4 - 11^2} \times 2.33756 \times 10^{-45} \times 2 \times 3.5751540638 \times 10^{44} +$$

$$3.5751540638 \times 10^{44} (4.83644 \times 10^{-45})^2 \times \frac{1}{(2.81794 \times 10^{-15})^9} +$$

$$3.5751540638 \times 10^{44} (2.81794 \times 10^{-15})^{24}$$

### Result

$$7.46381203823155408236753016758579130272232287132660325664314... \times 10^{86}$$

$$7.46381203823... \times 10^{86}$$

Dividing the above result, by the previous expression

$$9.1093837015 \times 10^{-31} \times \frac{\pi^{5.5} \times 2.81794 \times 10^{-15}}{\Gamma\left(\frac{11}{2} + 1\right)}$$

we obtain, after some calculations:

$$((89+2))/((1/((7.4638120382315 \times 10^{86} * ((9.1093837015 \times 10^{-31} * (\pi^{5.5} * 2.81794 * 10^{-15}) / (\Gamma(11/2+1))))^2)) - 2))$$

### Input interpretation

$$\frac{89 + 2}{\frac{1}{7.4638120382315 \times 10^{86} \left( 9.1093837015 \times 10^{-31} \times \frac{\pi^{5.5} \times 2.81794 \times 10^{-15}}{\Gamma\left(\frac{11}{2} + 1\right)} \right)^2} - 2}$$

$\Gamma(x)$  is the gamma function

### Result

$$1.6462233656208656404424677578458473725264955980396497720469853442$$

...

$$1.64622336562... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ... \text{ (trace of the instanton shape)}$$

(From: **Stability and minimality for a nonlocal variational problem** - *Nicola Fusco* - 1st Joint Meeting Brazil - Italy in Mathematics - Plenary Talk 7)

We have that:

**Theorem 1 (Choksi-Sternberg 2007)**  
If  $E$  is a critical point and  $X$  is as above, then

$$\begin{aligned} J''(E)[X] &= \int_{\partial E} \left( |D_\nu(X \cdot \nu)|^2 - |B_{\partial E}|^2 (X \cdot \nu)^2 \right) d\sigma \\ &+ 8\gamma \int_{\partial E} \int_{\partial E} G(x, y) (X \cdot \nu)(x) (X \cdot \nu)(y) d\sigma_x d\sigma_y \\ &+ 4\gamma \int_{\partial E} \partial_\nu \nu E (X \cdot \nu)^2 d\sigma \end{aligned}$$

From:

J. reine angew. Math. 611 (2007), 75—108 - DOI 10.1515/CRELLE.2007.074  
**On the first and second variations of a nonlocal isoperimetric problem**  
 by *Rustum Choksi* at Burnaby and *Peter Sternberg* at Bloomington

We have:

**Theorem 2.6.** *Let  $u$  be a stable critical point of  $\mathcal{E}_\gamma$  given by (2.7) such that  $\partial A$  is  $C^2$ . Let  $\zeta$  be any smooth function on  $\partial A$  satisfying the condition*

$$\int_{\partial A} \zeta(x) d\mathcal{H}^{n-1}(x) = 0.$$

*Then for  $v$  solving (2.2) one has the condition*

$$(2.20) \quad \begin{aligned} J(\zeta) := & \int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x) \\ & + 8\gamma \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\ & + 4\gamma \int_{\partial A} \nabla v \cdot v \zeta^2 d\mathcal{H}^{n-1}(x) \geq 0. \end{aligned}$$

*Here  $\nabla_{\partial A} \zeta$  denotes the gradient of  $\zeta$  relative to the manifold  $\partial A$ ,  $B_{\partial A}$  denotes the second fundamental form of  $\partial A$  so that  $\|B_{\partial A}\|^2 = \sum_{i=1}^{n-1} \kappa_i^2$  where  $\kappa_1, \dots, \kappa_{n-1}$  are the principal curvatures and  $v$  denotes the unit normal to  $\partial A$  pointing out of  $A$ .*

From:

$$(2.54) \quad \begin{aligned} \tilde{E}''(0) = & \int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x) \\ & + (n-1)^2 \int_{\partial A} (H - \bar{H}) H \zeta^2 d\mathcal{H}^{n-1}(x). \end{aligned}$$

$$\begin{aligned}
(2.72) \quad \tilde{F}''(0) &= 8 \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\
&\quad + 4 \int_{\partial A} \nabla v(x) \cdot v(x) \zeta^2(x) d\mathcal{H}^{n-1}(x) \\
&\quad + 4(n-1) \int_{\partial A} v(x) H(x) \zeta^2(x) d\mathcal{H}^{n-1}(x) \\
&\quad - 4(n-1) \left( \int_{\partial A} v(y) d\mathcal{H}^{n-1}(y) \right) \left( \int_{\partial A} H(x) \zeta^2(x) d\mathcal{H}^{n-1}(x) \right) \\
&= 8 \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\
&\quad + 4 \int_{\partial A} \nabla v(x) \cdot v(x) \zeta^2(x) d\mathcal{H}^{n-1}(x) \\
&\quad + 4(n-1) \gamma \int_{\partial A} \left[ v(x) - \int_{\partial A} v(y) d\mathcal{H}^{n-1}(y) \right] H(x) \zeta^2(x) d\mathcal{H}^{n-1}(x).
\end{aligned}$$

Thence:

$$\begin{aligned}
\tilde{E}''(0) &= \int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x) \\
&\quad + (n-1)^2 \int_{\partial A} (H - \bar{H}) H \zeta^2 d\mathcal{H}^{n-1}(x).
\end{aligned}$$

$\tilde{F}''(0)$

$$\begin{aligned}
&= 8 \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\
&\quad + 4 \int_{\partial A} \nabla v(x) \cdot v(x) \zeta^2(x) d\mathcal{H}^{n-1}(x) \\
&\quad + 4(n-1) \gamma \int_{\partial A} \left[ v(x) - \int_{\partial A} v(y) d\mathcal{H}^{n-1}(y) \right] H(x) \zeta^2(x) d\mathcal{H}^{n-1}(x).
\end{aligned}$$

we see that (2.54) and (2.72) combine to yield



$$\begin{aligned}
(2.73) \quad \frac{d^2 \mathcal{E}_\gamma(\tilde{U}(\cdot, t))}{dt^2} \Big|_{t=0} &= \tilde{E}''(0) + \gamma \tilde{F}''(0) \\
&= \int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x) \\
&\quad + 8\gamma \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\
&\quad + 4\gamma \int_{\partial A} \nabla v \cdot v \zeta^2 d\mathcal{H}^{n-1}(x).
\end{aligned}$$

Now, from:

$$\begin{aligned}
&\int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x) \\
&+ 8\gamma \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y) \\
&+ 4\gamma \int_{\partial A} \nabla v \cdot v \zeta^2 d\mathcal{H}^{n-1}(x).
\end{aligned}$$

Let  $\zeta$  be any smooth function on  $\partial A$

From:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned}
64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\
64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots,
\end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982 \dots$$

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

We consider  $\zeta = e^{\pi\sqrt{22}}$

$$\gamma > 0$$

$$\gamma = 8$$

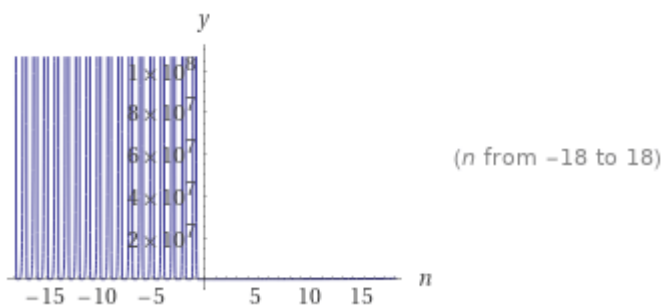
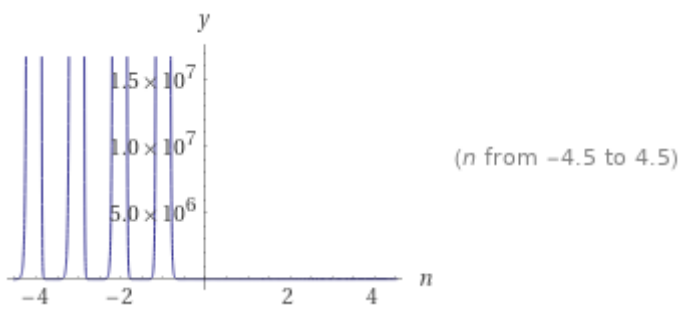
(HarmonicNumber(n))^10

**Input**

$$(H_n)^{10}$$

$H_n$  is the  $n^{\text{th}}$  harmonic number

### Plots



## Values

$n$	1	2	3	4	5
$(H_n)^{10}$	1	$\frac{59049}{1024}$	25 937 424 601 / 60 466 176	95 367 431 640 625 / 61 917 364 224	2 329 194 047 7563 391 944 849 / 604 661 760 000 000 000
approximation	1	57.665	428.958	1540.24	3852.06

## Alternate form

$$\psi^{(0)}(n+1)^{10} + 10\gamma\psi^{(0)}(n+1)^9 + 45\gamma^2\psi^{(0)}(n+1)^8 + 120\gamma^3\psi^{(0)}(n+1)^7 + 210\gamma^4\psi^{(0)}(n+1)^6 + 252\gamma^5\psi^{(0)}(n+1)^5 + 210\gamma^6\psi^{(0)}(n+1)^4 + 120\gamma^7\psi^{(0)}(n+1)^3 + 45\gamma^8\psi^{(0)}(n+1)^2 + 10\gamma^9\psi^{(0)}(n+1) + \gamma^{10}$$

$\psi^{(n)}(x)$  is the  $n^{\text{th}}$  derivative of the digamma function

$\gamma$  is the Euler-Mascheroni constant

## Numerical root

$$n \approx 0.0000704597843094207\dots$$

## Series expansion at $n=0$

$$\frac{\pi^{20} n^{10}}{60466176} + \frac{5\pi^{18} n^{11} \psi^{(2)}(1)}{10077696} + \frac{\pi^{16} n^{12} (2\pi^6 + 1215\psi^{(2)}(1)^2)}{181398528} + \frac{\pi^{14} n^{13} (36\pi^6 \psi^{(2)}(1) + 6480\psi^{(2)}(1)^3 + 5\pi^4 \psi^{(4)}(1))}{120932352} + \frac{1}{38093690880} \pi^{12} n^{14} (166\pi^{12} + 136080\pi^6 \psi^{(2)}(1)^2 + 10716300\psi^{(2)}(1)^4 + 42525\pi^4 \psi^{(2)}(1) \psi^{(4)}(1)) + O(n^{15})$$

(Taylor series)

## Series expansion at $n=\infty$

$$\begin{aligned}
 & (\log(n) + \gamma)^{10} + \frac{5 (\log(n) + \gamma)^9}{n} - \\
 & \frac{5 ((\log(n) + \gamma)^8 (2 \log(n) + 2 \gamma - 27))}{12 n^2} - \frac{15 ((\log(n) + \gamma - 4) (\log(n) + \gamma)^7)}{4 n^3} + \\
 & \frac{1}{48 n^4} (\log(n) + \gamma)^6 (4 \log^3(n) + 3 (5 + 4 \gamma) \log^2(n) + \\
 & 6 (-60 + 5 \gamma + 2 \gamma^2) \log(n) + 4 \gamma^3 + 15 \gamma^2 - 360 \gamma + 630) + \frac{1}{8 n^5} \\
 & (\log(n) + \gamma)^5 (3 \log^3(n) + (10 + 9 \gamma) \log^2(n) + (-70 + 20 \gamma + 9 \gamma^2) \log(n) + \\
 & 3 \gamma^3 + 10 \gamma^2 - 70 \gamma + 63) + O\left(\left(\frac{1}{n}\right)^6\right)
 \end{aligned}$$

(generalized Puiseux series)

## Derivative

$$\frac{d}{dn} ((H_n)^{10}) = \frac{5}{3} (H_n)^9 (\pi^2 - 6 H_n^{(2)})$$

$H_n^{(r)}$  is the generalized harmonic number

## Alternative representations

$$(H_n)^{10} = (H_n^{(1)})^{10}$$


---

$$(H_n)^{10} = (\gamma + \psi(1 + n))^{10}$$


---

$$(H_n)^{10} = (\gamma + \psi(1 + n))^{10}$$

$\psi(x)$  is the digamma function

## Series representations

$$(H_n)^{10} = n^{10} \left( \sum_{k=0}^{\infty} \frac{1}{(1+k)(1+k+n)} \right)^{10}$$


---

$$(H_n)^{10} = \left( \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} (-1)^j (1+k)^{-2-j} n^{1+j} \right)^{10} \quad \text{for } |n| < 1$$


---

$$(H_n)^{10} = \left( H_{z_0} + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} (-1)^j (n - z_0)^{1+j} (1+k+z_0)^{-2-j} \right)^{10} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 \geq 0)$$

From:

$$\int_{\partial A} (|\nabla_{\partial A} \zeta|^2 - \|B_{\partial A}\|^2 \zeta^2) d\mathcal{H}^{n-1}(x)$$

for  $\zeta = e^{\pi\sqrt{22}}$ , we obtain:

$$\text{Integrate}((\nabla * e^{\pi\sqrt{22}})^2 - B^2(e^{\pi\sqrt{22}})^2 * 1540.24) dx$$

For  $\nabla = \text{del } f(x) = (df(x))/(dx) e_x$

### Input interpretation

$$\nabla(e_x f'(x))$$

### Named operator form

$$\text{grad}(e_x f'(x))$$

### Result in 2D Cartesian coordinates

$$\nabla(e_x f'(x)) = \left( e \left( -x \frac{\partial^2 f(x)}{\partial x^2} + \frac{\partial f(x)}{\partial x} \frac{\partial x}{\partial x} \right), 0 \right)$$

(x: first Cartesian coordinate | y: second Cartesian coordinate)

$$\text{integrate}(((((((e (1 (d^2 f(x))/(dx^2) + (df(x))/(dx) (d)/(dx)))) * ((e^{\pi\sqrt{22}}))^2)))) - B^2(e^{\pi\sqrt{22}})^2 * 1540.24)))x$$

## Input interpretation

$$\int \left( \left( e \left( 1 \times \frac{d^2 f(x)}{dx^2} + \frac{df(x)}{dx} \times \frac{d}{dx} \right) e^{\pi\sqrt{22}} \right)^2 - B^2 \left( \left( e^{\pi\sqrt{22}} \right)^2 \times 1540.24 \right) \right) x dx$$

**From:**

$$(e (1 (d^2 f(x))/(dx^2) + (df(x))/(dx) (d)/(dx)))$$

**Input**

$$e \left( 1 \times \frac{d^2 f(x)}{dx^2} + \frac{df(x)}{dx} \times \frac{d}{dx} \right)$$

**Exact result**

$$e \left( \frac{df(x)}{x^2} + \frac{f(x)}{x^2} \right)$$

**Alternate form**

$$\frac{e(d+1)f(x)}{x^2}$$

**Expanded form**

$$\frac{e d f(x)}{x^2} + \frac{e f(x)}{x^2}$$

**Series expansion at x=0**

$$\frac{e(d+1)f(0)}{x^2} + \frac{e(d+1)f'(0)}{x} + \frac{1}{2} e(d+1)f''(0) + \frac{1}{6} e(d+1)f^{(3)}(0)x + \frac{1}{24} e(d+1)f^{(4)}(0)x^2 + O(x^3)$$

(Laurent series)

**Derivative**

$$\frac{\partial}{\partial x} \left( e \left( \frac{df(x)}{x^2} + \frac{f(x)}{x^2} \right) \right) = \frac{e(d+1)(xf'(x) - 2f(x))}{x^3}$$

## Definite integral over a square of edge length 2 L

$$\int_{-L}^L \int_{-L}^L e \left( \frac{f(x)}{x^2} + \frac{d f(x)}{x^2} \right) dx dd = \frac{2 e L f(x)}{x^2}$$

From:.

$$\frac{e (d+1) f(0)}{x^2} + \frac{e (d+1) f'(0)}{x} + \frac{1}{2} e (d+1) f''(0) + \frac{1}{6} e (d+1) f^{(3)}(0) x + \frac{1}{24} e (d+1) f^{(4)}(0) x^2 + O(x^3)$$

(Laurent series)

integrate((((((((e (d + 1) f(0))/x^2 + (e (d + 1) f'(0))/x + 1/2 e (d + 1) f''(0) + 1/6 e (d + 1) f^(3)(0) x + 1/24 e (d + 1) f^(4)(0) x^2 + O(2^3))) ((e^(pi\*sqrt(22))))^2))) - B^2(e^(pi\*sqrt(22)))^2 \* 1540.24))x

## Indefinite integral

$$\int \left( \left( \left( \frac{e (d+1) f(0)}{x^2} + \frac{e (d+1) f'(0)}{x} + \frac{1}{2} e (d+1) f''(0) + \frac{1}{6} e (d+1) f^{(3)}(0) x + \frac{1}{24} e (d+1) f^{(4)}(0) x^2 + O(2^3) \right) e^{\pi \sqrt{22}} \right)^2 - B^2 \left( e^{\pi \sqrt{22}} \right)^2 1540.24 \right) x dx =$$

$$-0.5 x^2 \left( 9.69556 \times 10^{15} B^2 + (-1.71111 \times 10^{13} d - 1.71111 \times 10^{13}) O(8) f''(0) - 1.16282 \times 10^{13} (d+1)^2 f''(0)^2 - 6.29484 \times 10^{12} O(8)^2 \right) - \frac{9.30259 \times 10^{13} (d+1)^2 f(0) f'(0)}{x} + x f'(0) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) + \log(x) \left( f(0) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) + 4.65129 \times 10^{13} (d+1)^2 f'(0)^2 \right) - \frac{2.32565 \times 10^{13} (d+1)^2 f(0)^2}{x^2} + \text{constant}$$

$\log(x)$  is the natural logarithm

## Alternate form assuming B, d, and x are real

$$\begin{aligned}
 & -4.84778 \times 10^{15} B^2 x^2 + f(0) \log(x) (4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} \\
 & d + 3.42223 \times 10^{13}) O(8)) - \frac{9.30259 \times 10^{13} (d+1)^2 f(0) f'(0)}{x} + 4.65129 \times 10^{13} (d+ \\
 & 1)^2 f'(0)^2 \log(x) + x (x ((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times \\
 & 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + f'(0) (4.65129 \times 10^{13} (d+1)^2 f''(0) + \\
 & (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8))) - \frac{2.32565 \times 10^{13} (d+1)^2 f(0)^2}{x^2} +
 \end{aligned}$$

constant

## Alternate forms of the integral

$$\begin{aligned}
 & -4.84778 \times 10^{15} B^2 x^2 + f(0) \log(x) (4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} \\
 & d + 3.42223 \times 10^{13}) O(8)) - \frac{9.30259 \times 10^{13} (d+1)^2 f(0) f'(0)}{x} + 4.65129 \times 10^{13} (d+ \\
 & 1)^2 f'(0)^2 \log(x) + x (x ((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times \\
 & 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + f'(0) (4.65129 \times 10^{13} (d+1)^2 f''(0) + \\
 & (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8))) - \frac{2.32565 \times 10^{13} (d+1)^2 f(0)^2}{x^2} +
 \end{aligned}$$

constant

---


$$\begin{aligned}
 & \frac{1}{x^2} (-4.84778 \times 10^{15} B^2 x^4 + 4.65129 \times 10^{13} d^2 x^2 f'(0)^2 \log(x) - 9.30259 \times 10^{13} d^2 \\
 & f(0) x f'(0) - 2.32565 \times 10^{13} d^2 f(0)^2 + 3.42223 \times 10^{13} d O(8) x^3 f'(0) + 9.30259 \times \\
 & 10^{13} d x^2 f'(0)^2 \log(x) - 1.86052 \times 10^{14} d f(0) x f'(0) + 3.42223 \times 10^{13} d f(0) O(8) \\
 & x^2 \log(x) - 4.65129 \times 10^{13} d f(0)^2 + 3.42223 \times 10^{13} O(8) x^3 f'(0) + 4.65129 \times 10^{13} \\
 & x^2 f'(0)^2 \log(x) - 9.30259 \times 10^{13} f(0) x f'(0) + 3.42223 \times 10^{13} f(0) O(8) x^2 \log(x) - \\
 & 2.32565 \times 10^{13} f(0)^2 + 3.14742 \times 10^{12} O(8)^2 x^4) + (5.81412 \times 10^{12} d^2 + 1.16282 \times \\
 & 10^{13} d + 5.81412 \times 10^{12}) x^2 f''(0)^2 + f''(0) (4.65129 \times 10^{13} d^2 x f'(0) + 4.65129 \times \\
 & 10^{13} d^2 f(0) \log(x) + 9.30259 \times 10^{13} d x f'(0) + 9.30259 \times 10^{13} d f(0) \log(x) + \\
 & 8.55557 \times 10^{12} d O(8) x^2 + 4.65129 \times 10^{13} x f'(0) + 4.65129 \times 10^{13} f(0) \log(x) + \\
 & 8.55557 \times 10^{12} O(8) x^2) + \text{constant}
 \end{aligned}$$


---



$$\begin{aligned}
& -\frac{1}{x^2} \left( 4.84778 \times 10^{15} B^2 x^4 - d^2 (5.81412 \times 10^{12} x^4 f''(0)^2 - 9.30259 \times 10^{13} f(0) x \right. \\
& f'(0) + 4.65129 \times 10^{13} x^3 f'(0) f''(0) + x^2 \log(x) (4.65129 \times 10^{13} f(0) f''(0) + \\
& 4.65129 \times 10^{13} f'(0)^2) - 2.32565 \times 10^{13} f(0)^2) - d(x^4 f''(0) (1.16282 \times 10^{13} f''(0) \\
& + 8.55557 \times 10^{12} O(8)) - 1.86052 \times 10^{14} f(0) x f'(0) + x^3 f'(0) (9.30259 \times 10^{13} \\
& f''(0) + 3.42223 \times 10^{13} O(8)) + x^2 \log(x) (f(0) (9.30259 \times 10^{13} f''(0) + 3.42223 \times \\
& 10^{13} O(8)) + 9.30259 \times 10^{13} f'(0)^2) - 4.65129 \times 10^{13} f(0)^2) - x^4 (8.55557 \times 10^{12} O( \\
& 8) f''(0) + 5.81412 \times 10^{12} f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + 9.30259 \times 10^{13} f(0) x \\
& f'(0) - x^3 f'(0) (4.65129 \times 10^{13} f''(0) + 3.42223 \times 10^{13} O(8)) - x^2 \log(x) (f(0) \\
& (4.65129 \times 10^{13} f''(0) + 3.42223 \times 10^{13} O(8)) + 4.65129 \times 10^{13} f'(0)^2) + 2.32565 \times \\
& 10^{13} f(0)^2) + \text{constant}
\end{aligned}$$

### Expanded form of the integral

$$\begin{aligned}
& -4847782280576693 B^2 x^2 + 5.81412 \times 10^{12} d^2 x^2 f''(0)^2 + 4.65129 \times 10^{13} d^2 f( \\
& 0) f''(0) \log(x) - \frac{9.30259 \times 10^{13} d^2 f(0) f'(0)}{x} + 4.65129 \times 10^{13} d^2 f'(0)^2 \log(x) + \\
& 4.65129 \times 10^{13} d^2 x f'(0) f''(0) - \frac{2.32565 \times 10^{13} d^2 f(0)^2}{x^2} + 8.55557 \times 10^{12} d O(8) \\
& x^2 f''(0) + 1.16282 \times 10^{13} d x^2 f''(0)^2 + 9.30259 \times 10^{13} d f(0) f''(0) \log(x) + \\
& 3.42223 \times 10^{13} d O(8) x f'(0) - \frac{1.86052 \times 10^{14} d f(0) f'(0)}{x} + 9.30259 \times 10^{13} d f'( \\
& 0)^2 \log(x) + 9.30259 \times 10^{13} d x f'(0) f''(0) + 3.42223 \times 10^{13} d f(0) O(8) \log(x) - \\
& \frac{4.65129 \times 10^{13} d f(0)^2}{x^2} + 8.55557 \times 10^{12} O(8) x^2 f''(0) + 5.81412 \times 10^{12} x^2 f''(0)^2 + \\
& 4.65129 \times 10^{13} f(0) f''(0) \log(x) + 3.42223 \times 10^{13} O(8) x f'(0) - \\
& \frac{9.30259 \times 10^{13} f(0) f'(0)}{x} + 4.65129 \times 10^{13} f'(0)^2 \log(x) + 4.65129 \times 10^{13} x f'(0) \\
& f''(0) + 3.42223 \times 10^{13} f(0) O(8) \log(x) - \frac{2.32565 \times 10^{13} f(0)^2}{x^2} + 3.14742 \times 10^{12} \\
& O(8)^2 x^2 + \text{constant}
\end{aligned}$$

### Series expansion of the integral at x=0

$$\begin{aligned}
& -\frac{2.32565 \times 10^{13} ((d+1)^2 f(0)^2)}{x^2} - \frac{9.30259 \times 10^{13} ((d+1)^2 f(0) f'(0))}{x} + \\
& \log(x) (f(0) (4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) \\
& O(8)) + 4.65129 \times 10^{13} (d+1)^2 f'(0)^2) + x f'(0) \\
& (4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8)) + O( \\
& x^2)
\end{aligned}$$

(Puiseux series)

## Series expansion of the integral at $x=\infty$

$$x^2 \left( -4.84778 \times 10^{15} B^2 + (8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + \right. \\ \left. 5.81412 \times 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2 \right) + x f'(0) \\ \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) + \\ \log(x) \left( f(0) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) \right. \right. \\ \left. \left. O(8) \right) + 4.65129 \times 10^{13} (d+1)^2 f'(0)^2 \right) - \\ \frac{9.30259 \times 10^{13} ((d+1)^2 f(0) f'(0))}{x} + O\left(\left(\frac{1}{x}\right)^2\right)$$

(generalized Puiseux series)

From:

$$-4.84778 \times 10^{15} B^2 x^2 + f(0) \log(x) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} \right. \\ \left. d + 3.42223 \times 10^{13}) O(8) \right) - \frac{9.30259 \times 10^{13} (d+1)^2 f(0) f'(0)}{x} + 4.65129 \times 10^{13} (d+ \\ 1)^2 f'(0)^2 \log(x) + x \left( x \left( (8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times \right. \right. \\ \left. \left. 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2 \right) + f'(0) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + \right. \right. \\ \left. \left. (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) \right) - \frac{2.32565 \times 10^{13} (d+1)^2 f(0)^2}{x^2} +$$

constant

For  $x = 2$ ,  $B = 4$  :

$$-4.84778 \times 10^{15} 4^2 2^2 + f(0) \log(2) \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + \right. \\ \left. (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) - (9.30259 \times 10^{13} (d+1)^2 f(0) \\ f'(0))/2 + 4.65129 \times 10^{13} (d+1)^2 f'(0)^2 \log(2)$$

## Input interpretation

$$-4.84778 \times 10^{15} \times 4^2 \times 2^2 + f(0) \log(2) \\ \left( 4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8) \right) - \\ \frac{1}{2} \left( 9.30259 \times 10^{13} (d+1)^2 f(0) f'(0) + 4.65129 \times 10^{13} (d+1)^2 f'(0)^2 \log(2) \right)$$

$\log(x)$  is the natural logarithm

## Result

$$f(0) \log(2) + (4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8)) + 3.22403 \times 10^{13} (d+1)^2 f'(0)^2 - 4.6513 \times 10^{13} (d+1)^2 f(0) f'(0) - 3.10258 \times 10^{17}$$

$$\log(2) (4.65129 \times 10^{13} (d+1)^2 + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) (8)) + 3.22403 \times 10^{13} (d+1)^2 - 4.6513 \times 10^{13} (d+1)^2 - 3.10258 \times 10^{17}$$

## Input interpretation

$$\log(2) (4.65129 \times 10^{13} (d+1)^2 + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) \times 8) + 3.22403 \times 10^{13} (d+1)^2 - 4.6513 \times 10^{13} (d+1)^2 - 3.10258 \times 10^{17}$$

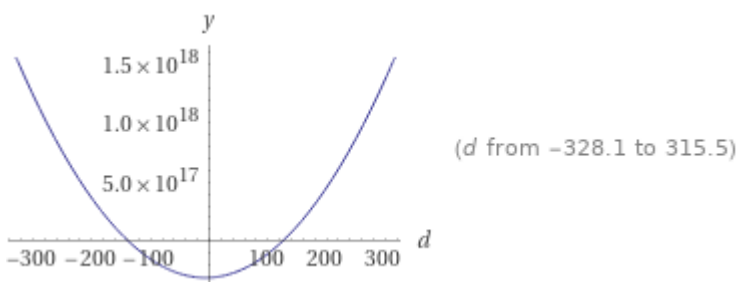
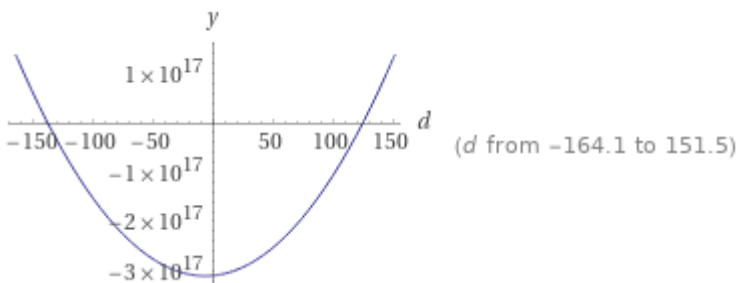
$\log(x)$  is the natural logarithm

## Result

$$-1.42727 \times 10^{13} (d+1)^2 + (4.65129 \times 10^{13} (d+1)^2 + 8 (3.42223 \times 10^{13} d + 3.42223 \times 10^{13})) \log(2) - 3.10258 \times 10^{17}$$

## Plots

(figures that can be related to the open strings)



From

$$-1.42727 \times 10^{13} (d+1)^2 + (4.65129 \times 10^{13} (d+1)^2 + 8(3.42223 \times 10^{13} d + 3.42223 \times 10^{13})) \log(2) - 3.10258 \times 10^{17}$$

For  $d = 151.5$  :

$$-1.42727 \times 10^{13} (151.5+1)^2 + (4.65129 \times 10^{13} (151.5+1)^2 + 8(3.42223 \times 10^{13} \times 151.5 + 3.42223 \times 10^{13})) \log(2) - 3.10258 \times 10^{17}$$

**Input interpretation**

$$-1.42727 \times 10^{13} (151.5 + 1)^2 + (4.65129 \times 10^{13} (151.5 + 1)^2 + 8(3.42223 \times 10^{13} \times 151.5 + 3.42223 \times 10^{13})) \log(2) - 3.10258 \times 10^{17}$$

$\log(x)$  is the natural logarithm

**Result**

$$1.36540... \times 10^{17}$$

$$2(2((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + f'(0)(4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8))) - (2.32565 \times 10^{13} (d+1)^2 f(0)^2 / 2^2)$$

**Input interpretation**

$$2(2(((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + f'(0)(4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8))) - \frac{2.32565 \times 10^{13} (d+1)^2 f(0)^2}{2^2})$$

**Result**

$$2(2(((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) O(8) f''(0) + 5.81412 \times 10^{12} (d+1)^2 f''(0)^2 + 3.14742 \times 10^{12} O(8)^2) + f'(0)(4.65129 \times 10^{13} (d+1)^2 f''(0) + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) O(8))) - 5.81413 \times 10^{12} (d+1)^2 f(0)^2)$$

$$2(2((8.55557 \times 10^{12} d + 8.55557 \times 10^{12})(8) + 5.81412 \times 10^{12} (d + 1)^2 + 3.14742 \times 10^{12} (8)^2) + (4.65129 \times 10^{13} (d+1)^2 + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13})(8))) - 5.81413 \times 10^{12} (d + 1)^2$$

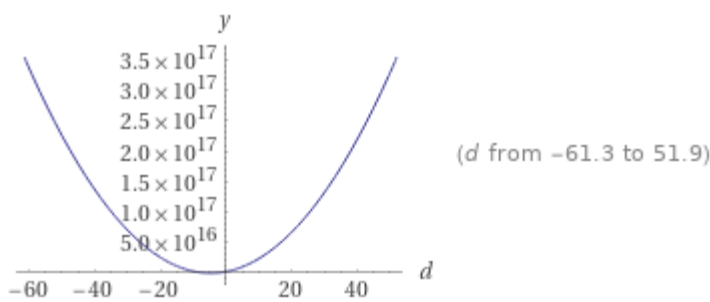
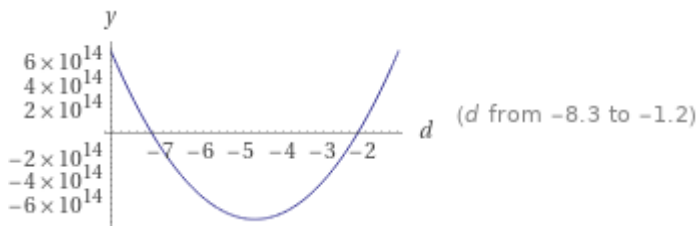
**Input interpretation**

$$2(2((8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) \times 8 + 5.81412 \times 10^{12} (d + 1)^2 + 3.14742 \times 10^{12} \times 8^2) + (4.65129 \times 10^{13} (d + 1)^2 + (3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) \times 8)) - 5.81413 \times 10^{12} (d + 1)^2$$

**Result**

$$2(4.65129 \times 10^{13} (d + 1)^2 + 8(3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) + 2(5.81412 \times 10^{12} (d + 1)^2 + 8(8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) + 2.01435 \times 10^{14})) - 5.81413 \times 10^{12} (d + 1)^2$$

**Plots** (figures that can be related to the open strings)



**From**

$$2(4.65129 \times 10^{13} (d + 1)^2 + 8(3.42223 \times 10^{13} d + 3.42223 \times 10^{13}) + 2(5.81412 \times 10^{12} (d + 1)^2 + 8(8.55557 \times 10^{12} d + 8.55557 \times 10^{12}) + 2.01435 \times 10^{14})) - 5.81413 \times 10^{12} (d + 1)^2$$

For  $d = 51.9$

$$2(2((8.55557 \times 10^{12} \cdot 51.9 + 8.55557 \times 10^{12})(8) + 5.81412 \times 10^{12} (51.9+1)^2 + 3.14742 \times 10^{12} (8)^2) + (4.65129 \times 10^{13} (51.9+1)^2 + (3.42223 \times 10^{13} \cdot 51.9 + 3.42223 \times 10^{13})(8))) - 5.81413 \times 10^{12} (51.9+1)^2$$

### Input interpretation

$$2(2((8.55557 \times 10^{12} \times 51.9 + 8.55557 \times 10^{12}) \times 8 + 5.81412 \times 10^{12} (51.9 + 1)^2 + 3.14742 \times 10^{12} \times 8^2) + (4.65129 \times 10^{13} (51.9 + 1)^2 + (3.42223 \times 10^{13} \times 51.9 + 3.42223 \times 10^{13}) \times 8)) - 5.81413 \times 10^{12} (51.9 + 1)^2$$

### Result

353389538777500000

### Scientific notation

$3.533895387775 \times 10^{17}$

$3.533895387775 \times 10^{17}$

Thence, from:

$$\int \left( \left( e \left( 1 \times \frac{d^2 f(x)}{dx^2} + \frac{df(x)}{dx} \times \frac{d}{dx} \right) e^{\pi\sqrt{22}} \right)^2 - B^2 \left( (e^{\pi\sqrt{22}})^2 \times 1540.24 \right) \right) x dx$$

$(1.36540 \times 10^{17} + 3.533895387775 \times 10^{17})$

### Input interpretation

$1.36540 \times 10^{17} + 3.533895387775 \times 10^{17}$

### Result

489929538777500000

### Scientific notation

$4.899295387775 \times 10^{17}$

$4.899295387775 \times 10^{17}$

For:

$$+ 8\gamma \int_{\partial A} \int_{\partial A} G(x, y) \zeta(x) \zeta(y) d\mathcal{H}^{n-1}(x) d\mathcal{H}^{n-1}(y)$$

for  $\gamma = 8$  :

$$64(((G(x, y) * (e^{\pi\sqrt{22}})^x * (e^{\pi\sqrt{22}})^y * (1540.24)^2)) dx dy$$

**Input interpretation**

$$\int \int 64 (G(x, y) e^{\pi\sqrt{22} x} e^{\pi\sqrt{22} y} \times 1540.24^2) dx dy$$

$G(z)$  is the Barnes G-function

**Result**

$$1.5183 \times 10^8 \int \int e^{\sqrt{22} \pi(x+y)} G(x, y) dx dy$$

For  $x = 2, y = 4$  :

$$1.5183 \times 10^8 \text{ integral integral } e^{(\text{sqrt}(22) \pi (2 + 4))} G(2, 4) dx dy$$

**Input interpretation**

$$1.5183 \times 10^8 \int \int e^{\sqrt{22} \pi(2+4)} G(2, 4) dx dy$$

**Result**

$$3.78714 \times 10^{46} x y G(2, 4)$$

**Alternate form**

$$3.78714 \times 10^{46} x y G(2, 4)$$

**Alternate form assuming x and y are real**

$$3.78714 \times 10^{46} x y G(2, 4) + 0$$

From:

$$3.78714 \times 10^{46} x y G(2, 4)$$

$$3.78714 \times 10^{46} * 8$$

**Input interpretation**

$$3.78714 \times 10^{46} \times 8$$

**Result**

302971200

**Scientific notation**

$$3.029712 \times 10^{47}$$

$$3.029712 * 10^{47}$$

From:

$$+ 4\gamma \int_{\partial A} \nabla v \cdot v \zeta^2 d\mathcal{H}^{n-1}(x).$$

32 integrate(((((((e (d + 1) f(0))/x^2 + (e (d + 1) f'(0))/x + 1/2 e (d + 1) f''(0) + 1/6 e (d + 1) f^(3)(0) x + 1/24 e (d + 1) f^(4)(0) x^2 + O(2^3)))))\*64\*64((e^(pi\*sqrt(22))))^2 \*1540.24))))))dx

**Input interpretation**

$$32 \int \left( \left( \frac{e (d + 1) f(0)}{x^2} + \frac{e (d + 1) f'(0)}{x} + \frac{1}{2} e (d + 1) f''(0) + \frac{1}{6} e (d + 1) (f^3 \times 0) x + \frac{1}{24} e (d + 1) (f^4 \times 0) x^2 + O(2^3) \right) \times 64 \times 64 e^{\pi \sqrt{22}} \right)^2 \times 1540.24 dx$$



## Result

$$5.20527 \times 10^{24} \left( \frac{e(d+1)O(8)(x^2 f''(0) - 2f(0))}{x} + \right. \\ \left. e(d+1)f'(0)\log(x)(e(d+1)f''(0) + 2O(8)) - \frac{1}{12x^3}e^2(d+1)^2(-3x^4 f''(0)^2 + 12x^2 f'(0)^2 + \right. \\ \left. 12f(0)x(xf''(0) + f'(0)) + 4f(0)^2) + O(8)^2 x \right)$$

## Alternate forms

$$\frac{1.41494 \times 10^{25} (d+1)O(8)(x^2 f''(0) - 2f(0))}{x} + \\ 1.41494 \times 10^{25} (d+1)f'(0)\log(x)(e(d+1)f''(0) + 2O(8)) - \frac{1}{x^3} 3.20517 \times 10^{24} \\ (d+1)^2(-3x^4 f''(0)^2 + 12x^2 f'(0)^2 + 12f(0)x(xf''(0) + f'(0)) + 4f(0)^2) + \\ 5.20527 \times 10^{24} O(8)^2 x$$

---


$$e^2 (1.30132 \times 10^{24} d^2 x + 2.60263 \times 10^{24} d x + 1.30132 \times 10^{24} x) f''(0)^2 + \\ \frac{1}{x^3} (-3.8462 \times 10^{25} d^2 x^2 f'(0)^2 - 3.8462 \times 10^{25} d^2 f(0) x f'(0) - \\ 1.28207 \times 10^{25} d^2 f(0)^2 + 2.82988 \times 10^{25} d O(8) x^3 f'(0) \log(x) - \\ 7.6924 \times 10^{25} d x^2 f'(0)^2 - 7.6924 \times 10^{25} d f(0) x f'(0) - \\ 2.82988 \times 10^{25} d f(0) O(8) x^2 - 2.56413 \times 10^{25} d f(0)^2 + \\ 2.82988 \times 10^{25} O(8) x^3 f'(0) \log(x) - 3.8462 \times 10^{25} x^2 f'(0)^2 - \\ 3.8462 \times 10^{25} f(0) x f'(0) - 2.82988 \times 10^{25} f(0) O(8) x^2 - \\ 1.28207 \times 10^{25} f(0)^2 + 5.20527 \times 10^{24} O(8)^2 x^4) + \frac{1}{x} \\ 2.71828 f''(0) (1.41494 \times 10^{25} d^2 x f'(0) \log(x) - 1.41494 \times 10^{25} d^2 f(0) + \\ 2.82988 \times 10^{25} d x f'(0) \log(x) - 2.82988 \times 10^{25} d f(0) + \\ 5.20527 \times 10^{24} d O(8) x^2 + 1.41494 \times 10^{25} x f'(0) \log(x) - \\ 1.41494 \times 10^{25} f(0) + 5.20527 \times 10^{24} O(8) x^2)$$


---

$$\begin{aligned}
& \frac{1}{x^3} (9.6155 \times 10^{24} d^2 x^4 f''(0)^2 - 3.8462 \times 10^{25} d^2 f(0) x^2 f''(0) - \\
& 3.8462 \times 10^{25} d^2 x^2 f'(0)^2 - 3.8462 \times 10^{25} d^2 f(0) x f'(0) + \\
& 3.8462 \times 10^{25} d^2 x^3 f'(0) f''(0) \log(x) - 1.28207 \times 10^{25} d^2 f(0)^2 + \\
& 1.41494 \times 10^{25} d O(8) x^4 f''(0) + 1.9231 \times 10^{25} d x^4 f''(0)^2 - \\
& 7.6924 \times 10^{25} d f(0) x^2 f''(0) + 2.82988 \times 10^{25} d O(8) x^3 f'(0) \log(x) - \\
& 7.6924 \times 10^{25} d x^2 f'(0)^2 - 7.6924 \times 10^{25} d f(0) x f'(0) + \\
& 7.6924 \times 10^{25} d x^3 f'(0) f''(0) \log(x) - 2.82988 \times 10^{25} d f(0) O(8) x^2 - \\
& 2.56413 \times 10^{25} d f(0)^2 + 1.41494 \times 10^{25} O(8) x^4 f''(0) + \\
& 9.6155 \times 10^{24} x^4 f''(0)^2 - 3.8462 \times 10^{25} f(0) x^2 f''(0) + \\
& 2.82988 \times 10^{25} O(8) x^3 f'(0) \log(x) - 3.8462 \times 10^{25} x^2 f'(0)^2 - \\
& 3.8462 \times 10^{25} f(0) x f'(0) + 3.8462 \times 10^{25} x^3 f'(0) f''(0) \log(x) - \\
& 2.82988 \times 10^{25} f(0) O(8) x^2 - 1.28207 \times 10^{25} f(0)^2 + 5.20527 \times 10^{24} O(8)^2 x^4)
\end{aligned}$$

### Series expansion of the integral at $x=\infty$

$$\begin{aligned}
& x \left( (1.41494 \times 10^{25} d + 1.41494 \times 10^{25}) O(8) f''(0) + \right. \\
& \quad \left. (3.10089 \times 10^{12} d + 3.10089 \times 10^{12})^2 f''(0)^2 + 5.20527 \times 10^{24} O(8)^2 \right) + \\
& 1.41494 \times 10^{25} (d + 1) f'(0) \log(x) (e (d + 1) f''(0) + 2 O(8)) + \frac{1}{x} \\
& (d^2 (-3.8462 \times 10^{25} f(0) f''(0) - 3.8462 \times 10^{25} f'(0)^2) + \\
& \quad d f(0) (-7.6924 \times 10^{25} f''(0) - 2.82988 \times 10^{25} O(8)) - \\
& \quad 7.6924 \times 10^{25} d f'(0)^2 + f(0) (-3.8462 \times 10^{25} f''(0) - 2.82988 \times 10^{25} O(8)) - \\
& \quad 3.8462 \times 10^{25} f'(0)^2) + O\left(\left(\frac{1}{x}\right)^2\right)
\end{aligned}$$

(generalized Puiseux series)

## Derivative

$$\frac{\partial}{\partial x} \left( 5\,205\,266\,588\,301\,298\,113\,708\,032 \left( \frac{e(d+1)O(8)(x^2 f''(0) - 2f(0))}{x} + e(d+1)f'(0)\log(x) \right. \right. \\ \left. \left. (e(d+1)f''(0) + 2O(8)) - \frac{1}{12x^3}e^2(d+1)^2(-3x^4 f''(0)^2 + 12x^2 f'(0)^2 + 12f(0)x(x f''(0) + f'(0)) + 4f(0)^2) + O(8)^2 x \right) \right) = \\ - \frac{14\,149\,381\,579\,264\,427\,481\,366\,528(d+1)O(8)(x^2 f''(0) - 2f(0))}{x^2} + \\ \frac{28\,298\,763\,158\,528\,854\,962\,733\,056(d+1)O(8)f''(0) + 14\,149\,381\,579\,264\,427\,481\,366\,528(d+1)f'(0)(e(d+1)f''(0) + 2O(8))}{x} + \\ \frac{1}{x^4}9\,615\,501\,707\,711\,911\,729\,037\,312(d+1)^2 \\ (-3x^4 f''(0)^2 + 12x^2 f'(0)^2 + 12f(0)x(x f''(0) + f'(0)) + 4f(0)^2) + \\ \frac{1}{x^3}38\,462\,006\,830\,847\,646\,916\,149\,248(d+1)^2 \\ (x f''(0)(x^2 f''(0) - 2f(0)) - 2x f'(0)^2 - f(0)f'(0)) + \\ 5\,205\,266\,588\,301\,298\,113\,708\,032O(8)^2$$

## Indefinite integral assuming all variables are real

$$\frac{7.07469 \times 10^{24} (d+1)O(8)x^2 f''(0) + 4.80775 \times 10^{24} (d+1)^2 x^2 f''(0)^2 + 3.8462 \times 10^{25} (d+1)^2 f(0)f'(0)}{x} - 1.41494 \times 10^{25} (d+1)x f'(0)(e d f''(0) + e f''(0) + 2O(8)) + 1.41494 \times 10^{25} (d+1)x f'(0)\log(x)(e d f''(0) + e f''(0) + 2O(8)) - 3.8462 \times 10^{25} (d+1)^2 \log(x)(f(0)f''(0) + f'(0)^2) - 2.82988 \times 10^{25} (d+1)f(0)O(8)\log(x) + \frac{6.41033 \times 10^{24} (d+1)^2 f(0)^2}{x^2} + 2.60263 \times 10^{24} O(8)^2 x^2 + \text{constant}$$

From:

$$5.20527 \times 10^{24} \left( \frac{e(d+1)O(8)(x^2 f''(0) - 2f(0))}{x} + e(d+1)f'(0)\log(x)(e(d+1)f''(0) + 2O(8)) - \frac{1}{12x^3}e^2(d+1)^2(-3x^4 f''(0)^2 + 12x^2 f'(0)^2 + 12f(0)x(x f''(0) + f'(0)) + 4f(0)^2) + O(8)^2 x \right)$$

For  $x = 2$  :

$$5.20527 \times 10^{24} (e(d+1) (8) (2^2 - 2))/2 + e (d+1) \log(2) (e(d+1)+2 (8) - (e^2 (d+1)^2 (-3 \cdot 2^4 + 12 \cdot 2^2 + 12 \cdot 2 (2+1) + 4))/(12 \cdot 8) + (8)^2 \cdot 2)$$

### Input interpretation

$$5.20527 \times 10^{24} \left( \frac{1}{2} \left( (e(d+1)) \times 8 (2^2 - 2) \right) \right) + e (d+1) \log(2) \left( e(d+1) + 2 \times 8 - \frac{e^2 (d+1)^2 (-3 \times 2^4 + 12 \times 2^2 + 12 \times 2 (2+1) + 4)}{12 \times 8} + 8^2 \times 2 \right)$$

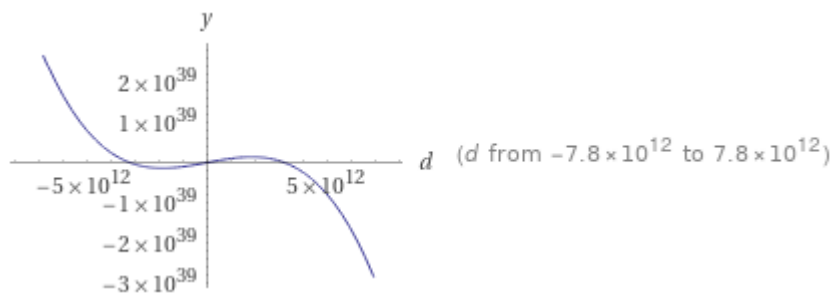
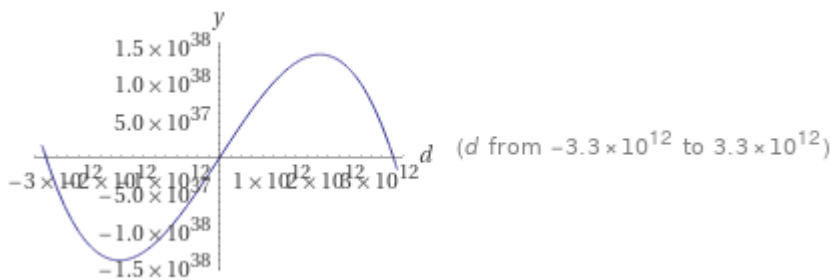
$\log(x)$  is the natural logarithm

### Result

$$1.13195 \times 10^{26} (d+1) + e \left( -\frac{19}{24} e^2 (d+1)^2 + e(d+1) + 144 \right) (d+1) \log(2)$$

### Plots

(figures that can be related to the open strings)



### Alternate forms

$$-\frac{1}{24} e (d+1) (19 e^2 d^2 + 38 e^2 d - 24 e d + 19 e^2 - 24 e - 3456) \log(2) + 1.13195 \times 10^{26} d + 1.13195 \times 10^{26}$$

---


$$\frac{1}{24} (-19 e^3 d^3 \log(2) - 3 e^2 (19 e - 8) d^2 \log(2) + 2.71668 \times 10^{27} d + 2.71668 \times 10^{27})$$


---

$$1.88417 (-5.84967 (d + 1)^2 + 2.71828 (d + 1) + 144) (d + 1) + 1.13195 \times 10^{26} (d + 1)$$

### Expanded form

$$-\frac{19}{24} e^3 d^3 \log(2) - \frac{19}{8} e^3 d^2 \log(2) + e^2 d^2 \log(2) + 1.13195 \times 10^{26} d - \frac{19}{8} e^3 d \log(2) + 2 e^2 d \log(2) + 144 e d \log(2) + 1.13195 \times 10^{26}$$

### Roots

$$d \approx -3.20471 \times 10^{12}$$


---

$$d = -1$$


---

$$d \approx 3.20471 \times 10^{12}$$

### Polynomial discriminant

$$\Delta = 6.39432 \times 10^{79}$$

### Integer root

$$d = -1$$

### Derivative

$$\frac{d}{dd} \left( 113\,195\,126\,825\,784\,097\,621\,671\,936 (d + 1) + e \left( -\frac{19}{24} e^2 (d + 1)^2 + e (d + 1) + 144 \right) (d + 1) \log(2) \right) = -33.0653 d^2 - 55.8872 d + 113\,195\,126\,825\,784\,097\,621\,671\,936$$

### Indefinite integral

$$\int \left( 1.13195 \times 10^{26} (1+d) + (1+d) e \left( 144 + (1+d) e - \frac{19}{24} (1+d)^2 e^2 \right) \log(2) \right) dd = -2.75544 d^4 - 9.31453 d^3 + 5.65976 \times 10^{25} d^2 + 1.13195 \times 10^{26} d + \text{constant}$$

### Definite integral area below the axis between the smallest and largest real roots

$$\int_{-3.20471 \times 10^{12}}^{3.20471 \times 10^{12}} \left( 1.13195 \times 10^{26} (1+d) + (1+d) e \left( 144 + (1+d) e - \frac{19}{24} (1+d)^2 e^2 \right) \log(2) \right) \theta \left( -1.13195 \times 10^{26} (1+d) - (1+d) e \left( 144 + (1+d) e - \frac{19}{24} (1+d)^2 e^2 \right) \log(2) \right) dd = -2.90633 \times 10^{50}$$

$\theta(x)$  is the Heaviside step function

### Definite integral area above the axis between the smallest and largest real roots

$$\int_{-3.20471 \times 10^{12}}^{3.20471 \times 10^{12}} \left( 1.13195 \times 10^{26} (1+d) + (1+d) e \left( 144 + (1+d) e - \frac{19}{24} (1+d)^2 e^2 \right) \log(2) \right) \theta \left( 1.13195 \times 10^{26} (1+d) + (1+d) e \left( 144 + (1+d) e - \frac{19}{24} (1+d)^2 e^2 \right) \log(2) \right) dd = 2.90633 \times 10^{50}$$

From:

$$1.13195 \times 10^{26} (d+1) + e \left( -\frac{19}{24} e^2 (d+1)^2 + e (d+1) + 144 \right) (d+1) \log(2)$$

For  $d = 7.8 * 10^{12}$  :

$$1.13195 \times 10^{26} (1 + 7.8 * 10^{12}) + (1 + 7.8 * 10^{12}) e (144 + (1 + 7.8 * 10^{12}) e - 19/24 (1 + 7.8 * 10^{12})^2 e^2) \log(2)$$

### Input interpretation

$$1.13195 \times 10^{26} (1 + 7.8 \times 10^{12}) + (1 + 7.8 \times 10^{12}) e \left( 144 + (1 + 7.8 \times 10^{12}) e - \frac{19}{24} (1 + 7.8 \times 10^{12})^2 e^2 \right) \log(2)$$

$\log(x)$  is the natural logarithm

### Result

$$-4.34748... \times 10^{39}$$

$$-4.34748 * 10^{39}$$

For  $d = 3.3 * 10^{12}$  :

$$1.13195 \times 10^{26} (1 + 3.3 * 10^{12}) + (1 + 3.3 * 10^{12}) e (144 + (1 + 3.3 * 10^{12}) e - 19/24 (1 + 3.3 * 10^{12})^2 e^2) \log(2)$$

### Input interpretation

$$1.13195 \times 10^{26} (1 + 3.3 \times 10^{12}) + (1 + 3.3 \times 10^{12}) e \left( 144 + (1 + 3.3 \times 10^{12}) e - \frac{19}{24} (1 + 3.3 \times 10^{12})^2 e^2 \right) \log(2)$$

$\log(x)$  is the natural logarithm

### Result

$$-2.25458... \times 10^{37}$$

$$-2.25458 * 10^{37}$$

Thence:

$$(4.899295387775 * 10^{17} + 3.029712 * 10^{47} - 1/2(-4.34748 * 10^{39} - 2.25458 * 10^{37}))$$

### **Input interpretation**

$$4.899295387775 \times 10^{17} + 3.029712 \times 10^{47} - \frac{1}{2} (-4.34748 \times 10^{39} - 2.25458 \times 10^{37})$$

### **Result**

30297120218501290000000000000000000000489929538777500000

### **Scientific notation**

$$3.02971202185012900000000000000000000000000004899295387775 \times 10^{47}$$

3.029712... \* 10<sup>47</sup>

From which:

$$16 \ln(-3.05053 \times 10^{17} + 3.029712 \times 10^{47} + 1.511314 \times 10^{22}) - 21 - \left( \frac{(\sqrt{10 - 2\sqrt{5}} - 2)}{(\sqrt{5} - 1)} \right)$$

where  $\left( \frac{(\sqrt{10 - 2\sqrt{5}} - 2)}{(\sqrt{5} - 1)} \right) = 0.28407904384 = \kappa = 8\pi G$ ;  $G = 0.011303146014$

### **Input interpretation**

$$16 \log(-3.05053 \times 10^{17} + 3.029712 \times 10^{47} + 1.511314 \times 10^{22}) - 21 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

$\log(x)$  is the natural logarithm

### **Result**

1727.99539...

1727.99539...  $\approx$  1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)



$$(64/1728((16\ln(-3.05053*10^{17}+3.029712*10^{47}+1.511314*10^{22})-21-(((\sqrt{(10-2\sqrt{5})-2}))/(\sqrt{5}-1))))))^2$$

**Input interpretation**

$$\left( \frac{64}{1728} \left( 16 \log(-3.05053 \times 10^{17} + 3.029712 \times 10^{47} + 1.511314 \times 10^{22}) - 21 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) \right)^2$$

log(x) is the natural logarithm

**Result**

4095.9782...

4095.9782...  $\approx$  4096 = 64<sup>2</sup>

$$(16\ln(-3.05053*10^{17}+3.029712*10^{47}+1.511314*10^{22})-21-(((\sqrt{(10-2\sqrt{5})-2}))/(\sqrt{5}-1))+1)^{1/15}$$

**Input interpretation**

$$\left( 16 \log(-3.05053 \times 10^{17} + 3.029712 \times 10^{47} + 1.511314 \times 10^{22}) - 21 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} + 1 \right)^{(1/15)}$$

log(x) is the natural logarithm

**Result**

1.643814937...

1.643814937...  $\approx$   $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  (trace of the instanton shape)

From:

**MINIMALITY VIA SECOND VARIATION FOR A NONLOCAL ISOPERIMETRIC PROBLEM - E.ACERBI, N.FUSCO, M.MORINI - 2013**

We have:

$$\begin{aligned} \eta \in (0, 1) \quad \varepsilon > 0. \quad \delta_1 > 0 \quad C > 0 \\ \eta &:= \|\psi\|_{W^{2,p}(\partial E)} + \|\psi\|_{L^2(\partial E)}. \\ \|\psi\|_{W^{2,p}(\partial E)} &\leq \eta_0, \end{aligned}$$

For  $\eta_0 > 0$ ;  $\eta_0 = 4$ ;  $\eta = 0.5$ ;  $\delta_1 = 8$ ;  $C = 2$ ;  $\varepsilon = 2/3$

$$0.5 = 4 + x; \quad x = -3.5 = \|\psi\|_{L^2(\partial E)}$$

From:

$$\left| \int_{\partial E} \psi \nu \, d\mathcal{H}^{N-1} \right| \leq \frac{\delta_1}{2} \|\psi\|_{L^2(\partial E)}.$$

we obtain:

$$8/2 * (-3.5)$$

**Input**

$$\frac{8}{2} \times (-3.5)$$

**Result**

-14

-14



For  $m_0 = 1$  and  $\|\psi\|_{L^2(\partial E)} = -3.5$

We have:

$$J(F) \geq J(E) + \frac{m_0}{8} \int_0^1 (1-t) \|X \cdot \nu^{E_t}\|_{H^1(\partial E_t)}^2 dt \geq J(E) + \frac{m_0}{8} \int_0^1 (1-t) \|X \cdot \nu^{E_t}\|_{L^2(\partial E_t)}^2 dt.$$

$$J(F) \geq J(E) + \frac{m_0}{32} \|\psi\|_{L^2(\partial E)}^2 \geq J(E) + C_0 |E \Delta F|^2.$$

From:

$$J(F) \geq J(E) + \frac{m_0}{32} \|\psi\|_{L^2(\partial E)}^2$$

$$y = x + 1/32 * (-3.5)^2$$

**Input**

$$y = x + \frac{1}{32} (-3.5)^2$$

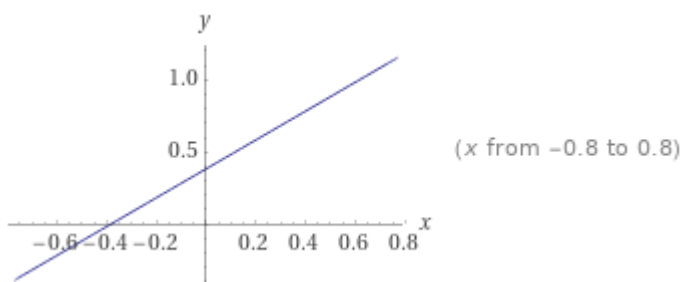
**Result**

$$y = x + 0.382813$$

**Geometric figure**

line

**Plot**



## Alternate forms

$$y = x + 0.382813$$

---

$$-x + y - 0.382813 = 0$$

## Root

$$x \approx -0.382813$$

## Properties as a real function

### Domain

$\mathbb{R}$  (all real numbers)

---

### Range

$\mathbb{R}$  (all real numbers)

---

### Bijectivity

bijjective from its domain to  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

## Partial derivatives

$$\frac{\partial}{\partial x}(x + 0.382813) = 1$$

---

$$\frac{\partial}{\partial y}(x + 0.382813) = 0$$

For  $x = J(E) = 0.61803398$  :

From:

$$y = x + 0.382813$$

$$0.382813 + \Phi$$

### Input interpretation

$$0.382813 + \Phi$$

$\Phi$  is the golden ratio conjugate

### Result

1.0008469887498948482045868343656381177203091798057628621354486227

...

$$1.00084698874989\dots = y = J(F)$$

As  $\nu_i, i = 1, \dots, N.$  ,  $\nu_E = 64$

$$X = \nu^E$$

$$X = 64$$

From:

$$I_t := \left| \int_{\partial E_t} (4\gamma\nu_{E_t} + H_t) \operatorname{div}_{\tau_t} (X_{\tau_t} (X \cdot \nu^{E_t})) d\mathcal{H}^{N-1} \right| \leq \frac{m_0}{4} \|X \cdot \nu^{E_t}\|_{H^1(\partial E_t)}^2$$

we obtain:

$$1/4 \|64 \times 64\|^2$$

### Input

$$\frac{1}{4} \|64 \times 64\|^2$$

$\|\text{expr}\|$  gives the norm of a number, vector, or matrix

### Result

4194304

4194304

From:

$$J(F) \geq J(E) + \frac{m_0}{32} \|\psi\|_{L^2(\partial E)}^2 \geq J(E) + C_0 |E \Delta F|^2.$$

$$J(E) + \frac{m_0}{32} \|\psi\|_{L^2(\partial E)}^2 \geq J(E) + C_0 |E \Delta F|^2.$$

$$0.61803398 + \frac{1}{32} (-3.5)^2 = 0.61803398 + x^2$$

### Input interpretation

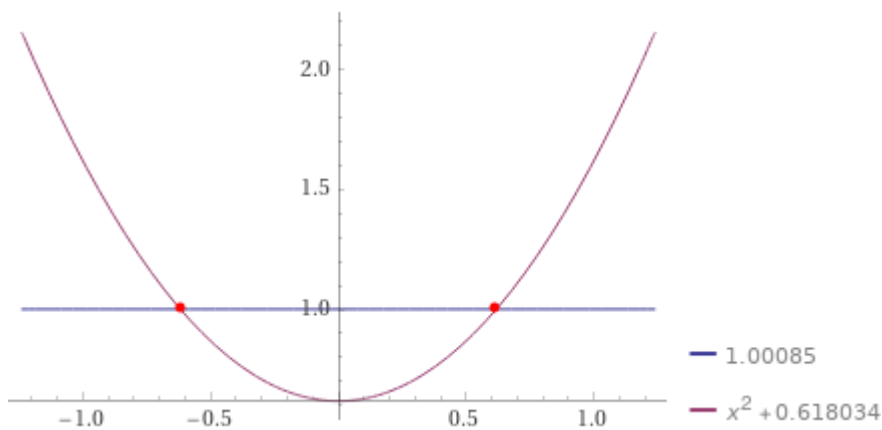
$$0.61803398 + \frac{1}{32} (-3.5)^2 = 0.61803398 + x^2$$

### Result

$$1.00085 = x^2 + 0.618034$$

### Plot

(figure that can be related to an open string)



### Alternate forms

$$1.00085 = x^2 + 0.618034$$

---

$$0.382812 - x^2 = 0$$

## Solutions

$$x \approx -0.618718$$


---

$$x \approx 0.618718$$

$$\left( C_0 |E \Delta F| \right) = 0.618718 \approx 0.61803398 = \text{golden ratio conjugate}$$

We have:

**Lemma 4.1.** *Let  $E \subset \mathbb{T}^N$  be of class  $C^2$  and let  $F \subset \mathbb{T}^N$  be a set of finite perimeter. Then there exists  $C = C(E) > 0$  such that*

$$P_{\mathbb{T}^N}(F) - P_{\mathbb{T}^N}(E) \geq -C|E \Delta F|.$$

*Proof.* Let  $X \in C^1(\mathbb{T}^N; \mathbb{R}^N)$  be a vector field such that  $\|X\|_\infty \leq 1$  and  $X = \nu^E$  on  $\partial E$ . Then,

$$\begin{aligned} P_{\mathbb{T}^N}(F) - P_{\mathbb{T}^N}(E) &\geq \int_{\partial^* F} X \cdot \nu^F d\mathcal{H}^{N-1} - \int_{\partial E} X \cdot \nu^E d\mathcal{H}^{N-1} \\ &= \int_F \operatorname{div} X dx - \int_E \operatorname{div} X dx \geq -C|E \Delta F|, \end{aligned}$$

where  $C := \|\operatorname{div} X\|_\infty$ . □

From:

$$\int_F \operatorname{div} X dx - \int_E \operatorname{div} X dx \geq -C|E \Delta F|,$$

we obtain:

$$\left( \int_F \operatorname{div} X dx - \int_E \operatorname{div} X dx \right) \geq -0.618718$$



$\Omega = \mathbb{T}^N$  is the  $N$ -dimensional flat torus of unit volume

$E$  is a subset of the flat torus

We consider (from: **Lipschitz functions on the infinite-dimensional torus** - Dmitry Faifman and Bo'az Klartag - November 7, 2014)

Let  $\omega_{n,p}$  denote the  $n$ -dimensional volume of the  $\ell_p$ -ball  $B_p^n = \{x \in \mathbb{R}^n ; \sum_{i=1}^n |x_i|^p \leq 1\}$ . In this note, all integrals on tori and subtori are carried out with respect to the the uniform probability measure on the torus. We will need the following variant of Morrey's inequality:

**Lemma 3.** *Let  $n \geq 1, p \in (1, \infty], 0 < \varepsilon < 1/2$  and let  $f : \mathbb{T}^n \rightarrow \mathbb{R}$  be 1-Lipschitz with respect to the metric  $\text{dist}_p$ . Denote  $q = p/(p-1)$ , with  $q = 1$  in case  $p = \infty$ . Assume that*

$$\int_{\mathbb{T}^n} \sum_{i=0}^{n-1} \frac{2^{i^2+qi}}{\omega_{i,p}\varepsilon^{i+q}} \cdot \left| \frac{\partial f}{\partial x_{i+1}} \right|^q \leq 1. \quad (3)$$

Then  $\text{Osc}(f; \mathbb{T}^n) < 8\varepsilon$ .

By an inductive argument, we see that the last  $n-i$  coordinates of the random point  $P_i$  are independent random variables that are distributed uniformly over the circle  $\mathbb{T}$ . Let  $A_{i+1} \in \mathbb{T}^i$  be the vector which consists of the first  $i$  coordinates of  $P_{i+1}$ . We also write  $B_p^i(A_{i+1}, r)$  for the  $\text{dist}_p$ -ball of radius  $r$  centered at  $A_{i+1}$  in the torus  $\mathbb{T}^i$ . Since  $\varepsilon < 1/2$ ,

$$\mathbb{E} \left| \frac{\partial f}{\partial x_{i+1}}(P_i) \right| = \frac{\mathbb{E} \int_{B_p^i(A_{i+1}, \frac{\varepsilon}{2^i}) \times \mathbb{T}^{n-i}} \left| \frac{\partial f}{\partial x_{i+1}} \right|}{\text{Vol}_i(B_p^i(A_{i+1}, \frac{\varepsilon}{2^i}))} = \frac{\mathbb{E} \int_{B_p^i(A_{i+1}, \frac{\varepsilon}{2^i}) \times \mathbb{T}^{n-i}} \left| \frac{\partial f}{\partial x_{i+1}} \right|}{\omega_{i,p} \left(\frac{\varepsilon}{2^i}\right)^i}. \quad (5)$$

From (4), (5) and the Hölder inequality, for  $i = 0, \dots, n-1$ ,

$$\begin{aligned} \mathbb{E}|f(P'_i) - f(P_i)| &\leq \left( \int_{\mathbb{T}^n} \left| \frac{\partial f}{\partial x_{i+1}} \right|^q \right)^{\frac{1}{q}} \left( \omega_{i,p} \left(\frac{\varepsilon}{2^i}\right)^i \right)^{-\frac{1}{q}} \\ &\leq \left( \frac{2^{i^2+qi}}{\omega_{i,p}\varepsilon^{i+q}} \right)^{-\frac{1}{q}} \left( \frac{\omega_{i,p}\varepsilon^i}{2^{i^2}} \right)^{-\frac{1}{q}} = \frac{\varepsilon}{2^i}, \end{aligned}$$

Thence, we consider:

$$\left( \text{Vol}_i(B_p^i(A_{i+1}, \frac{\varepsilon}{2^i})) \right) = \left( \omega_{i,p} \left(\frac{\varepsilon}{2^i}\right)^i \right)$$

Where  $i = n-1$

Thence:

$$V_n(R) = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2} + 1\right)} R^n,$$

For  $R = 1$ , multiplied by

$$\left(\frac{\varepsilon}{2^i}\right)^i$$

we obtain:

$$(\pi^{5.5})/(\text{gamma}(5.5+1))*(((2/3)/(2^{11})))^{11}$$

### Input

$$\frac{\pi^{5.5}}{\Gamma(5.5 + 1)} \left(\frac{2}{3 \cdot 2^{11}}\right)^{11}$$

$\Gamma(x)$  is the gamma function

### Result

$$8.19354... \times 10^{-39}$$

$$8.19354... * 10^{-39}$$

### Alternative representations

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5} \left(\frac{2}{3 \cdot 2^{11}}\right)^{11}}{e^{5.66256}}$$

---


$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5} \left(\frac{2}{3 \cdot 2^{11}}\right)^{11}}{\frac{733\,746.}{2548.75}}$$


---

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5} \left(\frac{2}{3 \cdot 2^{11}}\right)^{11}}{5.5!}$$

## Series representations

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5}}{229949952899717277477822958308088086528 \sum_{k=0}^{\infty} \frac{(6.5 - z_0)^k \Gamma^{(k)}(z_0)}{k!}}$$

for  $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

---

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{4.5} \sum_{k=0}^{\infty} (6.5 - z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}}{229949952899717277477822958308088086528}$$

## Integral representations

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5}}{229949952899717277477822958308088086528 \int_0^{\infty} e^{-t} t^{5.5} dt}$$


---

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\pi^{5.5}}{229949952899717277477822958308088086528 \int_0^1 \log^{5.5}\left(\frac{1}{t}\right) dt}$$


---

$$\frac{\left(\frac{2}{3 \cdot 2^{11}}\right)^{11} \pi^{5.5}}{\Gamma(5.5 + 1)} = \frac{\exp\left(-\int_0^1 \frac{5.5 - 6.5x + x^{6.5}}{(-1+x) \log(x)} dx\right) \pi^{5.5}}{229949952899717277477822958308088086528}$$

We note that  $2.612280 \cdot 10^{-70}$  is the Planck area, and:

$$2.612280 \cdot 10^{-70} < 8.19354 \cdot 10^{-39}$$

### Input interpretation

$$2.612280 \times 10^{-70} < 8.19354 \times 10^{-39}$$

### Result

True

Thence  $E = 2.612280 \cdot 10^{-70}$

We have:

**Theorem 4.3.** *Let  $E \subset \mathbb{T}^N$  be a smooth set and  $p > 1$ . Assume that there exists  $\delta > 0$  such that*

$$J(F) \geq J(E) \tag{4.1}$$

*for all  $F \subset \mathbb{T}^N$ , with  $|F| = |E|$  and such that  $\partial F = \{x + \psi(x)\nu(x) : x \in \partial E\}$  for some function  $\psi$  with  $\|\psi\|_{W^{2,p}(\partial E)} \leq \delta$ . Then there exists  $\delta_0 > 0$  such that (4.1) holds for all  $F \subset \mathbb{T}^N$  of finite perimeter, with  $|F| = |E|$  and  $\mathcal{I}_{-\delta_0}(E) \subset F \subset \mathcal{I}_{\delta_0}(E)$ .*

$$\Lambda > 0$$

$$\begin{aligned} & J(\tilde{F}_h) + \Lambda(|\tilde{F}_h| - |E|) - J(F_h) - \Lambda(|F_h| - |E|) \\ &= P_{\mathbb{T}^N}(\tilde{F}_h) - P_{\mathbb{T}^N}(F_h) + \gamma \int_{\mathbb{T}^N} (|\nabla v_{\tilde{F}_h}|^2 - |\nabla v_{F_h}|^2) dx - \Lambda(|\tilde{F}_h| - |F_h|) \\ &\leq \int_{\partial^* \tilde{F}_h} \nu \cdot \nu^{\tilde{F}_h} d\mathcal{H}^{N-1} - \int_{\partial^* F_h} \nu \cdot \nu^{F_h} d\mathcal{H}^{N-1} + (\gamma C - \Lambda)(|\tilde{F}_h| - |F_h|) \\ &\leq \int_{\tilde{F}_h \Delta F_h} |\operatorname{div} \nu| dx + (\gamma C - \Lambda)(|\tilde{F}_h| - |F_h|) \\ &\leq (\|\operatorname{div} \nu\|_\infty + \gamma C - \Lambda)(|\tilde{F}_h| - |F_h|). \end{aligned} \tag{4.5}$$

For

$$|F_h| > |E|, \quad |\tilde{F}_h| = |E|$$

$$F_h > E \quad F_h > 2.612280 \cdot 10^{-70} = 1.616255 \cdot 10^{-35} \quad \gamma = 1/4$$

$\Lambda > 0$  sufficiently large  
 $= 128$

From:

$$\int_{\tilde{F}_h \Delta F_h} |\operatorname{div} \nu| dx + (\gamma C - \Lambda)(|\tilde{F}_h| - |F_h|) \leq (\|\operatorname{div} \nu\|_\infty + \gamma C - \Lambda)(|\tilde{F}_h| - |F_h|)$$

$$(\|\operatorname{div} 64\| + 1/4 * 2 - 128)(2.612280e-70 - 1.616255e-35)$$

$$(((df(x))/(dx) e) (64) + 1/4 * 2 - 128)(2.612280e-70 - 1.616255e-35)$$

### Input interpretation

$$\left( \left( \frac{\partial f(x)}{\partial x} e \right) \times 64 + \frac{1}{4} \times 2 - 128 \right) (2.612280 \times 10^{-70} - 1.616255 \times 10^{-35})$$

### Result

$$-1.61626 \times 10^{-35} \left( 64 e f'(x) - \frac{255}{2} \right)$$

### Expanded form

$$2.06073 \times 10^{-33} - 2.8118 \times 10^{-33} f'(x)$$

### Series expansion at x=0

$$\begin{aligned} & (2.06073 \times 10^{-33} - 2.8118 \times 10^{-33} f'(0)) - \\ & 2.8118 \times 10^{-33} x f''(0) - 1.4059 \times 10^{-33} f^{(3)}(0) x^2 - \\ & 4.68633 \times 10^{-34} f^{(4)}(0) x^3 - 1.17158 \times 10^{-34} f^{(5)}(0) x^4 + O(x^5) \end{aligned}$$

(Taylor series)

### Derivative

$$\frac{d}{dx} \left( -1.61626 \times 10^{-35} \left( 64 e f'(x) - \frac{255}{2} \right) \right) = -2.8118 \times 10^{-33} f''(x)$$









## Input

$$a^2 D^2 x \left( \psi^{(0)}(12)^{10} + 10 \gamma \psi^{(0)}(12)^9 + 45 \gamma^2 \psi^{(0)}(12)^8 + 120 \gamma^3 \psi^{(0)}(12)^7 + 210 \gamma^4 \psi^{(0)}(12)^6 \right)$$

$\psi^{(n)}(x)$  is the  $n^{\text{th}}$  derivative of the digamma function

## Exact result

$$a^2 \left( 210 \left( \frac{83711}{27720} - \gamma \right)^6 \gamma^4 + 120 \left( \frac{83711}{27720} - \gamma \right)^7 \gamma^3 + 45 \left( \frac{83711}{27720} - \gamma \right)^8 \gamma^2 + 10 \left( \frac{83711}{27720} - \gamma \right)^9 \gamma + \left( \frac{83711}{27720} - \gamma \right)^{10} \right) D^2 x$$

$\gamma$  is the Euler-Mascheroni constant

$$a^2 (210(83711/27720-0.57721)^6 0.57721^4 + 120(83711/27720-0.57721)^7 0.57721^3 + 45(83711/27720 - 0.57721)^8 0.57721^2 + 10(83711/27720 - 0.57721)^9 0.57721 + (83711/27720 - 0.57721)^{10}) D^2 x$$

## Input

$$a^2 \left( 210 \left( \frac{83711}{27720} - 0.57721 \right)^6 \times 0.57721^4 + 120 \left( \frac{83711}{27720} - 0.57721 \right)^7 \times 0.57721^3 + 45 \left( \frac{83711}{27720} - 0.57721 \right)^8 \times 0.57721^2 + 10 \left( \frac{83711}{27720} - 0.57721 \right)^9 \times 0.57721 + \left( \frac{83711}{27720} - 0.57721 \right)^{10} \right) D^2 x$$

## Result

$$61358.8 a^2 D^2 x$$

$$a^2 D^2 x (252 \gamma^5 \text{polygamma}(0, 12)^5 + 210 \gamma^6 \text{polygamma}(0, 12)^4 + 120 \gamma^7 \text{polygamma}(0, 12)^3 + 45 \gamma^8 \text{polygamma}(0, 12)^2 + 10 \gamma^9 \text{polygamma}(0, 12) + \gamma^{10})$$

## Input

$$a^2 D^2 \left( x \left( 252 \gamma^5 \psi^{(0)}(12)^5 + 210 \gamma^6 \psi^{(0)}(12)^4 + 120 \gamma^7 \psi^{(0)}(12)^3 + 45 \gamma^8 \psi^{(0)}(12)^2 + 10 \gamma^9 \psi^{(0)}(12) + \gamma^{10} \right) \right)$$

$\psi^{(n)}(x)$  is the  $n^{\text{th}}$  derivative of the digamma function

### Exact result

$$a^2 \left( \gamma^{10} + 10 \left( \frac{83711}{27720} - \gamma \right) \gamma^9 + 45 \left( \frac{83711}{27720} - \gamma \right)^2 \gamma^8 + 120 \left( \frac{83711}{27720} - \gamma \right)^3 \gamma^7 + 210 \left( \frac{83711}{27720} - \gamma \right)^4 \gamma^6 + 252 \left( \frac{83711}{27720} - \gamma \right)^5 \gamma^5 \right) D^2 x$$

$$a^2(0.57721^{10}+10(83711/27720-0.57721)0.57721^9+45(83711/27720-0.57721)^2 0.57721^8+120(83711/27720-0.57721)^3 0.57721^7+210(83711/27720-0.57721)^4 0.57721^6+252(83711/27720-0.57721)^5 0.57721^5)D^2 x$$

### Input

$$a^2 \left( 0.57721^{10} + 10 \left( \frac{83711}{27720} - 0.57721 \right) \times 0.57721^9 + 45 \left( \frac{83711}{27720} - 0.57721 \right)^2 \times 0.57721^8 + 120 \left( \frac{83711}{27720} - 0.57721 \right)^3 \times 0.57721^7 + 210 \left( \frac{83711}{27720} - 0.57721 \right)^4 \times 0.57721^6 + 252 \left( \frac{83711}{27720} - 0.57721 \right)^5 \times 0.57721^5 \right) D^2 x$$

### Result

$$1721.39 a^2 D^2 x$$

$$(61358.8 a^2 D^2 x + 1721.39 a^2 D^2 x)$$

### Input interpretation

$$61358.8 a^2 D^2 x + 1721.39 a^2 D^2 x$$

## Result

$$63080.2 a^2 D^2 x$$
$$63080.2 a^2 D^2 x$$

For  $v_L = 64$  ;  $\gamma_h = 1/24$

From:

$$-4\gamma_h \|\nabla v_L\|_{L^\infty}$$

we obtain:

$$-4 \cdot 1/24 \cdot ((df(x))/(dx) e) \quad (64)$$

## Input interpretation

$$-4 \times \frac{1}{24} \left( \frac{\partial f(x)}{\partial x} e \right) \times 64$$

## Result

$$-\frac{32}{3} e f'(x)$$

$$-32/3 * e * f'(x)$$

## Series expansion at x=0

$$-\frac{32}{3} (e f'(0)) - \frac{32}{3} x (e f''(0)) - \frac{16}{3} (e f^{(3)}(0)) x^2 -$$
$$\frac{16}{9} (e f^{(4)}(0)) x^3 - \frac{4}{9} (e f^{(5)}(0)) x^4 + O(x^5)$$

(Taylor series)

## Derivative

$$\frac{d}{dx} \left( -\frac{32}{3} e f'(x) \right) = -\frac{32}{3} e f''(x)$$

From:

$$\int_{\partial L} \varphi^2 d\mathcal{H}^{N-1}.$$

we consider:

Integrate((a)^2\*(HarmonicNumber(n))^10)dx

### Indefinite integral

$$\int a^2 (H_n)^{10} dx = a^2 x (H_n)^{10} + \text{constant}$$

$H_n$  is the  $n^{\text{th}}$  harmonic number

### Alternate form of the integral

$$a^2 x (\psi^{(0)}(n+1)^{10} + 10 \gamma \psi^{(0)}(n+1)^9 + 45 \gamma^2 \psi^{(0)}(n+1)^8 + 120 \gamma^3 \psi^{(0)}(n+1)^7 + 210 \gamma^4 \psi^{(0)}(n+1)^6 + 252 \gamma^5 \psi^{(0)}(n+1)^5 + 210 \gamma^6 \psi^{(0)}(n+1)^4 + 120 \gamma^7 \psi^{(0)}(n+1)^3 + 45 \gamma^8 \psi^{(0)}(n+1)^2 + 10 \gamma^9 \psi^{(0)}(n+1) + \gamma^{10}) + \text{constant}$$

$$a^2 x (\text{polygamma}(0, 12)^{10} + 10 \gamma \text{polygamma}(0, 12)^9 + 45 \gamma^2 \text{polygamma}(0, 12)^8 + 120 \gamma^3 \text{polygamma}(0, 12)^7 + 210 \gamma^4 \text{polygamma}(0, 12)^6 + 252 \gamma^5 \text{polygamma}(0, 12)^5 + 210 \gamma^6 \text{polygamma}(0, 12)^4 + 120 \gamma^7 \text{polygamma}(0, 12)^3 + 45 \gamma^8 \text{polygamma}(0, 12)^2 + 10 \gamma^9 \text{polygamma}(0, 12) + \gamma^{10})$$

### Input

$$a^2 x (\psi^{(0)}(12)^{10} + 10 \gamma \psi^{(0)}(12)^9 + 45 \gamma^2 \psi^{(0)}(12)^8 + 120 \gamma^3 \psi^{(0)}(12)^7 + 210 \gamma^4 \psi^{(0)}(12)^6 + 252 \gamma^5 \psi^{(0)}(12)^5 + 210 \gamma^6)$$

$\psi^{(n)}(x)$  is the  $n^{\text{th}}$  derivative of the digamma function

### Exact result

$$a^2 \left( 210 \gamma^6 + 252 \left( \frac{83711}{27720} - \gamma \right)^5 \gamma^5 + 210 \left( \frac{83711}{27720} - \gamma \right)^6 \gamma^4 + 120 \left( \frac{83711}{27720} - \gamma \right)^7 \gamma^3 + 45 \left( \frac{83711}{27720} - \gamma \right)^8 \gamma^2 + 10 \left( \frac{83711}{27720} - \gamma \right)^9 \gamma + \left( \frac{83711}{27720} - \gamma \right)^{10} \right) x$$

$$a^2 (210 \gamma^6 + 252 (3.019877 - \gamma)^5 \gamma^5 + 210 (3.019877 - \gamma)^6 \gamma^4 + 120 (3.019877 - \gamma)^7 \gamma^3 + 45 (3.019877 - \gamma)^8 \gamma^2 + 10 (3.019877 - \gamma)^9 \gamma + (3.019877 - \gamma)^{10}) x$$

### Input interpretation

$$a^2 (210 \gamma^6 + 252 (3.019877 - \gamma)^5 \gamma^5 + 210 (3.019877 - \gamma)^6 \gamma^4 + 120 (3.019877 - \gamma)^7 \gamma^3 + 45 (3.019877 - \gamma)^8 \gamma^2 + 10 (3.019877 - \gamma)^9 \gamma + (3.019877 - \gamma)^{10}) x$$

$\gamma$  is the Euler-Mascheroni constant

### Result

$$62770.6 a^2 x$$

62770.6

$$a^2 x (\text{polygamma}(0, 12)^4 + 120 \gamma^7 \text{polygamma}(0, 12)^3 + 45 \gamma^8 \text{polygamma}(0, 12)^2 + 10 \gamma^9 \text{polygamma}(0, 12) + \gamma^{10})$$

### Input

$$a^2 x (\psi^{(0)}(12)^4 + 120 \gamma^7 \psi^{(0)}(12)^3 + 45 \gamma^8 \psi^{(0)}(12)^2 + 10 \gamma^9 \psi^{(0)}(12) + \gamma^{10})$$

$\psi^{(n)}(x)$  is the  $n^{\text{th}}$  derivative of the digamma function

### Exact result

$$a^2 \left( \gamma^{10} + 10 \left( \frac{83711}{27720} - \gamma \right) \gamma^9 + 45 \left( \frac{83711}{27720} - \gamma \right)^2 \gamma^8 + 120 \left( \frac{83711}{27720} - \gamma \right)^3 \gamma^7 + \left( \frac{83711}{27720} - \gamma \right)^4 \right) x$$

$$a^2 (\gamma^{10} + 10 (3.019877 - \gamma) \gamma^9 + 45 (3.019877 - \gamma)^2 \gamma^8 + 120 (3.019877 - \gamma)^3 \gamma^7 + (3.019877 - \gamma)^4) x$$

### Input interpretation

$$a^2 (\gamma^{10} + 10 (3.019877 - \gamma) \gamma^9 + 45 (3.019877 - \gamma)^2 \gamma^8 + 120 (3.019877 - \gamma)^3 \gamma^7 + (3.019877 - \gamma)^4) x$$

$\gamma$  is the Euler-Mascheroni constant

### Result

$$76.4236 a^2 x$$

$$(62770.6 a^2 x + 76.4236 a^2 x)$$

### Input interpretation

$$62770.6 a^2 x + 76.4236 a^2 x$$

### Result

$$62847. a^2 x$$

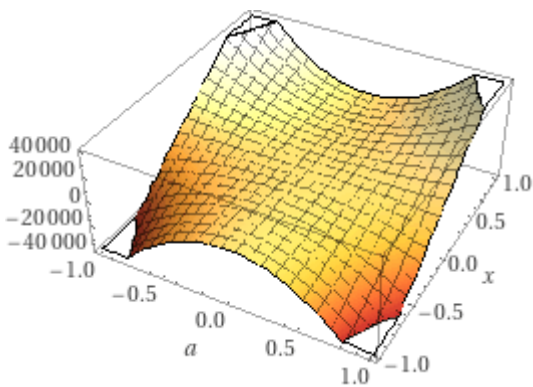
$$62847 a^2 x$$

### Geometric figure

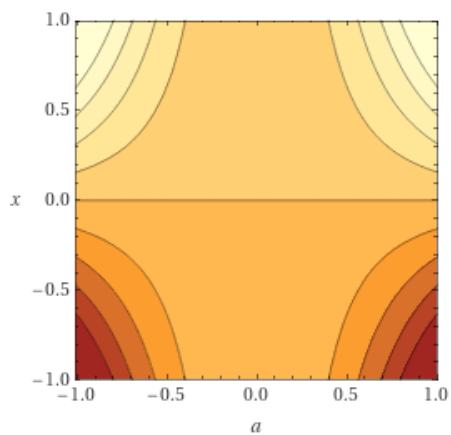
line

### 3D plot

(figure that can be related to a D-brane/Instanton)



### Contour plot



## Alternate form assuming a and x are real

$$62847. a^2 x + 0$$

### Roots

$$a = 0$$

---

$$x \approx 0$$

### Polynomial discriminant

$$\Delta = 0$$

### Properties as a real function

#### Domain

$\mathbb{R}$  (all real numbers)

---

#### Range

$\{y \in \mathbb{R} : a \neq 0 \text{ or } y = 0\}$

---

#### Injectivity

injective (one-to-one)

---

#### Parity

odd

$\mathbb{R}$  is the set of real numbers

## Derivative

$$\frac{\partial}{\partial x}(62847. a^2 x) = 62847. a^2$$

## Indefinite integral

$$\int 62847. a^2 x dx = 31423.5 a^2 x^2 + \text{constant}$$

## Definite integral over a disk of radius R

$$\iint_{a^2+x^2 < R^2} 62847. a^2 x da dx = 0$$

## Definite integral over a square of edge length 2 L

$$\int_{-L}^L \int_{-L}^L 62847. a^2 x dx da = 0$$

Thence, from:

$$\int_{\partial L} |D_{\tau} \varphi|^2 d\mathcal{H}^{N-1} - 4\gamma_h \|\nabla v_L\|_{L^\infty} \int_{\partial L} \varphi^2 d\mathcal{H}^{N-1}.$$

we obtain:

$$(63080.2 a^2 D^2 x) - ((-4 * 1/24 * ((df(x))/(dx) e) (64))) * (62847 a^2 x)$$

## Input interpretation

$$63080.2 a^2 D^2 x - \left( -4 \times \frac{1}{24} \left( \frac{\partial f(x)}{\partial x} e \right) \times 64 \right) (62847 a^2 x)$$

## Result

$$63080.2 a^2 D^2 x + 670368 e a^2 x f'(x)$$



## Alternate forms

$$63080.2 a^2 x (D^2 + 28.8878 f'(x))$$

---

$$a^2 x (63080.2 D^2 + 1.82225 \times 10^6 f'(x))$$

---

$$a^2 x (63080.2 D^2 + 670368 e f'(x))$$

## Alternate form assuming a, D, and x are positive

$$63080.2 a^2 x (D^2 + 28.8878 f'(x))$$

## Series expansion at x=0

$$x (63080.2 a^2 D^2 + 1.82225 \times 10^6 a^2 f'(0)) + 1.82225 \times 10^6 a^2 x^2 f''(0) + 911125. a^2 f^{(3)}(0) x^3 + 303708. a^2 f^{(4)}(0) x^4 + 75927. a^2 f^{(5)}(0) x^5 + O(x^6)$$

(Taylor series)

## Derivative

$$\frac{\partial}{\partial x} (63080.2 a^2 D^2 x + 670368 e a^2 x f'(x)) = 63080.2 a^2 (D^2 + 28.8878 x f''(x) + 28.8878 f'(x))$$

From:

$$63080.2 a^2 D^2 x + 670368 e a^2 x f'(x)$$

For  $x = 2$ ,  $a = 4$ :

$$63080.2 * 16 * D^2 * 2 + 670368 * e * 16 * 2$$

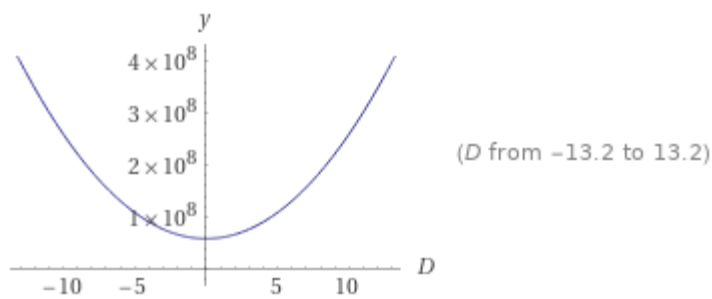
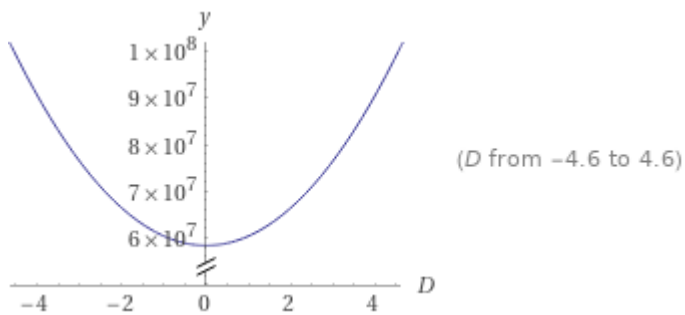
### Input interpretation

$$63\,080.2 \times 16 D^2 \times 2 + 670\,368 e (16 \times 2)$$

### Result

$$2.01857 \times 10^6 D^2 + 21\,451\,776 e$$

### Plots (figures that can be related to the open strings)



### Geometric figure

parabola

### Alternate forms

$$2.01857 \times 10^6 (D^2 + 28.8878)$$

$$2.01857 \times 10^6 D^2 + 5.8312 \times 10^7$$

---

$$6.4 (315\,401 D^2 + 3\,351\,840 e)$$

### Complex roots

$$D = -5.37474 i$$

---

$$D = 5.37474 i$$

### Polynomial discriminant

$$\Delta = -4.70826 \times 10^{14}$$

### Property as a function

#### Parity

even

### Derivative

$$\frac{d}{dD} (2.01857 \times 10^6 D^2 + 21\,451\,776 e) = 4.03713 \times 10^6 D$$

### Indefinite integral

$$\int (2.01857 \times 10^6 D^2 + 21\,451\,776 e) dD = 672855. D^3 + 5.8312 \times 10^7 D + \text{constant}$$

### Global minimum

$$\min\{2.01857 \times 10^6 D^2 + 21\,451\,776 e\} = 21\,451\,776 e \text{ at } D = 0$$

From:

$$63080.2 \times 16 D^2 \times 2 + 670368 e (16 \times 2)$$

For  $D = 4.6$  :

$$(63080.2 * 16 * 4.6^2 * 2 + 670368 * e 16 * 2)$$

### **Input interpretation**

$$63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e (16 \times 2)$$

### **Result**

$$1.01025... \times 10^8$$

$$1.01025... * 10^8$$

For  $D = 13.2$  :

$$(63080.2 * 16 * 13.2^2 * 2 + 670368 * e 16 * 2)$$

### **Input interpretation**

$$63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e (16 \times 2)$$

### **Result**

$$4.10027... \times 10^8$$

$$4.10027... * 10^8$$

From the sum and the mean of the two expressions, after some calculations, we obtain:

$$((1/2((((63080.2 * 16 * 4.6^2 * 2 + 670368 * e 16 * 2) + (63080.2 * 16 * 13.2^2 * 2 + 670368 * e 16 * 2))))))^{1/4+\varphi}$$

## Input interpretation

$$\left(\frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e (16 \times 2)) + \right. \right. \\ \left. \left. (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e (16 \times 2)) \right) \right)^{(1/4) + \phi}$$

$\phi$  is the golden ratio

## Result

128.051...

128.051....

## Alternative representations

$$\left(\frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e (16 \times 2)) + \right. \right. \\ \left. \left. (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e (16 \times 2)) \right) \right)^{(1/4) + \phi} = \\ -2 \cos(216^\circ) + \sqrt[4]{\frac{1}{2} (42903552 e + 2.01857 \times 10^6 \times 4.6^2 + 2.01857 \times 10^6 \times 13.2^2)}$$

---

$$\left(\frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e (16 \times 2)) + \right. \right. \\ \left. \left. (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e (16 \times 2)) \right) \right)^{(1/4) + \phi} = \\ 2 \cos\left(\frac{\pi}{5}\right) + \sqrt[4]{\frac{1}{2} (42903552 e + 2.01857 \times 10^6 \times 4.6^2 + 2.01857 \times 10^6 \times 13.2^2)}$$

---

$$\left(\frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e (16 \times 2)) + \right. \right. \\ \left. \left. (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e (16 \times 2)) \right) \right)^{(1/4) + \phi} = \\ \sqrt[4]{\frac{1}{2} (42903552 e + 2.01857 \times 10^6 \times 4.6^2 + 2.01857 \times 10^6 \times 13.2^2)} + \\ \boxed{\text{root of } -1 - x + x^2 \text{ near } x = 1.61803}$$

## Series representations

$$\left( \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e(16 \times 2)) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e(16 \times 2)) \right) \right)^\wedge$$

$$(1/4) + \phi = \phi + 68.0559 \sqrt[4]{9.19336 + \sum_{k=0}^{\infty} \frac{1}{k!}}$$


---

$$\left( \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e(16 \times 2)) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e(16 \times 2)) \right) \right)^\wedge$$

$$(1/4) + \phi = \phi + 57.228 \sqrt[4]{18.3867 + \sum_{k=0}^{\infty} \frac{1+k}{k!}}$$


---

$$\left( \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e(16 \times 2)) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e(16 \times 2)) \right) \right)^\wedge$$

$$(1/4) + \phi = \phi + 68.0559 \sqrt[4]{9.19336 + \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}}$$

Dividing the two previous expressions:

$$\frac{\frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e(16 \times 2)) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e(16 \times 2)) \right)}{(63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e(16 \times 2))}$$

And

$$(2.06073 \times 10^{-33} - 2.8118 \times 10^{-33}) - 2.8118 \times 10^{-33} \times 4 - 1.4059 \times 10^{-33} \times 4^2 - 4.68633 \times 10^{-34} \times 4^3 - 1.17158 \times 10^{-34} \times 4^4 + 4^5$$

$$= 1023.9999\dots \approx 1024$$

we obtain:

$$\left( \left( \frac{1}{2} \left( (63080.2 \cdot 16 \cdot 4.6^2 \cdot 2 + 670368 \cdot e \cdot 32) + (63080.2 \cdot 16 \cdot 13.2^2 \cdot 2 + 670368 \cdot e \cdot 32) \right) \right) / 1023.999999999 \right)^{1/5}$$

### Input interpretation

$$\left( \frac{1}{1023.999999999} \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e \times 32) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e \times 32) \right) \right)^{(1/5)}$$

### Result

12.0068...

12.0068....

From which:

$$\left( \left( \left( \frac{1}{2} \left( (63080.2 \cdot 16 \cdot 4.6^2 \cdot 2 + 670368 \cdot e \cdot 32) + (63080.2 \cdot 16 \cdot 13.2^2 \cdot 2 + 670368 \cdot e \cdot 32) \right) \right) / 1023.999999999 \right)^{1/5} \right)^{3-3}$$

### Input interpretation

$$\left( \frac{1}{1023.999999999} \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e \times 32) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e \times 32) \right) \right)^{(1/5)^3 - 3}$$

### Result

1727.94...

1727.94....  $\approx$  1728

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. ( $1728 = 8^2 \cdot 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

## Alternative representation

$$\begin{aligned} & \left( \left( (63\,080.2 \times 16 \times 4.6^2 \times 2 + 670\,368 e^{32}) + (63\,080.2 \times 16 \times 13.2^2 \times 2 + \right. \right. \\ & \quad \left. \left. 670\,368 e^{32}) \right) / (1023.9999999990000 \times 2) \right)^{(1/5)^3} - \\ & 3 = \left( \left( (63\,080.2 \times 16 \times 4.6^2 \times 2 + 670\,368 \exp(z) 32) + \right. \right. \\ & \quad \left. \left. (63\,080.2 \times 16 \times 13.2^2 \times 2 + 670\,368 \exp(z) 32) \right) / \right. \\ & \quad \left. (1023.9999999990000 \times 2) \right)^{(1/5)^3} - 3 \quad \text{for } z = 1 \end{aligned}$$

## Series representations

$$\begin{aligned} & \left( \left( (63\,080.2 \times 16 \times 4.6^2 \times 2 + 670\,368 e^{32}) + \right. \right. \\ & \quad \left. \left. (63\,080.2 \times 16 \times 13.2^2 \times 2 + 670\,368 e^{32}) \right) / \right. \\ & \quad \left. (1023.9999999990000 \times 2) \right)^{(1/5)^3} - 3 = \\ & -3 + 0.0103086555529192766 \left( 3.94428 \times 10^8 + 4.29036 \times 10^7 \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{3/5} \end{aligned}$$


---

$$\begin{aligned} & \left( \left( (63\,080.2 \times 16 \times 4.6^2 \times 2 + 670\,368 e^{32}) + \right. \right. \\ & \quad \left. \left. (63\,080.2 \times 16 \times 13.2^2 \times 2 + 670\,368 e^{32}) \right) / \right. \\ & \quad \left. (1023.9999999990000 \times 2) \right)^{(1/5)^3} - 3 = \\ & -3 + 0.0103086555529192766 \left( 3.94428 \times 10^8 + 2.14518 \times 10^7 \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{3/5} \end{aligned}$$


---

$$\begin{aligned} & \left( \left( (63\,080.2 \times 16 \times 4.6^2 \times 2 + 670\,368 e^{32}) + \right. \right. \\ & \quad \left. \left. (63\,080.2 \times 16 \times 13.2^2 \times 2 + 670\,368 e^{32}) \right) / \right. \\ & \quad \left. (1023.9999999990000 \times 2) \right)^{(1/5)^3} - 3 = \\ & -3 + 0.0103086555529192766 \left( 3.94428 \times 10^8 + 4.29036 \times 10^7 \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{3/5} \end{aligned}$$



$$(1/27((((([1/2((63080.2*16*4.6^2*2+670368*e *32)+(63080.2 *16* 13.2^2 *2+670368 *e *32)) / 1023.999999999))^1/5))^3-3))^2+\Phi/2$$

**Input interpretation**

$$\left(\frac{1}{27} \left( \left( \left( \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e \times 32) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e \times 32) \right) \right) / 1023.999999999 \right)^{\frac{1}{5}} \right)^3 - 3 \right)^2 + \frac{\Phi}{2}$$

Φ is the golden ratio conjugate

**Result**

4096.01...

4096.01... ≈ 4096 = 64<sup>2</sup>

$$((((([1/2((63080.2*16*4.6^2*2+670368*e *32)+(63080.2 *16* 13.2^2 *2+670368 *e *32)) / 1023.999999999))^1/5))^3-3)+1)^1/15$$

**Input interpretation**

$$\left( \left( \left( \left( \frac{1}{2} \left( (63080.2 \times 16 \times 4.6^2 \times 2 + 670368 e \times 32) + (63080.2 \times 16 \times 13.2^2 \times 2 + 670368 e \times 32) \right) \right) / 1023.999999999 \right)^{\frac{1}{5}} \right)^3 - 3 \right) + 1 \right)^{\frac{1}{15}}$$

**Result**

1.643811...

1.643811.... ≈ ζ(2) = π<sup>2</sup>/6 = 1.644934 ... (trace of the instanton shape)

Now, we have the following expression:

$$\begin{aligned}
\|D_{\tau_t} X_{\tau_t}\|_{L^2(\partial E_t)}^2 &\leq C \|D_{\tau_t}(X \cdot \nu^{E_t})\|_{L^2(\partial E_t)}^2 + C \int_{\partial E_t} |X \cdot \nu^{E_t}|^2 [ |D_{\tau_t} \nu_\sigma| + |D_{\tau_t} \nu^{E_t}| ]^2 d\mathcal{H}^{N-1} \\
&\leq C \|X \cdot \nu^{E_t}\|_{H^1(\partial E_t)}^2 + C \|X \cdot \nu^{E_t}\|_{L^{\frac{2p}{p-2}}(\partial E_t)}^2 \| |D_{\tau_t} \nu_\sigma| + |D_{\tau_t} \nu^{E_t}| \|_{L^p(\partial E_t)}^2 \\
&\leq C \|X \cdot \nu^{E_t}\|_{H^1(\partial E_t)}^2,
\end{aligned}$$

From:

$$C \|X \cdot \nu^{E_t}\|_{H^1(\partial E_t)}^2,$$

For :  $C = 2$  ;  $v_E = X = 64$  , we obtain:

$$2(64 \times 64)^2$$

**Input**

$$2(64 \times 64)^2$$

**Result**

$$33554432$$

**Scientific notation**

$$3.3554432 \times 10^7$$

$$3.3554432 * 10^7$$

Thence, from:

$$\begin{aligned}
\|D_{\tau_t} X_{\tau_t}\|_{L^2(\partial E_t)}^2 &\leq C \|D_{\tau_t}(X \cdot \nu^{E_t})\|_{L^2(\partial E_t)}^2 + C \int_{\partial E_t} |X \cdot \nu^{E_t}|^2 [ |D_{\tau_t} \nu_\sigma| + |D_{\tau_t} \nu^{E_t}| ]^2 d\mathcal{H}^{N-1} \\
&= 3.3554432 \times 10^7
\end{aligned}$$

From

$$2(64 \times 64)^2$$

we obtain also:

$$27 \times 2 \left( (2(64 \times 64)^2) \right)^{1/5} + 1$$

**Input**

$$27 \times 2 \sqrt[5]{2(64 \times 64)^2 + 1}$$

**Exact result**

1729

1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8<sup>2</sup> \* 3<sup>3</sup>) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$\left( (27 \times 2 \left( (2(64 \times 64)^2) \right)^{1/5} + 1) \right)^{1/15}$$

**Input**

$$\sqrt[15]{27 \times 2 \sqrt[5]{2(64 \times 64)^2 + 1}}$$

**Result**

$$\sqrt[15]{1729}$$

**Decimal approximation**

1.6438152287487281305800880313247695143292831436999401726452126788

...

1.6438152287....  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$  (trace of the instanton shape)

$$((2((2(64*64)^2))^{1/5}))^2$$

**Input**

$$\left(2\sqrt[5]{2(64 \times 64)^2}\right)^2$$

**Exact result**

4096

$$4096 = 64^2$$

## Observations

We note that, from the number 8, we obtain as follows:

$$8^2$$

$$64$$

$$8^2 \times 2 \times 8$$

$$1024$$

$$8^4 = 8^2 \times 2^6$$

True

$$8^4 = 4096$$

$$8^2 \times 2^6 = 4096$$

$$2^{13} = 2 \times 8^4$$

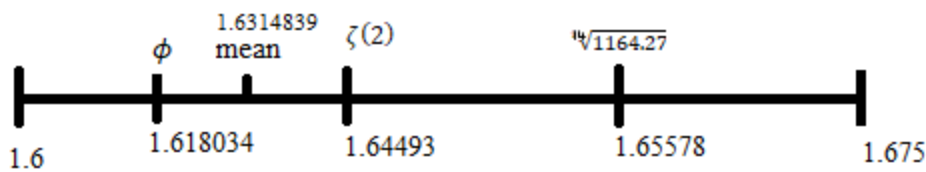
True

$$2^{13} = 8192$$

$$2 \times 8^4 = 8192$$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact  $8^2 = 64$ ,  $8^3 = 512$ ,  $8^4 = 4096$ . We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6, a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

## “Golden” Range



Finally we note how  $8^2 = 64$ , multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to  $\zeta(2)$  that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

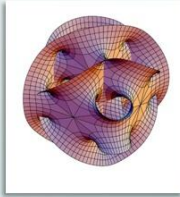
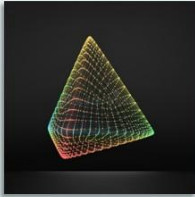
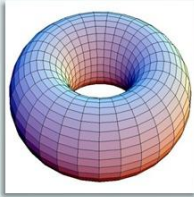
Furthermore for all the results very near to 1728 or 1729, adding  $64 = 8^2$ , one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

## Appendix

**Outlook**

Remarkably rich (apparently **UNIQUE**) framework

**BUT :**



Why a given **“shape” of the extra dimensions** ?  
[**CRUCIAL**, it determines the predictions for  $\alpha$ , ...]

A. Sagnotti – AstronomiAmo, 23.4.2020 21

From: A. Sagnotti – AstronomiAmo, 23.04.2020

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for  $\alpha$ ”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and  $\zeta(2)$ , (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally  $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

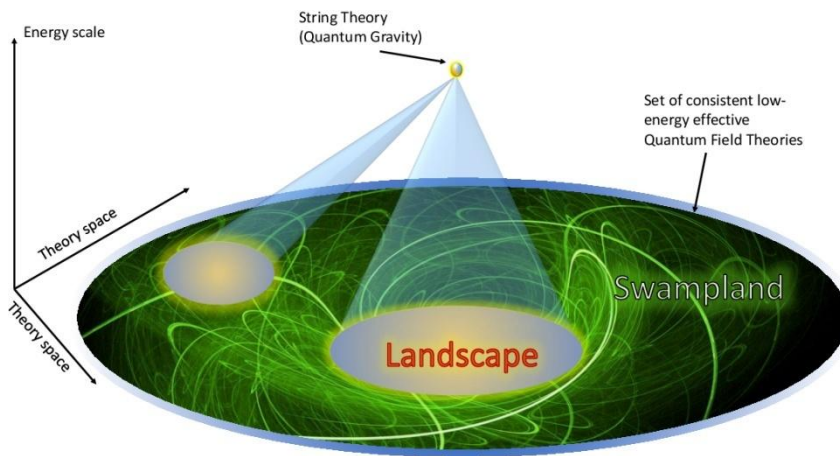
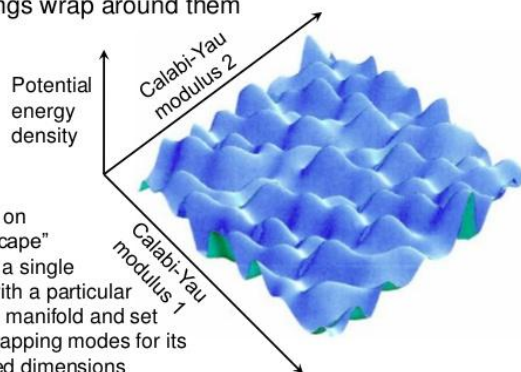


Fig. 1

### The String Theory “Landscape”

- Graph axes show only 2 out of hundreds of parameters (“moduli”) that determine the exact Calabi-Yau manifolds and how strings wrap around them



- Each point on the “Landscape” represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation “bubble”

Fig. 2



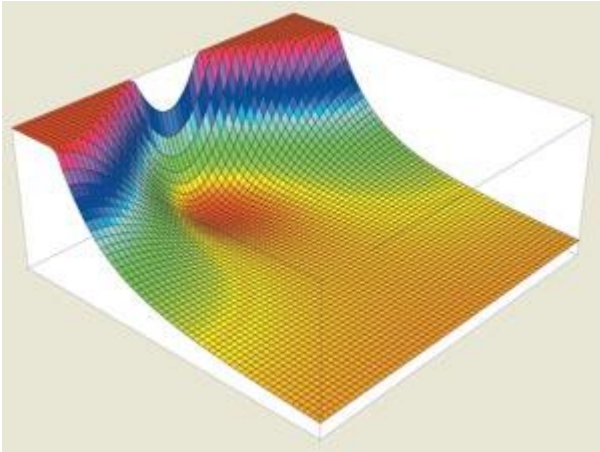
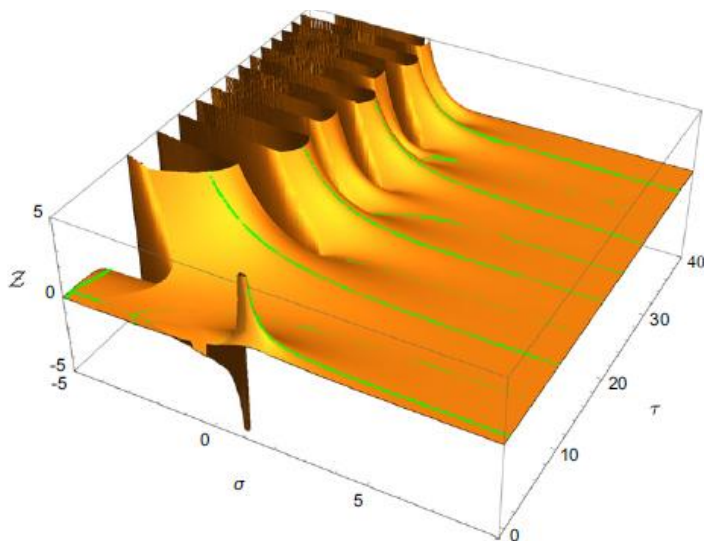


Fig. 3

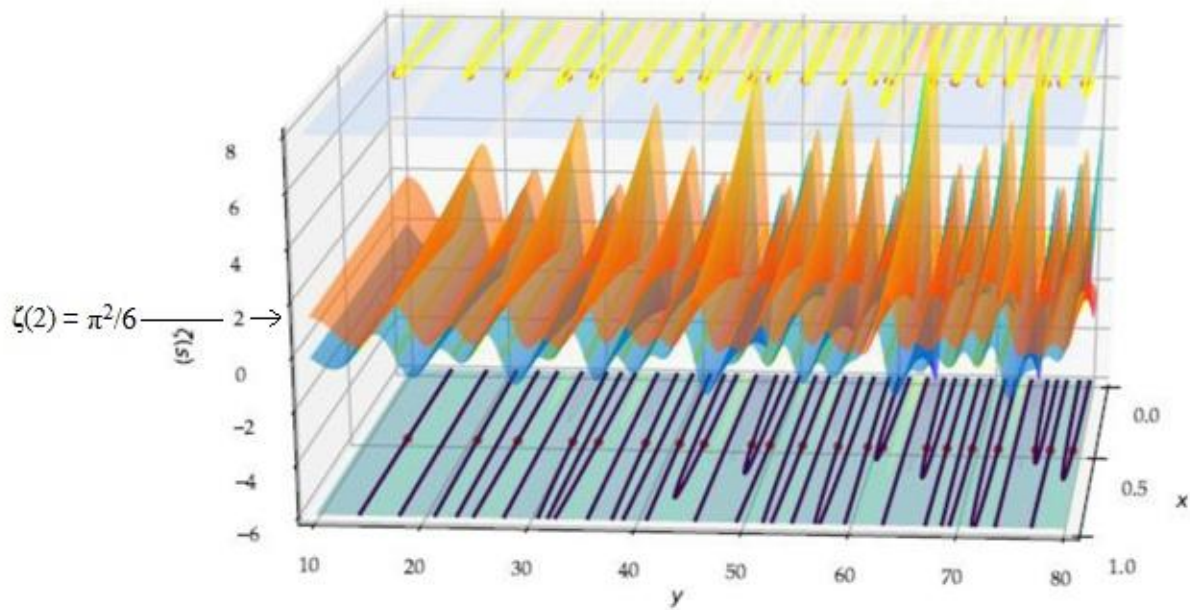
Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.



**Figure 2.** Lines in the complex plane where the Riemann zeta function  $\zeta$  is real (green) depicted on a relief representing the positive absolute value of  $\zeta$  for arguments  $s \equiv \sigma + i\tau$  where the real part of  $\zeta$  is positive, and the negative absolute value of  $\zeta$  where the real part of  $\zeta$  is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of  $2\pi$  run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: <https://www.mdpi.com/2227-7390/6/12/285/htm>



**Figure 1.**  $C(x, y)$  and  $S(x, y)$  surfaces of the Riemann  $\zeta(x, y) = C - iS$  function, in the critical strip  $\mathcal{S}$ :  $0 \leq x \leq 1$ ;  $10 \leq y \leq 80$ . On the top and bottom planes, the  $C$  and  $S$  common zeros are the red points.

Fig. 5

3D plot  $\zeta(2 + it)$

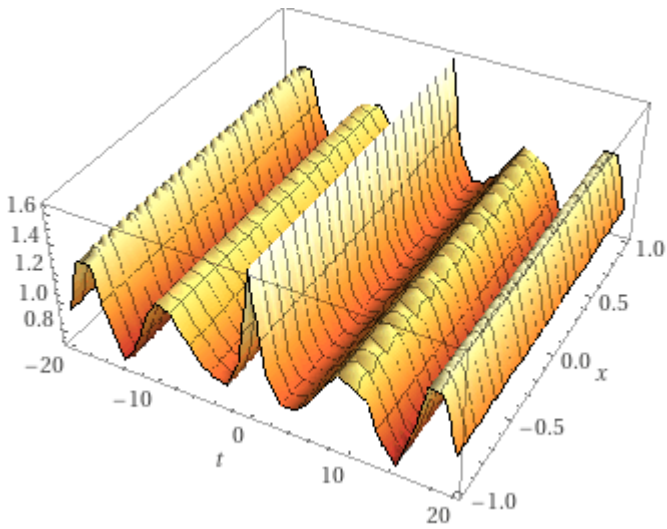


Fig. 6

Where  $\zeta(2+it)$  :

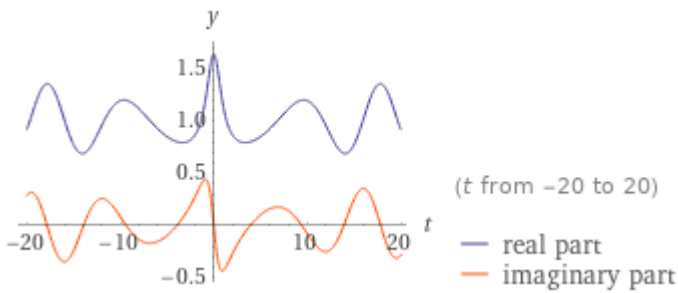
**Input**

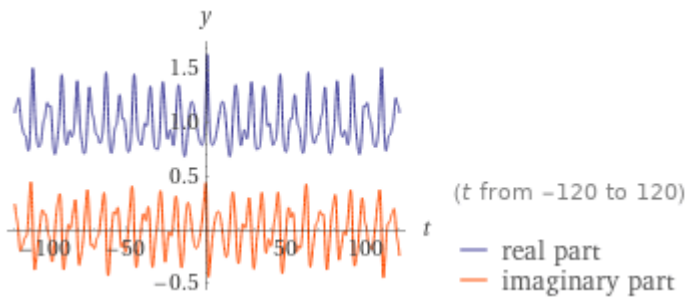
$\zeta(2 + it)$

$\zeta(s)$  is the Riemann zeta function

$i$  is the imaginary unit

**Plots**





## Roots

$$t = 2i(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$$


---

$$t = -i(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$$

$\mathbb{Z}$  is the set of integers

$\rho_n$  is the nontrivial  $n^{\text{th}}$  zero of the Riemann zeta function

## Series expansion at $t=0$

$$\frac{\pi^2}{6} + it \zeta'(2) - \frac{1}{2} t^2 \zeta''(2) - \frac{1}{6} i \zeta^{(3)}(2) t^3 + \frac{1}{24} \zeta^{(4)}(2) t^4 + O(t^5)$$

(Taylor series)

## Alternative representations

$$\zeta(2 + it) = \zeta(2 + it, 1)$$


---

$$\zeta(2 + it) = S_{1+it,1}(1)$$


---

$$\zeta(2 + it) = \frac{\zeta\left(2 + it, \frac{1}{2}\right)}{-1 + 2^{2+it}}$$

$\zeta(s, a)$  is the generalized Riemann zeta function

$S_{n,p}(x)$  is the Nielsen generalized polylogarithm function

## Series representations

$$\zeta(2 + it) = \sum_{k=1}^{\infty} k^{-2-it} \text{ for } \text{Im}(t) < 1$$

---

$$\zeta(2 + it) = \frac{\sum_{k=0}^{\infty} (1 + 2k)^{-2-it}}{1 - 2^{-2-it}} \text{ for } \text{Im}(t) < 1$$

---

$$\zeta(2 + it) = e^{\sum_{k=1}^{\infty} P(k(2+it))/k} \text{ for } \text{Im}(t) < 1$$

$\text{Im}(z)$  is the imaginary part of  $z$

$P(z)$  gives the prime zeta function

## Integral representations

$$\zeta(2 + it) = \frac{1}{\Gamma(2 + it)} \int_0^{\infty} \frac{\tau^{1+it}}{-1 + e^{\tau}} d\tau \text{ for } \text{Im}(t) < 1$$

---

$$\zeta(2 + it) = \frac{2^{1+it}}{\Gamma(3 + it)} \int_0^{\infty} \tau^{2+it} \text{csch}^2(\tau) d\tau \text{ for } \text{Im}(t) < 1$$

---

$$\zeta(2 + it) = \frac{2^{1+it}}{\Gamma(2 + it)} \int_0^{\infty} e^{-\tau} \tau^{1+it} \text{csch}(\tau) d\tau \text{ for } \text{Im}(t) < 1$$

$\Gamma(x)$  is the gamma function

$\text{csch}(x)$  is the hyperbolic cosecant function

## Functional equations

$$\zeta(2 + it) = -i 2^{2+it} \pi^{1+it} \Gamma(-1 - it) \sinh\left(\frac{\pi t}{2}\right) \zeta(-1 - it)$$


---

$$\zeta(2 + it) = \frac{\pi^{3/2+it} \Gamma\left(-\frac{1}{2} - \frac{it}{2}\right) \zeta(-1 - it)}{\Gamma\left(1 + \frac{it}{2}\right)}$$


---

$$\zeta(2 + it) = - \frac{i \sum_{k=0}^{\infty} \frac{\Gamma\left(k - \frac{it}{2}\right) \sum_{j=0}^k (-1)^j (1+2j) \binom{k}{j} \zeta(2+2j)}{k!}}{(-i + t) \Gamma\left(-\frac{it}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with  $k = \dots, -2, -1, 0, 1, 2, \dots$

we obtain:

$$2\pi/(\ln(2))$$

**Input:**

$$2 \times \frac{\pi}{\log(2)}$$

**Exact result:**

$$\frac{2\pi}{\log(2)}$$

**Decimal approximation:**

9.0647202836543876192553658914333336203437229354475911683720330958

...

9.06472028365....

**Alternative representations:**

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$


---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a) \log_a(2)}$$


---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 \coth^{-1}(3)}$$

**Series representations:**

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$


---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$


---

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

### Integral representations:

$$\frac{2\pi}{\log(2)} = \int_1^2 \frac{1}{t} dt$$

---

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

From which:

$$(2\pi / (\ln(2))) * (1/12 \pi \log(2))$$

**Input:**

$$\left(2 \times \frac{\pi}{\log(2)}\right) \left(\frac{1}{12} \pi \log(2)\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

1.6449340668482264364724151666460251892189499012067984377355582293

...

$$1.6449340668\dots = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$



From:

**Modular equations and approximations to  $\pi$  - Srinivasa Ramanujan**  
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64 \left\{ \left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

We note that, with regard 4372, we can to obtain the following results:

$$27((4372)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi$$

### Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

$\phi$  is the golden ratio

### Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2(\sqrt{5}-1)}\right)$$

### Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944....

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ( $1728 = 8^2 * 3^3$ ) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

### Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5(10-2\sqrt{5})}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2(5-\sqrt{5})}-374\right)$$

---


$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2(5+\sqrt{5})}\right)$$


---

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2(\sqrt{5} - 1)}$$

### Minimal polynomial

$$\begin{aligned} &256x^8 + 95744x^7 - 3248750080x^6 - \\ &914210725504x^5 + 15498355554921184x^4 + \\ &2911478392539914656x^3 - 32941144911224677091680x^2 - \\ &3092528914069760354714456x + 26320050609744039027169013041 \end{aligned}$$

### Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}$$


---

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2(\sqrt{5} - 1)}$$

### Series representations

$$\begin{aligned} &27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2}\right) + \phi = \\ &\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + \right. \\ &108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - \\ &\left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k} \right) / \left( 2 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \right) \end{aligned}$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

Or:

$$27((4096+276)^{1/2}-2-1/2(((\sqrt{(10-2\sqrt{5})}-2))/(\sqrt{5}-1))))+\phi$$

## Input

$$27 \left( \sqrt{4096 + 276} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi$$

$\phi$  is the golden ratio

## Result

$$\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)$$

## Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

...

1729.0526944.... as above

## Alternate forms

$$\frac{1}{8} \left( -27\sqrt{5(10 - 2\sqrt{5})} + 58\sqrt{5} + 432\sqrt{1093} - 27\sqrt{2(5 - \sqrt{5})} - 374 \right)$$

---

$$\phi - 54 + 54\sqrt{1093} + \frac{27}{4} \left( 1 + \sqrt{5} - \sqrt{2(5 + \sqrt{5})} \right)$$

---

$$\phi - 54 + 54\sqrt{1093} - \frac{27 \left( \sqrt{10 - 2\sqrt{5}} - 2 \right)}{2(\sqrt{5} - 1)}$$

## Minimal polynomial

$$\begin{aligned}
 &256x^8 + 95744x^7 - 324875080x^6 - \\
 &914210725504x^5 + 15498355554921184x^4 + \\
 &2911478392539914656x^3 - 32941144911224677091680x^2 - \\
 &3092528914069760354714456x + 26320050609744039027169013041
 \end{aligned}$$

## Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5(10-2\sqrt{5})}$$


---

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)}$$

## Series representations

$$\begin{aligned}
 &27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{(\sqrt{5}-1)2}\right)+\phi= \\
 &\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\right. \\
 &\quad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}- \\
 &\quad \left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)
 \end{aligned}$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)
\end{aligned}$$


---

$$\begin{aligned}
& 27 \left( \sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi = \\
& \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& \quad 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \\
& \quad 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \\
& \quad \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left( 2 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

From which:

$$(27((4372)^{1/2}-2-1/2((\sqrt{(10-2\sqrt{5})-2})/(\sqrt{5}-1))))+\phi)^{1/15}$$

### Input

$$\sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{1}{2} \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + \phi}$$

$\phi$  is the golden ratio

### Exact result

$$\sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

### Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124

...

$$1.64381856858\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

### Alternate forms

$$\sqrt[15]{\phi - 54 + 54\sqrt{1093} - \frac{27(\sqrt{10 - 2\sqrt{5}} - 2)}{2(\sqrt{5} - 1)}}$$

---


$$\sqrt[15]{\frac{1}{166 - 108\sqrt{5} - 108\sqrt{1093} + 108\sqrt{5465} - 27\sqrt{2(5 - \sqrt{5})}}}$$


---



$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 154983555492184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

### Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 154983555492184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

### Expanded forms

$$\sqrt[15]{\frac{1}{2}(1 + \sqrt{5}) + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5(10 - 2\sqrt{5})}}$$

All 15th roots of  $\phi + 27(-2 + 2\sqrt{1093} - (\sqrt{10 - 2\sqrt{5}} - 2)/(2(\sqrt{5} - 1)))$

$$e^0 \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.50170 + 0.6686i$$

$$e^{(4i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 1.0999 + 1.2216i$$


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$$e^{(2i\pi)/5} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx 0.5080 + 1.5634i$$


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$$e^{(8i\pi)/15} \sqrt[15]{\phi + 27 \left( -2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2(\sqrt{5} - 1)} \right)} \approx -0.17183 + 1.63481i$$


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### Series representations

$$\begin{aligned} & \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\ & \frac{1}{\sqrt[15]{2}} \left( \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \right. \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \right. \right. \\ & \quad \left. \left. \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (9 - 2\sqrt{5})^{-k} \right) / \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \right)^{\wedge (1/15)} \end{aligned}$$


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$$\begin{aligned}
& \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (9 - 2\sqrt{5})^{-k}}{k!} \right) / \right. \\
& \quad \left. \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right)^{\wedge (1/15)}
\end{aligned}$$


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$$\begin{aligned}
& \sqrt[15]{27 \left( \sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{(\sqrt{5} - 1)2} \right) + \phi} = \\
& \frac{1}{\sqrt[15]{2}} \left( \left( 162 - 108\sqrt{1093} - 2\phi - 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 108\sqrt{1093} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 2\phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \right. \right. \\
& \quad \left. \left. 27\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} \right) / \right. \\
& \quad \left. \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)^{\wedge (1/15)}
\end{aligned}$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

## Integral representation

$$(1 + z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From:

## An Update on Brane Supersymmetry Breaking

*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left( p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left( 7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$  instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning  $p$ ,  $C$ ,  $\beta_E$  and  $\phi$  correspond to the exponents of  $e$  (i.e. of exp). Thence we obtain for  $p = 5$  and  $\beta_E = 1/2$ :

$$e^{-6C + \phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to  $64^2$ , while  $-6C + \phi$  is equal to  $-\pi\sqrt{18}$ . From this it follows that it is possible to establish mathematically, the dilaton value.

For

$\exp((-Pi*\text{sqrt}(18))$  we obtain:

**Input:**

$$\exp\left(-\pi \sqrt{18}\right)$$

**Exact result:**

$$e^{-3\sqrt{2}\pi}$$

**Decimal approximation:**

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

**Property:**

$e^{-3\sqrt{2}\pi}$  is a transcendental number

**Series representations:**

$$e^{-\pi \sqrt{18}} = e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

And:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

**Input interpretation:**

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

**Result:**

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp((-Pi*\sqrt{18})))))) * 1/0.000244140625$$

**Input interpretation:**

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

**Result:**

0.00666501785...

0.00666501785...

**Series representations:**

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

From:

$$\ln(0.00666501784619)$$

**Input interpretation:**

$$\log(0.00666501784619)$$

**Result:**

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$



### Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

### Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2i\pi \left[ \frac{\arg(0.006665017846190000 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

### Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for  $C = 1$ , we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \phi$$

Note that the values of  $n_s$  (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \phi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(<http://www.bitman.name/math/article/102/109/>)

Also performing the 512<sup>th</sup> root of the inverse value of the Pion meson rest mass 139.57, we obtain:

$$((1/(139.57)))^{1/512}$$

**Input interpretation:**

$$\sqrt[512]{\frac{1}{139.57}}$$

**Result:**

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0.989117352243 =  $\phi$**  and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}} - \phi + 1$$

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