IL NUOVO CIMENTO **45 C** (2022) 25
DOI 10.1393/ncc/i2022-22025-8

Colloquia: QGRAV 2021

Quantum gravity, higher derivatives and nonlocality

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received 4 February 2022

Summary. — The aim of the workshop *Quantum Gravity, Higher Derivatives & Nonlocality (QGRAV-2021)* was to assess our current understanding of the interplay between gravity and quantum mechanics, with particular attention to what is the main role of higher derivatives and nonlocality in the formulation of a consistent theory of quantum gravity, and whether they are really necessary to describe Nature at a fundamental level. The workshop had brought together various schools of thought, and besides talks numerous discussions were also held. In addition to focusing on formal aspects of several approaches, applications of higher derivatives and nonlocality were also discussed in the context of cosmology and black-hole physics. In this preface article we present a concise explanation of the main problems underlying the quantization of gravity in the high-energy regime, briefly list several approaches that have been proposed so far, and also provide a gist of what was intensively discussed during the workshop.

1. – Introduction

A vast amount of observational data, including the recent detection of gravitational waves from black-hole binary mergers, has made Einstein's General Relativity (GR) the best current theory to describe classical aspects of the gravitational interaction. Its Lagrangian formulation is based on the Einstein-Hilbert action that is linear in the Ricci scalar,

$$S_{\rm GR}=\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R, \tag{1}$$

where $\kappa^2=8\pi G=1/M_p^2$, G being the Newton constant, M_p the reduced Planck mass, and we set the cosmological constant equal to zero for simplicity (we work in Natural Units $c=1=\hbar$, and adopt the mostly positive convention for the metric signature).

Despite its great success there are still fundamental questions that remain unanswered. GR predicts the existence of black-hole and cosmological big-bang singularities where a smooth description of spacetime fails and the theory eventually breaks down. Moreover, Einstein's theory lacks predictivity in the high-energy regime since it is perturbatively non-renormalizable. It is generally believed that a consistent theory of *quantum gravity* is necessary to deal with these issues and improve Einstein's GR in the ultraviolet (UV) regime, *i.e.*, in the short-distance and high-energy regimes.

In what follows we are going to illustrate the difficulties arising when trying to describe the gravitational interaction with the same tools that are normally used to analyse other forces of Nature, *i.e.*, electromagnetic, weak and strong interactions. After having understood the main problem, we will reflect on what is that needs to be quantized in a theory of quantum gravity, we will briefly list different point of views and several promising approaches that have been proposed in the last decades.

2. – Perturbative non-renormalizability of Einstein's general relativity

Quantum Field Theory (QFT) is the corner stone of our successful understanding of high-energy particle physics. The Standard Model of particle physics is formulated in the framework of perturbative QFT, in which the fundamental particles are described as quantized field fluctuations in Minkowski vacuum (*e.g.*, quantization of electromagnetic fields leads to photons). In perturbative QFT the interaction couplings must be sufficiently small for the perturbation theory to be valid, and in the Standard Model they turn out to be dimensionless which means that electromagnetic, weak and strong interactions can be described by means of *strictly renormalizable* QFTs. On a similar note, the first thing that can come to our mind is to check whether the gravitational field can also be quantized in the standard perturbative QFT framework. Since, as a starting point, we wish to use tools of QFT in Minkowski spacetime, we need to consider regimes in which the gravitational interaction is sufficiently weak so that we are allowed to introduce a spin-2 field fluctuation $h_{\mu\nu}$ defined as

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa\, h_{\mu\nu}, \tag{2}$$

where $\eta_{\mu\nu}$ is metric tensor of the Minkowski spacetime. From eq. (3) it should be clear that the gravitational coupling is $\kappa = 1/M_p$ and it has negative mass dimension. We can expand the Einstein-Hilbert action (1) in powers of $h_{\mu\nu}$, or in other words in powers of the coupling κ, and obtain

$$S_{\rm GR} = \int \mathrm{d}^4x \left\{ \frac{1}{2} h_{\mu\nu}\mathcal{K}^{\mu\nu\rho\sigma}h_{\rho\sigma} + \mathcal{O}(\kappa h^3) \right\}. \tag{3}$$

The first piece corresponds to the kinetic term, $\mathcal{K}^{\mu\nu\rho\sigma}$ being a two-derivative operator in Minkowski spacetime whose explicit form is not important for our discussion. The remaining pieces are the interaction terms of any order h^n with $n \geq 3$. Thus, the expanded action (3) can be quantized with standard QFT tools, and the corresponding Feynman rules can be derived.

Since the gravitational coupling κ is a dimensionful quantity, it follows that the quantized version of the action (3) leads to a *non-renormalizable* perturbative QFT in which an infinite number of counterterms is required in order to renormalize the theory. The quantum theory can still be renormalized but an infinite number of independent parameters must be introduced. This means that quantum GR *cannot be predictive* in the high-energy regime since no conceivable experiment would be able to measure all those infinite parameters.

By performing a detailed analysis of the perturbative loop expansion, one can actually show that in the absence of matter quantum fields, GR is one-loop finite. More precisely, the one-loop divergent contribution to the effective action can be computed, *e.g.*, in dimensional regularization with $\epsilon = 4 - D$, and it is given by [1]

$$\Gamma^{(1)}_{\rm div} = -\frac{\mu^{-\varepsilon}}{(4\pi)^2\varepsilon}\int {\rm d}^D x\sqrt{-g}\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu} + \frac{53}{90}{\rm GB}\right), \tag{4}$$

where μ is the usual renormalization scale, and ${\rm GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + R^2 - 4R_{\mu\nu}R^{\mu\nu}$ is the Gauss-Bonnet invariant which becomes a total derivative in four dimensions (*i.e.*, in the limit $D \to 4$, or $\varepsilon \to 0$). Interestingly, since in four dimensions the contribution (4) is proportional to the vacuum field equations we can make the field redefinition

$$g'_{\mu\nu} = g_{\mu\nu} - \frac{\kappa^2\mu^{-\varepsilon}}{10(4\pi)^2\varepsilon}\left(\frac{11}{3}Rg_{\mu\nu} - 7R_{\mu\nu}\right) \tag{5}$$

to cancel the one-loop divergence perturbatively [1], so that we obtain

$$S[g'] = S_{\rm GR}[g'] + \Gamma^{(1)}_{\rm div}[g'] = S_{\rm GR}[g] + \mathcal{O}(\kappa^2\mathcal{R}^3), \tag{6}$$

where $\mathcal{R}^3$ stands for cubic curvature terms. It is worth mentioning that if we had started with a non-zero cosmological constant, then the terms in $\Gamma^{(1)}_{\rm div}$ would not have been proportional to the field equations. However, also in this case the quantum theory turns out to be one-loop renormalizable because the one-loop divergence would be only responsible for the renormalization of the cosmological constant [2].

One might be brought to think or hope that similar cancellations can happen even beyond one-loop and also when couplings to matter are turned on; unfortunately that's not the case. First, one can show that GR coupled to matter is one-loop divergent [1]. Furthermore, the two-loop divergence of pure gravity cannot be eliminated by a field redefinition because it is not proportional to the field equations, and it reads [3, 4]

$$\Gamma^{(2)}_{\rm div} = \frac{\kappa^2\mu^{-\varepsilon}}{(4\pi)^4\varepsilon}\frac{209}{2880}\int {\rm d}^D x\sqrt{-g}\, R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu}. \tag{7}$$

Hence, a perturbative QFT framework of GR turns out to be pathological especially at energies larger than Planck's, *i.e.*, in the regime $E \gtrsim M_p$ where E is the energy scale characterizing some gravitational process.

Remark. It is sometimes stated that because of the failure of perturbative renormalizability, GR and quantum mechanics are not compatible. This statement is highly misleading and it undermines the fact that a quantum description of the gravitational interaction can be applied at energies smaller than some cut-off scale, *e.g.*, the Planck mass, and a consistent QFT description of the gravitational interaction can be achieved in that energy regime. Indeed, one can formulate an Effective Field Theory (EFT) of the gravitational interaction valid for energies $E \ll M_p$, and make very accurate quantum-gravity predictions [5]. In this regard, we should distinguish the two notions of low-energy quantum gravity and of a theory of quantum gravity that can describe the gravitational interaction at Planck scales and beyond.

However, we should also point out that observationally it still remains to be seen conclusively if our methods of quantizing gravity at low energies are respected by Nature because *no* quantum-gravity signature has been seen so far. See refs. [6, 7] for interesting experimental proposals to test low-energy quantum gravity in a laboratory.

2·1. *Effective field theory of gravity.* – The EFT approach to gravity is as powerful as any other application of EFT in particle physics, and can be used to make very accurate predictions up to errors of the order of $\mathcal{O}(E/M_p \ll 1)$. One takes into account the effects of higher-order local curvature operators that contribute to the divergent part of the effective action, but also nonlocal finite contributions [8-11], thus the resultant gravitational action turns out to be of the following form:

$$S_{\rm EFT} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,\Big\{R + a_1(\mu)R^2 + a_2(\mu)R_{\mu\nu}R^{\mu\nu} + \cdots$$

$$+b_1 R\log\left(\frac{-\Box}{\mu^2}\right)R + b_2 R_{\mu\nu}\log\left(\frac{-\Box}{\mu^2}\right)R^{\mu\nu} + b_3 R_{\mu\nu\rho\sigma}\log\left(\frac{-\Box}{\mu^2}\right)R^{\mu\nu\rho\sigma} + \cdots\Big\}, \tag{8}$$

where the ellipses stand for higher-order contributions. The exact values of the coefficients a_i depend on the particular ultraviolet (UV) completion, while those of b_i associated to the non-local operators can be computed precisely and are model independent (they depend on the number of fields of different spins coupled to gravity, including the graviton itself). Other type of nonlocal terms involving the operator $1/\Box$ are also expected to contribute to the finite part of the quantum effective action and, indeed, they have been computed in some cases [9-12].

One of the most interesting prediction of the EFT approach is the leading quantum correction to the static Newtonian potential between two massive scalar fields of masses m_1 and m_2 at one-loop order([1]) [14],

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3} + \cdots\right], \tag{9}$$

where we have restored c and $\hbar$ to identify explicitly the nature of each corrections. While the first term is just the standard post-Newtonian correction at order $\mathcal{O}(G^2)$([2]), the second one represent the true leading quantum correction to Newton's law. The ellipsis in eq. (9) stand for higher-order corrections coming from higher-loop contributions. For

([1]) Earlier attempts [5, 13] to derive the quantum leading correction gave a wrong result for the exact coefficients.

([2]) Although the post-Newtonian correction is purely classical, it comes out as the result of a one-loop computation. It is sometimes stated that loop corrections are expansions in $\hbar$; this is false and indeed we have just seen a counterexample. In fact, in the presence of at least two massless propagators classical contributions can arise from loop corrections due to the special infrared behavior of some Feynman diagram, and they are associated to non-analytic terms involving square roots of the momentum squared. See ref. [15] for more details.

instance, from dimensional analysis the two-loop contribution in the square brackets of eq. (9) should be proportional to $G^2\hbar^2/(c^6r^4)$ [18].

Surprisingly, there have been very few studies aimed at analysing theoretical and phenomenological aspects of the nonlocal operators in the curvature expansion in eq. (8), although recently more progress has been made both in the cosmological context [16] and in black-hole physics [17].

2˙2. *Beyond the effective field theory approach.* – It is true that an EFT treatment of gravity can be established, and indeed it can be predictive in its regime of applicability. However, ideally one wishes to describe gravitational phenomena also at energies higher than Planck mass, $E \gtrsim M_p$, where the EFT approach would break down. In other words, the general hope is to find a consistent UV completion of Einstein's GR that can be predictive at (possibly arbitrary) very high energies.

When trying to embark on this challenging path, the first difficulties arise from asking well-posed questions. For example, when we talk about quantum gravity we should understand what is that needs to be quantized, which are the guiding principles to follow, in which framework we are going to describe quantum-gravitational phenomena, what tools must be used to compute physical quantities and make accurate predictions.

In the what follows we will reflect on these fundamental aspects, and discuss several approaches to quantum gravity that are based on the foundations of QFT.

3. – Frameworks for quantum gravity

In the standard QFT approach one quantizes a classical field in some background, and usually the choice of what needs to be quantized uniquely selects the mathematical framework in which the quantum theory will be formulated. Hence, the important questions we should ask before formulating a complete theory of quantum gravity are: *what is it that needs to be quantized? In other words, what are the gravitational (and matter) degrees of freedom that needs to be quantized? And what is the background in which we quantize those degrees of freedom? Or can quantum gravity be background independent?* Several quests of answering these simple but non-trivial questions have landed in proposing new frameworks for quantum gravity that go beyond the standard EFT approach we discussed earlier.

- The gravitational interaction might be quite different from the others because at a full non-perturbative level the metric field $g_{\mu\nu}$ is a dynamical quantity that contains informations about both the classical background and its own (quantum) fluctuations around it, and it might not be possible to disentangle these two. In other words, gravity might just be a geometrical feature of spacetime and not a force. For example, this is more or less the reasoning behind the proposal of Loop Quantum Gravity [19] which is based on a fully non-perturbative canonical quantization of GR where the two physical degrees of freedom are quantized by using quantum mechanical tools involving, for instance, Hamiltonian constraints and the Wheeler-DeWitt equation. Being fully non-perturbative it is believed to not be plagued by an UV catastrophe, and it should be background independent by construction.

  However, there are still many open problems at both conceptual and technical level. For example, it is still not fully clear how to fully recover the Newtonian limit at low-energy, and a satisfactory understanding of how to couple quantum gravity to matter is also not yet fully available. See ref. [20] for a critical point of view on Loop Quantum Gravity.

- In the framework of Loop Quantum Gravity the quantized gravitational entity is the spacetime geometry itself. This might imply that the spacetime is *discrete* rather than continuum. Actually, in the past decades there have been several approaches whose starting assumption is the fundamental discreteness of the spacetime causal structure. Among these approaches the most known are: Causal Set Theory [21], Group Field Theory [22], and Non-Commutative Geometry [23]. A problem that seems to be common to all these approaches is the currently incomplete understanding of the transition from a discrete to a continuum description. Nevertheless, it is curious to note that in all these approaches higher derivatives or non-localities are understood to play a vital role.

- Standard Model of particle physics contains a number of free parameters that are experimentally determined rather than theoretically derived. In the view of treating gravity on equal footing with gauge theories, expecting that all the fundamental forces of Nature must have a common origin towards the Planck scales, assuming that the fundamental theory should be described by a few parameters, String Theory comes with an elegant and radical proposition that the fundamental objects at Planck scales are vibrating strings in higher dimensions. According to String Theory what we perceive in Nature such as GR and the Standard Model of particle physics are its low-energy limits [24-26]. In a way String Theory is a promising attempt to provide a UV-complete description of fundamental particles and interactions in terms of the modes of strings vibrations: for example gravitons are closed strings and the gauge fields are open strings. Thanks to the extended nature of the strings —expected to be of Planck size— the corresponding theory is greatly expected to be finite. Indeed, reasonable arguments point towards this direction. Interesting progress towards a rigorous proof of UV finiteness has been made in the string-field-theory formulation in which the dynamics of relativistic strings is described by means of a "nonlocal" QFT [27-29]. In a nutshell higher derivatives and non-locality are the inevitable integral part of string field theory.

  String Theory has had its ups and downs since its discovery, and went through several revolutions. It is built on the existence of supersymmetry and higher dimensions. The discovery of dualities has unified several branches of string theory but the major problem that one has is the large degeneracy in providing low-energy effective descriptions of Nature. One of the most studied branches of string theory in the last two decades is based on the conjectured AdS/CFT correspondence that relates gravity in the bulk spacetime with gauge theories on the boundary [30].

  One should appreciate that String Theory may be a promising proposal and that multitude of dualities may indeed help analysing aspects of quantum gravity. Moreover, in some cases it turns out that scattering-matrix calculations appear to be simplified when performed in the framework of String Theory, which may thus offer better computational tools than the standard QFT in some situations. However, one should also emphasize that a more satisfactory progress is still necessary, especially in terms of unique predictions that can be possibly test with future experiments. In fact, it needs supersymmetry, at least 10 dimensions, it seems to exclude the existence of a de Sitter vacuum; all features that are in contrast with what we do not observe and with what we do observe experimentally. See ref. [31] for a critical point of view on String Theory.

3˙1. *Insisting on the QFT framework.* – We have briefly commented on several interesting approaches and frameworks for quantum gravity that have been proposed as alternatives to the QFT treatment of the gravitational interaction. It is worth emphasizing that it is not the first time that the validity of QFT is called into question, indeed it went through at least two crises. In the forties many of the founding fathers of quantum mechanics did not believe in the theory of renormalization because they thought it was just a trick to hide the problem of UV divergences. But it turned out that the renormalization procedure was very accurate, and indeed its predictions have been experimentally confirmed at very high precision [32-34]. The requirement of (strict) renormalizability became the criteria to uniquely select the quantum theory. In the sixties QFT went again into troubles because it was not clear how to make sense of perturbation theory for the strong interaction, and several alternatives were proposed like the S-matrix approach [35]. However, thanks to a deeper understanding of QFT, the discovery of asymptotic freedom provided a reliable perturbative description of the strong force in the high-energy regime [36, 37].

We are now living in a historical period of Theoretical Physics in which the framework of QFT seems to be inadequate to describe gravity, and actually the failure of perturbative renormalizability of the gravitational interaction can be considered as the *third QFT crisis. But could it be that also this time an even deeper understanding of QFT is needed in order to achieve a more complete description of gravity in the UV regime?*

We believe that this question is very reasonable and well-posed. In fact, several interesting QFT-based approaches to quantum gravity have been proposed in the recent decades, and in our opinion they appear to be even more promising and predictive than the ones briefly mentioned above. It is clear that a description based on standard principles of perturbative QFT would lead to problems; this means that surely something must be changed in such a way that a QFT framework can still be consistently implemented for a more complete description of quantum gravity.

4. – QFT framework for quantum gravity

The standard framework of QFT is based on several guiding principles and fundamental ingredients: locality, unitarity, local and global gauge symmetries, Poincaré invariance, local commutativity, macro- and micro-causality, analyticity, perturbative renormalizability. Some of these properties are interconnected. For example, local commutativity is needed for the Poincaré invariance of the S-matrix if its elements are expressed in terms of time-ordered products; analyticity can be shown to follow from macrocausality under certain assumptions.

Furthermore, from the above properties several consequences can follow. The requirements of analyticity and macrocausality constrain the type of prescription that should be used to shift poles and circumvent pinching singularities in the complex energy plane, *e.g.*, the Feynman $i\epsilon$-prescription. By adding the assumption of locality then one also obtains that positive energy particles propagate forward in time, while negative energies propagate backward in time (antiparticles). Constraints on the Hilbert space that follows are the condition of positive norms (defined through the Dirac scalar product with bra and ket), and boundedness from below of Hermitian Hamiltonians. For instance, in gauge theories the full Hilbert space is unphysical because of the presence of

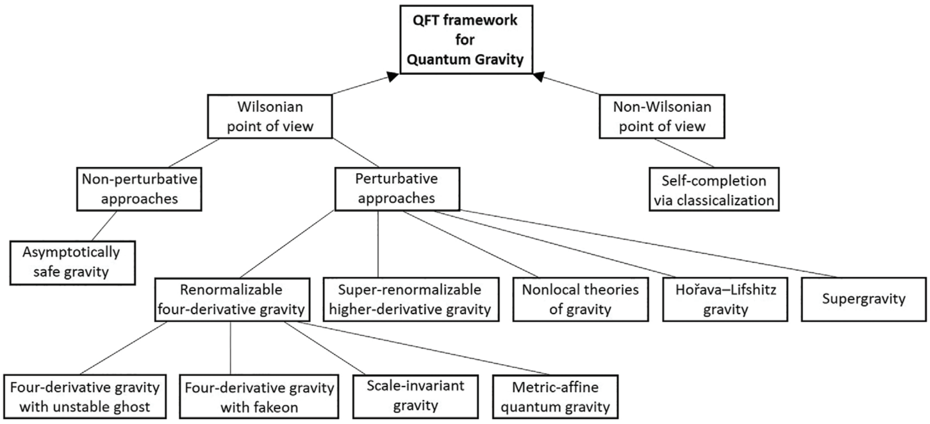

Fig. 1. – Schematic illustration of several QFT approaches to quantum gravity.

negative-norm states, but unitarity and gauge invariance are required to restrict the Hilbert space to a physical subspace in which only positive-norm states are present.

We already know that if we try to describe the gravitational interaction by quantizing Einstein's GR in the framework of QFT, then not all the nice properties above are satisfied, indeed perturbative renormalizability fails. What we would like to ask and understand is whether we can still make sense of a QFT of gravity by giving up some of the standard ingredients, or adding new ones (like new symmetries), without encountering pathologies and thus obtaining a more complete description even in the UV regime. To do this we should also decide on which point of view we want to adopt in order to extend/improve quantum GR at high energy. We can make the following distinction: *Wilsonian* and *non-Wilsonian* point of views, and we will discuss quantum-gravity approaches based on both possibilities. In fig. 1 we showed a schematic illustration of several approaches to quantum gravity based on the QFT framework.

4˙1. *Wilsonian point of view*. – The Wilsonian idea on how to improve the UV behavior of non-renormalizable theories consists in *integrating-in* new massive degrees of freedom whose dynamics decouples at low energy. One famous example is Fermi theory of weak interaction characterized by the non-renormalizable interaction term $(\bar{\psi}\psi)^2$. Indeed, by introducing $W^{\pm}$, Z^0, and Higgs bosons one goes from a non-renormalizable to a strictly renormalizable theory of the (electro)weak interaction.

We are now going to consider several QFTs of the gravitational interaction based on the Wilsonian point of view. We will discuss both *perturbative* and *non-perturbative* approaches, *i.e.,* QFTs in which interaction couplings are sufficiently small to allow perturbation theory, and others in which the usual perturbative expansion is not allowed. We will make a more extensive discussion for the four-derivative gravity and the related problem of unitarity, and after that we will only list other alternative approaches.

4˙1.1. Perturbative approaches.

Renormalizable four-derivative gravity. According to the Wilsonian point of view, a natural way to extend GR in the high-energy regime is to generalize the Einstein-Hilbert action (1) by adding higher powers of the curvature tensors and, indeed, in 1977 Stelle [38] showed that the following quadratic-curvature action was already sufficient to formulate a strictly renormalizable theory of quantum gravity:

$$S_{\rm 4th}=\frac{1}{2\kappa^2}\int {\rm d}^4x\sqrt{-g}\left[\gamma\, R+\alpha\, R^2+\beta\, R_{\mu\nu}R^{\mu\nu}\right], \tag{10}$$

where γ is a positive coefficient related to the Newton coupling, α and β are two parameters that are related to the masses of new degrees of freedom as we will see in a moment. The subscript 4th refers to the fact that the quadratic-curvature terms contain fourth order derivatives of the metric tensor. One may think to add other four-derivative curvature invariants like $\Box R$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$: the former is a total derivative, while the latter can be rewritten in terms of R^2 and $R_{\mu\nu}R^{\mu\nu}$ by using the Gauss-Bonnet identity in four dimensions.

The renormalizability of the theory can be easily shown via a power counting analysis. By expanding the metric tensor in terms of small fluctuations around Minkowski, $g_{\mu\nu}=\eta_{\mu\nu}+2\kappa h_{\mu\nu}$, we can build a QFT action for $h_{\mu\nu}$. The kinetic term will contain terms depending on γ, α and β, whereas the interaction terms will have the form $\gamma\kappa^n\partial^2h^{n+2}$, $\alpha\kappa^n\partial^4h^{n+2}$ and $\beta\kappa^n\partial^4h^{n+2}$. We can choose the mass dimensions of couplings and field as follows:

$$[\kappa]=0,\quad [h_{\mu\nu}]=0,\quad [\gamma]=2,\quad [\alpha]=0,\quad [\beta]=0, \tag{11}$$

so that all dimensions are non-negative which means that the theory is indeed strictly renormalizable. For the full proof of renormalizability one should also show that diffeomorphism invariance is respected at all order in perturbation theory [38]; moreover, since the bare Lagrangian is local we know that all counterterms will also be local in such a way that standard theorems of renormalization theory, like the BPHZ prescription [39], hold true. Renormalizability requires a non-vanishing cosmological constant is needed, but we set it to zero for simplicity.

It is worth mentioning that the quadratic curvature terms cannot be removed by a field redefinition similar to the one in eq. (5) that was used to cancel the one-loop divergence in GR. Indeed, the coefficients α and β in the action (10) are *not* necessarily small, and no perturbative cancellation can be achieved without introducing higher curvature terms that are as important as αR^2 and $\beta R_{\mu\nu}R^{\mu\nu}$. Moreover, the perturbation theory and the Feynman rules for the fourth order theory (10) is constructed in terms of a different expansion in which the propagator falls off as $1/k^4$ at high energies.

To analyse the spectrum of the theory we can compute the propagator and look at the pole structure. Recasting the action (10) in terms of a different basis of curvature invariants, *i.e.,* the Ricci scalar squared R^2 and the Weyl squared $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$, we can make the tensorial nature of the new degrees of freedom more manifest. By defining the coefficients $\lambda=\beta^{-1}$ and $\xi=(6\alpha+2\beta)^{-1}$ we can write

$$S_{\rm 4th}=\frac{1}{2\kappa^2}\int {\rm d}^4x\sqrt{-g}\left[\gamma\, R+\frac{1}{6\xi}R^2+\frac{1}{2\lambda}\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right]. \tag{12}$$

For this action we can compute the propagator around Minkowski, and its gauge-fixing independent part is given by [38]

$$i\Pi_{\mu\nu\rho\sigma}=i\Pi^{\rm GR}_{\mu\nu\rho\sigma}+\frac{1}{2\gamma}\frac{\mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma}}{k^2+m_0^2}-\frac{1}{\gamma}\frac{\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}}{k^2+m_2^2}, \tag{13}$$

where $\Pi^{\rm GR}_{\mu\nu\rho\sigma}$ is the standard massless spin-2 graviton in GR; $\mathcal{P}^{(0,s)}_{\mu\nu\rho\sigma}$ and $\mathcal{P}^{2}_{\mu\nu\rho\sigma}$ are two spin projector operators projecting along the spin-2 and one of the spin-0 (trace) components of the symmetric two-rank tensor $h_{\mu\nu}$, see refs. [38,40] for their expressions; whereas the two mass parameters are defined as $m_0^2\equiv\gamma\xi$ and $m_2^2\equiv-\gamma\lambda$. From eq. (13) it is clear that the higher order operators in the action imply the existence of additional propagating degrees of freedom: a massive spin-0 and a massive spin-2 particle. Note that to avoid tachyons we need to require

$$\xi>0 \quad \text{and} \quad \lambda<0. \tag{14}$$

Hence, in Wilsonian language we can say that the gravitational interaction can be described by a local, fourth order, diffeomorphism invariant, and renormalizable QFT by integrating-in 6 $(1+5)$ massive degrees of freedom.

It is worth mentioning that the quadratic curvature term R^2 in the action (12) can play a significant role in describing the physics of early Universe, the explanation of cosmic microwave background (CMB) and Large Scale Structure (LSS) of the Universe through initial quantum fluctuations which are measured to an unprecedented level by the recent Planck data [41]. Such a higher-curvature gravitational model of cosmic inflation (without the Weyl-squared term) is also known as Starobinsky model [42]. Furthermore, it has been realized that the nonlocal terms in the EFT action (8) can contribute to the particle production after the end of (Starobinsky) inflation [43, 44].

Ghost problem. Despite several nice features, the theory of quantum gravity described by the action (10) (or (12)) is pathological because of the presence of a "ghost" mode. Indeed, by inspecting the form of the propagator in eq. (13) we can notice that the spin-2 massive mode appears with a negative sign which would correspond to a negative kinetic term in the Lagrangian. Such a ghost particle causes classical instability by making the Hamiltonian unbounded from below (Ostrogradsky theorem [45,46]), and at the quantum level it breaks the unitarity condition on the S-matrix (optical theorem) [38]. We can explain why unitarity breaks down by using a simpler scalar propagator which contains all the information about the pole structure of the gravitational spin-2 component in eq. (13), namely

$$\Pi(k^2)=\frac{-i}{k^2(k^2+m^2)}=\frac{-i}{m^2}\left[\frac{1}{k^2}-\frac{1}{k^2+m^2}\right], \tag{15}$$

where $1/k^2$ and $-1/(k^2+m^2)$ can mimic the massless spin-2 graviton and the massive spin-2 ghost, respectively.

The unitarity condition $S^\dagger S=\mathbb{1}$ can be expressed in terms of the transfer matrix T defined through the relation $S=\mathbb{1}+iT$, so that one obtains the well known optical theorem $i(T^\dagger-T)=T^\dagger T$. By introducing in and out states $|a\rangle$ and $|b\rangle$, respectively, and using the completeness relation $\mathbb{1}=\sum_n|n\rangle\langle n|$, we can write

$$i\left[\langle b|T^\dagger|a\rangle-\langle b|T|a\rangle\right]=\sum_n\langle b|T^\dagger|n\rangle\langle n|T|a\rangle\,. \tag{16}$$

In the case of elastic scattering, $|b\rangle=|a\rangle$, it follows that the imaginary part of the diagonal elements of T must be positive, *i.e.,* $2{\rm Im}[\langle a|T|a\rangle]\geq0$, and this positivity condition must be satisfied by any unitary QFT.

If we consider a tree-level amplitude with the propagator in eq. (13) and constant vertexes $-ig$ (*e.g.*, a $2 \to 2$ scattering in ϕ^3-theory), we have([3])

$$\mathrm{Im}[\langle a|T|a\rangle] = \frac{g^2}{m^2}\mathrm{Im}\left[\frac{1}{k^2 - i\epsilon} - \frac{1}{k^2 + m^2 - i\epsilon}\right] = \frac{g^2\pi}{m^2}[\delta(k^2) - \delta(k^2 + m^2)], \tag{17}$$

where we have used the Feynman prescription, *i.e.*, $k^2 \to k^2 - i\epsilon$, and the limit $\epsilon \to 0^+$ is understood. The imaginary part in eq. (17) can be negative which implies that the optical theorem, and so the unitarity condition, is *not* respected. The main cause of this failure indeed comes from the "minus" sign in the ghost component of the propagator.

In presence of ghosts one can introduce a *pseudo-unitarity* equation that reads

$$i\left[\langle b|T^\dagger|a\rangle - \langle b|T|a\rangle\right] = \sum_n \sigma_n \langle b|T^\dagger|n\rangle \langle n|T|a\rangle\,, \tag{18}$$

where $\sigma_n = 1$ for normal healthy states, while $\sigma_n = -1$ for ghost states. Note that, in general, the pseudo-unitarity equation (18) is not equivalent to $S^\dagger S = \mathbb{1}$, but they can be made equivalent by changing something more in the QFT framework.

Let us now present two examples in which by altering some of the standard QFT properties one can recover (tree-level) unitarity.

- Negative norms: if we assume that ghost states have negative norms, then the completeness relation must be modified as follows:

$$\mathbb{1} = \sum_n \sigma_n |n\rangle\langle n|\,. \tag{19}$$

  By using this identity we can easily show that the unitarity condition $S^\dagger S = \mathbb{1}$ (*i.e.*, the optical theorem) written for the matrix elements corresponds to eq. (18), and it is respected.

- Negative energies: alternatively, we can assume that ghost fields propagate negative energies forward in time, and positive energies backward in time. This can be done by changing the Feynman prescription $k^2 \to k^2 - i\epsilon$ to $k^2 \to k^2 + i\epsilon$, which is sometime called Dyson prescription. Thus, one obtains

$$\mathrm{Im}\left[\frac{1}{k^2 - i\epsilon} - \frac{1}{k^2 + m^2 + i\epsilon}\right] = \pi[\delta(k^2) + \delta(k^2 + m^2)]\,, \tag{20}$$

  and tree-level unitarity is respected. In this case we only touched the left-hand-side of the optical theorem, and kept $\sigma_n = 1$ for all n.

([3]) To be more precise we should introduce the Feynman amplitude $\mathcal{M}$ whose elements are defined by the relation $\langle b|T|a\rangle = (2\pi)^4\delta^{(4)}(P_b - P_a)\langle b|\mathcal{M}|a\rangle$, where P_b and P_a are the outgoing and the ingoing momenta, respectively. However, to make the explanation simpler we intentionally work with T and do not write the Dirac delta.

Despite being interesting possibilities, both of them are still plagued by unphysical implications. In the former case, we can restore unitarity at the cost of negative probabilities whose reality seems to be difficult to accept; see refs. [47, 48] for some curious discussion on negative probabilities in quantum mechanics. In the latter case, instead, all probabilities are positive but the presence of propagators with different shifts (*i.e.,* $-i\epsilon$ and $i\epsilon$) running through loops can break unitarity at higher order in perturbation theory, and also spoil the nice high energy behavior because of nonlocal divergent contribution to the quantum effective action [49].

The same unitarity issues discussed above concern the renormalizable four-derivative theory of gravity in eq. (10) (or (12)). Such a failure of perturbative unitarity gave rise to several new ideas and attempts aimed at formulating a consistent QFT of gravity, and thus at solving such a conflict between renormalizability and unitarity in quantum gravity.

In what follows we are going to simply very briefly list some of the approaches without going into details; some of them have been discussed during the workshop and also presented as proceedings articles in this journal issue. Therefore, we are not going to mention pro and cons of each approach. The interested reader can either read the proceeding articles and/or the cited references in this article.

Four-derivative gravity with unstable ghost. In refs. [50,51] it was proposed that the ghost particle is unstable, so that it can decay and not appear in the physical spectrum of the theory. The reason for this is that loop computations can give rise to an imaginary part of the ghost propagator for time-like momenta, which would indeed correspond to a resonance. In this approach the particle spectrum is made up of the standard massless graviton, the massive spin-0 scalaron, and an unstable massive spin-2 ghost.

Four-derivative gravity with fakeon. In the previous approach unitarity can be saved by only reformulating the perturbation theory in terms of the dressed propagator with the ghost resonance. However, one may still wonder whether a way to achieve unitary already at the tree level exists. In fact, the authors in refs. [52-54] proposed a new quantization prescription through which the ghost can be converted into a purely virtual particle (fakeon) that never appears as an on-shell state. In this way they showed that the optical theorem is valid at any order in perturbation theory, without affecting the renormalizability of the theory.

Scale-invariant gravity. What if Nature is scale invariant at the fundamental level? In this case the fundamental bare action should only contain adimensional couplings. This point of view has been adopted in the context of quantum gravity by several people [55-57]. The starting point is the purely quadratic action(4) which is scale invariant as ξ and λ are dimensionless, and no mass or length scale appears. In this approach it has been proposed that a term proportional to R/κ^2 can be generated dynamically by means of specific couplings of matter to gravity. Obviously, also in this case the particle spectrum contains a spin-2 massive ghost, and all the problems concerning stability and unitarity are still present. One could tackle the ghost problem by employing the approaches reviewed

(4) Some authors proposed a conformally invariant action consisting only of the Weyl squared term [58]. However, this action is not renormalizable because terms proportional to the square of the Ricci scalar are generated at two loops [56].

above, *i.e.,* treating the ghost either as an unstable particle or as a fakeon. Moreover, for the purely quadratic gravity one can also implement an alternative prescription based on PT quantization [58].

Metric-affine quantum gravity. If the gauge structure is richer, then why not to make a full use of it. In this way, it naturally happens that other geometrical quantities like torsion and non-metricity only emerge at high energies (Planck scale), whereas at low energies we recover standard metric theory, either general relativity or Stelle gravity; see ref. [59] and references therein for more details. In this approach the gravitational action contains many more terms and couplings, and can still be renormalizable. Moreover, it may happen that specific values of the couplings can give rise to ghost-freeness, and one should then hope that these conditions are preserved by radiative corrections.

Super-renormalizable higher-derivative gravity. So far we have only discussed four-order derivative gravity. But, what if higher (than four) derivatives are needed to consistently describe Nature at the fundamental level? In the past decades, and also very recently, several authors [52,60-64] have considered sixth- and even higher-order derivative theories of gravity in which higher-order differential operators and higher order curvatures can appear in the action. Such theories can be super-renormalizable or even finite, and unitarity can be respected by employing a new elaborated Lee-Wick prescription which allows to circumvent singularities in the complex-energy plane, and consistently compute loop integrals and amplitudes.

Nonlocal theories of gravity. One can even go beyond local higher-order derivatives and consider infinite-derivative theories of gravity [65-72]. The gravitational Lagrangians are nonlocal and contain certain infinite-order differential operators acting on the curvature tensors. Also in this case one can achieve super-renormalizability and restore perturbative unitarity. Moreover, in the presence of infinite-order derivatives the Ostrogradsky theorem does not apply, and one may hope to simultaneously respect also classical stability without the need of extra efforts that instead seem to be required for the previous listed approaches. The nonlocal nature of the gravitational interaction may also be important to solve other problems like cosmological and black-hole singularities.

Hořava-Lifshitz gravity. All the approaches mentioned so far were compatible with diffeomorphism invariance and local Lorentz invariance. But, what if Nature does not respect such symmetries at very fundamental level (*i.e.,* at short distances or high energies)? We should emphasize that the problem of ghost, classical stability and unitarity is mainly related to the presence of higher-order time derivatives, while higher-order spatial derivatives are in principle not dangerous. In fact, in ref. [73] it was proposed a theory of quantum gravity in which diffeomorphism invariance is broken in such a way that time derivatives up to second order can be present, but higher-order spatial derivatives are allowed. There exists a wide class of this type of theories which are constructed in different backgrounds, and some of them have been shown to be renormalizable and unitarity [73-75].

Supergravity. Instead of losing symmetries at the fundamental level, could it be that Nature prefers to be more symmetric? This is what happens in Supergravity [76]. Thanks to the enhanced symmetry and to additional degrees of freedom it is been widely acknowledged that $N = 8$ supergravity in 4 dimensions is finite. This claim has been explicitly proven up to five loops [77]. However, more checks and general proofs are still needed. Indeed, the $E_{7(7)}$ duality symmetry that is responsible for special cancellations was claimed to be useful up to six loops, but it might not help at seven loops where divergent counterterms could appear [78, 79]. It is worth mentioning that supergravity can also emerge as a low-energy description of superstring theory [26].

4˙1.2. Non-perturbative approaches. All the approaches listed so far are perturbative, *i.e.,* they rely on a perturbative expansion in terms of some small interaction coupling. We know that quantum GR is perturbatively non-renormalizable, but could it be that instead it is renormalizable at the non-perturbative level? We are now going to mention one possible non-perturbative approach based on the QFT framework.

Asymptotically safe gravity. Weinberg [80] proposed that in the theory of gravity there might exist a nontrivial (non-Gaussian) fixed point of the renormalization group flow controlling the UV behavior of the interaction couplings. This means that: given some couplings g_i, in the UV regime they should tend to some critical values g_i^* for which all the beta functions vanish, *i.e.,* $\beta_j(g_i^*) = 0$. For this proposal to be physically consistent it is necessary that all the physical couplings are on the trajectory attracted to the fixed point g_i^*, and that the number of independent couplings is finite. Such a number is given by the dimensionality of the so-called *UV critical surface* formed by all trajectories attracted to the fixed point.

The existence of fixed points has been shown in several cases: in $2 + \epsilon$ dimension [80], using the truncated functional renormalization group equation [81, 82], through lattice techniques and Causal Dynamical Triangulation methods [83, 84], and via a $1/N$ approximation with N being the number of matter fields [85]. In all these studies a finite number of independent couplings (*i.e.,* a finite dimensional UV critical surface) has been found. See also refs. [59, 86-92] and references therein for more works and details on asymptotically safe gravity.

The problem of unitarity is still unsolved also in this case, and it is believed that non-perturbative effects can play a crucial role. For instance, it might happen that the trajectories on which the Weyl squared term and its ghost are relevant do not approach the fixed point, so that the physical trajectory will be free from pathological degrees of freedom. See also ref. [93] for a preliminary study on non-perturbative unitarity.

4˙2. *Non-Wilsonian point of view.* – Let us now consider a point of view that is different from the standard Wilsonian one, according to which Einstein's GR can be completed in the ultraviolet regime without the need of integrating-in many new massive degrees of freedom. In particular, we are going to briefly discuss a non-Wilsonian approach still based on the QFT framework.

4˙2.1. Self-completion via classicalization. In the Wilsonian approaches mentioned so far, the hope is that a UV-completed gravitational theory can be obtained by introducing higher-order curvature operators which bring-in new massive degrees of freedom. But, could it be that Einstein's GR complete itself through some collective phenomenon involving standard weakly interacting gravitons as the only degrees of freedom?

An approach of this type was proposed in refs. [94, 95] where it was claimed that Einstein's GR can self-complete itself via classicalization. This idea is based on the fact that at some high energy —*e.g.*, Planck scale— black holes must be produced. Black-hole states are seen as the result of a collective phenomenon involving weakly interacting gravitons, and their formation weakens gravitational amplitudes in the UV regime. In other words, black holes are responsible of softening the UV behavior of gravity, and in this sense Einstein's GR might self-complete.

Despite being very interesting, a full rigorous and more concrete formulation of this idea is still lacking.

5. – Discussion and concluding remarks

All these approaches to quantum gravity share the same hope to eventually provide a satisfactory explanation to the origin and evolution of the Universe, and to the formation and evaporation of black holes. Cosmology and black-hole physics are theoretical and phenomenological playgrounds for our fundamental description of spacetime and matter quantum fields. The problem of avoiding black-hole and cosmological singularities has inspired several proposals to extend Einstein's GR in the short-distance regime. The black-hole information paradox has led to a slew of enthusiasm and motivated lots of efforts towards a UV completion of quantum gravity. On the other hand, finding signatures of high-energy physics in CMB and LSS is another one of the biggest driving forces in probing physics at short distance and time scales. Last but not least, the elusive nature of dark matter and dark energy can also be important to the understanding of physical phenomena at the fundamental scales. It is worth mentioning that the technological developments have given unprecedented hope to test our theories and find a reliable path towards quantum gravity. Especially, the recent detection of gravitational waves from black-hole mergers has opened a new window of opportunities to probe unexplored regimes of the gravitational interaction.

During the workshop several aspects of early Universe cosmology were addressed, from resolving big-bang singularity to diverse explanations for CMB through cosmic inflation in higher-derivative gravity, bouncing and emergent Universes in string theory. We had also elaborate discussions on the recent conjectures in string theory and assessed their role in structuring of quantum gravity. Also problems of quantum cosmology and the initial condition problem were discussed. In the context of black-hole physics the workshop particularly focused on the role of higher-derivatives and its implications to singularity resolution, and the question of black-hole formation from high-energy scattering was addressed. Last but not least, the workshop posed interesting fundamental questions for QFT on curved spacetime, which has aroused great interest in recent times in relation to the problem of unitarity in curved spacetime [96, 97].

In short, the workshop was not mainly restricted to discussing formal aspects of quantum gravity approaches, but also to addressing fundamental questions related to physical applications and possible experiments: *what can cosmology and black-hole physics tell us about the nature of quantum gravity, and vice-versa?*

* * *

During the workshop different schools of thought were brought together, in such a way that everyone could benefit from fruitful discussions and learn from each other. In addition to individual talks, discussion sessions helped making the meeting more exciting and productive, and they gave rise to constructive debates and new insights. It is our great pleasure to thank all the speakers, advisory board members, and all the participants for helping to make it very successful. We hope that this workshop helped pushing all

of us towards deeper understanding, and that it made our quantum gravity community more solid, united and collaborative.

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