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Several aspects of nonlocal field theory and gravity

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Summary. — In this mini review we consider nonlocal field theory including nonlocal quantum gravity. We discuss gauge models with γ_5-anomalies and show how nonlocal field theory can help to make such models meaningful. We consider the field theory with infinite number of local fields with local interactions and show that an account of an infinite number of local fields leads to effective ultraviolet finite nonlocal field theory. We also discuss nonlocal generalization of nonsupersymmetric $SU(5)$ Georgi-Glashow GUT and show that it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum of the theory. Nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale $M_{GUT} \approx 3 \cdot 10^{16}$ GeV.

1. – Introduction

It is well known that $d = 4$ local field theories have ultraviolet divergences in perturbation theory. The simplest way to deal with ultraviolet divergent Feynman diagrams is the introduction of regularization. For instance, in Pauli-Willars regularization [1] for scalar ϕ^4-model the replacement of scalar propagator

$$D(p^2) = \frac{1}{m^2 - p^2 - i\epsilon} \rightarrow D^{reg}(p^2) = \frac{1}{m^2 - p^2 - i\epsilon} - \frac{1}{M^2 - p^2 - i\epsilon} \tag{1}$$

makes all Feynman integrals ultraviolet finite except vacuum diagrams. The main drawback of Pauli-Willars regularization is that the introduction of the second term $-\frac{1}{M^2-p^2-i\epsilon}$ in (1) is equivalent to the introduction of negative norm state with a mass M in the spectrum. The ϕ^4-model with Pauli-Willars propagator (1) is local and unitary but it contains negative norm states that make reasonable physical interpretation

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of such model impossible. Many years ago Efimov [2-5] proposed to use nonlocal field theory to make Feynman diagrams ultraviolet finite. The main idea of nonlocal field theory consists in the replacement of the local propagator $D(p^2)$ to nonlocal propagator, namely

$$D(p^2) \to D^{nl}(p^2) = \frac{V(p^2)}{m^2 - p^2 - i\epsilon}, \tag{2}$$

where the nonlocal formfactor $V(p^2)$ is an entire function on p^2 decreasing in the Euclidean region at $p^2 \to -\infty$, for instance $V(p^2) = \exp[l^2(p^2 - m^2)]$. Another interesting formfactor is

$$V(p^2) = V_{PW}(p^2)\left[1 - \sin\left(1 - \frac{p^2}{M^2}\right) \cdot \left(1 - \frac{p^2}{M^2}\right)^{-1}\right], \tag{3}$$

where

$$V_{PW}(p^2) = 1 - \frac{m^2 - p^2}{M^2 - p^2 - i\epsilon}. \tag{4}$$

The formfactor $V(p^2)$ (3) is an entire function on p^2 and $V(p^2) \to V_{PW}(p^2)$ at $p^2 \to -\infty$. Efimov considered [2-5] nonlocal scalar field theory with formfactor satisfying the following requirements:

1) $V(z)$ is an entire function on z of the growth $\rho \leq 1/2$.
2) $V(z) \leq C\exp(b|z|^{\rho})$.
3) $V(z) = O(z^{-2})$ at $\mathrm{Re}z \to -\infty$.
4) $V(z) = O(\exp(b|z|^{\rho})$ at $\mathrm{Re}z \to \infty$.
5) $V(z) = V^*(z^*)$.
6) $V(m^2) = 1$.

On the example of scalar ϕ^4-model Efimov proved that nonlocal field theory is unitary and causal. Nonlocal propagator (2) corresponds to nonlocal free scalar action

$$S_{onl} \int \mathrm{d}^4x \left[-\frac{1}{2}\phi(x)(\Box + m^2)V^{-1}(-\Box)\phi(x)\right], \tag{5}$$

where $\Box = \partial^{\mu}\partial_{\mu}$. The use of nonlocal propagator (2) allows to cure bad ultraviolet properties of the theory. One can obtain nonlocal formfactor $V(p^2)$ by the generalization of Pauli-Willars regularization (1), namely one can consider the regularization with [5]

$$D(p^2) \to D^{reg}(p^2) = \frac{1}{m^2 - p^2 - i\epsilon} + \sum_{n=1}^{n=\infty} c_n(\delta)\frac{(-1)^n}{M_n^2(\delta) - p^2 - i\epsilon}. \tag{6}$$

Here parameter δ is the regularization parameter. One can choose [5] the $c_n(\delta)$, $M_n(\delta)$ in such a way [5] that in the limit of the regularization $D^{reg}(p^2) \to \frac{V(p^2)}{m^2-p^2-i\epsilon}$. The regularization (6) allows to prove [5] that nonlocal scalar ϕ^4-theory is unitary, causal, ultraviolet finite and nonlocal.

In this mini review I discuss the applications of Efimov nonlocal field theory to gauge theories including γ_5-anomalous theories and quantum gravity. Also I consider nonlocal generalization of nonsupersymmetric $SU(5)$ Georgi-Glashow GUT and show that it is possible to solve the problems with the proton lifetime and the Weinberg angle without introduction of additional particles in the spectrum of the theory. In nonlocal $SU(5)$ GUT nonlocal scale Λ responsible for ultraviolet cutoff coincides (up to some factor) with GUT scale $M_{GUT} \approx 3 \cdot 10^{16}$ GeV. I discuss possible value of nonlocal scale in Nature and give some arguments (not proof) that nonlocal scale $O(10^{14})$ GeV $\leq \Lambda \leq O(10^{18})$ GeV. Also I consider the field theory with infinite number of local fields with local interactions and show that an account of an infinite number of local fields leads to effective ultraviolet finite nonlocal field theory.

2. – Local field theory with infinite number of local fields as origin of nonlocality

In this section we discuss the possible origin of nonlocality related with the introduction of infinite number of local scalar fields $\phi_n(x)$ [6, 7]. Consider the model with the Lagrangian

$$L_{tot} = L_0 + L_I, \tag{7}$$

where

$$L_0 = \frac{1}{2}\sum_{n=o}^{n=\infty}(\partial^\mu\phi_n\partial_\mu\phi_n - M_n^2\phi_n^2), \tag{8}$$

$$L_I = -g\phi_{eff}^4(x), \tag{9}$$

$$\phi_{eff}(x) = \sum_{n=0}^{n=\infty} c_n(-\Box)^{n/2}\phi_n(x). \tag{10}$$

The propagator for the effective field $\phi_{eff}(x)$ is the infinite sum of local propagators, namely

$$D_{eff}(p^2) = \sum_{n=0}^{n=\infty} c_n^2\frac{(p^2)^n}{M_n^2 - p^2 - i\epsilon}. \tag{11}$$

The imaginary part of the propagator (11) is nonnegative and it coincides with the imaginary part of the propagator for local field $\phi^0_{eff}(x) = \sum_{n=0}^{n=\infty} c_n(M_n^2)^{n/2}\phi_n(x)$

$$D^0_{eff}(p^2) = \sum_{n=0}^{n=\infty} c_n^2\frac{(M_n^2)^n}{M_n^2 - p^2 - i\epsilon}. \tag{12}$$

For the model [6, 7] with $M_n^2 = M_0^2 + n \cdot M_1^2$ and $c_n^2 = \frac{a^l}{l!}$, $a > 0$ the effective propagator (11) can be represented in the form

$$D_{eff}(p^2) = \frac{1}{M_1^2}\int_0^1 \mathrm{d}x \, \exp(axp^2) x^{-\left[\frac{p^2}{M_1^2}+\frac{M_0^2}{M_1^2}-1\right]}. \tag{13}$$

In the Euclidean region $q^2 = -p^2 > 0$ the propagator

$$D_{eff}(-q^2) \sim \sqrt{\frac{1}{2\pi q^2 M_1^6 a^2}} \exp\left[-\frac{q^2}{M_1^2} - \frac{q^2}{M_1^2}\log(M_1^2 a)\right] \text{at } q^2 \to \infty \quad \text{for } aM_1^2 > 1. \tag{14}$$

As a consequence we find that all Feynman diagrams for the model (7)–(10) are ultraviolet finite. So the model (7)–(10) describes an infinite number of local fields $\phi_n(x)$ with local interactions containing higher-order derivatives for each local field. An account of infinite number of local fields leads to nonlocal and ultraviolet finite theory. So we see that the introduction of infinite number of local fields is an origin of nonlocality([1]).

3. – Nonlocal gauge theories

The simplest nonlocal generalization of QED [5] consists in the replacement of local free photon Lagrangian

$$L_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \to L_{nl,A} = -\frac{1}{4}F^{\mu\nu}V^{-1}(-\partial^\mu\partial_\mu)F_{\mu\nu}. \tag{15}$$

For nonlocal QED (15) the Feynman rules coincide with standard QED Feynman rules except the replacement $\frac{1}{p^2+i\epsilon} \to \frac{V(p^2)}{p^2+i\epsilon}$ for local photon QED propagator. For formfactor

$$V(p^2) = O\left(\frac{1}{p^2}\right) \quad \text{at } p^2 \to -\infty \tag{16}$$

all Feynman diagrams except one-loop correction to the photon propagator are ultraviolet finite [5]. In nonlocal QED the interaction between two charges e_1 and e_2 is

$$W_{nl}(r) = \frac{e_1 e_2}{(2\pi)^3}\int \frac{V(-\vec{k}^2)}{\vec{k}^2}\exp(-i\vec{k}\vec{x}). \tag{17}$$

For local QED $W(r) = \frac{e_1 e_2}{r}$ and $W(0) = \infty$ while for nonlocal QED with the formfactor (16) the value $W_{nl}(r = 0)$ is finite. The straightforward generalization of nonlocal QED to nonAbelian gauge theories consists in the replacement [8]

$$L_{YM} = -\frac{1}{2g^2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) \to -\frac{1}{2g^2}\mathrm{Tr}(F^{\mu\nu}V^{-1}(-\Delta^2)F_{\mu\nu}), \tag{18}$$

([1]) The model (7)–(10) with finite number of scalar fields $\phi_n(x)$ with $n \le N_0$ is nonrenormalizable. An acount of the infinite number of local fields $\phi_n(x)$ leads to ultraviolet finite theory.

where $\Delta^2 = (\partial^\mu - iA^\mu)(\partial_\mu - iA_\mu)$. Nonlocal Lagrangian (18) is the generalization of Slavnov [9, 10] regularization with higher-order derivatives. Slavnov regularization is the gauge invariant generalization of Pauli-Willars regularization. Slavnov regularization corresponds to the formfactor $V^{-1}_{Slavnov}(-l^2\Delta^2) = 1 + c_k(l^2)^k(\Delta^2)^k$. Slavnov has proved [9, 10] that in his regularization all diagrams are ultraviolet finite except some finite number of diagrams. For instance, for $k = 2$ all diagrams are finite except one-loop propagators, three and four vertices. The increase of parameter k in Slavnov regularization does not help to cure the remaining ultraviolet divergences. However in odd dimensions ($d = 5$ for instance) one-loop diagrams are well defined and Slavnov regulization leads to ultraviolet finite diagrams in all loops.

4. – γ_5-anomaly and nonlocal theories

It is well known that triangle γ_5-anomalies spoil the gauge invariance at one-loop level [11, 12]. As a consequence longitudinal and transverse photons interact at one-loop level, which makes physical interpretation of γ_5-anomalous models impossible. Consider axial QED with the interaction

$$L_{int} = e\bar{\psi}\gamma^\mu\gamma_5\psi A_\mu. \tag{19}$$

The Lagrangian of axial QED is invariant under gauge transformations

$$\psi \to \psi \exp(ie\gamma_5\alpha), \quad A_\mu \to A_\mu + \partial_\mu\alpha. \tag{20}$$

Due to γ_5-anomaly the effective Lagrangian is not invariant under gauge transformations, namely [11, 12]

$$L_{eff}(A_\mu + \partial_\mu\alpha) - L_{eff}(A_\mu) = \frac{\alpha e^3}{12\pi^2}\epsilon^{\mu\nu\gamma\beta}F_{\mu\nu}F_{\gamma\beta}. \tag{21}$$

To restore the gauge invariance (20) at quantum level let us add additional scalar field ϕ which transforms as $\phi \to \phi + \alpha$ under the gauge transformations (20) and add the term ΔL, namely

$$L \to L + \Delta L, \tag{22}$$

$$\Delta L = -\frac{\phi e^3}{12\pi^2}\epsilon^{\mu\nu\gamma\beta}F_{\mu\nu}F_{\gamma\beta} + \frac{1}{2}m^2(A_\mu - \partial_\mu\phi)V_2^{-1}(-\partial^\mu\partial_\mu)(A^\mu - \partial^\mu\phi). \tag{23}$$

The Lagrangian ΔL restores the gauge invariance at quantum level. Due to nonlocal formfactor V_2 and formfactor V for the free vector field Lagrangian $L_A = -\frac{1}{4}F^{\mu\nu}V^{-1}(-\partial^\mu\partial_\mu)F_{\mu\nu}$ the modified axial electrodynamics becomes a superrenormalizable model and it describes the interaction of massive vector field with fermions [13-16]. In unitary gauge $\phi = 0$ nonlocal vector field propagator has the form

$$D_{\mu\nu}(k) = V(k^2)\cdot\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2V(k^2)V_2^{-1}(k^2)}\right]\cdot(k^2 - m^2V(k^2)V_2^{-1}(k^2))^{-1}. \tag{24}$$

For γ_5-nonanomalous models like QED the longitudinal part of the propagator (24) does not contribute at mass shell. For $V(k^2)V_2^{-1}(k^2) = \frac{k^2}{m^2}$ the gauge propagator (24) is transverse and triangle γ_5-anomaly does not contribute. So in axial electrodynamics it is possible to restore gauge invariance by the introduction of additional scalar field, however due to γ_5-anomaly the local version of the model with $V_2(k^2) = V(k^2) = 1$ is nonrenormalizable. The introduction of nonlocal formfactors $V(k^2)$, $V_2(k^2)$ makes the model superrenormalizable. It should be stressed that the limit $m \to 0$ does not exist, *i.e.*, axial QED with massless gauge field does not exist.

5. – Nonlocal gravity

The action of Einstein gravity without Λ-term and matter is([2])

$$S[g] = \int \mathrm{d}^4x \frac{1}{16\pi G} R\sqrt{-g}, \tag{25}$$

where $g = \mathrm{Det}(g_{\mu\nu})$, $G^{-1/2} \equiv M_{PL} = 1.2 \cdot 10^{19}$ GeV and R is the curvature. The use of perturbative expansion $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ around flat metric $\eta_{\mu\nu} = \mathrm{Diag}(-1,1,1,1)$ leads to([3])

$$S[g] = \sum_{n=2}^{n=\infty} \kappa^{n-2} S_n, \tag{26}$$

where S_2 is the quadratic action for graviton field $h_{\mu\nu}$ and higher-order terms S_n describe selfinteractions of gravitons. Off shell Einstein gravity (25) is nonrenormalizable at one-loop level [17]. On shell Einstein gravity (25) is nonrenormalizable at two-loop level [18]. The gravity action

$$S_{St}[g] = \int \mathrm{d}^4x \frac{1}{16\pi G}\sqrt{-g}\left[R + \frac{a_1}{2}R^2 + \frac{a_2}{2}R^{\mu\nu}R_{\mu\nu}\right] \tag{27}$$

describes renormalizable but nonunitarity theory [19]. For the model (27) free graviton propagator is

$$\begin{aligned} &(k^2)^{-1}[P^2_{\mu\nu\alpha\beta} - \frac{1}{2}P^0_{\mu\nu\alpha\beta}] - P^2_{\mu\nu\alpha\beta}(k^2+m_2^2)^{-1} + (1/2)P^0_{\mu\nu\alpha\beta}(k^2+m_0^2)^{-1} \\ &+\text{gauge-dependent terms.} \end{aligned} \tag{28}$$

Here $P^{2,0}_{\mu\nu\alpha\beta}$ are Nieuwenhuizen-Riverse operators [20,21]. The propagator (28) describes spin 2 masslesss graviton, spin 2 ghost with a mass $m_2 = (\frac{-2}{a_2})^{1/2}$ and spin zero scalar with a mass $m_0 = (3a_1 + a_2)^{-1/2}$. The existence of ghost state in the spectrum makes reasonable physical interpretation of the model impossible. To get rid of nonphysical

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([2]) In this section we use metric $(-,+,+,+)$.
([3]) Here $\kappa^{-1} \equiv m_{PL} = (\frac{1}{8\pi G})^{1/2} = 2.4 \cdot 10^{18}$ GeV is the scale of quantum gravity.

ghost states and improve ultraviolet properties of the theory it was proposed to deal with nonlocal gravity [8]. The nonlocal gravity action has the form [8, 22-28]

$$S_{St}[g] = \int \mathrm{d}^4x \frac{1}{16\pi G}\sqrt{-g}[R + Rf_1(-\Box_{cov})R + R^{\mu\nu} f_2(-\Box_{cov})R_{\mu\nu} + R^{\mu\nu\alpha\beta} f_3(-\Box_{cov})R_{\mu\nu\alpha\beta}], \tag{29}$$

Here $\Box_{cov}$ is the covariant d'Alembertian. Consider the simplest case $f_3 = 0$, $f_2 = -2f_1$ where only spin 2 propagates. The graviton propagator has the form

$$(k^2)^{-1}(P^2_{\mu\nu\alpha\beta} - \frac{1}{2}P^0_{\mu\nu\alpha\beta}) \cdot \left(1 - \frac{1}{2}k^2 f_2(k^2)\right)^{-1} + \text{gauge-dependent terms.} \tag{30}$$

We can choose the formfactor $f_2(k^2)$ in the form of the increasing function on k^2 in the Euclidean region, say $f_2(k^2) \sim -(k^2)^{n-1}$, $n > 1$. One can find that the theory with such formfactor is superenormalizable [8, 22-28]. It is interesting to note that for the model with nonlocal formfactor

$$(1 - \frac{1}{2}k^2 f_{2,nl}(k^2))^{-1} = 2(k^2l^2)^{-n}\left[1 - \frac{\sin((k^2l^2)^n)}{(k^2l^2)^n}\right] \tag{31}$$

ultraviolet behaviour is determined by the ultraviolet behaviour of the model with local formfactor $f_{2,l}(k^2) = -l^2(k^2l^2)^{n-1}$ and the infrared behaviour of the graviton propagator (28) reproduces Einstein gravity. Probably especially interesting is the case $n = 1$ for the formfactor (31) that corresponds to the renormalizable Stelle gravity [19] in the ultraviolet region and to the absence of the problems with indefinite metric due to nonlocal propagator (31).

Careful discussion of the renormalization in nonlocal gravity model (29) is contained in the brilliant review [26].

As was mentioned before in nonlocal QED the interaction between the charges could be finite at $r = 0$. Absolutely the same situation takes place in nonlocal gravity. Classical aspects of nonlocal gravity are discussed in review [29].

A very important question naturally arises: what about the scale of nonlocality $\Lambda \equiv \frac{1}{l}$? It is clear that nonlocal scale Λ has to be smaller or equal to the Planck scale m_{PL} because in the opposite case we shall have the problems with tree level unitarity for graviton amplitudes. The most natural assumption is that $\Lambda \sim O(m_{PL})$ but we cannot exclude the case $\Lambda \ll m_{PL}$. Current experimental data support Starobinsky R^2 inflation model [30]. There are attempts [31, 32] to use nonlocal gravity for the generalization Starobinsky R^2 model. It is interesting to mention that in Starobinsky model the free parameter is the scalar mass $M = 1.3 \cdot 10^{-5} m_{PL}$ and the nonlocal scale Λ has to be much larger than the scalar mass Λ [31, 32], namely $\Lambda \gg M$. So if we believe in Starobinsky model we find that nonlocal scale $\Lambda \geq 10^{14}$ GeV.

6. – Nonlocal $SU(5)$ GUT

The remarkable success of the supersymmetric $SU(5)$ grand unified theory (GUT) [33-50] was considered by many physicists as the first hint in favour of the existence of low energy broken supersymmetry in Nature. However the nonobservation of supersymmetry at the LHC is probably the opposite hint that the supersymmetry concept and in particular the supersymmetric $SU(5)$ GUT is wrong. It is well known that the standard $SU(5)$ GUT [51, 52] is in conflict with experimental data [46, 47]. So a natural question arises: is it possible to invent nonsupersymmetric generalizations of the standard $SU(5)$ GUT non contradicting to the experimental data? The answer is positive, in particular, in the $SO(10)$ GUT the introduction of the intermediate scale $M_I \sim 10^{11}$ GeV allows to obtain the Weinberg angle θ_w in agreement with experiment [53]. In refs. [54, 55] the introduction of the additional split multiplets $5 \oplus \overline{5}$ and $10 \oplus \overline{10}$ in the $SU(5)$ model has been proposed. In ref. [56] the extension of the standard $SU(5)$ GUT with light scalar colour octets and electroweak triplets has been considered.

In this section we point out that in nonlocal generalization of Georgi-Glashow $SU(5)$ GUT it is possible to solve the problems with the proton lifetime and the Weinberg angle by the introduction of additional nonlocal terms to the $SU(5)$ GUT Lagrangian [57]. Additional nonlocal terms lead to the modification of the GUT condition $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT})$ for the effective coupling constants. Nonlocal scale Λ responsible for the ultraviolet cutoff coincides (up to some factor) with GUT scale M_{GUT}. In the simplest nonlocal modification of the standard renormalizable $SU(5)$ GUT the value of the GUT scale is $M_{GUT} \approx 3 \cdot 10^{16}$ GeV.

Let us start with the observation that in standard $SU_c(3) \otimes SU_L(2) \otimes U(1)$ gauge model the effective coupling constants $\alpha_3(\mu)$ and $\alpha_2(\mu)$ cross each other ($\alpha_3(M_{GUT}) = \alpha_2(M_{GUT})$) at the scale $M_{GUT} \approx O(10^{17}$ GeV). At one-loop level the effective coupling constants $\alpha_i(\mu)$ obey the equations

$$\mu \frac{\mathrm{d}\alpha_i(\mu)}{\mathrm{d}\mu} = \frac{b_i}{2\pi}\alpha_i^2(\mu), \tag{32}$$

where for the SM model with 3 generations $b_3 = -7$, $b_2 = -3\frac{1}{6}$ and $b_1 = 4.1$. As a consequence we find that

$$\frac{1}{\alpha_2(m_t)} - \frac{1}{\alpha_3(m_t)} = \frac{b_2 - b_3}{2\pi} \ln\left(\frac{M_{GUT}}{m_t}\right). \tag{33}$$

Numerically $M_{GUT} = (0.9 \pm 0.2) \cdot 10^{17}$ GeV and $\frac{1}{\alpha_3(M_{GUT})} = 46.9 \pm 0.2$([4]).

The unification scale $M_{GUT} = (0.9 \pm 0.2) \cdot 10^{17}$ GeV is safe for the current proton decay bound [58]. Really, in the standard $SU(5)$ model the proton lifetime due to the massive vector exchange is determined by the formula [59]

$$\Gamma(p \to e^+\pi^o)^{-1} = 4 \cdot 10^{29\pm0.7} \left(\frac{M_v}{2 \cdot 10^{14}\,\mathrm{GeV}}\right)^4 \mathrm{yr}, \tag{34}$$

where $M_v \equiv M_{GUT} = \sqrt{\frac{5}{24}} g_5 \Phi_0$ is the mass of vector bosons responsible for proton decay([5]). From the current experimental limit [58] $\Gamma(p \to e^+\pi^o)^{-1} \geq 1.67 \cdot 10^{34}$ yr we

([4]) In our estimates we use $\alpha_3(m_Z) = 0.118 \pm 0.001$, $\sin^2(\theta_W)(m_Z) = 0.231 \pm 0.001$ and $\alpha_{em}^{-1}(m_Z) = 127.8 \pm 0.1$.

([5]) Here Φ_0 is the vacuum expectation value of the $SU(5)$ scalar 24-plet $\langle\Phi\rangle = \frac{\Phi_0}{\sqrt{15}}\mathrm{Diag}(1, 1, 1 - 3/2, -3/2)$ responsible for $SU(5) \to SU_c(3) \otimes SU_L(2) \otimes U(1)$ gauge symmetry breaking and g_5 is the $SU(5)$ gauge coupling at the GUT scale M_{GUT}.

conclude that $M_{GUT} \geq 2.5 \cdot 10^{15}$ GeV. The main problem of the standard $SU(5)$ GUT with the unification scale $M_{GUT} \approx 10^{17}$ GeV is that the experimental values of $\alpha_3(m_Z)$, $\sin^2(\theta_W)(m_Z)$, $\alpha_{em}^{-1}(m_Z)$ lead to non equal values of the effective coupling constants $\alpha_3(M_{GUT})$ and $\alpha_1(M_{GUT})$, namely $\alpha_1^{-1}(M_{GUT}) = 36.0 \neq \alpha_3^{-1}(M_{GUT}) = 46.9$.

Our main observation is that the use of nonrenormalizable interaction([6])

$$\Delta L_{F\Phi F\Phi} = \frac{1}{4\Lambda_{\Phi 1}^2}(Tr(F_{\mu\nu}\Phi))(Tr(F^{\mu\nu}\Phi)) \tag{35}$$

leads to the additional term for the effective coupling constant $\alpha_1(\mu)$ at GUT scale, namely

$$\frac{1}{\alpha_1(M_{GUT})} = \frac{1}{\alpha_3(M_{GUT})} - \Delta, \tag{36}$$

where

$$\Delta = \frac{\pi\Phi_o^2}{\Lambda_{\Phi 1}^2} = \frac{1}{\alpha_3(M_{GUT})}\frac{6M_v^2}{5\Lambda_{\Phi 1}^2}. \tag{37}$$

Numerically we find $\Delta = 10.9 \pm 0.2$ and $\Lambda_{\Phi 1} \approx 2.3 \cdot M_v$. So additional nonrenormalizable interaction (35) can modify GUT unification condition in such a way that the GUT unification scale $M_{GUT} \approx 10^{17}$ GeV is nondangerous for proton decay bound and the unification scale M_{GUT} does not contradict to the experimental values of $\sin^2(\theta_W)(M_Z)$ and $\alpha^{-1}(M_Z)$. The appearance of additional arbitrary parameter Δ in the relation (36) means that we cannot predict the value of $\sin^2(\theta_W)$. Here the untrivial fact is that the unifcation of $\alpha_2(\mu)$ and $\alpha_3(\mu)$ effective coupling constants takes place at the scale $M_{GUT} = O(10^{17}\,\text{GeV})$ which is safe for the proton lifetime bound. An account of two-loop effects for the evolution of the effective couplings $\alpha_k(\mu)$ leads [44] to the replacement

$$\frac{1}{\alpha_k(m_Z)} \to \frac{1}{\alpha_k(m_Z)} - \theta_k, \tag{38}$$

where

$$\theta_k = \frac{1}{4\pi}\sum_{j=1}^{3}\frac{b_{kj}}{b_j}\ln\left[\frac{\alpha_j(M_{GUT})}{\alpha_j(m_Z)}\right]. \tag{39}$$

Here b_{ij} are the two-loop β-functions coefficients([7]). An account of two-loop corrections leads to the decrease of M_{GUT} by a factor 3. The parameter Δ in (36) is not small. Really, $\Delta/(\frac{1}{\alpha_2(M_{GUT})}) \approx 0.24$ and $\Lambda_{\Phi 1} \approx 2.3 \cdot M_v$. It means that at the scale M_{GUT} we

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([6]) In refs. [60,61] the influence of nonrenormalizable interaction $L_{nl} = \frac{c}{M_{PL}}\text{Tr}(F_{\mu\nu}\Phi F^{\mu\nu})$ with $c = O(1)$ has been studied. It was realized that this interaction allows to increase the GUT scale but cannot solve the problem with wrong Weinberg angle prediction.

([7]) At two-loop level the renormalization group equations for $\alpha_i(\mu)$ effective coupling constants are $\mu\frac{d\alpha_i}{d\mu} = \frac{b_i}{2\pi}\alpha_i^2 + \sum_{j=1}^{j=3}\frac{b_{ij}}{4\pi^2}\alpha_i^2\alpha_j$, see refs. [62,63].

must have some ultraviolet cutoff (regulator) to make sence to the nonrenormalizable interaction (35) at quantum level. The most promising way to deal with nonrenormalizable theories is the use of nonlocal field theory [2-5]. The simplest nonlocal generalization of the renormalizable Yang-Mills Lagrangian has the form

$$L_{YM,nl} = -\frac{1}{2g_5^2}\mathrm{Tr}(F_{\mu\nu}V(-\Delta_\mu\Delta^\mu)F^{\mu\nu}), \tag{40}$$

where $F_{\mu\nu} = \Delta_\mu A_\nu - \Delta_\nu A_\mu$, $\Delta_\mu = \partial_\mu - iA_\mu$, $A_\mu = A^a_\mu T_a$ (8) and the formfactor $V(x)$ is an entire function on x. The use of nonlocal formfactor $V(p^2)$ with increasing behaviour in the Euclidean region at $p^2 \to -\infty$, for instance $V(p^2) = \exp(-p^2/\Lambda^2_{\Phi 1})$ makes the Yang-Mills model superrenormalizable. A possible nonlocal generalization of nonrenormalizable interaction (35) is

$$\Delta L_{F\Phi F\Phi,nl} = -\frac{1}{4\Lambda^2_{\Phi 1}}(\mathrm{Tr}(F_{\mu\nu}\Phi)V_{\Phi 1}(-\partial^\mu\partial_\mu)(\mathrm{Tr}(F^{\mu\nu}\Phi)) \tag{41}$$

with $V_{\Phi 1}(p^2) \sim \exp(p^2/\Lambda^2_{\Phi 1})$ The use of nonlocal formfactors V and $V_{\Phi 1}$ cures bad ultraviolet properties of nonrenormalizable interaction (35) and make it superrenormalizable. For nonlocal Lagragian (41) the parameter Δ in formula (36) depends on the scale μ

$$\Delta(\mu) = \frac{\pi\Phi_o^2}{\Lambda^2_{\Phi 1}}V_{\Phi 1}(-\mu^2). \tag{42}$$

We can use the normalization condition $V_{\Phi 1}(-M^2_{GUT}) = 1$. In this case formula (37) and numerical estimate for Δ are valid.

7. – Conclusions

1) The use of nonlocal field theory for gravity can cure bad ultraviolet properties of the theory and make quantum gravity superrenormalizable and unitary theory.
2) The use of nonlocal field theory allows to make γ_5-anomalous models meaningful.
3) There are ultraviolet finite field theory models with infinite number of local fields with local interactions. An account of infinite number of local fields leads to ultraviolet finite nonlocal field theory.
4) Nonlocal generalization of Georgi-Glashow $SU(5)$ GUT allows to overcome the problems with fast proton decay and wrong Weinberg angle prediction. The price of such modification is the absence of predictive power for Weinberg angle θ_W. For nonlocal $SU(5)$ GUT the nonlocal scale is $\Lambda = M_{GUT} \approx 3 \cdot 10^{16}$ GeV.

* * *

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(8) Here T_a are the $SU(5)$ matrices with $\mathrm{Tr}(T_aT_b) = \frac{1}{2}\delta^a_b$ and g_5 is the $SU(5)$ gauge coupling constant.

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