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Towards a global and multi-purpose equation of state: Quasi-clusters for an effective treatment of short-range correlations

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Summary. — The formation of nuclear clusters at subsaturation densities constitutes an essential feature for the construction of the equation of state (EoS). Phenomenological models that make use of energy density functionals (EDFs) offer a convenient approach to account for the presence of these bound states of nucleons when clusters are introduced as additional degrees of freedom. However, in these models clusters dissolve when the density approaches the saturation density, so that only nucleons survive at higher densities, in contrast with recent evidences of sizeable short-range correlations (SRCs) even at a larger density. We propose a novel approach which allows, within the EDF framework, incorporating SRCs at supra-saturation densities, by using effective clusters immersed in dense matter as a surrogate for correlations. Our idea is to improve the modelization of the EoS, by embedding the SRCs within generalized relativistic energy density functionals through the introduction of suitable in-medium modifications of the cluster properties. As a first exploratory step, the example of a quasi-deuteron in a relativistic mean-field model with density dependent couplings is explored. Implications for astrophysical applications and for general aspects of reactions dynamics, such as the clustering processes emerging in heavy-ion collisions, are envisaged.

1. – Introduction

The formation of nuclear clusters, whose emergence as many-body correlations is attributed to the properties of the nucleon-nucleon (NN) interaction, constitutes an essential feature for developing reliable models for the nuclear equation of state (EoS). A plethora of different theoretical models was employed in the last decades for the construction of global and multi-purpose tables with EoS data, which are fundamental ingredients for astrophysical simulations [1]. However, most of these models take into account only nucleons as basic constituents and fail in describing in an appropriate way the formation of clusters at densities below nuclear saturation.

Some progress was recently achieved by using phenomenological approaches, which explicitly introduce clusters as additional degrees of freedom at low densities [2]. These phenomenological models are generally based on energy density functionals (EDFs), as derived in self-consistent mean-field approximation. An effective in-medium interaction, which has no direct link to the NN interaction in free space, is then usually employed [3]. Relativistic formulations should be generally preferred, although non-relativistic Skyrme or Gogny-type functionals are also widely adopted.

The description of dilute matter as a mixture of nucleons and nuclei in thermodynamic equilibrium, by neglecting the interaction among the constituents, is at the basis of the model known as nuclear statistical equilibrium (NSE) [4]. Such a model is largely used in astrophysics where it has revealed successful to describe, *e.g.*, the isotopic composition of the proto-neutron star crust [5].

A common feature of the phenomenological models with clusters is that they are constructed in such a way that clusters dissolve when the density approaches the nuclear saturation density n_0. In traditional NSE models, the cluster dissolution, *i.e.*, the so-called Mott effect, is usually realized through the geometric excluded-volume mechanism, for which a finite size is assigned to every cluster. So they cannot exist above a certain density, since the maximum packing fraction is reached [4, 6-8]. On the other hand, more recent approaches aim at more microscopically addressing in-medium effects, by assuming that the mass of the clusters can change, realized through the introduction of proper binding-energy shifts. This is the case for the generalized relativistic density functional (GRDF), which is based on an extension of the relativistic mean-field (RMF) model, where light clusters are added as explicit degrees of freedom. In this approach, the effective interaction between nucleons, free or bound in clusters, is described by the exchange of mesons with density dependent couplings. A cluster will dissolve when its effective binding energy vanishes, since no bound state exists anymore [2].

Regardless of the dissolution mechanism, above saturation cluster-free nuclear matter is described in a mean-field framework as a free Fermi gas of quasi-particles with the usual step function in the single-particle momentum distribution at zero temperature. Such a picture is in contrast with recent experimental findings from nucleon knock-out on nuclei using inelastic electron scattering [9, 10]. These works evidence the smearing of the Fermi surface and a high momentum tail approximately decreasing with the inverse fourth power of the momentum in the single-nucleon distribution of cold nucleonic matter [10-12]. This feature is ascribable to the existence of nucleon-nucleon short-range correlations (SRCs), attributed to the tensor components and/or the repulsive core of nuclear forces. Interestingly, the height of the tail is practically identical for neutrons and protons even in a neutron-proton asymmetric system, producing a strong enhancement of the tail of the minority species as compared to the majority one [13-16]. Significant efforts have been made to constrain the isospin-dependence of this distribution, bringing to light that these correlations make the kinetic symmetry energy considerably different from the free Fermi gas prediction [17, 18]. This highlights once again the importance of incorporating SRCs in established models for the EoS.

In the present work, we sketch the idea of effectively treating SRCs at supra-saturation densities, by using effective resonances or quasi-clusters immersed in dense matter as a surrogate. Our aim is to estimate the modification one should expect on the high-density behavior of the mass-shift introduced in the GRDF to account for the cluster dissolution. Since two-body correlations in the np $^3S_1$ channel are much more important than others, the quasi-deuteron is chosen in this pioneering work and the zero-temperature case, when these deuterons form a boson condensate, is investigated.

The paper is organized as follows. In sect. **2** some general properties of the mass-shift parameterization are illustrated, also invoking some instructive remarks on the GRDF. Preliminary results for the symmetric nuclear matter (SNM) case at zero temperature are then shown in sect. **3**, by assuming the boson condensation condition at high-density. Conclusions and an outlook are delineated in sect. **4**.

2. – Theoretical formalism

The fundamental principles of the GRDF and the RMF model with density-dependent couplings may be found in [2, 19-21]. For the purpose of the present work, it shall be therefore sufficient to recall only the definition of the quantities directly involved in the mass-shift evaluation, referring the reader to the cited works for more details.

Let us consider the simplified case of isospin symmetric matter composed only of nucleons (labeled as *nuc* in the following) and deuterons (labeled as d), which are characterized by their particle number A_i $(i = nuc, d)$ and mass m_i. Such a system is completely determined by specifying the baryon density $n_b = n_{nuc} + 2n_d$ and the deuteron mass fraction X_d, such that the nucleon n_{nuc} and the deuteron n_d particle number density would be defined as

$$n_{nuc} = n_b\,(1 - X_d)\,, \qquad n_d = n_b \frac{X_d}{2}, \tag{1}$$

respectively, while its chemical composition would be fixed by the equilibrium condition $\mu_d = 2\mu_{nuc}$ among the chemical potentials μ_{nuc} and μ_d of the degrees of freedom involved. By assuming the boson condensation condition for the deuterons, the latter relation can also be expressed as

$$m_d^* + V_d = 2\left[\sqrt{k_{nuc}^2 + \left(m_{nuc}^*\right)^2} + V_{nuc}\right], \tag{2}$$

where the nucleon Fermi momentum is defined as $k_{nuc} = [(3\pi^2/2)n_{nuc}]^{1/3}$. The effective mass m_i^* and the effective chemical potential μ_i^*

$$m_i^* = m_i - S_i, \qquad \mu_i^* = \mu_i - V_i \tag{3}$$

involve scalar S_i and vector V_i potentials. In the case of symmetric nuclear matter, two different types of mesons are considered, namely a scalar σ meson to describe the attraction between nucleons and a vector ω to reproduce the repulsion. A minimal coupling between baryons and mesons is assumed and the scalar and vector potentials are defined as

$$S_i = \chi_i A_i C_\sigma n_\sigma - \Delta m_i, \qquad V_i = \chi_i A_i C_\omega n_\omega + A_i V_i^{(r)} + W_i^{(r)}, \tag{4}$$

where χ_i is a scaling factor and the coefficients $C_j = \Gamma_j^2/m_j^2$ $(j = \sigma, \omega)$ are connected to the density-dependent nucleon-meson coupling strengths Γ_j with the meson masses m_j. Moreover, a density dependent mass-shift Δm_i appears in the scalar potential of eq. (4) as an extra contribution only for the deuterons. The definition of the source densities n_j of the σ and ω meson fields, of the rearrangement potential $V_i^{(r)}$ and of the mass-rearrangement contribution $W_i^{(r)}$ are provided in refs. [20, 21].

The natural choice to assume that the nucleons inside the deuteron couple to the mesons with the same unitary scaling factor as the free nucleons ($\chi_{nuc} = \chi_d = 1$) simplifies the theoretical framework depicted above. Indeed, in that case, eq. (2) directly returns a useful expression for the deuteron-mass-shift

$$\Delta m_d = -m_d + 2\left[C_\sigma n_\sigma + \sqrt{k_{nuc}^2 + (m_{nuc} - C_\sigma n_\sigma)^2}\right], \tag{5}$$

which holds under the specific assumption of the condensation condition at high-density.

However, in general, the mass-shift Δm_d could be calculated explicitly by solving the in-medium many-body Schrödinger equation and would present several contributions, coming from the Pauli blocking or, in case of stellar matter with Coulomb interaction, due to the screening of the electronic background. The results can then be approximated by suitable parameterizations. For example, in the zero temperature symmetric nuclear matter case we are focusing on, by neglecting the momentum of the cluster with respect to the medium, a linear increase of the mass-shift is predicted at low baryon densities. This is consistent with the results obtained from recent calculations [2, 22-24]. However, beyond dissociation density, a heuristic extrapolation is usually adopted. It assumes a stronger density dependence, which is introduced to prevent the clusters to reappear.

The possibility of using quasi-deuterons to effectively embed nuclear SRCs at supra-saturation density requires thus a proper change of the usual parameterization adopted in the GRDF model, allowing the clusters to survive even beyond saturation density. Such a parameterization should be appropriately chosen to interpolate between the low-density limit constrained by microscopic many-body calculations and the high-density behavior described by (5) under the assumed boson condensation condition. In sect. **3** we finally explore the possibility to find such a parametrization.

3. – Results

In fig. 1, the mass-shift parameterization of the original GRDF model, as assumed by ref. [21], is compared to those calculated according to eq. (5), by employing the DD2 parameterization [2] of the nucleon-meson couplings and by fixing different values for the deuteron fraction X_d. The results show that a substantial modification is required by the deuteron mass-shift parameterization at high densities, where Δm_d is calculated by eq. (5) and turns out to be much lower than the heuristic GRDF parametrization [21]. As one intuitively expects, fig. 1 shows that the largest deuteron fraction generally corresponds to the lowest mass-shift, at least below a crossing point which is observed at $n_c \simeq 0.55$ fm^{-3}. A suitable interpolation of the mass-shift between the low-density limit from the microscopic Pauli blocking calculation and the high-density limit of the condensate model could then in principle be introduced, to get an extended GRDF model with quasi-clusters at supra-saturation densities.

When performing such an interpolation to have at high-density a fixed deuteron fraction value, an overall extra binding is predicted in the density dependence of the energy per particle of SNM with respect to the original DD2 parameterization without deuterons, as one can observe in fig. 2. Although the considered interpolation allows recovering the correct low-density behavior, *i.e.*, the deuteron binding energy at zero density, the over binding observed when increasing the deuteron fraction implies that a proper refit of the

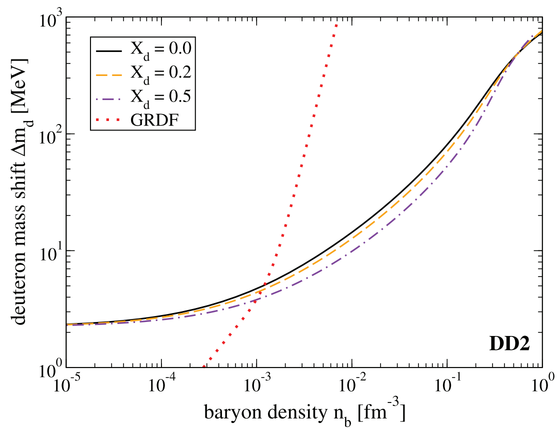

Fig. 1. – Baryon density dependence of the deuteron mass-shift Δm_d, as obtained in SNM at zero temperature by eq. (5), for fixed values of the deuteron mass fraction X_d. The DD2 parameterization of the nucleon-meson couplings, as provided in ref. [2], is used. The parameterization adopted in the original GRDF model (red dashed line) is also shown for comparison.

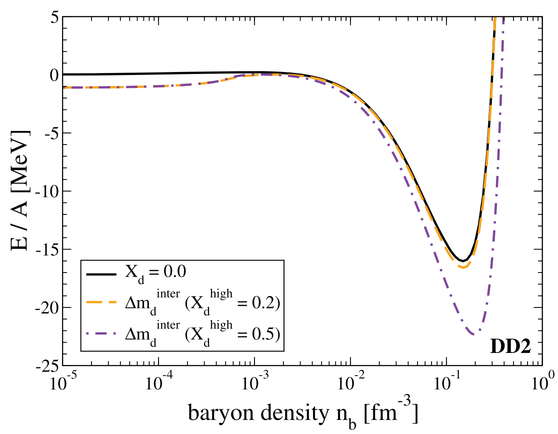

Fig. 2. – Baryon density dependence of the energy per particle E/A in cold symmetric nuclear matter. The case without deuteron (full black curve) is compared with the ones obtained by assuming deuteron mass-shifts which produce at high-density different (fixed) deuteron fraction values. The DD2 parameterization of the nucleon-meson couplings, as provided in ref. [2], is used.

nucleon-meson couplings to the SNM saturation properties is mandatory if one wants to keep nuclear matter quantities around n_0 well constrained.

Moreover, constraints around saturation might be imposed also on the mass-shift from the SRC fraction as found from recent experimental investigations in various measured nuclei by extrapolating to infinite matter. They assess that SRCs pairs amount to approximately 20% of the nucleon density [13, 14, 25].

Some possible mass-shift parameterizations, as experimentally constrained at saturation and in the low-density limit by microscopic many-body calculations, are shown in fig. 3, panel (a). Then, in fig. 3, panel (b), the same parameterizations are employed to deduce predictions for the high-density behavior of the deuteron mass fraction.

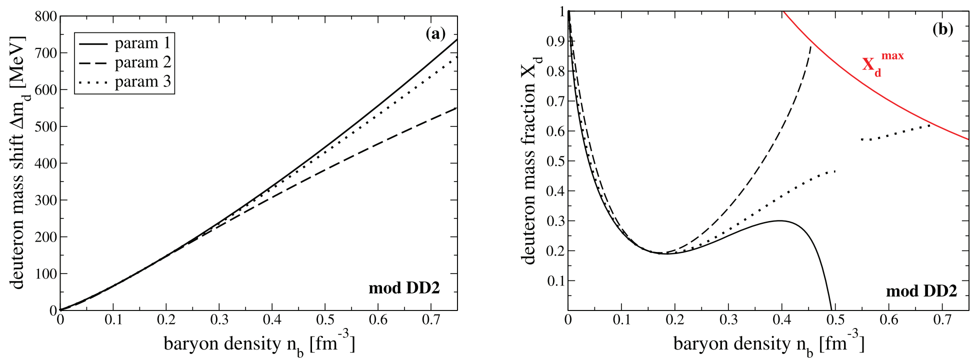

Fig. 3. – (a) Baryon density dependence of three exemplary deuteron mass-shift parameterizations. These mass-shifts are constrained in the low-density limit by microscopic many-body calculations and nuclear matter parameters at saturation. (b) Baryon density dependence of the deuteron mass fraction X_d as determined with the parameterizations considered in panel (a). The maximum allowed value for the deuteron mass fraction $X_d^{\max}$ (see text) is also indicated. In both panels, the DD2 nucleon-meson effective interaction is employed, with properly rescaled σ and ω coupling strengths.

As one observes in fig. 3, panel (b), three main alternative scenarios emerge. On the one hand, one could have a strong increase of the deuteron mass fraction, such that X_d tends to overcome the maximum allowed value $X_d^{\max}$ before the crossing point. However, this behavior can not be accepted since it implies a negative Dirac effective mass of the nucleons. On the other hand, one could have a solution with a sudden disappearance of the cluster, again before n_c. This solution produces interesting variations in the thermodynamical quantities. In particular, a discontinuity would appear in the matter incompressibility, as the signature for the possible emergence of a second order phase transition. This feature is in analogy to what was observed in previous works at low-density [26, 27] owing to the diseappearance of pairing correlations. Work is in progress to better clarify this analogy and the observed features.

A third scenario manifests itself when the cluster survives up to the crossing point. In that case, a critical behavior for X_d emerges around n_c, owing to the strong sensitivity of X_d to the mass-shift around this point as observed in fig. 1. This behavior is connected to the emergence of a pole in the density dependence of the deuteron fraction, which might be removed only with a fine tuning of the density dependence of the mass-shift. Investigating the physical reason behind the appearance of this pole goes actually beyond the scope of the present work. However, it is worthwhile to mention that, according to our recent analysis [28], the pole does not occur with a proper reduction of the scaling factor χ_d related to the coupling to the mesons of the nucleons bound inside the deuterons. Quite interestingly, a similar reduction was also suggested in some recent works [29,30] to take into account in-medium effects in describing results from the INDRA Collaboration [31].

4. – Conclusions and outlooks

In this paper, we focused on a phenomenological model based on nucleon and cluster degrees of freedom. It aims at providing a global and multi-purpose EoS that accounts for the dissolution of bound states of nucleons at high-density, through the introduction of proper mass-shift parameterizations of cluster degrees of freedom. We propose a novel idea to effectively account for the existence of SRCs, evidenced by recent experiments, by using quasi-deuterons as surrogates. In such a way, we aim to improve the description of nuclear matter at supra-saturation densities in EDFs by effectively considering correlations.

The possibility to perform an interpolation of the deuteron mass-shift between the constraints existing at low-density and the high-density condensate model proposed here is explored. Our analysis permits to recover the correct low-density limit of the EoS and suggests the emergence of a critical behavior at high-density, which calls for further investigations. This first exploratory step requires of course further developments, to extend the present calculations to asymmetric nuclear matter and finite temperatures, even including the presence of heavier clusters. More generally, we aim at achieving a more comprehensive description of correlations and clustering phenomena, which still represent a challenge from a theoretical point of view, despite the importance of these features in the widest scope of astrophysical applications and for general aspects of reactions dynamics emerging in heavy-ion collisions [32].

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