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# Role of $^6$Li non-sphericity in nuclear reactions below the Coulomb barrier

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**Summary.** — We investigate the impact of the $^6$Li ground-state deformation on Coulomb-barrier penetrability in nuclear reactions between $^6$Li and a structureless projectile. The $^6$Li ground state is described through a quantum di-cluster model, including a quadrupolar component which induces a tensor term in the projectile-target interaction. The corresponding ground-state form factor, for each possible $^6$Li orientation, is employed as a potential barrier in a Wentzel-Kramers-Brillouin-Jeffreys (WKB) radial penetrability calculation. The formalism was applied to the $^6$Li–p scattering. Throughout the sub-Coulomb energy range, we find no significant influence of the tensor interaction on the overall penetrability.

## 1. – Introduction

In a nuclear reaction between charged particles at energies below the Coulomb barrier, the process of overcoming the reactants electrostatic repulsion through quantum tunnelling plays a paramount role, hence the need to study it in detail. In heavy-ion fusion reactions, correlations between the reactants internal degrees of freedom are known to increase the sub-barrier cross sections [1]. In light-ion reactions, the role of clustering in the barrier penetration process has been studied through semi-classical models [2], in the attempt to explain a set of experimental anomalies known as the electron screening problem, which is also important for its possible astrophysical implications. The goal of this work is to analyse the effects of the quadrupolar deformation of the $^6$Li ground state on the Coulomb barrier seen by another nucleus, which is a proton in the present application. In particular, a $^6$Li wave-function including both $s$ and $d$ components is constructed phenomenologically as in ref. [3] (sect. 5.3), then employed to generate a form factor for the $^6$Li–p interaction, in analogy with the procedure in refs. [3, 4]. Finally, the transmission coefficient for barrier penetration is computed in the WKB approximation as in ref. [2]. The model and the performed calculations are covered in greater detail in ref. [5].

 

## 2. – Construction of the $^6$Li di-cluster-model deformed state

The $^6$Li nucleus is here modelled as a bound state of two inert clusters, an $\alpha$-particle and a deuteron, each found in the ground state of the corresponding isolated nuclide, with spin-parity $0^+$ and $1^+$ respectively. Let $|\Psi_{1,\sigma}\rangle$ be a state for the internal motion of both clusters, in which the deuteron has definite spin projection equal to $\sigma$. Similarly, let $|\mathrm{Li}_{1,M}\rangle$ be the ground state of $^6$Li with spin-parity $1^+$ and spin projection $M$. To correctly couple both angular momentum and parity, the orbital-angular-momentum modulus quantum number for the inter-cluster motion, $L$, must be 0 or 2. The $^6$Li state, projected on a specific inter-cluster displacement $\boldsymbol{r}$, expressed in spherical coordinates as $\boldsymbol{r} = (r, \theta, \phi)$, can thus be written as

$$\langle \boldsymbol{r} \,|\, \mathrm{Li}_{1,M}\rangle = \sum_{L,m} \langle (L,m),(1,M-m)\,|\,1,M\rangle\, c_L \phi_L(r) Y_{L,m}(\theta,\phi)\,|\Psi_{1,M-m}\rangle \tag{1}$$

where $Y_{L,m}$ is a spherical harmonic, $\langle (j_1,m_1),(j_2,m_2)\,|\,J,M\rangle$ is a Clebsch-Gordan coefficient, the $c_L$ are a set of weights, and each $\phi_L(r)$ is a radial wave-function normalised to 1. Since only bound states are considered, all $\phi_L$ and $c_L$ are taken to be real, further setting that each $\phi_L(r)$ is non-negative at infinity and $c_0 = \sqrt{1-|c_2|^2}$. The radial wave-functions are the same employed in ref. [3] (sect. 5.3). $c_2$ can be adjusted with respect to the di-cluster-model predictions for the charge electric quadrupole moment [3] (eq. (5.4)) or the magnetic dipole moment (deduced from ref. [6], eq. (A.9)), finding respectively $c_2 = -0.0909$ or $-0.257$, using experimental data in [7]. The agreement may possibly be improved using a more microscopical construction of the radial wave-functions (as in ref. [4]). Within the present model, the conclusions of the study do not change by adjusting $c_2$ on either observable. The figures shown here refer to $c_2 = -0.257$.

## 3. – Construction of the projectile-target potential

Let $V_{1p}$ and $V_{2p}$ be phenomenological potentials, taken as central for simplicity, for the interaction between the structureless projectile, a proton, and each cluster composing the $^6$Li target. The potentials can be expressed in terms of the $\alpha$–d displacement, $\boldsymbol{r}$, and the $^6$Li–p distance, $\boldsymbol{R}$. The complete projectile-target potential, $V_{tp}(\boldsymbol{r},\boldsymbol{R})$, is then $V_{1p}(\boldsymbol{r},\boldsymbol{R}) + V_{2p}(\boldsymbol{r},\boldsymbol{R})$. To focus on the impact of ground-state deformations, all couplings to excited states are neglected. Then, the eigenvalue problem for the $\alpha$–d motion can be decoupled from the $^6$Li–p scattering problem. To solve the latter, it is sufficient to consider the form factor $\langle \mathrm{Li}_{1M'}|V_{tp}|\mathrm{Li}_{1M}\rangle$, denoted as $V(M,M',\boldsymbol{R})$. Performing the integration over $\boldsymbol{r}$, it is $V(M,M',\boldsymbol{R}) = \langle 1,M'|\tilde{V}_{tp}|1,M\rangle$, where $|J,M\rangle$ is a state describing only the $^6$Li spin degree of freedom, and $\tilde{V}_{tp}$ acts only on such spin and on $\boldsymbol{R}$.

$V_{tp}$ can be decomposed in multipoles of both coordinates ($\boldsymbol{r}$ and $\boldsymbol{R}$). In a classical cluster model, as in ref. [2], all multipoles would be retained. Here, the form factor selects only those compatible with the state under study ($|\mathrm{Li}_{1M}\rangle$ in eq. (1)), namely the monopole and the quadrupole. These multipoles of $V_{tp}$ generate, respectively, a central and a tensor component in $\tilde{V}_{tp}$, which can thus be written as

$$\tilde{V}_{tp} = U_C(R) + U_T(R)\frac{1}{\hbar^2}\left[\left(\widehat{\boldsymbol{J}}_t \cdot \boldsymbol{R}/R\right)^2 - \frac{1}{3}\widehat{J}_t^2\right] \tag{2}$$

where $\widehat{\boldsymbol{J}}_t$ is the $^6$Li spin vector operator and $R = |\boldsymbol{R}|$. The central functions $U_C(R)$ and $U_T(R)$ were computed as in ref. [4] (eqs. (9), (11)), but without truncating to first order in $c_2$. A sample of the radial profiles of the form factor computed in this manner is shown in fig. 1. The adopted $\alpha$–p interaction is the central part of the potential in ref. [8] (eq. (4.3)), while the d–p potential was obtained from the Fr2in code [9].

## 4. – The $^6\text{Li} + \text{p}$ barrier penetrability

In analogy with the approach followed in ref. [2], consider a fixed centre-of-mass collision energy $E$, spin projection $M$ of $^6$Li, and orientation of $\boldsymbol{R}$. The corresponding $s$-wave radial transmission coefficient, $T_{M,\theta}(E)$, was calculated in WKB approximation [10] (sect. VI.10), adopting a central potential barrier $V_{M,\theta}(R)$ equal to the form factor $V(M, M, \boldsymbol{R})$. Treating the complete non-central problem phenomenologically as a set of many central problems (as a function of the direction) greatly simplifies the computations, but may destroy some features of the system which are relevant for the process of interest. This appears to be the most severe approximation performed in the calculation discussed here.

Let $m$ be the reactants reduced mass and $Z$ the product of their charge numbers. The angle-integrated cross-section for barrier penetration in $s$-wave, $\sigma(E)$, can be expressed as $\sigma(E) = \hbar^2\pi/(2mE)\,T(E)$. Furthermore, let $S(E)$ be the corresponding astrophysical $S$-factor, defined as $S(E) = E\exp(2\pi\eta)\sigma(E)$, where $\eta = \alpha_e Z\sqrt{mc^2/(2E)}$, with $\alpha_e$ being the fine-structure constant and $c$ the speed of light in vacuum. Figure 2 displays, as orange points, the average over all orientations and spin projections of the computed astrophysical factor. The black solid line is instead the astrophysical factor calculated using the average of $V_{M,\theta}$ (shown as the black solid line in fig. 1) as potential barrier. The difference between the two calculations is seen to be negligible. Note that taking the average of the deformed potential is slightly different than considering a spherical $^6$Li, but the deviation is comparable with the uncertainty introduced by the phenomenological radial wave-functions employed in eq. (1).

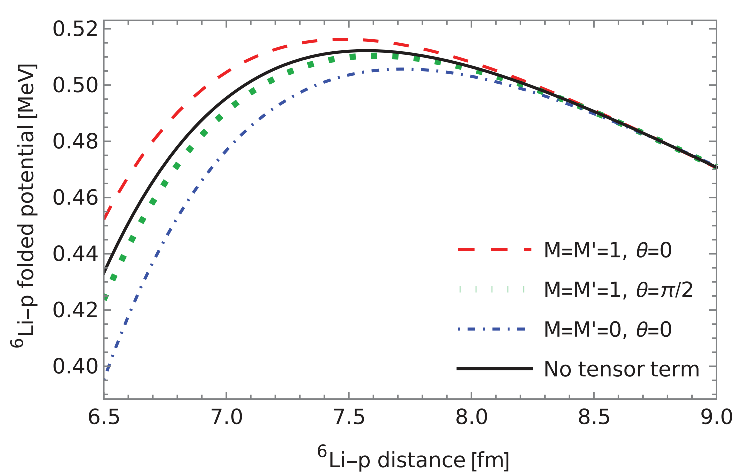


Fig. 1. – Radial profiles of the form factor $V(M, M', \boldsymbol{R})$, for different $^6$Li spin projections, $M$ and $M'$, and angles $\theta$ between projectile-target momentum and $^6$Li quantization axis. The black solid line is the average over all orientations (*i.e.*, just $U_C$ in eq. (2)).

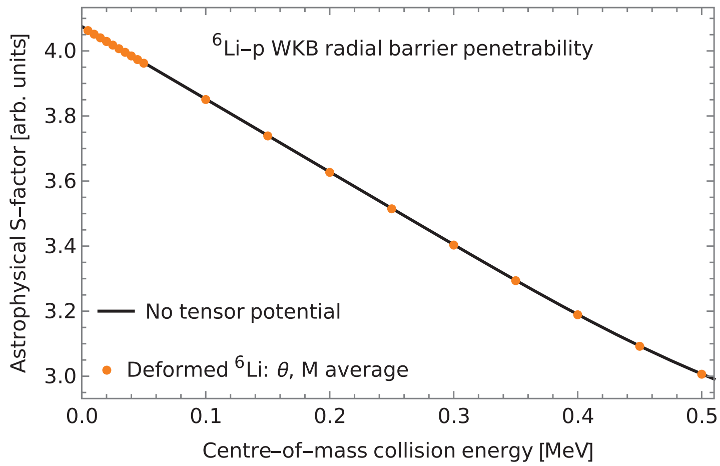


Fig. 2. – Preliminary astrophysical factor for $^{6}$Li–p WKB radial barrier penetration in arbitrary units. Orange points refer to the average of the penetrability over all angles and spin projections. The black solid line was computed using the potential shown as the black solid line in fig. 1. See text for details.

## 5. – Conclusions

The role of the ground-state quadrupole deformation of $^{6}$Li on Coulomb barrier penetrability in reactions with a proton was explored by modelling this system as a quantum bound state of $\alpha$ and d, expanding the semi-classical model proposed in ref. [2]. Within this preliminary calculation, the tensor interaction generated by the deformations does not affect the overall barrier penetrability. We stress that the possible impact of dynamic deformations or reorientations was not investigated here. The present formalism would benefit by an improved treatment of the cross-section calculation, for instance using some of the ideas discussed in ref. [1]. It may also be relevant to provide a more microscopic form for the $^{6}$Li wave-function, following the construction in ref. [4], and refine the adopted projectile-cluster interactions.

REFERENCES


[1] Hagino K. and Takigawa N., *Prog. Theor. Phys.*, **128** (2012) 1061.
[2] Spitaleri C. *et al.*, *Phys. Lett. B*, **755** (2016) 275.
[3] Nishioka H. *et al.*, *Nucl. Phys. A*, **415** (1984) 230.
[4] Merchant A. C. and Rowley N., *Phys. Lett. B*, **150** (1985) 35.
[5] Perrotta S. S., PhD Thesis (Università degli Studi di Catania) 2022.
[6] Mason A. *et al.*, *Eur. Phys. J. A*, **39** (2008) 107.
[7] International Atomic Energy Agency, `www-nds.iaea.org/nuclearmoments`.
[8] Bang J. and Gignoux C., *Nucl. Phys. A*, **313** (1979) 119.
[9] Brown B. A. and Thompson I. J., `people.nscl.msu.edu/brown/reactioncodes`.
[10] Messiah A., *Quantum Mechanics*, 2nd edition (Dover Publications) 2014.