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Magnetic field screening in strong crossed electromagnetic fields(*)

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Summary. — The gravitomagnetic interaction of a rotating black hole (BH) with a surrounding magnetic field, aligned and parallel to the BH axis, leads to an electric field with strength proportional to the magnetic field. Here, we study the magnetic field screening process in proportional electric and magnetic fields, operated by the combination of the motion of a huge number of electron-positron pairs, the production of synchrotron photons by the pairs, and the magnetic pair production process. We simulate this process for an initial magnetic field of the order of 10^{12} G and a seed of 10^{10} pairs. We obtain a reduction of the magnetic field strength of a few percent in a timescale shorter than femtoseconds. These results might have consequences on the high-energy (GeV) emission of astrophysical systems like gamma-ray bursts (GRBs). The reduction of the magnetic field strength implies a less efficient magnetic pair production, increasing the probability of GeV photons to leave the system.

1. – Introduction

The screening of a strong electric field by $e^\pm$ pairs created by QED processes has been studied for years (see *e.g.*, [1] for a recent study). No similar process has been developed for the magnetic field. In this paper, we propose a model for the magnetic field screening (MFS). Differently from the electric field case, the MFS is based on a combination of a series of processes: 1) an initial number of pairs is present or injected([1]) in a region with electric field $\mathbf{E} = E\hat{y}$ and magnetic field $\mathbf{B} = B\hat{z}$, where $E(t) = \Upsilon B(t)$, with $0 < \Upsilon \le 1$ and $B < B_{cr} = m_e^2 c^3/e\hbar \approx 4.4 \times 10^{13}$ G; 2) the pairs are accelerated (by $\mathbf{E}$) and start to radiate photons via curvature/synchrotron (or their combination) processes (by $\mathbf{B}$); 3) these photons interact with the background magnetic field via the magnetic pair production (MPP) process, generating new pairs; 4) the initial and created pairs, during their motion, generate a counter current that induces a magnetic field that opposes the original one, hence screening it. We show that a reduction of a few percent of the field strength effectively occurs if a huge number of pairs is present ($\ge 10^{10}$) and they have a dominant perpendicular velocity component.

This model can be applied to astrophysical systems like GRBs. Following the "*inner engine*" model [2,3] for binary-driven hypernova (BdHN) [4-6], the gravitomagnetic interaction between a rotating BH and a magnetic field induces an electric field([2]) that accelerates electrons which emit synchrotron GeV photons. If the MFS is efficient, the MPP optical depth for these photons decreases and they could escape from the system explaining the observed GeV emission.

2. – The theoretical model

2.1. *Assumptions.* – We describe the particles motion as a fluid since: 1) for the high strength of the fields all the particles follow the same dynamics and trajectory; 2) the particles flux obeys the continuity equation with a flux equal to 0 through the lateral surface of the tube flux, while it has the same amount through the upper and lower surfaces (e^- and e^+ move in opposite directions). We neglect quantum-mechanical effects, so we are in a regime where the $e^\pm$ cyclotron radius $R_L = cp/eB$ is larger than their de Broglie wavelength $\lambda = \hbar/p$ (see [7]), where $p = m\gamma vc$([3]) is the e^- momentum, which implies that $B \le B_{cr} v^2 \gamma^2$, and the work exerted by the electric force onto an electron/positron over a de Broglie wavelength satisfies, $eE\lambda < mc^2$, which translates into $E < \gamma v E_{cr} = \gamma v B_{cr}$. These conditions set the initial values (see sect. (**3**)) of B_0 and γ_0. The short characteristic time of the MFS, $10^{-21} \le \Delta t \le 10^{-15}$ s, which implies $10^{-11} \lesssim \Delta r \lesssim 10^{-5}$ cm, validates the assumption of Minkowski spacetime and the constancy of E/B (see [8]).

2.2. *Particles dynamics.* – The dynamics of the particles is driven by the eqs. of motion eq. (1a) and energy balance eq. (1b). In eq. (1b), I is the energy loss per unit time due to the emission of radiation by an accelerated particle while $\overline{H}(\chi)$ (whose expression is defined in [9]), with $\chi \equiv \varepsilon_*/2\,\varepsilon_e$ and $\varepsilon_* = \hbar\omega_* = \frac{3e\hbar\gamma^2}{2mc}\sqrt{(\mathbf{E} + \mathbf{v} \times \mathbf{B})^2 - (\mathbf{v} \cdot \mathbf{E})^2}$ the *critical photon energy*, determines the energetic regime of the emission (quantum or classical). The photon production rate is $\dot{N}_\gamma(t, \phi) = N_\pm(t, \phi)\,[I(t)/\varepsilon_\gamma(t)]$, where ϕ is

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([1]) At this stage we are not interested in the process of pair creation since it has no effect on the model.
([2]) This field polarizes the vacuum leading to the seed pairs that we are considering.
([3]) Through all the paper v ($\mathbf{v}$) is the particle velocity normalized to the light velocity c.

the angle between the particle/photon emission direction and the magnetic field, ε_γ the synchrotron photon energy and $N_\pm$ the number of new created pairs, whose production rate is $\dot{N}_\pm(t,\phi) = N_\gamma(t,\phi) R^{\rm e}_{\rm A}(t,\phi) c$, where $R^{\rm e}_{\rm A}$ is the attenuation coefficient for the MPP process and c the velocity of light:

$$
\text{(1a)} \qquad \frac{{\rm d}\mathbf{v}}{{\rm d}t} = \frac{e}{mc\gamma} \left[\mathbf{E} + \mathbf{v}\times\mathbf{B} - \mathbf{v}(\mathbf{E}\cdot\mathbf{v})\right],
$$

$$
\text{(1b)} \qquad \frac{{\rm d}\gamma}{{\rm d}t} = \frac{e}{mc}(\mathbf{E}\cdot\mathbf{v}) - \frac{I}{mc^2}, \quad \text{where } I \equiv \left|-\frac{{\rm d}E}{{\rm d}t}\right| = \frac{e^2\, m^2\, c^3}{\sqrt{3}\,\pi\,\hbar^2}\overline{H}(\chi),
$$

2˙3. MPP rate. – Let us define $\zeta \equiv R^{\rm e}_{\rm A}(t,\phi) c$. Following [10], the MPP rate in the observer frame at infinity (where we work) is, defining $\xi \equiv \sqrt{n_y^2\left(1 - \frac{E_y^2}{B_z^2}\right) + \left(n_x - \frac{E_y}{B_z}\right)^2}$

$$
\text{(2)} \qquad \zeta = \frac{0.23\alpha_{\rm f}c}{\lambda_{\rm c}}\frac{B_z}{B_{\rm cr}}\left(1 - \frac{E_y^2}{B_z^2}\right)\frac{\xi}{1 - \frac{E_y}{B_z}n_x}\exp\left[-\frac{8}{3}\frac{mc^2}{\varepsilon_\gamma}\frac{B_{\rm cr}}{B_z}\frac{1}{\xi}\right].
$$

with $\alpha_{\rm f}$ the fine structure constant and $\lambda_{\rm c} = \hbar/m\,c = 3.86\times10^{-11}$ cm the Compton length. This expression is valid until the following condition is satisfied (see [10-12] for details)
(3)

$$
\Psi = \frac{3}{4}\frac{e\hbar}{m\,c}\frac{B^2}{B_{\rm cr}}\gamma^2\sqrt{v_y{}^2\left(1 - \frac{E^2}{B^2}\right) + \left(\frac{E}{B} - v_x\right)^2}\sqrt{n_y{}^2\left(1 - \frac{E^2}{B^2}\right) + \left(n_x - \frac{E}{B}\right)^2} \ll 1.
$$

In eqs. (2), (3), the vector $\mathbf{n}$ is the photon momentum director cosine (in the co-moving particle frame) $\mathbf{n} = \left\{\mathbf{u} + \mathbf{v}\left[\gamma + \frac{(\gamma-1)}{v^2}\nu\right]\right\}[\gamma(1+\nu)]^{-1}$, where γ and $\mathbf{v}$ are the $e^\pm$ Lorentz factor and velocity, $\nu = v_x\sin\Theta\cos\Phi + v_y\sin\Theta\sin\Phi + v_z\cos\Theta$, $\mathbf{u} = (\sin\Theta\cos\Phi,\ \sin\Theta\sin\Phi, \cos\Theta)$, Θ and Φ the photons polar and azimuthal emission angles, respectively (we set $\Theta = \Phi = \frac{\pi}{2}$).

2˙4. Induced magnetic field. – The motion of a particle is a combination between acceleration along z-direction, and a circular motion, in the (x,y)-plane. The related current densities are: $\mathbf{J}_\perp = e\,\mathbf{v}_\perp d_\lambda c$ and $\mathbf{J}_\parallel = e\,\mathbf{v}_\parallel d_\lambda c$, with $v_\perp = (v_x^2 + v_y^2)^{1/2}$, $v_\parallel = v_z$ and $d_\lambda = \frac{{\rm d}N_\pm}{{\rm d}l}$ the particle linear number density on a path ${\rm d}l$. Then, from the Biot-Savart law, we derive the time evolution of the induced magnetic field created by $\mathbf{J}_\perp$ that, at the coil centre ($z = 0$), is written as

$$
\text{(4)} \qquad\qquad\qquad \frac{{\rm d}B_{z,{\rm ind}}}{{\rm d}t} = e\frac{v_\perp(t)}{R_{\rm c}(t)^2}\frac{{\rm d}N_\pm}{{\rm d}t},
$$

where $R_{\rm c}(t) = c/|{\rm d}\mathbf{v}/{\rm d}t|$ is the curvature radius of the particle's trajectory [9].

3. – Results

We show the integration of the system of equations for three particle emission directions: 1) along the $\hat{y}$-axis; 2) along the $\hat{z}$-axis; 3) along the "*generic*" direction characterized by $\theta = 75°$ and $\phi = 30°$. For each direction, we chose a value of B_0 and,

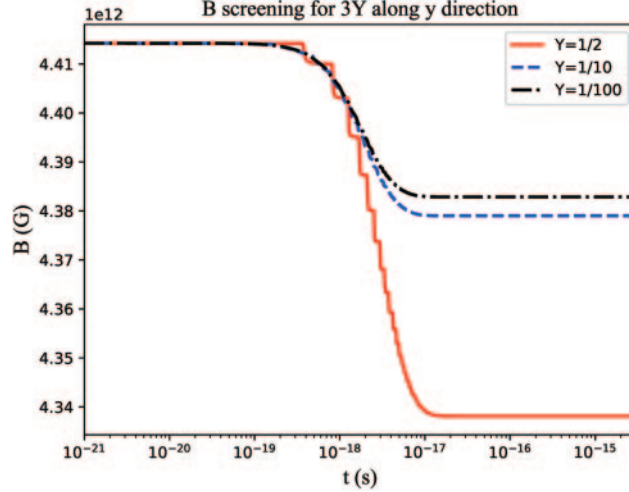Fig. 1. – MFS for selected values of Υ, $B_0 = 0.1\ B_{cr}$, $N_{\pm,0} = 10^{10}$ pairs emitted initially along the $\hat{y}$ direction.

consequently, the maximum value of particles Lorentz factor γ_0 that satisfies eq. (3) and the assumptions in sect. **2**˙1. The maximum initial upper values for B_0 and γ_0, for the directions (*y, z, generic*) are, respectively, $B_0 = 0.1\ B_{cr}$ and $\gamma_0 = (3.66,\ 7.098,\ 6.48)$ for $\Upsilon = 1/2$; $(3.71,\ 22.66,\ 4.18)$ for $\Upsilon = 1/10$; $(3.71, -,\ 3.81)$ for $\Upsilon = 1/100$. We also vary $N_{\pm,0} = 1,\ 10^3,\ 10^6,\ 10^{10}$, with $N_{\gamma,0} = 0$. In fig. 1, we show an example of the MFS for the three values of Υ, $B_0 = 0.1\ B_{cr}$ and $N_{\pm,0} = 10^{10}$ emitted along the $\hat{y}$-direction.

Summarizing, we have built a model to describe the MFS assuming specific approximations that make the problem tractable and allow to extract the physics behind it. We have shown that the screening of a few percent of a strong magnetic field occurs when a huge number of pairs are emitted with a major perpendicular component of their velocity and is more effective decreasing the value of Υ. This result is relevant for astrophysical systems like BdHN (*e.g.*, [3,9]) where the consequent reduction of the MPP due to the MFS could explain the observed GeV emission of long GRBs. We refer the reader to [11,12] for further details and results of the model.

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