IL NUOVO CIMENTO **45 C** (2022) 166
DOI 10.1393/ncc/i2022-22166-8

COMMUNICATIONS: SIF Congress 2021

Magnetically excited breather modes in an overlap-geometry Josephson tunnel junction

D. DE SANTIS(1)(*), C. GUARCELLO(2)(3), B. SPAGNOLO(1)(4), A. CAROLLO(1) and D. VALENTI(1)

(1) *Dipartimento di Fisica e Chimica "E. Segrè", Group of Interdisciplinary Theoretical Physics, Università degli Studi di Palermo - I-90128 Palermo, Italy*
(2) *Dipartimento di Fisica "E. R. Caianiello", Università degli Studi di Salerno I-84084 Fisciano, Salerno, Italy*
(3) *INFN, Sezione di Napoli, Gruppo Collegato di Salerno - Complesso Universitario di Monte S. Angelo, Napoli I-80126, Italy*
(4) *Radiophysics Department, Lobachevsky State University - 603950 Nizhniy Novgorod, Russia*

received 28 January 2022

Summary. — A scheme for the controlled generation of single traveling sine-Gordon breathers in a long Josephson junction is described. In the presence of dissipation, a direct current source, and thermal fluctuations, the theoretical analysis shows that, through the nonlinear supratransmission effect, tailored magnetic pulses at the junction's edge can effectively yield breather modes only.

1. – Introduction

Solitonic wave objects, whose emergence is due to a fascinating mechanism of compensation between dispersion and nonlinearity, constitute a topic of never-ending interest for the scientific community, both from a fundamental and an applicative point of view [1]. In particular, an excellent environment for the exploration of soliton-related phenomena is provided by the sine-Gordon (SG) equation, which arises in contexts such as pendula, superconductivity, gravity, high-energy physics, biophysics, and seismology [2].

Two classes of elementary SG solitons exist: kinks and breathers. Being topological excitations, the former manifest exceptional robustness against perturbations, while the latter are space-localized, time-pulsating bound states formed by kinks and antikinks. In contrast to kinks, breathers are rather elusive, and their detection in certain experimental realms can represent a tough challenge for researchers. In this regard, recent results include the tracking of optically excited breathers in cuprate superconductors [3] and the occurrence of breather-type oscillations of the global tectonic shear stress fields [4].

Here, a long Josephson junction (LJJ) is considered [5]. In this framework, a kink (also termed fluxon) describes a quantum of magnetic flux Φ_0, and it is both readily

(*) E-mail: duilio.desantis@unipa.it

trackable and of great practical use [6]. Instead, mainly due to its rapid oscillations and the dissipation-induced radiative decay, a definitive experimental proof of the Josephson breather's existence is still missing [7]. On the application side, such a mode offers some intriguing possibilities, *e.g.*, in information transmission [8] and quantum computation [9]. To these ends, studies focusing on breather excitation and manipulation techniques are vital.

It is worth mentioning that topologically neutral nonlinear waves, such as fluxon-antifluxon couples and breathers, can play a main role in fluctuation-governed escape processes [10] as well as in a Kibble–Zurek–like scenario [11]. Also, breather-like regimes can often be observed at higher propagation speeds in Josephson junction chains [12,13].

In the present work, nonlinear supratransmission (ST), *i.e.*, the phenomenon of energy propagation in the forbidden band gap of a nonlinear medium [14, 15], is shown to be suitable for the controlled generation of single traveling breathers in a magnetic-pulse–driven LJJ. More specifically, taking into account dissipative effects, a possible dc current term, and a thermal noise source, the frequency/amplitude space of the external oscillating magnetic field is numerically explored to demonstrate the existence of significant regions in which the exclusive emergence of breather modes takes place.

2. – The model

In an overlap-geometry LJJ, the phase difference $\varphi(x,t)$ between the pair wave functions of the two superconductors is governed by the equation

$$\varphi_{xx} - \varphi_{tt} - \alpha\varphi_t = \sin\varphi - \gamma - \gamma_T(x,t). \tag{1}$$

Throughout the paper, the subscripts indicate partial differentiation, space is normalized to the Josephson penetration depth λ_J, and time is normalized to the inverse of the Josephson plasma frequency ω_p [5]. In eq. (1), α is a damping parameter accounting for the tunneling of quasiparticles, γ is the bias term in units of the critical Josephson current density J_c, and $\gamma_T(x,t)$ is a Gaussian, zero-average noise source with $\langle\gamma_T(x_1,t_1)\gamma_T(x_2,t_2)\rangle = 2\alpha\Gamma\delta(x_1-x_2)\delta(t_1-t_2)$, in which $\Gamma = 2ek_BT/(\hbar J_c\lambda_J)$ (T is the absolute temperature, e is the electron charge, k_B is the Boltzmann constant, and $\hbar$ is the reduced Planck constant).

If an alternating magnetic field is applied to the $x=0$ end of the LJJ, the boundary conditions read $\varphi_x(0,t) = \widetilde{A}(t)\sin(\Omega t)$ and $\varphi_x(l,t)=0$, where l is the device's normalized length. The forcing signal is chosen with a frequency laying in the junction's plasma gap, *i.e.*, $\Omega < 1$, and Gaussian switching-on/off regimes, *i.e.*, $\widetilde{A}(t) = Af(t)$ with $f(t) = \exp\left[-\frac{(t-t_\pm)^2}{2\sigma_\pm^2}\right]$ for $t \lessgtr t_\pm$ and $f(t)=1$ otherwise. Here, $t_+ = 3\sigma_+$ is fixed, whereas t_- is adjusted by monitoring the phase's evolution at a position $x=\bar{x}$, such that $0<\bar{x}\ll l$, to obtain single breathers.

The numerical integration of eq. (1), with initial conditions $\varphi(x,0) = \arcsin\gamma$ and $\varphi_t(x,0)=0$, is performed through an implicit finite-difference scheme. The discretization steps are $\Delta x = \Delta t = 0.005$, the junction length is set to $l=100$, which usually implies negligible reflection effects at $x=l$, and the observation time is $t_{\max} \geq 150$ to track the entire evolution of the induced breathers. Also, in what follows, $\alpha = 0.03$, $\gamma = 0$, $\sigma_+ = 10$, and $\sigma_- = 2.5$. The main panels of fig. 1 show the typical simulated profiles.

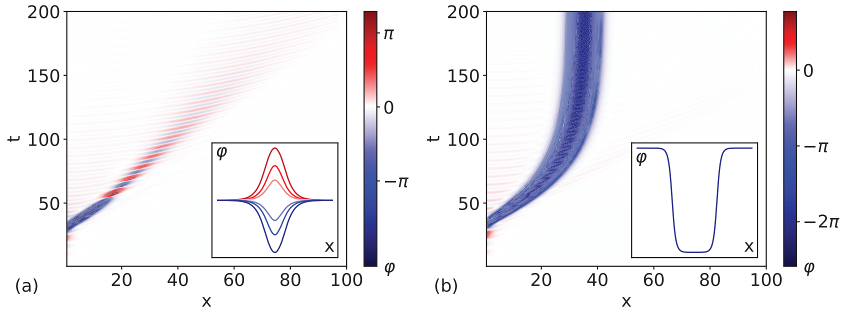

Fig. 1. – Phase profiles of a breather (panel (a)) and a fluxon-antifluxon pair (panel (b)). Two numerical runs with ST-induced traveling modes are displayed in the main panels. Parameter values: $t_{\max} = 200$, $\Gamma = 0$, $\Omega = 0.5$, and $A = 2.3$ in panel (a) ($\Omega = 0.45$ and $A = 2.25$ in panel (b)). The insets illustrate the corresponding analytical solutions of the pure SG equation.

3. – Results and discussion

In analyzing the (Ω, A) space, three cases can be distinguished. First, since the driving frequency Ω belongs to the plasma gap, within the linear approximation the medium supports evanescent waves only. According to the ST effect, there exists an Ω-dependent amplitude threshold for unlocking energy propagation in the form of purely nonlinear modes, such as kinks and breathers. One can identify the situations where at least a kink (or antikink) is present in the LJJ's final state and separate these cases from those in which just breathers remain from some point of the simulation onwards. In fact, in the presence of dissipation, breather modes radiatively decay within a time interval $\sim 1/\alpha$ (as illustrated in fig. 1(a), the breather waveform both fades and widens in time, see also ref. [16]), while kinks and antikinks persist as 2π phase twists (fig. 1(b)). Then, assuming an observation time much larger than the expected breather lifetime, *i.e.*, $t_{\max} \gg 1/\alpha$, the condition

$$\max_{x\in[0,l]} |\varphi(x, t_{\max})| \geq 2\pi \tag{2}$$

can only be satisfied if at least a kink (or antikink) is present at $t = t_{\max}$.

In the main panel of fig. 2, the noise amplitude is $\Gamma = 0$ and the (Ω, A) parameter space is explored by varying the driving frequency Ω in $[0, 1]$ and the amplitude A in $[0, 2.5]$, with increments $\Delta\Omega = \Delta A = 0.01$. Remarkably, vast breather-only (purple) areas are displayed, whereas the white color indicates (Ω, A) couples for which practically no energy transmission is sustained, and the yellow color corresponds to regions with at least one kink (or antikink) left in the junction at $t = t_{\max} = 200$ (see eq. (2)).

To test the approach's robustness against the thermal background, the fraction of breather-only cases out of N total realizations can be computed (by means of eq. (2)). As shown in the inset plot in fig. 2, that examines the (Ω, A) subsection $[0.4, 0.55] \times [2, 2.4]$ $(\Delta\Omega = \Delta A = 0.01)$ for $t_{\max} = 150$, $\Gamma = 0.03$, and $N = 520$, the breather-only region clearly survives in the noisy environment, and a sort of fluctuation-induced widening emerges as well. Thus, a nontrivial, positive effect due to the stochastic term is found.

Previous literature [7] has established that a direct current source drives a breather towards the breakup into a kink-antikink pair. Therefore, the breather-only areas (such

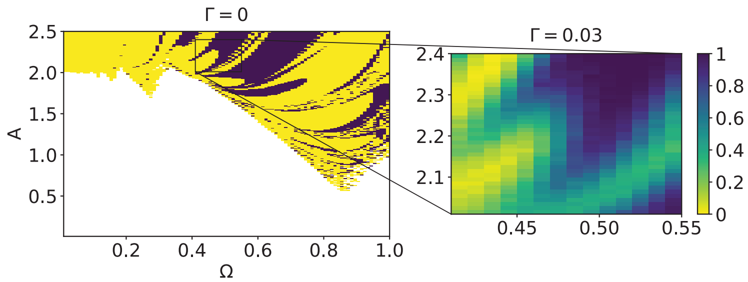

Fig. 2. – Main plot: Bifurcation diagram in the (Ω, A) plane. The white color indicates (Ω, A) couples for which practically no energy transmission is sustained, the yellow color corresponds to regions with at least one kink (or antikink) left in the junction at $t = t_{\max} = 200$, and the purple color is exclusively associated with breathers. The noise amplitude is $\Gamma = 0$. Inset plot: Fraction of breather-only cases out of $N = 520$ total realizations, calculated for an (Ω, A) subsection, in the presence of thermal fluctuations. Parameter values: $t_{\max} = 150$ and $\Gamma = 0.03$.

as those of fig. 2) are progressively seen to disappear upon increasing the value of γ (not shown here). Lastly, it should be mentioned that the proposed technique works well under a wide set of conditions, *e.g.*, for different values of the parameters α, Γ, σ_+, and σ_-. These aspects will be addressed in future papers [15].

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This work was financially supported through Project R4D08-P3PGDAR4_MARGINE, Università degli Studi di Palermo.

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