

Optimized state transfer in systems of ultrastrongly coupled matter and radiation

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Summary. — Ultrastrong coupling may allow faster operations for the development of quantum technologies at the expenses of increased sensitivity to new kind of intrinsic errors. We study state transfer in superconducting circuit QED architectures in the ultrastrong coupling regime. By using optimal control methods we find a protocol resilient to the main source of errors, coming from the interplay of the dynamical Casimir effect with cavity losses.

1. – Introduction: circuit-QED and adiabatic state transfer

Circuit-QED solid-state systems [1] made of artificial atoms (AA) and resonator modes [2] are paradigm models for studying fundamental physics from measurement theory [1] to quantum thermodynamics [3] and quantum communication [4] besides being one of the most promising platforms for quantum hardware [5].

Recently, solid-state ultrastrongly coupled (USC) AA-cavity systems have been fabricated [6,7] where the coupling constants g_i are comparable to the natural frequencies of the AAs (ϵ^i) and of the cavity (ω_c). These structures may in principle implement ultrafast quantum operations. The USC regime exhibits new physical effects of great fundamental interest but detrimental for quantum processing as the highly entangled nature of the eigenstates dressed by virtual photons [6-8] and the triggering of photon pairs by the

dynamical Casimir effect (DCE) when coupling constants are time-dependent. Multi-photon effects deteriorate the fidelity of quantum operations [9] in USC architectures even in absence of decoherence.

1.1. *Basic equations.* – To overcome this problem, a communication channel implemented by an adiabatic protocol similar to STIRAP [10] has been proposed [11]. The system of two qubits (eigenstates $|\sigma^i\rangle \in \{|g^i\rangle, |e^i\rangle\}$ for $i = 1, 2$), coupled to a single resonator mode is described by the Rabi Hamiltonian [2]

$$(1) \quad H_R = \omega_c a^\dagger a + \sum_{i=1,2} \epsilon^i \sigma_+^i \sigma_-^i + \sum_{i=1,2} g_i (a + a^\dagger) (\sigma_-^i + \sigma_+^i),$$

where a (a^\dagger) is the annihilation (creation) operator of the resonator mode satisfying $[a, a^\dagger] = 1$, $\sigma_+^i = |e^i\rangle\langle g^i|$ and $\sigma_-^i = |g^i\rangle\langle e^i|$ are the qubit rising and lowering operators. We consider resonant subsystems, $\omega_c = \epsilon^1 = \epsilon^2$. The Hilbert space is spanned by the factorized basis $\{|n\sigma^2\sigma^1\rangle\}$, where $|n\rangle$ are the oscillator's number eigenstates.

The Rabi Hamiltonian is adopted when the couplings g_i are large enough to overcome the decoherence rates of the qubits (γ^i) and of the cavity (κ). If in addition $g_i \ll \omega_c$ the Hamiltonian conserves approximately the number of excitations as described by the rotating wave approximation (RWA). In the ultrastrong USC regime, where $0.1\omega_c \lesssim g_i \lesssim \omega_c$, this is no longer true and the full Rabi Hamiltonian H_R has to be taken into account.

We consider time-dependent couplings $g_i(t)$. In the regime where the RWA holds, a STIRAP-like process implemented by turning on $g_2(t)$ before $g_1(t)$ yields the state transfer [11] $|0\rangle|g^2\rangle|\alpha g^1 + \beta e^1\rangle = |\psi_{\text{initial}}\rangle \rightarrow |0\rangle|\alpha g^2 + \beta e^2\rangle|g^1\rangle = |\psi_{\text{target}}\rangle$. We seek the performance of this protocol in the USC regime, allowing in principle much faster operations. The figure of merit is the transfer efficiency

$$(2) \quad \mathcal{F} = |\langle \psi_{\text{target}} | \psi_{\text{final}} \rangle|^2,$$

where $|\psi_{\text{final}}\rangle$ is the state of the system at the end of the process carried with the Hamiltonian equation (1). As in ref. [11], $|\psi_{\text{final}}\rangle$ is obtained by solving the Schrödinger equation $i\partial_t|\psi(t)\rangle = (H_R(t) - \frac{1}{2}\kappa a^\dagger a)|\psi(t)\rangle$ with $|\psi(-\infty)\rangle = |\psi_{\text{initial}}\rangle$, the extra non-Hermitian term describing cavity losses.

Figure 1(a) reports the transfer efficiency as a function of the inverse speed $(\omega_c T)^{-1}$ and the maximal coupling g_0/ω_c for Gaussian time dependence. We assume a cavity decay $\kappa = 0.005\omega_c$, which is a large figure, compensating the oversimplified description of decoherence sources [12-14]. The red lines refer to what could be obtained if terms non conserving the number of excitations were dropped, *i.e.*, the RWA. For the Rabi model the efficiency is still remarkably large even in the presence of cavity losses up to values $g \sim 0.3\omega_c$. Here operations are already much faster than for standard circuit-QED architectures where $g \sim 10^{-2}\omega_c$ and the RWA is applicable. The efficiency is smaller than with the RWA in what could suggest that an important loss mechanism are virtual and DCE photons appearing during the protocol, which are irreversibly lost by cavity decay.

2. – Results by optimal control

In order to improve the results of [11] in what follows, we investigate the optimal time-dependence of $g_i(t)$ using the Quantum Optimal Control (QOC) [15, 16] tools developed in [17].

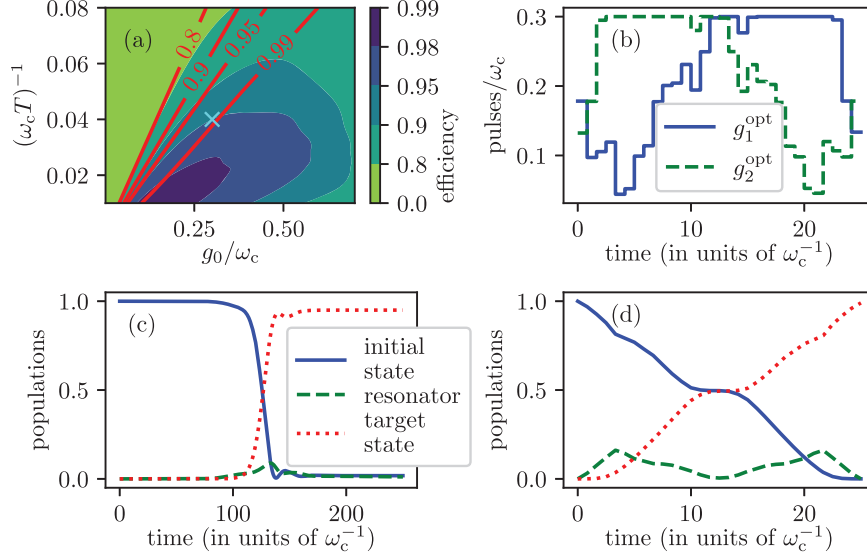


Fig. 1. – (a) Transfer efficiency \mathcal{F} for Gaussian pulses $g_1(t) = g_0 e^{-[(t-\tau)/T]^2}$ and $g_2(t) = g_0 e^{-[(t+\tau)/T]^2}$ and $\kappa = 0.005 \omega_c$. The red lines represent constant \mathcal{F} lines in the RWA where ideal STIRAP is implemented. The point $(\omega_c T)^{-1} = 0.04, g_0/\omega_c = 0.3$ (cyan x) is analyzed in detail in the other panels: (b) shows the optimal shape of the couplings found by QOC; (c) shows the population histories for Gaussian pulses ($\mathcal{F} \simeq 0.95$); (d) shows the population histories obtained with QOC ($\mathcal{F} \simeq 0.99$). Notice the different time scales in panels (c) and (d) showing that the QOC protocol is faster by a factor of ~ 3 .

We consider the point parameters $(\omega_c T)^{-1} = 0.04, g_0/\omega_c = 0.3$ of fig. 1(a). The optimized $g_i^{\text{opt}}(t)$ are step functions shown in fig. 1(b). The bottom panels of fig. 1 report the evolution of the populations for Gaussian pulses of width T (c) and for the QOC solution $g_i^{\text{opt}}(t)$ (d). For the former protocol the duration is $\sim 3T$ and $\mathcal{F} \simeq 0.95$ while the optimized pulses g_i^{opt} allow for $\mathcal{F} \simeq 0.99$ (even slightly better than RWA) in a time interval T , thus being also ~ 3 times faster. Notice that the optimized pulses (fig. 1(c)) are counter-intuitively ordered, thus this process is still STIRAP-like, with pulse shapes performing better. For the QOC case it is apparent that the mode is more populated during the protocol (see fig. 1(d)) but this happens for a shorter time so the impact of losses is reduced.

3. – Conclusions

We have shown that QOC techniques can improve state transfer in a two-qubit circuit-QED architecture in the USC regime. We found optimized $g_i^{\text{opt}}(t)$ yielding larger efficiency with a time duration shorter than for Gaussian pulses with the same maximal strength g_0 . It is likely that QOC, machine learning techniques [17,18] or superadiabatic driving [19] could further improve the transfer efficiency of such operations exploiting also modulation of detunings [20-22].

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