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Radiation squeezing in interacting quantum Hall edge channels

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Summary. — A mesoscopic system emits microwaves when subject to a periodic drive in the GHz range. The quantum features of this emitted radiation, such as squeezing, can be accessed by measuring the finite frequency photo-assisted noise in a quantum point contact geometry. In this context, we theoretically investigate the robustness of these quantum properties against electron-electron interaction using quantum Hall edge channels at $\nu = 2$ as the testbed.

1. – Introduction

In recent years, electron quantum optics emerged as a very active topic in the condensed matter agenda [1-3]. It aims at transposing conventional quantum optics concepts to describe electrons propagating in mesoscopic channels. However, electrons strongly differ from photons due to their statistics and their charged nature. This dramatically emerges in Quantum Hall (QH) systems at $\nu = 2$, where screened Coulomb interaction leads to phenomena such as charge fractionalization and energy relaxation [4, 5]. In this direction, Levitons, purely electronic wave-packets generated through properly quantized Lorentzian voltage pulses in time [6], show a major robustness as indicated by current fluctuations (noise) analysis [7-9].

The study of finite frequency noise at the output of periodically driven quantum point contacts (QPC) has also revealed a deep connection with the fluctuations of the microwave radiation emitted by driven mesoscopic devices [10, 11]. This quantum radiation shows squeezing in the frequency domain, which can be properly engineered by controlling the form of the applied drive [12]. A key ingredient to obtain such squeezed light is the presence of non-linearities which are typically observed in the current-voltage characteristics of a QPC geometry in the presence of interaction [13-15].

Accordingly, in this theoretical work, we will show that the squeezing of the radiation generated by a periodic train of Levitons remains relevant also in the presence of interaction, even if suppressed with respect to the free electron case [16].

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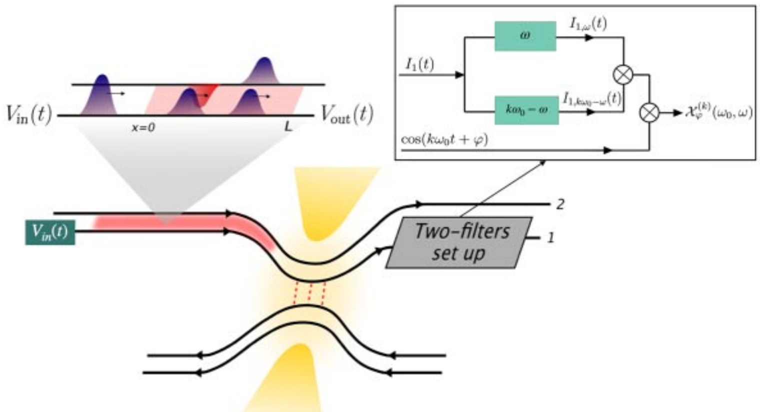

Fig. 1. – Schematic view of a QH bar at filling factor $\nu = 2$ in the QPC geometry. Top left: zoom of interacting region of length L where the injected charge is fractionalized. Top right: schematic inset view of the two-filters set-up.

2. – Model

We consider a QH bar at filling factor $\nu = 2$ (see fig. 1). The electronic excitations are injected through a time-dependent voltage source $V_{in}(t)$ modeled as an ohmic contact coupled to the inner channel. For the rest of this work we will focus on a periodic train of Lorentzian voltage pulses with period $\mathcal{T}$ and width $w = \eta\mathcal{T}$. Its AC contribution is

$$V_{in}^{(AC)}(t) = \frac{\tilde{V}}{\pi}\sum_{l=-\infty}^{+\infty}\frac{\eta}{\eta^2 + (\frac{t}{\mathcal{T}} - l)^2} - \tilde{V}, \tag{1}$$

with $\tilde{V}$ the amplitude of the AC part, while with $V_{in}^{(DC)}$ we indicate its DC part.

After the injection, the excitations flow along the inner channel which interacts with the outer one through a short range Coulomb repulsion over a region of finite length L. Following Wen's hydrodynamical description [17] the behaviour of these edge states is described by the Hamiltonian density

$$\mathcal{H} = \sum_{i=1,2}\frac{v_i}{4\pi}(\partial_x\phi_i)^2 + \frac{u}{2\pi}\partial_x\phi_1\partial_x\phi_2, \tag{2}$$

where the the chiral bosonic field ϕ_i(1) is related to the particle density operator ρ_i through $\rho_i(x) = (1/2\pi)\partial_x\phi_i(x)$ [18, 19]. The first term describes the bare propagation along the channels, while the second is the coupling contribution. It is possible to diagonalize eq. (2) through a rotation of an angle θ in the field space such that $\tan(2\theta) = 2u/(v_1 - v_2)$. This angle is related to the strength of interactions and is limited to the $0 \le \theta \le \pi/4$ interval. Physically, this diagonalization leads to a fractionalization process where a fast charged bosonic mode is separated from a slow dipolar one. These fractionalized charges enter the QPC region, which is assumed as non interacting

(1) Label $i = 1, 2$ denotes inner and outer edge channels as indicated in fig. 1 after the two-filters set up.

and whose tunneling process involves only the inner channels. After the QPC, a two-filter set-up is used in order to access the current fluctuations at finite frequency [20].

3. – Current fluctuations and electromagnetic quadratures

In the two-filter set-up shown in the right inset of fig. 1, the current of inner channel I_1 outgoing from the QPC is filtered at two different frequencies ω and $k\omega_0 - \omega$([2]). These contributions are multiplied among themselves and further multiplied with an external sinusoidal signal of the form $\cos(k\omega_0 t + \varphi)$. From this point on, we consider the driving frequency (ω_0) and the measurement one (ω) in the GHz range. According to this, it is possible to access the dynamical response of the noise at finite frequency ω defined as

$$\chi_{\varphi}^{(k)}(\omega_0, \omega) = \lim_{\mathcal{T}_m \to +\infty} \frac{1}{\mathcal{T}_m} \int_{-\frac{\mathcal{T}_m}{2}}^{+\frac{\mathcal{T}_m}{2}} \langle I_{1,\omega}(t) I_{1,k\omega_0-\omega}(t) \rangle \cos(k\omega_0 t + \varphi)\, \mathrm{d}t, \tag{3}$$

where the brackets indicate the zero temperature quantum mechanical correlator.

Furthermore, the current operator can be related to the emitted electromagnetic field annihilation operator through the simple relation $a(\omega) = -iI_1(\omega)/2\mathcal{A}(\omega)$, where $\mathcal{A}(\omega) = GF\hbar\omega$ with G is the conductance of the channel and F is the Fano factor. Through this relation, the electromagnetic field quadratures can be written as

$$A_{\varphi}(\omega) = \frac{1}{\sqrt{2}}[e^{i\varphi} I_1(\omega) + e^{-i\varphi} I_1(-\omega)]. \tag{4}$$

Then, the finite frequency noise is linked to the quadratures' fluctuations of the emitted electromagnetic field through

$$\Delta A_{2\varphi} = \sqrt{\chi_0^{(0)}(2\omega, \omega) + \chi_{2\varphi}^{(1)}(2\omega, \omega)}, \tag{5}$$

where $\Delta A_j = \sqrt{\langle A_j^2 \rangle - \langle A_j \rangle^2}$. Finally, due to the Heisenberg principle, the quadratures' fluctuations satisfy the uncertainty relation which states that

$$\Delta A_{2\varphi} \Delta A_{2\varphi+\pi} \geq \mathcal{A}(\omega). \tag{6}$$

4. – Results and conclusions

The effects of radiation squeezing results from eq. (6). Indeed, when the value of the fluctuations of one quadrature decreases below the quantum vacuum, the other must increases in order to preserve the uncertainty relation. In fig. 2 we show how Coulomb interactions affect squeezing of the two orthogonal quadratures when Levitons are injected into the channel through a periodic drive in the GHz range. They lead to a reduction of the minima of the two orthogonal quadratures with respect to the non-interacting case. However, it is worth noticing that, even for an interacting system, the squeezing effect still remains remarkable since it is well below the quantum vacuum.

([2]) Where $k \in \mathbb{N}$ and $\omega_0 = 2\pi/\mathcal{T}$.

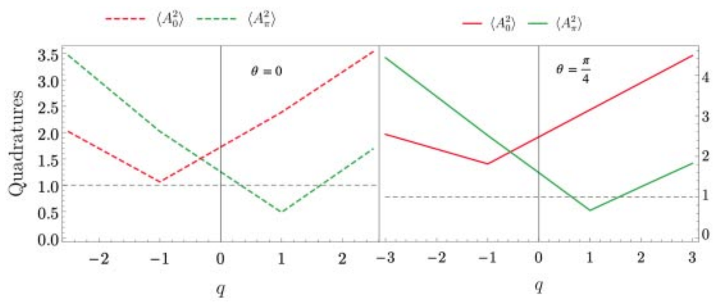

Fig. 2. – Quadratures of the emitted e.m. field in units of $\mathcal{A}(\omega)$ as a function of the number of injected electrons $q = eV_{\text{in}}^{(DC)}/\hbar\omega$. The two panels describe the orthogonal quadratures $\langle A_0^2 \rangle$ (red) and $\langle A_\pi^2 \rangle$ (green) for a Lorentzian voltage drive with $\eta = 0.1$ in the non-interacting case $\theta = 0$ (left) and in the interacting one $\theta = \pi/4$ (right). The dashed horizontal line indicates vacuum fluctuations. Other parameters are: $e\tilde{V}/\hbar\omega_0 = 0.856$, $L = 2.5\,\mu$m, $v_2 = 2.8 \times 10^4$ m/s, $v_1/v_2 = 2.1$ and $\omega_0/\omega = 2$.

In this work we have shown that the outgoing radiation from a mesoscopic set-up is strongly non-classical presenting single-photon squeezing even in the presence of interactions when Levitons are injected into the system. This quantum feature is reduced with respect to the non-interacting case but it is still evident for a Lorentzian voltage drive.

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