IL NUOVO CIMENTO **45 C** (2022) 223
DOI 10.1393/ncc/i2022-22223-4

Communications: SIF Congress 2021

Starquakes in millisecond pulsars and gravitational waves emission

E. Giliberti(1) and G. Cambiotti(2)

(1) *Dipartimento di Fisica, Università degli Studi di Milano - Via Celoria 16, 20133, Milano, Italy*
(2) *Dipartimento di Scienze della Terra, Università degli Studi di Milano*
Via Cicognara 7, Milano, 20129, Italy

received 31 January 2022

Summary. — Accreting neutron stars (NSs) have long been considered potential continuous gravitational-wave (GW) emitters. Centrifugal forces in these rapidly rotating objects can be so strong to break the neutron star crust (causing starquakes), deforming the star in a non-axially symmetric way, leading to the emission of GWs. At equilibrium, the angular momentum gained by accretion and the one lost via GWs emission should balance each other, stopping the stellar spin-up. We investigate the above physical picture by modeling the NS as a Newtonian model describing compressible, non-magnetized and self-gravitating object. In particular, we calculate the rotational frequency necessary to break the crust and estimate an upper limit for the consequent ellipticity. Depending on the stellar mass and its equation of state, we find that the maximum starquake-induced ellipticity ranges from 10^{-9} to 10^{-5}. Furthermore, the corresponding equilibrium frequency is below the higher NS frequency observed of 716.36 Hz for all the scenarios. Finally, we also discuss possible observational constraints on the ellipticity upper limit.

1. – Introduction

The astrophysical community is looking for observing gravitational wave signals (GWs) from rapidly rotating neutron stars (NSs), generated by permanent non-axisymmetric deformation called *mountain*. Recently it has been claimed [1] that starquakes [2, 3] can only occur on accreting, rapidly rotating stars, where the centrifugal force is large enough to break the crust. They also introduced the hypothesis that these events might produce mountains. In [4], within the framework of the self-consistent elastic model described in [5], we study the deformation of rotating, compressible, non-magnetized, self-gravitating NS that evolves along the following path: the star accretes mass from a companion, gaining angular momentum; the increasing rotation bring the crust to a breaking condition, causing a starquake. This event alters the stellar axial symmetry, creating a non-null ellipticity, making possible GWs emission. The angular momentum gained from the infalling material will balance the one lost by emission, bringing the star, through a sequence of starquakes, to dynamical equilibrium. Here we will briefly present the main results of our work. In our model, the star has a fluid core, extending from the origin to the radius r_c and an elastic crust extending in the layer from r_c to the stellar surface $r = a$. We choose a polytrope $n = 1$ for the description of the pressure-density relation. In this case, the Newtonian equilibrium configuration as a radius that is independent of the mass of the star. We can thus assign, for a given mass M, the corresponding (Relativistic) radius given by the solution of a more realistic equation of state (EoS). In particular, we evaluate the impact of two unified equations of state, SLy [6] and stiffer BSk21 [7] and we analyze the star's behaviour in two different scenarios. The first is when the dynamical timescale is large compared to the microscopic reactions: in that case, matter is described by the *equilibrium* adiabatic index, namely $\gamma^* = 2$. In the opposite case, one has to use the *frozen* adiabatic index γ_f. Here, we will consider the two cases of $\gamma_f = 2.1$ (small departure from equilibrium case) and of $\gamma_f = \infty$ (incompressible limit). For a deeper discussion on these parameters we refer to [5].

2. – Breaking frequency

Our initial configuration is the one of an unstressed, spherical symmetric, non-rotating star, *i.e.*, the usual configuration given by hydrostatic equilibrium. The elastic model developed in [5] allows calculating the corresponding perturbed configuration rotating at a given frequency. Accretion will make the star rotate faster and faster, deforming the object from the spherical to an oblate shape. This deformation stresses the crust, until it reaches a breaking condition; in our work we used the Tresca criterion [8], which states that the crust fails when the *strain angle* α, defined as the difference between the maximum and the minimum eigenvalues of the strain tensor, is half of breaking strain σ_{max}. For the value of σ_{max} we used the recent result of 10^{-1} by [9]. Since the deformation due to rotation is proportional to the frequency squared, the Tresca criterion gives a threshold frequency, called *breaking frequency* ν_b. In fig. 1, on the left, we show the results of this first analysis, where the curves for ν_b are plotted for different EoSs and different values of the adiabatic index. Focusing on a typical $M = 1.4M_\odot$ NS, we can say that the breaking frequency is in the range 200–600 Hz, well below the maximum observed rotational frequency [10] $\nu_o = 716.36$. Furthermore, we can see that softer EoSs lead to larger breaking frequencies, since they produce more compact stars and, consequently, give larger breaking frequency values. On the other hand, a small change in the adiabatic index value gives large changes in the breaking frequency curve.

3. – Ellipticity and equilibrium frequency

As we have seen, sufficiently fast rotating NSs can reach the condition for the rupture of the crust, and thus we can now study the consequence of starquakes. A break of the crust causes a local deformation (altering the star's pure axial symmetry), that can be evaluated by ellipticity ϵ, defined as $\epsilon = (A - B)/C$. Here A and B are the principal moments of inertia along the equatorial axes, while C is the one along the rotation axis. Starquakes can be seen as an attempt of a rotating NS to achieve the corresponding equilibrium fluid shape despite the constraining action of its elastic crust. Whatever the angular velocity, the star is stressed: starquakes bring the star towards the fluid stress-free configuration. These failure events create mountains thus giving ellipticity different from zero to the star. The idea is to calculate the maximum ellipticity's value that an NS could achieve at a given angular velocity, by comparing the principal moment of inertia of two different configurations, rotating at the same frequency. One represents an NS with a solid unbroken crust, while the second is the one of a pure fluid star. By making the difference of these two configurations we obtained the maximum value of

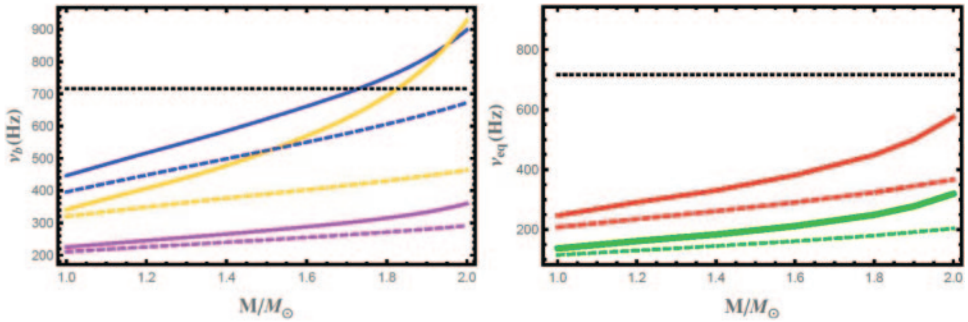

Fig. 1. – Left: study of breaking frequency ν_b as a function of NS's mass for different adiabatic indices (blue for $\gamma^* = 2$, yellow for $\gamma_f = 2.1$ and purple for $\gamma_f = \infty$) and EoSs: the dashed lines represent stars described by the BSk21 Eos, while the solid ones represent stars described by the SLy one. The dotted black line is the rotation frequency of the fastest rotating pulsar observed so far $\nu_o = 716.36$ Hz. Right: the equilibrium frequency ν_{eq} as a function of NS's mass for different adiabatic indices and mass accretion rates. The solid lines are used for the SLy EoS, the dashed ones for BSk21 EoS, fixed $\gamma^* = 2$. We used two colours to distinguish two mass accretion rates: $\dot{M} = 2 \times 10^{-8} M_\odot/\mathrm{yr}$ (red) and $\dot{M} = 10^{-10} M_\odot/\mathrm{yr}$ (green). The dashed black line on the top represents the actual maximum observed rotational frequency ν_o. These are taken from our main article [4], see figs. 2 and 4.

ϵ, called ϵ_{max}. In the previous section we have shown that typical $1.4 M_\odot$ NSs break when their frequencies are in the range 200–600 Hz; therefore, one can calculate the dynamical equilibrium frequency ν_{eq} reached by a fast rotating star that has developed maximum ellipticity ϵ_{max} through a sequence of quakes. In fig. 1, on the right, we show our study ν_{eq} as a function of the stellar mass, where, following [11] we assumed $10^{-10} M_\odot \leq \dot{M} \leq \times 10^{-8} M_\odot$. As expected, more compressible stars have larger equilibrium frequencies than incompressible ones. Furthermore, in all cases $\nu_{eq} < \nu_o$. Thus, from our analysis we expect that rapidly rotating stars will exceed the crust breaking threshold, even in the case of very high (10^{-1}) breaking strain; moreover, through the development of large ellipticity, they could also reach a rotational frequency below 700 Hz. We think that mountains on these stars could be long-living, since the mechanism that creates mountains is active until the equilibrium frequency is achieved. Interestingly, a recent work [12] has shown that GWs emission produced by a permanent active quadrupole could explain the observed spin distribution of accreting stars.

4. – Observational constraints on ellipticity

In the previous section we calculated an upper limit for ellipticity due to starquakes, using it to estimate a corresponding equilibrium frequency value. However, we do not expect all the fast rotating NSs to reach that limit, but, instead, only a fraction of it. Because of our actual poor knowledge of the crust and its failure, the estimation of how large this fraction can be is really challenging. Therefore, we followed a somewhat reverse path: using observational data coming from electromagnetic observations to constraint ϵ, we calculated the fraction β that the NSs have developed as $\epsilon = \beta\epsilon_{max}$. In particular, here we show our results on LMXBs; we refer to [4] for a deeper discussion. We assumed that the measured rotational frequency of an observed LMXB is its equilibrium frequency, that we call ϵ_{acc} where the subscript *acc* stands for *accretion*. Then we studied $\beta_{acc} =$

$\epsilon_{acc}/\epsilon_{max}$, as a function of observed rotational frequency for a group of LMXB. We concluded that rapidly rotating pulsars could develop a value of ϵ that is about 10^{-2}–10^{-4} times our upper limit. In this sense we expect that fast rotating neutron stars could have an ellipticity in the range 10^{-7}–10^{-10}, depending on their mass and EoSs. These values are partially compatible with the recent estimations of [13], which calculate the maximum size of mountains produced on NS using different quadrupolar perturbing forces in General Relativity.

5. – Conclusion

In our paper [4], to the best of our knowledge, it is presented for the first time a realistic and consistent calculation of a new mechanism to produce GWs emission: starquakes. Using the model introduced in [5] we showed that centrifugal force acting on an accreting NS can break the crust when the object's frequency is in the range 200–900 Hz, depending on the EoS and on the mass of the star. In general, the equilibrium frequency is found to be smaller than the breaking one and, therefore, it is also below the actual observable threshold of 716.36 Hz. The maximum value of ellipticity caused by starquakes lies between 10^{-9} and 10^{-5}; in particular, stiffer EoSs produce larger ellipticities, leading the star to lower equilibrium frequencies. The comparison of LMXB observational data with our ellipticity's upper limit, lead us to expect that the value of ϵ developed by NSs is typically 10^{-2}–10^{-4} times ϵ_{max}. In summary, we can say that the investigated NSs' evolutionary path is robust and can be used to understand the evolution of highly rotating NSs and their observed frequency distribution. In particular, our model expects that, with the right combination of distance, mass and frequency, using LIGO-Virgo detectors we could detect GWs emitted by accreting NSs.

* * *

The authors are grateful to B. Haskell, M. Antonelli and P. M. Pizzochero for suggestions and useful discussions.

REFERENCES

[1] Fattoyev F., Horowitz C. and Lu H., arXiv:1804.04952 (2018).
[2] Ruderman M., *Nature*, **223** (1969) 597.
[3] Baym G., Pethic C., Pines D. and Ruderman M., *Nature*, **224** (1969) 872.
[4] Giliberti E. and Cambiotti G., *Mon. Not. R. Astron. Soc.*, **511** (2022) 3365.
[5] Giliberti E., Antonelli M., Cambiotti G. and Pizzochero P., *Mon. Not. R. Astron. Soc.*, **491** (2020) 1064.
[6] Douchin F. and Haensel P., *Astron. Astrophys.*, **380** (2001) 151.
[7] Goriely S., Chamel N. and Pearson J. M., *Phys. Rev. C*, **82** (2010) 035804.
[8] Christensen R. M., *The Theory of Materials Failure* (Oxford University Press) 2013.
[9] Horowitz C. and Kadau K., *Phys. Rev. Lett.*, **102** (2009) 191102.
[10] Hessels J. W. T., Ransom S. M., Stairs I. H., Freire P. C. C., Kaspi V. M. and Camilo F., *Science*, **311** (2006) 1901.
[11] Ushomirsky G., Cutler C. and Bildsten L., *Mon. Not. R. Astron. Soc.*, **319** (2000) 902.
[12] Gittins F. and Andersson N., *Mon. Not. R. Astron. Soc.*, **488** (2019) 99.
[13] Gittins F. and Andersson N., *Mon. Not. R. Astron. Soc.*, **507** (2021) 116.