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CKM measurements and CP violation in charm and beauty at LHCb

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Summary. — The excellent LHCb detector and data quality enable numerous measurements with world-leading precision and searches in various decays. The latest results on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and CP violation in charm and beauty decays at the LHCb experiment are presented, including the first observation of a nonzero mass difference in the D^0 meson system, a more precise measurement of y_{CP}, a new measurement of the CKM unitarity triangle γ and the latest γ combination with all LHCb measurements from both charm and beauty sectors, as well as two searches of CP violation in baryon decays.

1. – Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2] describes the mixing between the three different families of quarks and CP violation in the mixing and decays within the Standard Model (SM). Precise measurements of the CKM matrix elements provide a strict test of the SM and allow for indirect searches of new physics in the quark sector up to very high mass scales. The currently running experiments Belle(II) and BESIII on the electron-positron colliders [3, 4], and the LHCb experiment [5] on the proton-proton collider are the main contributors to the related measurements in charm and beauty decays.

This proceeding discusses the recent progress on the measurement of mixing and CP violating parameters in charm and beauty decays at the LHCb experiment, focusing on the mixing parameters in neutral charm mesons, the CKM angle γ as well as CP violation searches in beauty baryon decays. The LHCb detector at the Large Hadron Collider is described in detail in ref. [6] and the corresponding performance in ref. [7]. It has collected about 9 fb^{-1} of proton-proton collision data at the centre-of-mass energies of 7, 8 and 13 TeV during 2011-2018. The results presented below are obtained by analysing all or part of the full LHCb dataset.

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2. – Mixing and CP violation in neutral charm mesons

Neutral charm mesons can oscillate into their anti-mesons freely before they decay, as the mass eigenstates are linear combinations of the flavor eigenstates. These flavor-changing neutral currents do not occur at tree level in the SM and are sensitive to contributions from new particles of arbitrarily high mass. Therefore, measuring these processes can be a probe for physics beyond the SM [8]. The mass eigenstates of neutral charm mesons can be expressed as $|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle$, where p and q are complex parameters satisfying $|p|^2 + |q|^2 = 1$, and $|D_1\rangle$ ($|D_2\rangle$) is defined as the CP even (odd) eigenstates. The $D^0 - \bar{D}^0$ oscillations are described by the dimensionless parameters $x \equiv (m_1 - m_2)/\Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2)/2\Gamma$, where $m_{1(2)}$ and $\Gamma_{1(2)}$ are the mass and decay width of the $D_{1(2)}$ state, respectively, and Γ is the average decay width [9]. In D^0 and $\bar{D}^0$ decays to common final state, f, CP violation manifests itself in mixing if $|q/p| \neq 1$ or in the interference between mixing and decay if $\phi_f \equiv arg[(q\bar{A}_f/pA_f)] \neq 0$, where A_f ($\bar{A}_f$) denotes the amplitude of the decay process $D^0 \to f$ ($\bar{D}^0 \to f$).

The world average of the mixing and CP-violating parameters before June 2021 yields $x = (3.7\pm1.2)\times10^{-3}$, $y = (6.8^{+0.6}_{-0.7})\times10^{-3}$, $|q/p| = 0.951^{+0.053}_{-0.042}$ and $\phi = -0.092^{+0.085}_{-0.079}$ [10]. Precise measurements of y with decays such as $D^0 \to K^+\pi^-$ have allowed for the observation of mixing [11], while the measurements of x remain compatible with zero and consistent with CP symmetry. Theoretical predictions for the mixing parameters are of similar magnitude but less precise, while predictions for the CP-violating phase are around 0.002 [12] and well below the current experimental precision. With high production rates at the proton-proton collision, the LHCb detector collected huge amount of charm meson candidates, providing an ideal platform for the study of neutral charm meson. Usually, the typical mode $D^{*+} \to D^0\pi^+$ is selected to tag the production flavor of D^0 according to the charge of soft pion. This applies to all results discussed in this section.

2·1. *Observation of a non-zero mass difference in $D^0 \to K^0_S\pi^+\pi^-$ decays.* – The self-conjugate decay process $D^0 \to K^0_S\pi^+\pi^-$ is sensitive to all mixing and CP-violating parameters, as the same final states from both D^0 and $\bar{D}^0$ decays lead to interference between the mixing and decay amplitudes. The dynamics of the three-body decay are expressed as a function of a pair of two-body invariant masses following the Dalitz-plot formalism [13]. The squared invariant mass $m^2(K^0_S\pi^\pm)$ is denoted as $m^2_\pm$ for D^0 decays and $m^2_\mp$ for $\bar{D}^0$ decays. A mixture of doubly-Cabibbo-suppressed and Cabibbo-favored decay amplitudes results in large variations of the strong phase and, with mixing, causes a decay-time evolution of the density of decays across the phase space. Therefore, the mixing parameters can be determined with a joint analysis of the Dalitz-plot and decay-time distributions.

A model-independent approach, the so-called "bin-flip" method [14], is applied to obviate the need for detailed models of the efficiency, resolution and contributing amplitudes. As shown in the left plot of fig. 1, about 30 million signal candidates, selected from 5.4 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV, are partitioned into two sets of eight disjoint regions (bins) of the Dalitz plot, which are defined to preserve nearly constant strong-phase differences $\Delta\delta(m^2_-, m^2_+)$ between the D^0 and $\bar{D}^0$ amplitudes within each bin [14]. The data are further split into 13 bins of decay time with approximately equally number of events [15]. For each decay-time interval, the ratio of the number of decays in each negative Dalitz-plot bin ($-b$) to its position counterpart ($+b$) is measured, as illustrated in the right plot of fig. 1. The mixing parame-

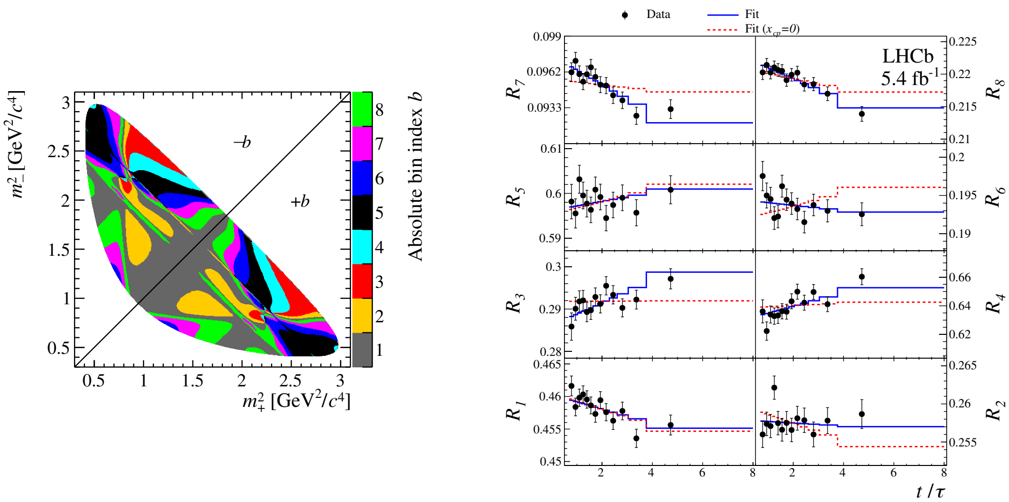

Fig. 1. – Left: "Binning" of the $D^0 \to K^0_S \pi^+\pi^-$ Dalitz plot, where colors indicate the absolute value of the bin index b. Right: CP-averaged yield ratios as a function of t/τ for each Dalitz-plot bin with fit projections overlaid.

ters are determined by minimizing a least-squares function which takes into account all decay-time intervals and Dalitz-plot bins. Deviations from constant values are due to mixing. The fit projection with x_{CP} fixed to zero indicates the inability of a nonzero y_{CP} value to produce the deviations on its own. The differences of ratios between D^0 and $\bar{D}^0$ decays are also checked where no sign of CP violation is observed. The measured values of $x_{CP} = (3.97 \pm 0.46 \pm 0.29) \times 10^{-3}$, $y_{CP} = (4.59 \pm 1.20 \pm 0.85) \times 10^{-3}$, $\Delta x = (-0.27 \pm 0.18 \pm 0.01) \times 10^{-3}$ and $\Delta y = (0.20 \pm 0.36 \pm 0.13) \times 10^{-3}$ are converted to the commonly defined mixing parameters x, y, $|q/p|$ and ϕ by forming a likelihood function and assuming the observed correlations to be independent of the true parameter values. This results in $x = (3.98^{+0.56}_{-0.54}) \times 10^{-3}$, $y = (4.6^{+1.5}_{-1.4}) \times 10^{-3}$, $|q/p| = 0.996 \pm 0.052$ and $\phi = 0.056^{+0.047}_{-0.051}$, where the nonzero value of the mass difference x of neutral charm meson mass eigenstates is observed for the first time with a significance of more than seven standard deviations and the limit on mixing-induced CP violation in the charm sector is significantly improved.

2·2. *Measurement of the mixing parameter* $y_{CP} - y_{CP}^{K\pi}$ *with two-body* D^0 *decays.* – The ratios of the effective decay widths of $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ decays over that of $D^0 \to K^-\pi^+$ decays is measured via the observable:

$$y_{CP}^f - y_{CP}^{K\pi} = \frac{\hat{\Gamma}(D^0 \to f) + \hat{\Gamma}(\bar{D}^0 \to f)}{\hat{\Gamma}(D^0 \to K^-\pi^+) + \hat{\Gamma}(\bar{D}^0 \to K^+\pi^-)} - 1 \tag{1}$$

where, $y_{CP}^f = y_{12} cos\phi_f^\Gamma$, f denotes the final state ($f = \pi^+\pi^-$ or K^+K^-) and $\phi_f^\Gamma = \arg(\Gamma_{12} A_f/\bar{A}_f)$ describes the CP-violating phase difference of the interference between decay amplitudes with and without mixing. Any deviation of y_{CP}^f from y_{12} would be a sign of CP violation. The parameter y_{12} is equal to $|y|$ up to second order CP violation effects [12]. Therefore, an accurate measurement of y_{CP} would provide an important constraint on y_{12}. The world average value of $y_{CP} - y_{CP}^{K\pi}$ was measured to be $(7.19 \pm 1.13) \times 10^{-3}$ before June 2021 [10].

With 6 fb^{-1} of proton-proton collisions data collected at a centre-of-mass energy of 13 TeV, the LHCb collaboration measured $y_{CP}^{\pi\pi} - y_{CP}^{K\pi}$ and $y_{CP}^{KK} - y_{CP}^{K\pi}$ from the decay-time ratios of $D^0 \to f$ over $D^0 \to K^-\pi^+$ signal yields as a function of the reconstructed D^0 decay time, where the kinematics of $D^0 \to f$ are matched with $D^0 \to K^-\pi^+$ to ensure equal acceptance of kinematic phase space [16]. The ratios give directly access to the charm mixing parameters $y_{CP}^{\pi\pi} - y_{CP}^{K\pi}$ and $y_{CP}^{KK} - y_{CP}^{K\pi}$, which are measured to be $(6.57 \pm 0.53(stat.) \pm 0.16(syst.)) \times 10^{-3}$ and $(7.08 \pm 0.30(stat.) \pm 0.14(syst.)) \times 10^{-3}$, respectively. Assuming negligible dependence of final states contribution to y_{CP} with current experimental sensitivity [12], the combination of the two measurements yields $y_{CP} - y_{CP}^{K\pi} = (6.96 \pm 0.26(stat.) \pm 0.13(syst.)) \times 10^{-3}$, which is four times more precise than the previous world average [10].

3. – Measurement of γ and the latest combination

The CP violating phase $\gamma \equiv arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$, where $V_{qq'}$ is the relevant CKM matrix element, is the only angle of the unitarity triangle that can be determined using solely measurements of tree-level B-meson decays with negligible theoretical uncertainty, assuming no sizeable new physics effects present at tree level [17]. Any deviations between the direct measurements of γ and the value derived from global CKM fits, which assume validity of the SM, would be a clear hint of new physics. Moreover, measurements and comparisons of γ with decays of different B-meson species provide sensitivity to possible new physics effects at tree level considering the different decay topologies involved. The experimental uncertainty of the world average for direct measurements of $\gamma = (66.2^{+3.4}_{-3.6})^\circ$ [10] is larger than these obtained from global CKM fits, $\gamma = (65.6^{+0.9}_{-2.7})^\circ$ using a frequentist method [18] and $\gamma = (65.8 \pm 2.2)^\circ$ with a Bayesian approach [19]. Further improving the experimental precision of γ is one of the key physics goals of the LHCb experiment and the comparison between direct and indirect determination of γ would be important to test the SM.

The typical decay modes $B \to Dh$, where D is an admixture of D^0 and $\bar{D}^0$ flavor states and $h^\pm$ is either a kaon or pion, are commonly used to measure γ as it is sensitive to the interference between $b \to c$ and $b \to u$ quark transition amplitudes. The ratio of these two amplitudes is $A_{b\to c}/A_{b\to u} = r_B e^{i\delta_B \pm \gamma}$, where + or − sign corresponds to the initial flavor of B containing $\bar{b}$ or b, r_B is the ratio of the amplitude magnitudes and δ_B their CP-conserving strong-phase difference. The sensitivity to γ and CP violation varies with different decays of D mesons [20]. For D decays to non-charge-conjugate states, known as ADS modes [21], larger CP violation is possible and they can be particularly sensitive to γ for the favoured (suppressed) $B \to Dh$ decay followed by a suppressed (favoured) D meson decay. For D decays to CP eigenstates, known as GLW modes [22], such as $D \to hh$, the amplitude magnitude of D decay r_D equals 1 and strong phase difference δ_D is 0. For D decays to multi-body final states, coherence factors of B and D decays will be introduced to account for a dilution of the interference term due to strong phase variation between different contribution of intermediate resonances. While hadronic parameters are specific to each B decay and subsequent D decay, the CP-violating weak phase difference between B^+ and B^- amplitudes, γ, is shared by all such decays. Therefore, combination of all these measurements is expected to provide the best precision of γ and δ_B. Previously, the hadronic parameters of D decays in the combination were taken as external inputs using dedicated charm-meson measurements.

However, the large B-meson samples now constrain γ and δ_B so precisely that $\delta_D^{K\pi}$, the strong phase difference between $D^0 \to K^-\pi^+$ and $\bar{D}^0 \to K^-\pi^+$ decays, can be measured with similar precision as γ and δ_B, a factor about two better than the previous world average [10]. This improved precision on $\delta_D^{K\pi}$ can then help to improve the knowledge of charm mixing parameters. In addition, it was found that not accounting for the effect of D-meson mixing in the combination results in a bias on γ of approximately 1.8° and even larger bias for the hadronic parameters r_B and δ_B, as the effect of D mixing for some decays like $B^\pm \to D\pi^\pm$, where $r_B \sim x, y$, is significant [23]. Therefore, a simultaneous combination using both beauty and charm observables from LHCb is performed for the first time [24] to ensure an unbiased determination of γ and improved precision for the charm mixing parameters.

The full list of LHCb measurements that are used in this simultaneous combination is listed in ref. [24], including decay-rate ratios and charge asymmetries of $B^\pm \to D^{(*)}h^\pm$, $B^\pm \to DK^{*\pm}$, $B^\pm \to Dh^\pm\pi^+\pi^-$, $B^0 \to DK^{*0}$, $B^0 \to D^\mp\pi^\pm$, $B_s^0 \to D_s^\mp K^\pm$ and$B_s^0 \to D_s^\mp K^\pm\pi^+\pi^-$ decays, as well as the time-dependent measurements of charm sectors: $D^0 \to h^+h^-$, $D^0 \to K^+\pi^-$, $D^0 \to K^\pm\pi^\mp\pi^+\pi^-$ and $D^0 \to K_S^0\pi^+\pi^-$ decays. All the results are combined using a frequentist treatment [25] with a total of 151 input observables to determine 52 free parameters, along with auxiliary information from other experiments. The value of γ is determined to be $(65.4^{+3.8}_{-4.2})^\circ$, providing the most precise measurement from a single experiment. The p-value (1-CL) distribution as a function of γ is shown in the left plot of fig. 2 for the total combination and for subsets in which the input observables are split by the species of the initial B meson. A moderate tension of 2.2 standard deviations between the charged and neutral B states is seen. The larger uncertainty in B^0 and B_s^0 is expected to be reduced by a factor of 2 with the update of analyses using full LHCb data sample. The charm mixing parameters x and y are determined simultaneously along with γ, as shown in the right plot of fig. 2, where the result $y = 0.603^{+0.033}_{-0.030}$ is more precise by a factor of two than the world average [10], driven entirely by the improved measurement of $\delta_D^{K\pi}$ from the beauty system and the simultaneous averaging methodology. Note that this combination does not include the latest measurement of y_{CP} discussed in sect. **2**˙2. A preliminary combination of all charm measurements at LHCb including the latest y_{CP} further improves the precision of $y = (6.46^{+0.24}_{-0.25})$ by a factor of 1.4 [16].

Apart from the latest simultaneous combination of γ and charm mixing parameters, another new measurement of γ using $B^\pm \to Dh^\pm$ with $D^0 \to h^\pm h'^\mp\pi^0$ is performed by analysing the full LHCb data with an integrated luminosity of 9 fb^{-1}, where $h^{(\prime)}$ is either a kaon or a pion [27]. The interference effects that are sensitive to γ in the quasi-ADS modes $D \to \pi K\pi^0$, vary over the phase space of the D decay due to different contribution of intermediate resonances. Integration over the D decay phase space dilutes the sensitivity. For the suppressed mode $B^- \to [\pi^-K^+\pi^0]_D K^-$, the dilution factor $\kappa_D = 0.79 \pm 0.04$ [28] is large so that it is still sensitive to γ. The quasi-GLW modes $D \to h^-h^+\pi^0$, are admixtures of CP-even and CP-odd eigenstates. Integration over phase space dilutes the overall CP asymmetry as the CP-even and CP-odd states have opposite CP asymmetries. Eleven CP observables are formed from ratios of partial decay rates and CP asymmetries, and measured with the world-best precision. The suppressed $B^- \to [\pi^-K^+\pi^0]_D K^-$ decay is observed for the first time, with a significance of 7.8 standard deviations. The results are interpreted in terms of the fundamental parameters γ, r_B and δ_B, which are determined to be $\gamma = (56^{+24}_{-19})^\circ$, $r_B = (9.3^{+1.0}_{-0.9}) \times 10^{-2}$ and $\delta_B = (122^{+19}_{-23})$, and are consistent with the latest combination at LHCb.

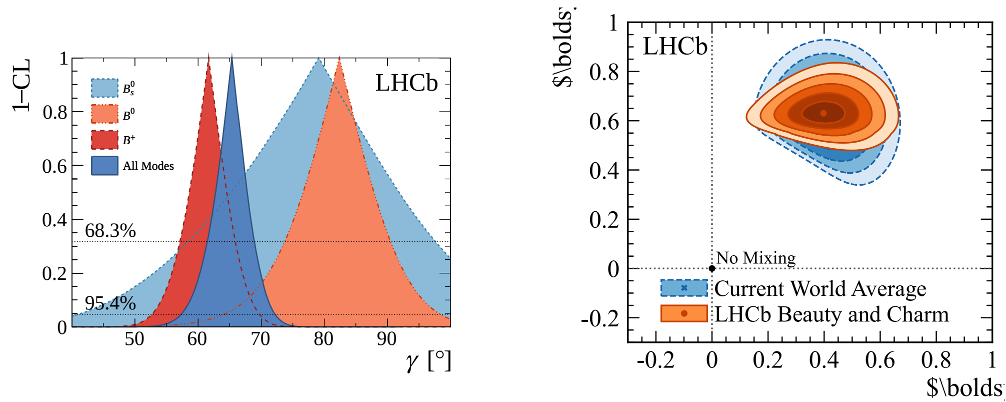

Fig. 2. – Left: One dimensional 1-CL profiles for γ from the combination using inputs from B_s^0 (light blue), B^0 (orange), B^+ (red) mesons and all species together (dark blue). Right: Two-dimensional profile likelihood contours for the charm mixing parameters x and y, where the blue contours show the current charm world average from ref. [10], the brown contours show the result of this combination. Contours are drawn out from 1 to 5 standard deviations.

4. – CP violation searches in b baryon decays

All effects of CP violation observed so far are consistent with the SM prediction, owing its origin to a single irreducible complex phase in the CKM matrix [1,2]. However, the degree of CP violation permitted in the SM is not enough to explain the observed matter-antimatter asymmetry in the Universe [29]. Further searches for sources of CP violation beyond the SM is therefore motivated, especially in the baryon sector, where no breaking of CP asymmetry has been observed yet.

Few studies of beauty-baryon decays to final states involving a single open-charm meson exist, but they are nonetheless promising for measurement of CP violation. A measurement of Λ_b^0 decays to final states including a D meson yielded the first observation of the singly Cabibbo-suppressed $\Lambda_b^0 \to [K^-\pi^+]_D pK^-$ decay, where D represents a D^0 or $\bar{D}^0$ meson [30]. This motivates the search for the suppressed decay $\Lambda_b^0 \to [K^+\pi^-]_D pK^-$, which is of particular interest since its decay amplitude receives contributions from $b \to c$ and $b \to u$ amplitudes of similar magnitude, given the CKM suppression between the two D decays. The interference between these two amplitudes, which depends upon the CKM angle γ, is expected to be large, but the different strong phases associated with the various configurations of polarization states for the Λ_b^0 and intermediate resonances complicate the determination of γ. The suppressed $\Lambda_b^0 \to [K^+\pi^-]_D pK^-$ is observed for the first time by analysing the full LHCb data [31]. The ratio of branching fractions for the $\Lambda_b^0 \to [K^-\pi^+]_D pK^-$ and $\Lambda_b^0 \to [K^+\pi^-]_D pK^-$ and the CP asymmetry, are measured in the full phase space to be $R = 7.1 \pm 0.8(stat.)^{+0.4}_{-0.4}(syst.)$ and $A_{CP} = 0.12 \pm 0.09(stat.)^{+0.02}_{-0.03}(syst.)$, respectively. The ratio of branching fractions is consistent with the estimate based on the relevant CKM matrix elements and the measured asymmetry is consistent with zero. With larger samples to be collected by LHCb in the coming years, further study of this mode will contribute to the overall determination of γ.

In light of the large CP violation effects observed in three-body charmless decays of B mesons [32], it is of great interest to extend the range of searches in beauty-baryon decays. The recently observed $\Xi_b^- \to pK^-K^-$ decay [33] provides an interesting new opportunity to search for the CP violation effect. The first amplitude analysis of $\Xi_b^- \to pK^-K^-$ decays is performed with data taken between 2011-2016 at LHCb, corresponding to integrated luminosities of 1 fb^{-1} at a center-of-mass energy of $\sqrt{s} = 7$ TeV, 2 fb^{-1} at $\sqrt{s} = 8$ TeV and 2 fb^{-1} at $\sqrt{s} = 13$ TeV [34]. A good description of the data is obtained with an amplitude model containing contributions from $\Sigma(1385)$, $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(1670)$, $\Sigma(1775)$ and $\Sigma(1915)$ resonances. The CP asymmetry for each contributing component is evaluated and no significant CP violation effect is observed.

5. – Summary

Precise measurements of the CKM matrix elements and CP violating phases provide a good opportunity to test the SM prediction and to search for the possible new physics phenomenologies indirectly. The LHCb collaboration made leading contributions to the measurements of observables in beauty and charm sectors, dominating the world average values of the CKM phase γ and charm mixing parameters x and y, as well as many others that are not discussed in this proceeding. As LHCb Run 3 is going to start data taking soon this year, many more exciting measurements with improved precisions and new searches are expected to be performed in the near future.

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