Colloquia: La Thuile 2022

Gauge flavour unification: From the flavour puzzle to stable protons

J. Davighi

University of Zürich - Zürich, Switzerland

Summary. — The idea of unification attempts to explain the complicated structure of the Standard Model (SM) in terms of fewer fundamental forces and/or matter fields. However, traditional GUTs based on $SU(5)$ and Spin(10) shed no light on the existence of three generations of fermions, nor the distinctive pattern of their Yukawa couplings to the Higgs. In this proceeding I discuss two routes for unifying the SM gauge symmetry with its 3-family flavour symmetries: firstly, unifying flavour with electroweak symmetries via the group $SU(4) \times Sp(6)_L \times Sp(6)_R$; secondly, unifying flavour and colour via an $SU(12) \times SU(2)_L \times SU(2)_R$ gauge theory. In either case, all three generations of SM fermions are unified into just two fundamental fields. In the larger part of the talk, I describe how the former model of "electroweak flavour unification" offers a compelling new explanation of hierarchical fermion masses and CKM angles. As a postscript, we will see that gauge flavour unification can have unexpected spin-offs not obviously related to flavour. In particular, the $SU(12) \times SU(2)_L \times SU(2)_R$ symmetry, when broken, typically leaves behind remnant discrete gauge symmetries that exactly stabilize protons to all orders.

1. – A unified theory of flavour

The Standard Model (SM), while familiar, is a complicated quantum field theory. The gauge algebra $G_{\text{SM}} := \mathfrak{su}(3) \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ entails three independent gauge couplings, corresponding to both non-abelian and abelian forces. There are five (or six, if including right-handed neutrinos) fermion fields transforming in a curious set of irreducible representations (henceforth *irreps*), a structure that is repeated across three generations. The generations are themselves a source of yet more unexplained structure, through the distinctive pattern of their Yukawa couplings to the Higgs field. The dynamics of the SM are just as rich; the Higgs mechanism gives rise to weak, short-range forces as well as a long-range electromagnetic force, while QCD becomes strongly coupled and confines. Unification of forces and/or matter attempts to explain all (or part of) this structure as a consequence of something simpler at high energies.

Famously, there are just two grand unified theories that do not require extra fermions (beyond a right-handed neutrino), based on the gauge algebras $\mathfrak{su}(5)$ [1] and $\mathfrak{so}(10)$ [2,3].

The SM fermions, including a right-handed neutrino, transform in the representations $[\mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}]^{\oplus 3}$ and $\mathbf{16}^{\oplus 3}$ respectively. But these models shed no light on flavour. Intriguingly, there is no simple gauge algebra that further packages together the three generations into a gauge-flavour unified model (at least not without requiring extra fermions). It seems that, if one wishes to unify the three generations of matter, one must forgo the complete unification of forces.

Here we explore possible routes for unifying the gauge and flavour symmetries of the SM [4,5]. As proven by exhaustion in ref. [6], there are three semi-simple gauge algebras that unify all the fermions of the SM+$3\nu_R$ into just two representations. All three are flavour enrichments of the quark-lepton-unifying $\mathfrak{ps} := \mathfrak{su}(4) \oplus \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R$ of Pati-Salam [7], for which the fermions transform as $\psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}$ and $\psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$. The first option is to unify electroweak symmetry with a chiral $\mathfrak{so}(3)_L \oplus \mathfrak{so}(3)_R$ flavour symmetry. This can be done in two ways, either preserving left-right symmetry with $\mathfrak{g}_{\mathrm{EWF}} := \mathfrak{su}(4) \oplus \mathfrak{sp}(6)_L \oplus \mathfrak{sp}(6)_R$ (see also [8]), or not, with $\mathfrak{g}_{\mathrm{EWF}'} := \mathfrak{su}(4) \oplus \mathfrak{sp}(6)_L \oplus \mathfrak{so}(6)_R$. Here, $\mathfrak{sp}(6)$ denotes the Lie algebra of $Sp(6)$, which is the Lie subgroup of $SU(6)$ matrices U that further satisfy $U^T \Omega U = \Omega$, where $\Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$. In both these cases, the fermions transform in the two representations $\psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1})$ and $\psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$. The second option is to unify the $\mathfrak{su}(4)$ "colour" symmetry with a complex $\mathfrak{su}(3)$ family symmetry, to arrive at $\mathfrak{g}_{\mathrm{CF}} := \mathfrak{su}(12) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$, with fermions transforming as a left-handed $(\mathbf{12}, \mathbf{2}, \mathbf{1})$ plus a right-handed $(\mathbf{12}, \mathbf{1}, \mathbf{2})$.

Each of the three gauge algebras $\mathfrak{g}_{\mathrm{CF}}$, $\mathfrak{g}_{\mathrm{EWF}}$ or $\mathfrak{g}_{\mathrm{EWF}'}$ intricately intertwines the flavour symmetry with the SM gauge symmetries, unifying all three families together in the same multiplet, thereby explaining why there are three families in terms of an underlying gauge symmetry. In this proceeding, we further show that electroweak flavour unification [4], based on $\mathfrak{g}_{\mathrm{EWF}}$, can provide an elegant model for the hierarchical structure of fermion masses and CKM mixing angles, with no ingredients needed beyond a set of scalars that break the symmetry down to the SM. In sect. **3** we briefly discuss the other route, namely colour flavour unification, remarking that this symmetry contains discrete gauge subgroups $\Gamma \cong \mathbb{Z}_{3n_f} = \mathbb{Z}_9$ that conserve baryon number mod n_g, serving to exactly stabilize the proton to all orders [5].

2. – Electroweak flavour unification

2˙1. *Embedding the SM*. – We here describe the basics of the electroweak flavour unification model, based on the anomaly-free UV gauge group

$$(1) \qquad G_{\mathrm{EWF}} = SU(4) \times Sp(6)_L \times Sp(6)_R \,,$$

the Lie algebra of which is $\mathfrak{g}_{\mathrm{EWF}}$. The SM chiral fermions are embedded in two fields. All 24 left-handed Weyl fermions are embedded in the left-handed field

$$(2) \qquad \Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}) \sim \begin{pmatrix} u_1^r & u_2^r & u_3^r & d_1^r & d_2^r & d_3^r \\ u_1^g & u_2^g & u_3^g & d_1^g & d_2^g & d_3^g \\ u_1^b & u_2^b & u_3^b & d_1^b & d_2^b & d_3^b \\ \nu_1 & \nu_2 & \nu_3 & e_1 & e_2 & e_3 \end{pmatrix} ,$$

where the subscript indices indicate family number. All 24 right-handed fields are similarly embedded in a field $\Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$.

TABLE I. – *Field content of the electroweak flavour unification model. For the symmetry breaking scalars, we also indicate whether they are real (ℝ) or complex (ℂ) scalars.*

Type	Field	G_{EWF} irrep
SM fermions	Ψ_L	$(\mathbf{4}, \mathbf{6}, \mathbf{1})$
	Ψ_R	$(\mathbf{4}, \mathbf{1}, \mathbf{6})$
Higgs	H_1	$(\mathbf{1}, \mathbf{6}, \mathbf{6})$
	H_{15}	$(\mathbf{15}, \mathbf{6}, \mathbf{6})$
SSB scalars	S_L (ℝ)	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	S_R (ℂ)	$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{6})$
	Φ_L (ℝ)	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	Φ_R (ℂ)	$(\mathbf{1}, \mathbf{1}, \mathbf{14})$

Recall that family-blind models with $SU(4)$ quark lepton unification, such as those based on the Pati-Salam group, typically require two Higgs fields, in the singlet and adjoint of $SU(4)$, in order to split quark and lepton masses. The same is true here except now the electroweak indices of each Higgs field must extend to $Sp(6)$ indices, meaning that the Higgs necessarily picks up flavour quantum numbers. The minimal embedding of Higgs fields is in representations $H_1 \sim (\mathbf{1}, \mathbf{6}, \mathbf{6})$ and $H_{15} \sim (\mathbf{15}, \mathbf{6}, \mathbf{6})$. We can write down renormalisable Yukawa couplings with these fields,

$$(3) \qquad \mathcal{L} = \sum_{a \in \{1,15\}} y_a \mathrm{Tr}[\overline{\Psi}_L \Omega H_a \Omega^T \Psi_R] + \overline{y}_a \mathrm{Tr}[\overline{\Psi}_L \Omega H_a^* \Omega^T \Psi_R] \,,$$

where recall Ω denotes the constant matrix $\Omega = \left(\begin{smallmatrix} 0 & I_3 \\ -I_3 & 0 \end{smallmatrix} \right)$ throughout.

The symmetry G_{EWF} must ultimately be broken down to $G_{\mathrm{SM}} = SU(3) \times SU(2)_L \times U(1)_Y$. We do so using an (almost) minimal set of scalars S_L, S_R, Φ_L and Φ_R, whose quantum numbers are recorded in table I which summarizes the entire field content of the model. The scalars S_L and Φ_L are real, while S_R and Φ_R are complex.

2˙2. *Symmetry breaking pattern.* – The UV symmetry G_{EWF} is broken sequentially down to G_{SM}, as summarized in fig. 1. At the high scale, the gauge-flavour-unified symmetries are first *deconstructed* to products of family specific symmetries:

$$(4) \qquad \langle S_L \rangle : Sp(6)_L \qquad\qquad \longrightarrow SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \,,$$

$$(5) \qquad \langle S_R \rangle : SU(4) \times Sp(6)_R \qquad \longrightarrow SU(3) \times Sp(4)_{R,12} \times U(1)_R \,,$$

where $U(1)_R$ acts as $B - L$ on the light right-handed fermions, and acts as hypercharge on everything else. In the "right sector" the first two families remain unified. Also note that the scalar S_R simultaneously breaks the quark-lepton unification([1]). The result of

---

([1]) We remark that similar family-deconstructed gauge symmetries have been used recently in flavour model-building [9], in models designed to explain the recent anomalies in semileptonic B meson decays, together with fermion masses.
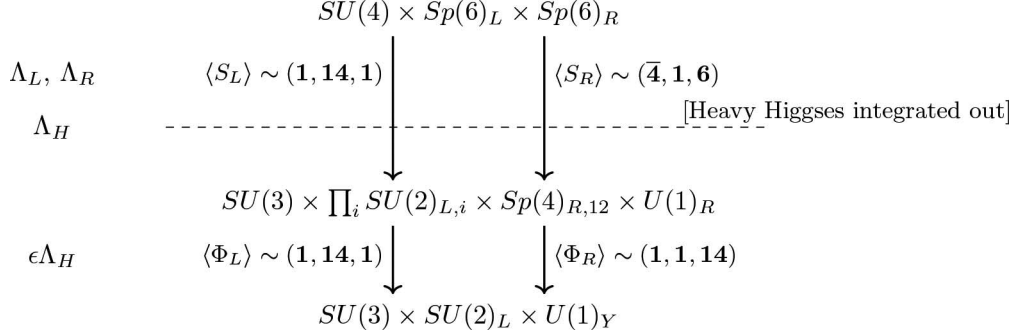
$$SU(4) \times Sp(6)_L \times Sp(6)_R$$

$\Lambda_L, \, \Lambda_R$ $\quad\quad \langle S_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ $\quad\quad\quad \langle S_R \rangle \sim (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{6})$

Λ_H $\quad\quad$ - [Heavy Higgses integrated out]

$$SU(3) \times \prod_i SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

$\epsilon \Lambda_H$ $\quad\quad \langle \Phi_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ $\quad\quad\quad \langle \Phi_R \rangle \sim (\mathbf{1}, \mathbf{1}, \mathbf{14})$

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Fig. 1. – The symmetry breaking scheme in our model.

these high scale symmetry breakings is an intermediate gauge symmetry

$$(6) \qquad G_{\text{int}} = SU(3) \times \prod_{i=1}^{3} SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R \,.$$

Upon breaking to this intermediate symmetry, the fundamental Higgs fields H_1 and H_{15} split into a set of flavour-specific components, with a component that couples to every pair of SM families (one left-handed, one right-handed). While we save a proper investigation of the scalar potential for future work, it is reasonable for the Higgs vev to fall into a small number of these family-aligned components, which we take to be the components $\mathcal{H}_a \sim [\mathbf{1}, (\mathbf{1}, \mathbf{1}, \mathbf{2}), \mathbf{1}]_3$ and $\overline{\mathcal{H}}_a \sim [\mathbf{1}, (\mathbf{1}, \mathbf{1}, \mathbf{2}), \mathbf{1}]_{-3}$, for each of $a = 1, 15$.

We assume all other components of the fundamental Higgs fields are heavy, and are integrated out at a high scale $\Lambda_H \lesssim \Lambda_{L,R}$. The fact that the light Higgs components transform under one $SU(2)_{L,i}$ factor singles out one family of fermions that couples directly to the Higgs via the fundamental Yukawa interaction (3). This *defines* the third family (until this point, the theory is invariant under permuting the family labels of left-handed fermions). The other fermions are massless at the renormalisable level.

Lastly, the remaining two scalar fields Φ_L and Φ_R break the intermediate symmetry (6) down to the SM. These scalars start life in 2-index antisymmetric irreps of $Sp(6)_L$ and $Sp(6)_R$ respectively, which means that they decompose into sets of "link fields" under (6). In the case of Φ_L, which is a real scalar,

$$(7) \qquad \Phi_L \sim \mathbf{14}_L \mapsto \mathbf{1}^{\oplus 2} \oplus (\mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2})$$

under (6), where we only record the representations under $SU(2)_{L,i}$ factors on the RHS. We thus get a link field charged under each pair of $SU(2)_{L,i}$ symmetries. Let ϕ_L^{ij} denote the link field transforming as a bidoublet of $SU(2)_{L,i} \times SU(2)_{L,j}$. If any two of these three link fields acquire non-zero vevs, then the deconstructed electroweak symmetry is broken to the diagonal,

$$(8) \qquad \langle \Phi_L \rangle : SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \longrightarrow SU(2)_L$$

which is the SM electroweak symmetry.

We take the vevs to be in the ϕ_L^{12} and ϕ_L^{23} components (while we could have chosen otherwise, this is the choice that naturally reproduces the observed hierarchy in the three CKM angles), with magnitudes of order

$$(9) \qquad\qquad \langle \phi_L^{12} \rangle \sim \epsilon_L^{12} \Lambda_H \, , \qquad \langle \phi_L^{23} \rangle \sim \epsilon_L^{23} \Lambda_H \, .$$

These vevs can be taken as definitions of the real parameters $\epsilon_L^{ij} < 1$, which are ratios of scales. We will see (sect. **2**˙3) that mild ratios between scales, $\epsilon_L^{12} \sim 1/5$ and $\epsilon_L^{23} \sim (1/5)^2$, are needed to explain the observed hierarchies in the SM Yukawas.

In the case of Φ_R, which is a complex scalar,

$$(10) \qquad\qquad \Phi_R \sim \mathbf{14}_R \mapsto \mathbf{1} \oplus \mathbf{5}_0 \oplus \mathbf{4}_{-3} \oplus \mathbf{4}_3$$

under (6), where we only record the representations under $Sp(4)_R \times U(1)_R$ on the RHS. The $\mathbf{5}$ is the antisymmetric 2-index irrep of $Sp(4)_R$ and serves as the link field ϕ_R^{12} for the first two families, while the $\mathbf{4}_{\pm 3}$ components $\phi_R^{13,23}$ link the light families with the third. Suitably chosen vevs in all these components (see ref. [4] for the details) achieve the breaking

$$(11) \qquad\qquad \langle \Phi_R \rangle : Sp(4)_R \times U(1)_R \longrightarrow U(1)_Y \, ,$$

and we are left with the SM. As for the "left-sector", we introduce parameters ϵ_R^{12} and ϵ_R^{23}, which set the vevs of ϕ_R^{12} and ϕ_R^{23} respectively in units of the heavy scale Λ_H.

2˙3. *Effective field theory for the light Yukawas.* – Recall that the light Higgs fields $\mathcal{H}_a$ and $\overline{\mathcal{H}}_a$ have renormalisable Yukawa couplings only to one family of fermions, which defines the third. Effective Yukawa couplings for all the light flavours are in fact generated in the steps described in sect. **2**˙2, with in-built hierarchies and enough parametric freedom to account for the fermion mass and quark mixing data.

We here illustrate the mechanism with the simplest example, which is the generation of the effective Y_{23} Yukawa couplings. (The full details of the effective field theory (EFT) matching are laborious, so we refer to ref. [4].) Let us assume the presence of the following cubic scalar interaction in the UV,

$$(12) \qquad\qquad V(\phi) \supset \beta_L^1 \Lambda_H \, \mathrm{Tr} \left(\Omega^T H_1^\dagger \Omega \Phi_L \Omega H_1 \right) \, .$$

Then, upon breaking $G_{\mathrm{EWF}} \to G_{\mathrm{int}}$ and integrating out the heavy Higgs components, the diagram on the left of fig. 2 generates dimension-5 operators

$$(13) \qquad\qquad \mathcal{L}_{\mathrm{int}} \supset \frac{y_1 \beta_L^1}{2\Lambda_H} \phi_L^{23} \left(\overline{Q}_2 \mathcal{H}_1 D_3 + \overline{Q}_2 \overline{\mathcal{H}}_1 U_3 + \overline{L}_2 \mathcal{H}_1 E_3 \right)$$

in the intermediate effective theory. These dimension-5 operators then contribute to the Yukawa couplings Y_{23} after Φ_L acquires its vev (there are also contributions from the $\mathcal{H}_{15}$ and $\overline{\mathcal{H}}_{15}$ Higgses), *i.e.* upon breaking $G_{\mathrm{int}} \to G_{\mathrm{SM}}$. The effective Yukawa couplings Y_{32} are also generated at dimension-5, this time by operators involving Φ_R link fields (fig. 2, right), which require cubic interactions between H_a and Φ_R.

Continuing to higher dimensions in the G_{int}-invariant EFT, the Yukawa couplings Y_{13}, Y_{22}, and Y_{31} originate from dimension-6 operators involving *two* insertions of the
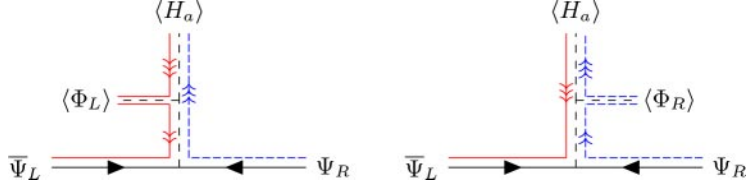
Fig. 2. – Feynman diagrams that contributes to the Y_{23} (left) and Y_{32} (right) elements of the Yukawa matrices. The red and blue lines depict the flow of family indices embedded in $Sp(6)_L$ and $Sp(6)_R$ respectively. We see that Φ_L plays the role of a "link field", converting a third family left-index to a second family left-index.

symmetry breaking scalars $\Phi_{L,R}$. The Y_{12} and Y_{21} Yukawa couplings arise at dimension-7, and lastly the Y_{11} couplings at dimension-8. The result of all these contributions is the following hierarchical texture for each fermion mass matrix,

$$(14) \qquad \frac{M^f}{v} \sim \begin{pmatrix} \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23} \\ \epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{23} \\ \epsilon_R^{12}\epsilon_R^{23} & \epsilon_R^{23} & 1 \end{pmatrix} , \qquad f = u, d, e .$$

Each entry in these matrices is accompanied by a coefficient which is some explicit function of the fundamental dimensionless couplings of the model, namely, the fundamental Yukawa couplings (3) plus the parameters in the scalar potential (only one of which we have written explicitly in eq. (12)).

The physical observables, *i.e.* the fermion masses and quark mixing angles, can then be extracted using matrix perturbation theory. They depend on the scale ratios $\epsilon_{L,R}^{ij}$ as

$$(15) \qquad y_{u,d,e} \sim \epsilon_L^{12}\epsilon_R^{12}\epsilon_L^{23}\epsilon_R^{23} ,$$

$$(16) \qquad y_{c,s,\mu} \sim \epsilon_L^{23}\epsilon_R^{23} ,$$

$$(17) \qquad y_{t,b,\tau} \sim 1 ,$$

$$(18) \qquad V_{us} \sim \epsilon_L^{12} ,$$

$$(19) \qquad V_{cb} \sim \epsilon_L^{23} ,$$

$$(20) \qquad V_{ub} \sim \epsilon_L^{12}\epsilon_L^{23} .$$

The hierarchies in the CKM angles then favour scale suppressions of the order

$$(21) \qquad \epsilon_L^{12} \sim \lambda \approx 0.23, \qquad \epsilon_L^{23} \approx \lambda^2,$$

where λ is the Cabibbo angle. The remaining ϵ_R^{ij} ratios can then be adjusted to reproduce the average mass hierarchies between the three generations. Since no pair of symmetry breaking scales are separated by more than an order of magnitude, it is expected that this ladder of scales is at least radiatively stable [10]. Finally, and crucially, it is shown in [4] that there is enough freedom in the particular EFT coefficients to actually fit all the data (beyond just reproducing the observed hierarchies), despite some interrelations between elements of the CKM matrix.

2˙4. *Outlook*. – To summarize, the electroweak flavour unification model offers a new explanation for the origin of three generations of fermions and the hierarchical structure of fermion masses and quark mixing angles, in terms of a flavour-enriched version of the quark-lepton unifying symmetry of Pati and Salam. We emphasize that no extra ingredients, such as vector-like fermions, are required to generate the Yukawa structure, beyond the scalar fields needed to break G_{EWF} down to the SM.

Beyond reproducing these known facts of Nature, we have not discussed the phenomenological consequences of such a framework —which, if the unification scales are not too far off, would be multitudinous. Indeed, the $SU(4) \times Sp(6)_L \times Sp(6)_R$ gauge model does not give rise to proton decay, and so the mass scale of the heavy gauge bosons can in principle be low. The lightest gauge bosons in the model will be those arising from the final breaking steps, $G_{\mathrm{int}} \to G_{\mathrm{SM}}$, which transform as a selection of *flavoured* W' and Z' bosons, some coupled to left-handed fermions and others to right-handed fermions. We save an explanation of the low-energy phenomenology signatures of the model, which are complicated, for future work. For now, we remark only that the extra states will provide extra sources of quark flavour violation and lepton universality violation, for which there is intriguing evidence already in the decays of B mesons.

3. – Postscript: colour flavour unification

I conclude this proceeding with some comments about the other route to unifying gauge and flavour symmetries, namely "colour flavour unification" based on the (admittedly rather large) gauge group $G_{\mathrm{CF}} = SU(12) \times SU(2) \times SU(2)$ [5]. The key observation we wish to make is that there is a sequence of subgroup embeddings,

$$(22) \qquad\qquad G_{\mathrm{SM}} \times \mathbb{Z}_9 \hookrightarrow G_{\mathrm{SM}} \times U(1)_{\mathrm{LFUV}} \hookrightarrow G_{\mathrm{CF}} .$$

Here the discrete symmetry $\mathbb{Z}_9$ acts on all quarks with a charge of 1 mod 9, and on leptons with a charge of 0 mod 9. A family of possible $U(1)_{\mathrm{LFUV}}$ symmetries that fit in this sequence are generated by the following combinations of baryon number and lepton family numbers

$$(23) \qquad\qquad X = 3B + 18r(L_e + L_\tau) - 9(4r + 1)L_\mu ,$$

where $r \in \mathbb{Z}$. These $U(1)$ symmetries violate lepton flavour universality, hence the label "LFUV"([2]).

Interestingly, the symmetry breaking $\pi : U(1)_{\mathrm{LFUV}} \to \mathbb{Z}_9$ can be triggered by the vevs of a pair of scalars ϕ_1 and ϕ_2 with precisely the right $U(1)_{\mathrm{LFUV}}$ charges, namely $q_1 = 36r$ and $q_2 = 9(2r + 1)$, needed to sufficiently populate (up to a single zero entry) the 3×3 neutrino Majorana matrix via renormalisable couplings $\sim \phi_a \overline{\nu}_i^c \nu_j$. Thus, if the breaking π occurs at the traditional see-saw scale, of order 10^{14} GeV, then realistic neutrino masses and mixings are generated, and one is left with the gauged $\mathbb{Z}_9$ symmetry that acts only on quarks (and thence on baryons, thence nuclei). This remnant $\mathbb{Z}_9$ gauge symmetry persists all the way to the deep IR.

([2]) In [5], building on refs. [11-13], these $U(1)_{\mathrm{LFUV}}$ symmetries were used to justify why scalar leptoquarks might have 'muophilic' couplings, to explain the evidence for new physics in muons (notably, the B decay anomalies and the anomaly in $(g - 2)_\mu$) without also causing charged lepton flavour violation.

While the existence of this discrete gauge symmetry would likely evade positive confirmation by any conventional experiment, it can certainly be falsified because it leads to the strict selection rule,

$$\Delta B = 0 \bmod 3 \,, \tag{24}$$

which would be *exact* to all orders in the SM EFT. Protons are therefore exactly stable in a model with this $\mathbb{Z}_9$ symmetry, regardless of whatever new physics might be lurking in the UV (be it colour flavour unification, or anything else). Nor do neutrons oscillate, for example, since $\Delta B = 2$ processes are forbidden. High scale leptogenesis *is* possible, on the other hand, because the $\Delta B = 3$ electroweak sphaleron is allowed.

The fact that such a "fine" $\mathbb{Z}_9$ discrete gauge symmetry, under which the leptons are neutral, emerges as a remnant in the IR is a direct consequence of gauge flavour unification in the UV. If there were only one generation, then adapting the preceding argument (starting this time from the Pati-Salam symmetry) gives us at most a mod 3 quark symmetry, from breaking $B - L$. This gives no protection of baryon number, and indeed forbids nothing that is not already forbidden by QCD. If there were two generations, then colour flavour unification can spit out a mod 6 gauge symmetry capable of stabilizing the proton —but it would not prevent neutrons from oscillating. It is only with three generations (at least) that both $\Delta B = 1$ and $\Delta B = 2$ processes are elegantly forbidden in this framework. We arrive at the notion that the existence of multiple generations might be responsible for the stability of the proton.

$$* \; * \; *$$

REFERENCES

[1] GEORGI H. and GLASHOW S. L., *Phys. Rev. Lett.*, **32** (1974) 438.
[2] FRITZSCH H. and MINKOWSKI P., *Annals Phys.*, **93** (1975) 193.
[3] GEORGI H., *AIP Conf. Proc.*, **23** (1975) 575.
[4] DAVIGHI J. and TOOBY-SMITH J., *JHEP*, **09** (2022) 193, arXiv:2201.07245.
[5] DAVIGHI J., GRELJO A. and THOMSEN A. E., *Phys. Lett. B*, **833** (2022) 137310, arXiv:2202.05275.
[6] ALLANACH B. C., GRIPAIOS B., and TOOBY-SMITH J., *Phys. Rev. D*, **104** (2021) 035035, arXiv:2104.14555.
[7] PATI J. C. and SALAM A., *Phys. Rev. D*, **10** (1974) 275.
[8] KUO T.-K. and NAKAGAWA N., *Nucl. Phys. B*, **250** (1985) 641.
[9] BORDONE M., CORNELLA C., FUENTES-MARIN J. and ISIDORI G., *Phys. Lett. B*, **779** (2018) 317, arXiv:1712.01368.
[10] ALLWICHER L., ISIDORI G. and THOMSEN A. E., *JHEP*, **01** (2021) 191, arXiv:2011.01946.
[11] DAVIGHI J., KIRK M. and NARDECCHIA M., *JHEP*, **12** (2020) 111, arXiv:2007.15016.
[12] GRELJO A., STANGL P. and THOMSEN A. E., *Phys. Lett. B.*, **820** (2021) 136554, arXiv:2103.13991.
[13] GRELJO A. *et al.*, *JHEP*, **04** (2022) 151, arXiv:2107.07518.