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On a theory of the Antikythera Mechanism

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Summary. — In this paper we sketch some elements of a new reconstruction of the theory underlying the design of the Antikythera Mechanism based on the mathematical knowledge available at the time of its construction.

1. – Introduction

The *Antikythera Mechanism* (hereafter AM) was a portable device (33 cm in height, 18 cm in breadth and 8 cm in depth), composed by more than thirty bronze gears arranged in different trains. All of these started from a central wheel and ended on pointers moving along dials. In total there were (at least) twelve pointers, seven on the front side and five on the back side, all moving simultaneously and set in motion by the user through an external knob connected via a crown gear to the main wheel. Outside the dials there were inscriptions giving additional informations (including a *parapegma*) and both sides were protected by two cover plates. These were engraved in the interior side with a sort of user manual, explaining the phenomena shown by the device and the meaning of the different dials. In short, the AM was a compact and highly sophisticated *astronomical computer*. But *what* did it *compute*?([1])

On the front side, the motion of the seven pointers simulated the observable longitudinal motions of Sun, Moon and five planets along the zodiac, represented by a scale divided in 360 parts. Outside of this there was a moveable ring representing a lunar calendar divided in 12 months. So, on the internal zodiac scale one could read the *angular positions* of the seven heavenly bodies, and on the outer calendar scale the *date* on which the shown configuration could be observable. The date was probably indicated by a separate date pointer, that could also serve the purpose to indicate the time elapsed between any two configurations. The phases of the Moon were also displayed via a rotating sphere colored in black and white, and according to recent reconstructions an additional pointer indicated also the position of the Moon's orbit nodes, a very relevant information for eclipse predictions.

On the back side, two spiral dials were engraved, with two pointers moving along them in the outward direction with a mechanism similar to that of modern vinyl players. The upper scale was essentially a lunisolar calendar based on the *Metonic Cycle* (19 tropical

([1]) Reference [1] is a quick review of the current understanding of the AM, including the relevant bibliography.

years = 235 synodic *months*). The lower spiral was an eclipse-possibility calculator based on the *Saros Cycle* (1 *saros* = 223 synodic *months*) approximating the time necessary for the Sun and the Moon to return to the same relative position and in the same points of the ecliptic. Both these cycles are of Babylonian origin and were common knowledge for Greek astronomers well before the middle of II century BC.

Notice that the surviving fragments of the AM contain only the Sun and Moon gear trains, whereas all the planetary gears are lost. The surviving *pin-slot device* used to alter the mean motion of the Moon makes quite likely that also in the case of planets variations of velocity were shown. Apart from this clue and a few numbers representing planetary cycles readable in the inscriptions (of unprecedented accuracy and not found in other sources), the reconstruction of the planetary gearings remains for the most part conjectural.

Another matter of conjecture is the authorship. Who *designed* the AM? Generally speaking, the AM is a highly advanced product of the ancient Greek tradition of *sphere-making* or *sphairopoiia*, the art of building scale-sized objects exhibiting the configuration and/or the movements of heavenly bodies. The simplest examples of this mathematical art were *celestial globes* and *armillary spheres*, while the AM is a way more sophisticated exemplar. The most revered Greek *sphere-maker* was Archimedes (III century BC), who devoted a full treatise to this art. In Cicero (*De re publica*, I, xiv, 21-22) there is mention of a physical model that Archimedes would have constructed himself, and his description, albeit vague, fits well with the features of the AM. In particular, Cicero emphasizes that Archimedes' *sphaira* (which seems to have been *tridimensional* rather than *plane*) represented the simultaneous motions of Sun, Moon and planets "by a single *conversio*". For chronological reasons the AM cannot have been manufactured by Archimedes himself, but it is undoubtedly part of his scientific legacy, probably the result of a further refinement of his work. Many contextual elements indicate as a strong candidate Hipparchus (II century BC), the last and maximum astronomer of the Hellenistic age. Among these, the connections of the AM with the island of Rhodes, an important center for mechanical studies where Hipparchus was active in the estimated period of construction of the AM.

Since it is unreasonable to think that such a complex device could be the result of trial-and-error, it is only natural to ask: *on what mathematical theory was the design of the AM based*? The question appears legitimate, since the AM is a brilliant (and unique) specimen of Hellenistic *scientific technology*, an artifact whose design *must* be based on some *theoretical* model of the heavenly motions, that, in some way, matched the possibility of plane mechanization via parallel and multi-axial trains of toothed wheels.

The AM is also, among other things, the only extant astronomical source dating back to the golden age of Hellenistic science. Therefore, its relevance for the reconstruction of the most mature developments of Greek mathematical astronomy is *immense*, its very same survival being a miracle in the general loss of Hellenistic scientific works. However, the *theoretical* problem posed by the AM has not received much attention in the existing literature, and the impact of the AM on the current views about Greek astronomy has been practically null. We are therefore confronted with the paradoxical circumstance that the current understanding of the AM has rewritten the history of ancient *technology* without even touching upon the history of ancient *science*. How could it happen?

Indeed, for the back side the theoretical problem is relatively simple and the general question appears to be settled (albeit there are still controversies on the details of the eclipse scheme). All the informations shown on the spiral dials depend on the proper *synchronization* of the motions of Sun and Moon, and the only astronomical knowledge involved is that of the already mentioned lunisolar cycles. These have been reconstructed from the counting of tooth numbers and on the basis of the readable inscriptions.

For the front side the problem is far more complex([2]). Its design involves the theoretical problem of representing via a single rotatory input the simultaneous motions of Sun, Moon and planets *as they are observable from the Earth*. In other terms, the front dial of the AM answered the question: *when the Sun is there at a given date, where should I look to see the body x?* To answer such a question is equivalent to solve the theoretical problem of *synchronizing all the heavenly motions with the motion of the Sun*, and to represent them as they appear to an observer on the Earth. Such a task appears very ambitious also by modern standards.

Also, if we agree that the AM was a follow-up of an Archimedean tradition of *sphere-making*, the theoretical question asked above overlaps with another, that, as far as I know, has never been really addressed: *what kind of theory was exposed in Archimedes' lost treatise on sphere-making?* A conjectural answer to this question, if not of historical value by itself, may well be relevant for the *not less* conjectural reconstruction of the planetary gear-work of the AM. Conversely, it seems reasonable that the AM itself can indicate the path for the restoration of Archimedes' lost *theory of sphere-making*.

To the first question, the answer one finds in the existing literature about the AM is that it embedded some cruder version of Ptolemaic models, *i.e.*, some simple eccentric/epicycle models. Such interpretation fits well with Ptolemy's testimony about the results achieved by previous astronomers (namely Hipparchus), and the analogy between the *pin-slot device* found in the Moon gear-train and a suitably calibrated epicycle/eccentric construction has been taken as a confirmation of this view [3]. Therefore, according to this interpretation, the accuracy of the astronomical predictions given by the AM was inferior to that of Ptolemaic models, whose predictive power comes mostly from the introduction of the equant point [4]. This view of the AM is coherent with the general historical narrative which regards the *Almagest* as the culmination point of a continuous, homogeneous and uninterrupted development of Greek mathematical astronomy, going from its origins up to Ptolemy's time. In this framework, the *Almagest* would be the only extant astronomical treatise of its kind because it superseded all the preceding works treating the same subjects, just as those of Euclid are the only surviving *Elements* of Greek geometry [5, 6]. Such a *continuist* view seems to be shared by most of the scholars who actively worked on the problems posed by the AM.

A different reconstruction of the general history of Greek astronomy has been proposed by those who emphasized the relevance of the scientific breakdown that occurred in the Mediterranean world at the middle of II century BC, when the Romans expanded their dominions to the Hellenistic kingdoms of Greece, Egypt and Mesopotamia. A side effect of this huge event was the end of the the golden age of Hellenistic science, a fact clearly shown by the subsequent decline of Alexandria's Museum [7, 8]. According to this reconstruction, which was much more fashionable among historians until the beginning of the XX century, Ptolemy *cannot* be considered as a direct successor of scientists like Euclid, Archimedes and Apollonius, but rather as a (very skilled) mathematician heavily relying on the work of his predecessors, although animated by very different (pseudo-Aristotelian) conceptions of mathematical astronomy. In fact, a direct comparison between Ptolemy's *Mathematical Syntaxis* and any one of Archimedes' extant works is sufficient to convince any scientific reader of the abyss that separates the two authors.

In this latter view, to which we totally adhere, the *Almagest* clearly *cannot* be taken as

([2]) The most credited reconstruction of the planetary part of the AM is in [2].

a paradigm for the reconstruction of the mathematical theory underlying the mechanical design of the AM, which is the ripe fruit of a scientific tradition interrupted three centuries before Ptolemy. After all, the very same *existence* of a device like the AM is at odds with Ptolemy's claim that, before him, no one had *even began* to establish a planetary theory ([3]).

In any case, the problem of the relationship between Ptolemy and his Hellenistic sources (notably Hipparchus) is a very complex one, and the origins of Ptolemaic models are, to say the least, matter of debate. Historical research and fact-checking on the *Almagest* brought convincing evidence that Ptolemy is not always a reliable source about the results achieved by his predecessors. Well-known examples are the misappropriation of Hipparchus' star catalogue (recently found in a palimpsest), the absence from the *Almagest* of important ideas dating back at least to III century BC (like the heliocentric hypothesis, despite traces of it have been recognized [10]) and the numerous inconsistencies between Ptolemy's claims and the actual astronomical observations he could have made. These were first noted by Jean-Baptiste Delambre at the beginning of the XIX century, and after him many others have questioned the authenticity of Ptolemy's astronomical observations as they are reported in the *Almagest*([4]).

Another cautionary argument to the *exclusive* use of the Ptolemy's testimony for the reconstruction of the astronomical theory underlying the AM is that he probably had no access to Hipparchus' latest works, that in the three intervening centuries of turbulent events never found their way to the declining Library of Alexandria. In 1994 Lucio Russo first proposed a conjectural reconstruction of the astronomical knowledge available at the time of Hipparchus, based on the study of pre-Ptolemaic sources and therefore independent from Ptolemy's *Almagest* [12]. Russo's study is sound and, in our opinion, his conclusions very convincing. His analysis suggests the possibility that at the time of Hipparchus a *heliocentric* and *dynamical* theory of heavenly motions had been developed, similar in its essential features to *classical dynamics*. According to Russo's reconstruction, this theory (used, in particular, to account for planetary motions) was based on a principle of inertia and on the idea that deviations from such *natural motion* are due to the mutual interactions of bodies. Since the time of Hipparchus is also the period of construction of the AM, Russo's result seems to be very relevant for the *not less* conjectural reconstruction of the planetary gear-work of the AM. However, Russo's paper seems to have been ignored by all those who worked on this problem.

In short, even if Ptolemy's *Almagest* is undoubtedly an important source of informations about the work of previous astronomers, there is a considerable risk of being misled in the interpretation of the AM if one sticks too close to Ptolemy's *own* conceptions of mathematical astronomy.

If, on the other hand, we take the extant Greek mathematical works up to II century BC as a leading guide in the necessary guesswork involved in the reconstruction of

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([3]) Elsewhere I set forth the conjecture that the *Almagest* could be the result of a reverse-engineering based on a device not too far from the AM. For more details see [9], where an explanation is also attempted of why a device like the AM could not be considered by Ptolemy's own criteria an actual proof of the existence of a planetary theory, reconciling his primacy claim with his familiarity with *sphairopoiia*.

([4]) Reference [11] is worth mentioning, where an extensive study of Ptolemy's reported observations is carried out. The title of the book is sufficient to indicate the level reached by the debate, but, everything considered, the factual results of Newton's analysis seem hardly contestable.

the planetary theory underlying the front side of the AM, we find ourselves in a completely different world: geometric algebra, numerical progressions, theory of proportions, exhaustion methods, theory of conic sections and a huge arsenal of highly sophisticated mathematical techniques that are *completely absent* from Ptolemy's works become available. Obviously, the mathematics we find in the extant works of Euclid, Archimedes and Apollonius is only a lower boundary to what could have been involved in the original design of the AM. To this, we should add what we are still learning about Babylonian astronomy of the Seleucid period, since the decipherment of cuneiform tablets keeps revealing an unexpected complexity of numerical computational techniques that Hellenistic mathematicians of the II century BC incorporated in their astronomical practice. As well known, Hipparchus in particular played a prominent role in this assimilation process, which began already at Euclid's time and took the general form of a *geometrical reinterpretation of numerical algorithms*([5]).

From this perspective, the historical and theoretical problem posed by the discovery of the AM becomes much more difficult (and, of course, way more interesting), but it is my conviction that only in this way we can hope to restore the original *form* and *meaning* of such an extraordinary device. In my PhD dissertation, I tried to answer the questions asked above looking at the AM through the eyes of Archimedes rather than through those of Ptolemy, and taking into account the possibility that a dynamical theory of celestial motion was developed in II century BC. In the present paper I just anticipate some of the arguments there developed, limiting myself to some general remarks about the role of *sphere-making* in the context of Hellenistic astronomy and on a simple hermeneutical proposal about eccentric/epicycle diagrams that, as far as I know, has never been advanced.

2. – On Hellenistic mathematics

Generally speaking, Greek mathematics (like all ancient mathematics) was much more *problem-oriented* than ours. In particular, in their ripest form Hellenistic mathematical sciences were explicitly conceived as a collection of *problem-solving* disciplines, *scientific arts* covering a wide range of domains and sharing a common methodology. The intertwinement between "pure" and "applied" sciences became so tight that most branches of mathematics borrowed their very name from the *class of problems* they aimed to solve, *i.e.*, from their intended "application" (like *scenography*, *catoptrics* or *mechanics*). In all these mathematical sciences a transversal role was played by geometry and the connected activity of drawing *diagrams*.

The three fundamental postulates of Euclid's *Elements*, for example, are nothing but the abstractions of the three elementary operations one can perform with a straight-edge and a compass —*draw* a line, *produce* a line, *draw* a circle—, and every proposition of the *Elements* is essentially a *list of commands* involving such operations. Every proposition is thus the *analysis* or *synthesis* of a certain diagram, that in principle may be done in any way, the only constraint being the adherence to the postulates. As Russo has emphasized, it is exactly this *operational* character of the postulates that in Euclid's

([5]) For example, according to Berggren and Thomas the main purpose of Euclid's *Phenomena* was to *exhibit geometrically the symmetry assumptions* underlying Babylonian numerical methods for computing the length of daylight at any given day of the year. For more details see [13].

system of geometry *explicitly* guarantees an unbreakable connection between the *abstract* deductive model and something *tangible* existing in the real world, *i.e.*, a *diagram* drawn according to some pre-established rules.

For Greek mathematicians drawing worked also (but *crucially*) as a form of *computing*. After Eudoxus' invention of a general theory of continuous *magnitudes* (IV century BC) every mathematical problem could be framed in terms of a diagram, in which the *data* of the problem were represented by the *length* of a given line, or *area* of a given figure, or *volume* of a given solid, and the *solution* was obtained by the explicit *construction* of a line/figure/solid whose length/area/volume had a required *ratio* to that of the given line/figure/solid. In this way, drawing instruments and geometrical constructions became extremely powerful tools for *analog computing*, and any problem was generally reduced to that of devising the most convenient construction to achieve a given purpose. So, in short, in the context of Hellenistic science geometry was, first and foremost, a *theory of diagrams*, which, as Russo beautifully summarized, entangled three activities that in modern mathematics are often regarded as independent of each other: drawing, computation and deductive reasoning. As such, it could be put to the service of *any* discipline, abstract or practical, which used diagrams to express the relationship between magnitudes of any kind. The more the range of geometry extended, from Eudoxus' *theory of proportions*, to Euclid's *Elements*, through Archimedes' *Method* up to Apollonius' *Conics*, the more extensive, expressive and powerful such diagrammatic language became for the solution of any kind of problem. Notice, in particular, how far this conception is from the modern idea of geometry as the "science of space".

In their mature form, all Greek mathematics shared a similar conceptual structure, with the construction and analysis of diagrams occupying a central role in every domain. What changed was the *meaning* attributed to the magnitudes whose reciprocal proportions were encoded in a certain drawing, *i.e.*, what diagrams *represented*, which depended of course on the context. This point was already emphasized by Plato (*Republic*, VI, 510C-E), and the possible meanings and interpretations of a geometrical diagram extended as long as "the method of mathematicians" (as Plato called it, but we could safely say the *scientific method*) was applied to different classes of problems.

Despite the loss of all the advanced astronomical treatises of the Hellenistic period, it is sure that astronomy was no exception to this epistemological framework. In particular, also astronomy worked in strict connection with specific *instruments*. Gemino (I century BC), quoted by Proclus, describes the subject matter of astronomy as follows ([14], p. 249):

> There remains astronomy, which treats the cosmic motions, the sizes and shape of the heavenly bodies, their illuminations and their distances from the Earth, and all such questions. [...] Its parts are: gnomonics, which is engaged with the measurement of the hours through the placement of gnomons; meteoroscopy, which discovers the different altitudes and the distances of the stars and teaches many complex matters from astronomical theory; and dioptrics, which examines the positions of the Sun, Moon, and the other stars by means of such instruments.

So, all the three sub-branches of astronomy were identified with a specific *class of problems* and with the instruments employed in their solution: *gnomonics* with sundials, *meteoroscopy* with the *meteoroskopeion* (a kind of astrolabe) and *dioptrics* with the *dioptra*. To these, one should add the fascinating practice of *sphairopoiia*.

3. – On *sphairopoiia*

Immediately before the passage quoted above, Proclus-Gemino classifies *sphairopoiia* (literally *construction of spheres*) as a sub-branch of mechanics "in imitation of celestial motions, as Archimedes practiced". In different forms, this art evolved side by side with Greek mathematical astronomy, from the first steps of the Pythagorean school (archaic period, VI–V century BC), through its shaping at the time of Plato's Academy (Hellenic period, V–IV century BC), up to the golden age of Alexandria's Museum (Hellenistic period, III–II century BC). After II century BC there are mentions of *sphairopoiia* as a *received* practice, but devices such as the AM were never constructed again in antiquity.

Despite in modern times the construction of mechanical devices incorporating astronomical models has been a charming but mostly secondary activity, the importance of *sphairopoiia* for Greek astronomy can hardly be overestimated. As Aujac has justly emphasized ([15], p. 7), in the hands of Greek mathematicians this art became a powerful tool for the *construction*, *visualization* and *validation* of astronomical models, and the AM shows that in its most mature form it became also a tool for *automated computation* specifically adapted to astronomical problems. Working like drawing as an intermediate *operational* layer between *theory* and *phenomena*, it was this blending of *kinematic geometry*, *mechanics* and *astronomy* that guaranteed a solid grounding to the theoretical activity of Hellenistic astronomers.

The idea we wish to put forward is that, in its most mature developments, *sphairopoiia* and theoretical astronomy were actually *the very same thing*, in the sense that mechanical devices were designed to be an *exact* realization of (theoretical) astronomical models. More specifically, we propose to regard *sphairopoiia* as the branch of Hellenistic astronomy specifically dealing with the *motions* of heavenly bodies. In this regard, it is remarkable that Proclus-Gemino defines astronomy as the science dealing with "cosmic *motions*", but then in the description of its different sub-branches —*gnomonics*, *meteoroscopy* and *dioptrics*— the word *motion* does not appear at all. We think that such class of problems was the specific object of *sphairopoiia*.

Since it is likely that in Archimedes' lost mechanical works a rigorous *theory of machines* was developed, which necessarily involved *dynamical* notions, through the medium of *sphairopoiia* an astronomical theory could well have arisen as an *application* of this theory to the reproduction of the observable celestial motions. We think that *this* was, loosely speaking, the subject matter of the lost treatise on *sphairopoiia*: a *mechanical-astronomical theory*, *i.e.*, a theory of celestial motions that *at the same time* grounded the design of mechanical devices intended to *imitate* such motions. Since the decipherment of the famous Codex C we know that Archimedes explicitly used mechanics —namely *statics*— to *induce* results that he later *deduced* by standard geometrical methods. His *theory of sphairopoiia* would have been, in essence, an extension/application of his *mechanical method* to astronomical (kinematical) problems.

I believe that the key ingredient of this Archimedean *aufhebung* between astronomy and mechanics in the art of *sphairopoiia* lies in the central role that *circular motion* played for the Greeks as a *theoretical* cornerstone of both disciplines. Such theoretical homogeneity grounded the possibility to treat in a unified way *astronomical problems*, involving the motions of stars, Sun, Moon and planets, and *mechanical problems*, involving the motions of the parts of a machine. In this way, enabling *in principle* to incorporate exactly an astronomical theory in a mechanical device, and coherently with the general epistemological framework of Hellenistic mathematics, *sphairopoiia* came to overlap and perhaps even *coincide* with the most mature developments of Greek mathematical astronomy.

Depite only fragments of his works survive, it is sure that in this intertwined development of mechanics and astronomy a crucial role was played by the already mentioned Eudoxus, probably the most important mathematician of the Hellenic period.

Eudoxus, pupil of Plato and Archytas (one of the founders of the Greek mechanical tradition), was the first to use a combination of circular motions to account in a relatively simple way for the irregularities observed in planetary motions. His model, exposed in a lost work entitled *On Speeds* ($\Pi\varepsilon\rho\iota\ \tau\alpha\varkappa\acute{\omega}\nu$), was based on a system of concentric spheres for each planet, rotating around different axes and with different speeds, so that a point placed on the equator of the external sphere and *carried* by all these *simultaneous motions* traced a spherical lemniscate called *hyppopede*. In this way, identifying the point with the observable sidereal position of a planet, the model gave an account of planetary stations and retrogradations, that could be studied through the medium of the *progressive kinematical generation* of the *hyppopede*.

Eudoxus' kinematical approach provided the basis for much of the further developments of Greek mathematical astronomy, which became more and more a byword for *kinematical geometry*. If in Eudoxus' *Phaenomena* (of which some fragments are extant) there were still references to the actual astronomical bodies, these progressively disappeared in favor of the study of a general and "abstract" *revolving sphere* carrying with it *tracing points* drawing circles on its surface. In this way, the simplest version of *sphairopoiia*, *celestial globes*, allowed, on one hand, to visualize the *celestial sphere*, *i.e.*, the "abstract" or *theoretical* model used to account for the phenomena; and, on the other, they became the very same *instrument* used to frame and solve problems of *spherical astronomy*. In particular, it seems that in this domain diagrams were drawn *directly* on celestial globes by the use of appropriate instruments [16]. This progressive transformation can be clearly seen in all the surviving Greek astronomical works dealing with the *daily motions* of Sun and stars and the connected problems of *rising and setting times* ([14], pp. 4–8).

The point we wish to emphasize is the effective *vanishing* of the distinction between the "abstract" theory and the "concrete" model: the turning sphere *is* the theory, in the sense that it *is*, by itself, the *model* of a certain set of *phenomena*. A globe with the proper inscribed circles, properly inclined on its support to match the local latitude, and rotating at the proper frequency, is the *exact* realization of such *theory of the celestial sphere*, an *analog computer* of some well-definite celestial phenomena and the simplest form of *sphairopoiia* in the broader sense we outlined above.

4. – Hipparchus' diagrams

At some point during the III century BC, *plane* astronomical models made their appearance in Greek astronomy, under the well-known form of *eccentric circles* (literally *off-center circles*) and *epicycles* (literally *circles upon circles*). The origin of these models is a complete mystery, but the angular equivalence between eccentric/epicycle models and the *pin-slot device* found in the Moon gear train of the AM has suggested a new and evidence-based track to the solution to this problem, *i.e.*, that such constructions could have a mechanical origin in the context of *sphairopoiia* [3]. My proposal is that, just as happened in the context of *spherical astronomy*, also for the problems involving the motions of Sun, Moon and planets the evolution of theoretical models eventually coincided with that of concrete models which embodied them. Like with Eudoxus' *concentric spheres*, I find no reason to ascribe any physical character to *eccentric circles* (apart from Ptolemy's later view of them) or to the AM's *pin-slot device*. All these constructions can and should be regarded as mere *computational tools* internal to a well definite *theory*. Drawn on papyri, they were theoretical diagrams representing the graphical solution of some astronomical problem. Mechanized with gears, they became *analog computers* of such solutions. The AM was, in my view, a highly sophisticated example of this kind of *astronomical computer*. Since the names usually associated to eccentric/epicycle models are those of Apollonius and Hipparchus, I will call the whole class of eccentric/epicycle/pin-slot constructions *Hipparchus' diagrams*.

Now, the question is: if we reject Ptolemy's interpretation and use of such constructions, what could have been the original *meaning* of Hipparchus' diagrams?

During the III century BC two other crucial and interconnected ideas appear in the same span of time: *relativity of observable motions*, clearly demonstrated by Euclid in the *Optics* as a *theorem* (proposition 51), and the *heliocentric hypothesis*, used by Aristarchus to account for planetary phenomena (building upon ideas already circulating in the Pythagorean School). These two facts by themselves imply as a *logical* consequence that astronomical observations *may not* give any information about the *positions in space* of celestial bodies, but only about their *relative motions*. This observation, that hardly could have escaped someone like Archimedes' (also our main source about Aristarchus' heliocentric hypothesis), leads naturally to regard *motions* instead of *positions*, as the *primary* magnitude to deal with in theoretical astronomy.

Diagrams of velocity are explicitly used in the oldest surviving Greek mechanical treatise, the pseudo-Aristotelian *Mechanical Problems* (IV–III century BC), where figures representing *simultaneous displacements* of lines and points are used in the arguments. In particular, rules are given instructing on how motions can be theoretically *analyzed* in their components and *combined* by the so-called *parallelogram rule*. Even if this work was not a rigorous mathematical treatise, the key idea is already there: *to compare simultaneous motions and study the ratios between the lengths/areas described in the same time*. Notice that this is also a natural approach for astronomical problems, where the only thing one can do is to compare the simultaneous motions of different bodies, picking one among them and using it as a *clock* to track the motion of all the others. This idea of *simultaneous motions* is also used by Archimedes to define the *spiral* and solve the celebrated problem of *squaring the circle*.

Indeed, *all* the existing sources agree in indicating as the subject matter of astronomy the study of the *motions* of the heavenly bodies, rather than of their *positions*. This is of course a natural consequence of the simple observation that, in the skies, *everything moves at the same time*, and the very same title of Eudoxus' seminal work *On Speeds* (*Περι ταϰών*) is significant in this regard. Since Eudoxus himself developed a general notion of *magnitude* which applies to *motions* or *displacements* as well as to *distances* or *positions*, we find no difficulty in the idea that *diagrams of velocity* could have been considered by Hellenistic mathematicians working on astronomical problems. Notice that Archimedes' word for *uniform* motion is *ισοταχέος* (literally *with the same speed*), a technical term which implicitly *defines* motion with constant speed and recalls directly the title of Eudoxus' work.

Other indications come from the already mentioned Greek assimilation of Babylonian astronomical practice, which, overall, seems to be the real characteristic element of the more mature stages of Hellenistic astronomy and of Hipparchus' work in particular.

It was common practice of Babylonian astronomers in the Seleucid period to describe the motion of celestial bodies not in terms of *positions* and *time*, but rather in terms of *daily motions*, where one *day* was defined as the time employed by the Sun to describe one *degree* on the zodiac. An example is the tablet labeled ACT 190, a listing of Moon daily velocities over a period of 248 days, not attached to any specific date ([17], p. 179). This was a *template of displacements* that could be used to generate day-by-day positions of the Moon over any desired period.

Moreover, from the decipherment of cuneiform texts reporting calculations relative to the motion of Jupiter, Mathieu Ossendrijver made the groundbreaking discovery that, starting from these observed pairs of time and velocity, the position of Jupiter at any given date was computed by a process equivalent to the so-called *Merton Rule* or *mean-speed theorem*, *i.e.*, by *integration* of the area under a trapezoidal time-velocity diagram. Ossendrijver concludes ([18], p. 484)

> The Babylonian trapezoid procedures are geometrical in a different sense than the methods of the mentioned Greek astronomers, since the geometrical figures describe configurations not in physical space but in an abstract mathematical space defined by time and velocity (daily displacement).

We find no difficulty in imagining that also Greek mathematicians, and Archimedes in particular, could have made use of such "abstract mathematical spaces defined by time and velocity", computing future celestial positions by *quadrature* of velocity diagrams.

My proposal is therefore to interpret Hipparchus' diagrams not as diagrams of *relative positions*, but as diagrams of *relative velocities*, *i.e.*, as diagrams expressing the *kinematical relationship* between two bodies instead of their *spatial relationship*.

With a little help from William Rowan Hamilton (1805–1865), in my PhD dissertation it will be shown how these diagrams, if properly interpreted, give a *complete solution* to the astronomical problem that in modern times was called *Kepler problem*. Therefore, if the view I propose will be accepted, the AM will appear to be a direct evidence that a theory of heavenly motions mathematically equivalent to Newton's theory of inverse-square gravity was developed by Hellenistic mathematicians around II century BC.

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