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# Logic circuits and optical circuits: A teaching learning sequence to build quantum computation for high school students

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**Summary.** — The possibility to realize experimental devices from the circuit representation of protocols and algorithms for quantum computation and quantum information is embedded in their mathematical formulation, and such opportunity becomes extremely relevant in a teaching context where the formal logical aspect need to be supported by ideal physical realizations. We present a part of the teaching learning sequence constructed for this purpose and the first outcomes of two experiments conducted by teachers from the *Liceo* Volta in Castel San Giovanni and *Liceo Scientifico* Gramsci in Florence. The approach followed aims to build a useful dialectic between diagrammatic representations and ideal experimental setups with optical devices, involving students through inquiry-based teaching strategies with the goal of promoting understanding and construction of the formal language and logic of quantum protocols.

## 1. – Introduction

In recent works, a number of authors have proposed courses, tools and strategies in an effort to advance the scope of education to quantum mechanics (QM) in secondary school to include topics related to the "second quantum revolution" [1]. For example, Walsh *et al.* [2] have designed and tested a one-year high-school course on quantum computing based on classical wave optics, with a focus on hands-on experiments and simulation activities adopting an inquiry-based approach, and the contextual introduction of new topics and competencies (such as the matrix formalism, or Python programming skills) when needed for the completion of students' inquiry projects. Satanassi *et al.* [3] developed a quantum computing course for high school students based on the general idea of leading students to follow the evolution of computational thinking in human history, from the most primitive computing machines, and ending with quantum computers and algorithms. The final part of their course uses a spin first approach, with the re-interpretation of Stern-Gerlach experiments in terms of information input (the state preparation), information processing (the state evolution) and information output (the measurement) playing a central role as a bridge from basic QM to quantum computation. Pospiech [4] proposes a course on QM for the German high school, in which quantum computing and quantum cryptography are introduced as rich technological contexts in which the fundamental concepts of quantum theory (*e.g.*, superposition, entanglement, incompatibility, measurement) find their full development and application. According to the author, teaching QM in the context of quantum technologies has positive reflexes on conceptual understanding, on students' ability to construct consistent mental models, and on the epistemological acceptance of QM as an ordinary physical theory. Research-based course proposals based on a hands-on approach for different targets, ranging from secondary school students [5] to undergraduates with little or no physics background [6] have very recently appeared in the educational literature. However, while research on the teaching and the learning of quantum physics is a well-developed field within physics education [7, 8] and student difficulties at different levels, both in general and in connection with different teaching approaches, have undergone significant clarification, quantum technology and information science represents still a largely uncharted territory. There is a need to build effective programs and to design curricula for diverse student populations and educational levels, identifying goals and challenges according to the context at hand. Recently, quantum computation experts from both academia and industry signed an open letter [9] calling for an earlier start of education in quantum computer science in the academic career and recommending the involvement of education experts in the curriculum development.

Based on the research described and following the teacher professional development course designed and implemented by us in 2020 (see [10]) we built two curricular paths for high school students, stemming from the same general framework but tailored to the needs and preferences of teachers, focusing on the topics related to the second quantum revolution. The approach we have followed is diagrammatic in nature, in the sense that it is possible to describe physical processes in general and experiments, algorithms or protocols in particular, by means of diagrams made up of wires (physical systems) and boxes (transformations); these diagrams (logic circuits) can also be interpreted from the perspective of optical devices that implement the transformations (optical circuits). The educational aim is to provide a diagrammatic model through which students achieve an integrated perspective of the dialectic between physics, logic and computer science.

## 2. – The teaching-learning sequence

**2**˙1. *Educational reconstruction.* – To realize the construction of a diagrammatic model of quantum computation in the teaching learning sequence (TLS), the educational reconstruction for instruction, according to the model of educational reconstruction (MER) [11] was based on a content analysis focusing on the theoretical perspective, the students perspective and teachers perspeactive.

From the theoretical side we considered two aspects as most significant for our educational reconstruction: the history of physical information theory starting with the work of Bennett [12] and the diagrammatic approach linked to category theory [13]. The history of quantum computation and information provides the first conceptually relevant element: the extension of the semantic field of the word computation from the area of logic mathematics to that of physics. More precisely, what is brought to light is the need to consistently problematize, when talking about computation, whether we are referring to hardware or software, to physics or logic. Therefore, we need a language that can do this as deeply as possible, and to do it in the same way whether we are dealing with classical or quantum computation. The second aspect, the diagrammatic language, serves precisely this purpose. In some of the more recent axiomatic formulations of quantum theory, the use of category theory and its possible diagrammatic circuit representations is deeply embedded even when not explicitly declared [14], and we find it adopted in several more application-oriented works, such as in computer sciences, and the physics of computation [15]. These works show the possibility of using appropriate monoidal categories to describe any kind of processes: be they physical, chemical, linguistic (texts), musical (compositions), or otherwise [16]. The unifying attempt of these works translates in our research into the use of a language able to create a unified model for logic, the physical theory of computation, and the corresponding experimental realizations in the quantum case using optical devices. Therefore, we have introduced useful categorical tools, the diagrams, appropriate for defining a unifying language for computational theory, physical theory and implementation using optical devices [17].

As for the students' and teachers' perspective, in addition to the recent literature reported in the "Introduction", we used data from previous explorative tests carried out by our research group, from online courses realised in collaboration with a group of Italian universities [18] and the course for teacher professional development realized in 2020 [10]. There are some aspects that seem to characterize the analyzed data: on average, the educational path about quantum technologies was useful to familiarize students with fundamental aspects of quantum mechanics, and student understanding of the basic quantum mechanics formalism benefits from the use of multiple representations [19] (formal, graphical, diagrammatic); in particular the diagrammatic language is appreciated as a tool for conveying different meanings. However, a strong need also emerged to make the concepts introduced more concrete and physically grounded.

In view of the above, when starting work on the co-design with teachers of the classroom implementation we proposed a reconstruction taking into account both content and design characteristics according to the following design hypotheses which served as a guide for the design of a TKS in quantum information physics:

DH1: Students can master mathematical formalism if supported by multiple representations (algebraic, geometric, diagrammatic).

DH2: Constantly explaining the relationship between classical and quantum elements helps to exceed the classical approach and grasp the quantum characteristics proper.

DH3: Students, if properly guided through specially designed materials, can construct the computational model using optical devices (half-wave plates, phase shifters, beam splitters, polarising beam splitters).

DH4: The diagrammatic model appears to the students in its entirety, in the sense that models are artefacts created to solve scientific problems in practice [20].

We include in table I the basic structure of our TLS including the learning goals for each step.

**2·2.** *Methods and detailed sequence account.* – Instruction proceeds through a variety of activities, including lectures based on slides, but also inquiry based and modelling tasks described in two-three–page worksheets. Worksheets are to be completed by students step by step in suitable short pauses of the lesson flow, and are designed to emphasize

TABLE I. – *Structure of the teaching-learning sequence.*

| Content | Learning goals |
| --- | --- |
| Introduction to QP | Introducing quantum physical quantity, state, vector, superposition, interference, measurement |
| Computational approach to problems: classical computation | Interpreting a problem and its solution from a logic-computational point of view. Linking logical to physical aspects (software to hardware). |
| From bit to qubit (1): one-qubit computation | Introducing and developing quantum computation: Dirac's vector formalism and its geometric interpretation for new single-qubit computation. |
| From wave model to single-photon model for computation (1): encode information. | Describing the transition from the known wave model of polarization via Jones vectors and use it to build the polarization qubit. |
| Single-photon model for computation (2) | Building the single-photon model (from logical to optical circuits) of polarization-encoded computation. |
| Spatial model for computation | Building the single-photon model based on the spatial mode in an interferometer. |
| From bit to qubit (2): two qubit computation. | Introducing and developing quantum computation using the Dirac vector formalism for new two-qubit computation. Differentiating separable states from entangled states. |
| Two-qubit computation: complete model. Logic and optical circuits. | Correctly solve logic circuits and, transformed into optical circuits, propose correct ideal experimental setups. |

written explanations of student reasoning. Therefore, the worksheets that are used in our work have multiple uses. First and foremost, they are designed to get students to work independently to become personally active in constructing knowledge. Since the worksheets are carried out in the classroom, it is the teacher's task to support the work, and their use also allows teachers to understand the difficulties their students may be having. In particular, the micro-steps in which the worksheets are structured allow them to grasp specifically where the significant difficulties lie. The third use of these worksheets is closely linked to the data collection and analysis. Thanks to the collection of the worksheets and their analysis, it was possible to monitor students' learning, identify possible changes to the worksheets themselves and modify some parts of the TLS.

The common part of the TLS summarised in table I had two different developments for the two educational experiments: in one case the study of quantum algorithms; in the other entanglement and the teleportation protocol. Here we present only the central part of the TLS, relating to the encoding of two qubits, to be used for all basic quantum information and computation application, as polarization and spatial mode of a single photon [21].

*Introdution to QP:* The learning path here described is preceded by an introduction lasting about 5–6 hours on basic quantum theory based on a two state approach, with a structure similar to the one of ref. [22]. Such introduction will not be discussed in this article.

*From classical to quantum computation:* In this section, the diagrammatic language is introduced and interpreted both logically and physically: wires represent physical systems, boxes represent transformations. The preparation-transformation-measurement tripartition is reinterpreted in informational terms as coding-processing-decoding. The proposal of a problem solvable by a classical algorithm has the function to support understanding of the model and its natural extension to the quantum case.

*Polarization encoding of qubits:* The fundamental tools needed to build polarization-based logic gates by means of materials already familiar from the introductory part of the course (*i.e.*, birefringent crystals) are phase shifting materials. We initially introduce the electromagnetic description of light in an elementary form. Since the direction of the linear polarization of light is identified by the electric field vector, we focus only on the mathematical expression for such quantity. We recall the concepts of global phase, of phase difference and its role in wave interference. Finally we present students with linear isotropic dielectrics, *i.e.*, for our purpose, phase shifting materials that do not change the direction of polarization. Since in the course we only work with real numbers, the basic phase shifting device will be a sheet of refractive material, whose refractive index and thickness are designed to obtain, for waves of the chosen wavelength, a phase shift of $\pi$. In order to make precise the analogy between the quantum and classical polarization states, we express the electric field vector as a polarization vector. Since we are interested only in the direction of linear polarization and the relative phase of the orthogonal components of the wave, we use a representation in terms of Jones vectors, *i.e.*, we omit the spatiotemporal elements from the cosine, normalize the amplitude of the vector and set the global phase to zero. For a field oscillating in an arbitrary direction, the result is a normalized Jones vector: $(a\mathbf{i} + b\mathbf{j})$, with $\sqrt{a^2 + b^2} = 1$. Since we restrict us to linear polarization, the coefficients of the Jones vector are real; if the value of only one coefficient is negative, this corresponds to a phase difference of $\pi$ between the two components. The mathematical expression is identical to that of a generic quantum state of linear polarization of a photon. This first part ends with a worksheet in which students are asked to describe the similarities and differences between classical and quantum descriptions (see table II).

By encoding the horizontal state of polarization of a photon as $|0\rangle$ and the vertical one as $|1\rangle$, we need only a system composed of two calcite analyzers with a phase shifter in the extraordinary ray to design a Z logic gate, *i.e.*, a symmetry around the horizontal axis (see fig. 1).

Actually, this setup can be used for implementing an infinite number of gates. As a matter of fact, by rotating a birefringent crystal around its ordinary axis, we obtain a beam separation on different couples of perpendicular directions of polarization. It follows that every gate which can be described as an axial symmetry of the state plane is realizable in this way. In particular, if the ordinary axis is associated with a polarization angle $\theta = 45°$, we obtain a $X$ (*i.e.*, NOT) gate, if $\theta = 22,5°$, an Hadamard gate. Next, we present students with half-wave plates, a more realistic device producing the same transformation which can also be interpreted as an axial symmetry around the slow axis.

The construction of logic gates using crystals and phase shifters is carried out by students using a worksheet structured in three consecutive requests: the first question requires to determine the action of a phase shifter of $\pi$ on the state vector of photons

TABLE II. – *Activity on the comparison of classical-ondulatory and quantum description of polarization mode. Students are required to fill the table which is presented as blank; here the expected answers are reported.*

| | Polarization of the classical plane electromagnetic wave $\mathbf{E} = a\mathbf{i} + b\mathbf{j}$ | Photon polarization $\|\psi\rangle = a\|0°\rangle + b\|90°\rangle$ |
|---|---|---|
| Physical interpretation and unit of measure of the vector in the left-hand side of the equation | The physical quantity vector electric field, whose unit of measurement is V/m. | An abstract vector representing the polarization state of the photon. Since it is not measurable, it has no units. |
| Space to which vector belongs | The plane in physical space where we identify the direction of polarization of light, *i.e.*, the direction of oscillation of the electric field. This plane is orthogonal to the direction of propagation. | The state plane, an abstract vector plane in which the polarization state vectors of the photon are defined. |
| Interpretation of coefficients and their square | Relative electric field amplitudes on the two chosen orthogonal axes; their square is proportional to the fraction of energy associated with each. | Probability amplitudes of the chosen observable (here 0°–90°); their squares are the probabilities that in a measurement of 0°–90° we detect the photon at 0° or 90°. |
| Physical interpretation of the superposition sign | Oscillation phase of the components of the electric field on the chosen axis system. A change of sign corresponds to a $\pi$ phase change of a component and the polarisation changes. | Phase (sign) of the basis vectors of the observable on which the state is represented. A sign change corresponds to a $\pi$ phase change of a basis vector, and the state vector changes. |

prepared in $|1\rangle$ and $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$; the second is a question in which the students, in small groups, try to construct the Z port with the optical devices introduced; in the last question we ask students to construct the other logic gates from the correspondence between the geometric interpretation of the logic gates (symmetries in the plane of states) and the role of the ordinary propagation path.

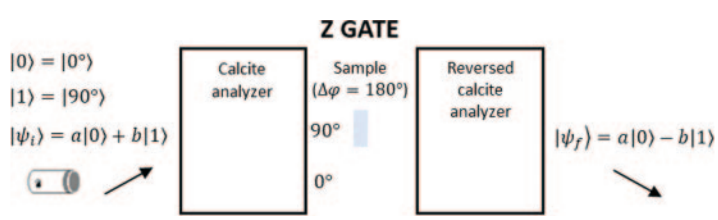


Fig. 1. – Idealized design of a Z gate on a polarization-encoded qubit.

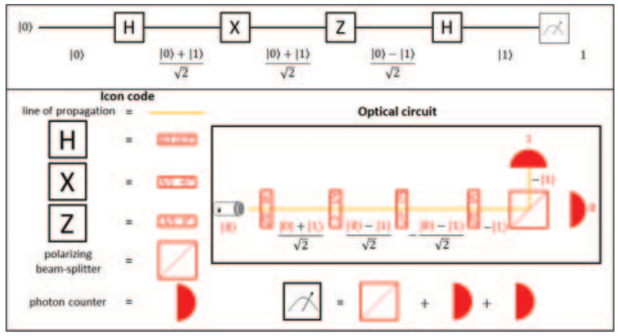


Fig. 2. – Top: an example of simple logic circuit proposed to students. Bottom: ideal realization of the circuit on an optical bench. For maximizing educational effectiveness, we introduce a color code (fig. 5) to identify visual elements pertaining to the polarization encoding, which are represented in red. For instance, half-wave plates are red rectangles with the caption $\lambda/2$ and the angle of the slow axis.

After this sequence, students have all they need to implement logic circuits with one polarization- encoded qubit, as can be seen in an exercise assigned to students to translate a logic circuit into an optical circuit (see fig. 2).

*Spatial mode encoding of qubits:* The basic device we need to prepare a qubit and act as logic gate in a spatial mode encoding is a non-polarizing beam splitter. The analysis of the action of a beam splitter on a classical light beam starts with a 50 : 50 device (half of the light is transmitted, half reflected). Since we are interested only in the fraction of amplitude of the two outgoing beams and in their relative phases, we simplify the expression of the field vector in a similar way as in the previous unit and label the versor of the field as **0** or **1** according to the label of the path taken by the beam. For the quantum description, in a rigorous treatment one should pass through a representation in terms of photon numbers in which the outputs of the beam splitter with a photon in either input (and the vacuum state on the other) is expressed as different linear combinations of states with one photon at one output, and no photon at the other. Since these two states, $|0\rangle|1\rangle$ and $|1\rangle|0\rangle$ are orthogonal, they can be relabeled as $|0\rangle$ and $|0\rangle$, where the labels may now be thought as referring to the two different possible paths available to the photon (see fig. 3).

As with polarization, at the end of this part we propose a worksheet in which students are asked to describe the similarities and differences between classical and quantum descriptions (see table III).

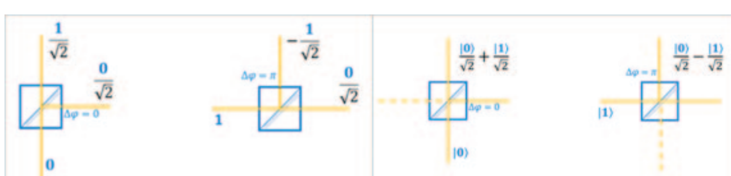

Fig. 3. – On the left, the classical description. The versors **0** and **1** can be seen as labeling different field directions, orthogonal to the direction of wave propagation. On the right, the simplified quantum description in which state labels refer to different possible paths available to one photon. For identifying visual elements pertaining to the spatial mode coding, we represent them in blue.

TABLE III. – *Activity on the comparison of classical-ondulatory and quantum description of spatial mode. Students are required to fill the table which is presented as blank; here the expected answers are reported.*

| | Description of the relative amplitude and phase of the field on the two paths $(a, b)$ | Description of the spatial state of the photon on the two paths $\vert\psi\rangle = a\vert 0\rangle + b\vert 1\rangle$ |
|---|---|---|
| Interpretation of coefficients and their square | Relative amplitudes of the field in the two arms, their square is proportional to the fraction of energy associated with each. | Probability amplitudes of the chosen observable (here position on the arms), their squares being the probabilities that in a position measurement we detect the photon on arm 0 or 1. |
| Physical interpretation of the superposition sign | Field oscillation phase on the two arms. A sign change corresponds to a $\pi$ phase change of a component on one branch and the polarization changes. | Phase (sign) of the components of the state vector for the chosen observable. A change of sign corresponds to a change of phase of pi of a basis vector and the state vector changes. |
| Is the angle between the components fixed f yes, specify its physical interpretation; if no, explain why. | No, it can be zero or 180° depending on the phase, then if the polarization is changed it can take other values. | The components of the state vector are always orthogonal, as they correspond to mutually exclusive properties. |
| Does it make sense to talk about superposition components f yes, specify its meaning; if no, explain why. | NO because the two components are vectors applied at different points in space. | Always, each vector in the space of states can be expressed in superposition with respect to a basis to obtain the transition probability. |

In the context of spatial mode encoding, the construction of the qubit is not as immediate as in the case of polarization since identifying physical properties that can correspond to the states $|0\rangle$ and $|1\rangle$ is a necessary but not sufficient condition to encode information in a qubit. We must be capable of preparing arbitrary superpositions of the basis states on which devices implementing logic gates can act. The key to the solution is preparing quantum states by means of a custom-designed beam splitter, with transmission and reflection coefficients chosen in accordance with the goals of the designer. In this case, the sign of the superposition can be established in two ways: either by choosing the incoming path (0 or 1), or by placing a phase shifter in one outgoing path.

Again, students are directly involved in the construction of the logic gates required for the prosecution of the TLS, culminating in the discussion of a logic circuit with two Hadamard gates. A circuit formed by two H gates and a measurement device corresponds to the basic setup of a Mach-Zehnder interferometer (fig. 4): a source of single photons (omitted in the figure), two 50:50 beam-splitters, two mirrors with no phase shift and photon counters. As in the case of polarization, students are asked to represent the implementation of single-qubit circuits in a spatial mode encoding, one of which is the Mach-Zehnder interferometer.

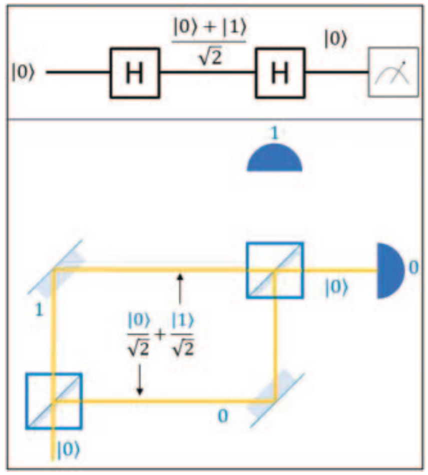


Fig. 4. – Top: an example of logic circuit. Bottom: its ideal implementation using spatial mode encoding.

*Two-qubit computation:* The last part of the circuit model construction aims at two-qubit computation. For the introduction of two-qubit gates, we rely on the conceptual description of spatial and polarization modes of a photon and of their entanglement, and on their mathematical representation in terms of product states. The implementation of non-entangling gates on spatial modes and polarization-encoded qubits is quite straightforward. Entangling gates, such as CX (controlled NOT), may be more or less easy to implement, depending on which encoding is used for the control and which for the target.

## 3. – Context and results

Our research on teaching-learning quantum computation and information topics in secondary school proceeded gradually by running, in parallel, courses for teacher professional development (see for example ref. [15]) and experimentations with students. Here we discuss two experimentations both involving an active role of the researchers. Both experimentations were conducted with around 40 students of a final year classes of Liceo Scientifico (science-oriented high school) in spring 2022 (Liceo Volta in Castel San Giovanni (PC) and Liceo Gramsci in Florence).

With regard to the data collected, the analysis of the worksheets allowed us to re-evaluate the design hypotheses:

DH1: Data shows that students manage to master the formalism with the help of one or more representations. In general, the aspect of mathematical formalism was the one that caused the least difficulty.

DH2: The transition from classical to quantum was extremely difficult when building the model (see tables II and III). When it came to applying the model in the algorithms and the teleportation protocol, however, the classical-quantum dialectic enabled the students to grasp the quantum advantages. This leads us to two evaluations, one intrinsic to TLS and one of a general nature: the first concerns the need for more time to be devoted to certain phases of construction: in particular those of polarisation and dual rail coding in relation to the concept of superposition first of all. Secondly, students are often not adequately supported by previous knowledge of classical physics.

DH3: Considering the difficulty and the innovation of the proposed educational pathway, we find the results of the construction of optical circuits encouraging. In simpler designs, students operate well; they struggle more if the optical circuit to be realized involves many devices, implements two registers and the students are not guided. We think that the possibility of proposing laboratory activities, or possibly the construction of a specially designed simulation, can greatly facilitate these results.

DH4: This is, to all intents and purposes, the hypothesis that we do not feel able to confirm. The proposed experiments still lack a strong experimental approach in the laboratory for the model to be complete. However, what emerges from the two experiments is that about half the class is able to interpret and design diagrams both logically and experimentally and to connect their meaning, thanks to the designed worksheets, to the solution of real problems.

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