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Fits of unpolarized proton and pion TMDs at N^{3}LL accuracy

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Summary. — We present the most recent global extraction of unpolarized proton Transverse Momentum Dependent Parton Distribution Functions (TMD PDFs) and Fragmentation Functions from experimental data sets of Semi-Inclusive Deep-Inelastic Scattering, Drell-Yan and Z-boson production. We also present an extraction of the pion TMD PDFs from all available data of unpolarized pion-nucleus Drell–Yan processes. Both fits are performed at next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy.

1. – Introduction

A fundamental step to understand the phenomena of confinement and hadronization is the mapping of the multi-dimensional internal structure of hadrons. Transverse-Momentum-Dependent Parton Distribution Functions (TMD PDFs) and Fragmentation Functions (TMD FFs) are examples of multidimensional maps, as they encode information about the distribution of quarks in three-dimensional momentum space. Based on currently available data, we can reconstruct the TMD functions of protons (and equivalently neutrons) and pions.

In recent years, the extractions of proton TMD functions have reached ever higher levels of sophistication, and we obtained a very good knowledge of the unpolarized TMD functions, see refs. [1-3]. In the first part of this contribution, we summarize the results obtained in ref. [4], in which our group performed a fit of Semi-Inclusive Deep-Inelastic Scattering (SIDIS) and Drell-Yan (DY) data at next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy by obtaining an excellent description of data, with a global $\chi^2 = 1.06$.

Just as we have a great deal of knowledge about the proton TMD functions, much less is known about the pion ones [5]. For this reason, we also made an extraction of the

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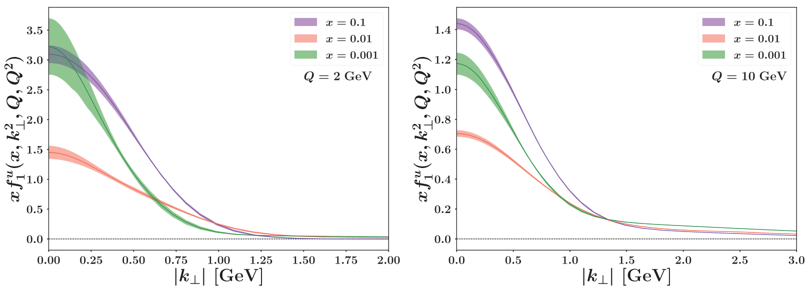

Fig. 1. – The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for different values of x [4]. The uncertainty bands represent the 68% confidence level (CL).

pion TMD functions at the same accuracy as the proton ones. In the second part of this contribution, we summarize the results of this work, see ref. [6], in which we made a fit of all the existing pion data obtaining a fairly good description of them, with a global $\chi^2 = 1.55$.

2. – Formalism

In the TMD factorization region [7], the differential cross section for the Drell-Yan process $h_A(P_A) + h_B(P_B) \to \gamma^*(q)/Z + X \to \ell^+(l) + \ell^-(l') + X$, can be written in the following form:

$$\frac{\mathrm{d}\sigma^{DY}}{\mathrm{d}|\boldsymbol{q}_T|\mathrm{d}y\mathrm{d}Q} \simeq \frac{16\pi^2\alpha^2}{9Q^3}|\boldsymbol{q}_T|\frac{x_A x_B}{2\pi}\mathcal{H}^{DY}(Q;\mu) \\ \times \sum_a c_a(Q^2) \int \mathrm{d}|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0(|\boldsymbol{q}_T||\boldsymbol{b}_T|)\hat{f}^a_{1A}(x_A, \boldsymbol{b}_T^2; \mu, \zeta_A)\hat{f}^{\bar{a}}_{1B}(x_B, \boldsymbol{b}_T^2; \mu, \zeta_B), \tag{1}$$

where $\boldsymbol{q}_T$ and y are respectively the transverse momentum and the pseudorapidity of the vector boson, Q is the invariant mass of the lepton pair, $x_{A,B} = \frac{Q}{\sqrt{s}}e^{\pm y}$ and $\mathcal{H}^{DY}$ is the hard function of the process. We denote with $\hat{f}^f_{1H}$ the Fourier transform of the unpolarized TMD PDF of a quark with flavour f in a hadron H, which depends on the longitudinal momentum fraction x_H, the Fourier-conjugate variable $\boldsymbol{b}_T$ (with respect to $\boldsymbol{q}_T$) and on the renormalization and rapidity scales μ and ζ. When proton-proton data are considered, two proton TMD PDFs are involved. In the case of pion-nucleon data, one proton and one pion TMD PDFs are involved.

Similarly, the differential cross section for unpolarized SIDIS at small transverse momentum [1, 8] can be written in the following form:

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}|\boldsymbol{q}_T|\,\mathrm{d}Q} = \frac{8\pi^2\,\alpha^2\,z^2\,|\boldsymbol{q}_T|}{x\,Q^3}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]\frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q, \mu) \\ \times \sum_a e_a^2 \int_0^{+\infty} \mathrm{d}|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0(|\boldsymbol{b}_T||\boldsymbol{q}_T|)\,\hat{f}^a_1(x, \boldsymbol{b}_T^2; \mu, \zeta_A)\,\hat{D}_1^{a\to h}(z, \boldsymbol{b}_T^2; \mu, \zeta_B), \tag{2}$$

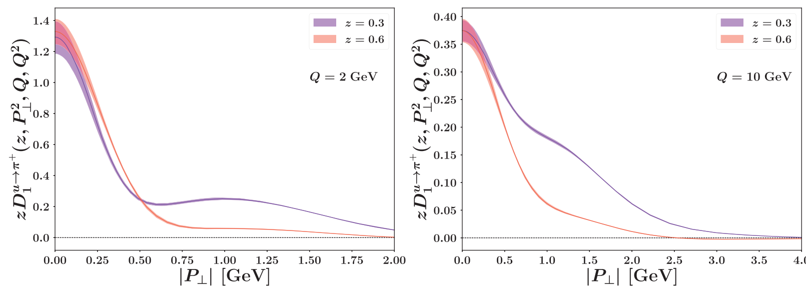

Fig. 2. – The TMD FF for an up quark in a proton fragmenting into a π^+ at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the hadron transverse momentum $|P_\perp|$ for different values of z [4]. The uncertainty bands represent the 68% confidence level (CL).

where x, z are the light-cone fractions of the incoming and outgoing quarks collinear momenta. We denote with $\hat{D}_1^{f\to h}$ the Fourier transforms of the TMD FFs for a quark with flavor f fragmenting into a hadron h, which has the same variable dependencies as $\hat{f}_{1H}^{f}$ with the only difference that it does not depend on x but on z.

3. – Data

Since the TMD formalism works only in the small $\boldsymbol{q}_T$ region, we impose the following cuts for the proton and pion fits respectively:

$$|\boldsymbol{q}_T|\,|_{\mathrm{DY/Z}} < 0.20Q, \quad |\boldsymbol{P}_{hT}|\,|_{\mathrm{SIDIS}} < \min[\,\min[0.2\,Q, 0.5\,zQ] + 0.3\,\mathrm{GeV},\ zQ],$$
$$|\boldsymbol{q}_T|\,|_{\mathrm{DY\ pion}} < 0.30\,Q + 0.60. \tag{3}$$

For the proton, the number of point passing the cuts is 2031, of which 484 are from Drell-Yann processes, coming from FERMILAB fixed-target experiments, Tevatron, LHC, STAR and PHENIX, and 1547 from SIDIS process, coming from COMPASS and HERMES datasets. Unfortunately, for pions we have only a small number of points passing the cut, *i.e.*, 138, but this is due to the scarcity of pion-induced DY experimental data, which come only from two fixed-target FERMILAB experiments (E537 and E615).

4. – Results

We can see the high quality of the fit involving the proton TMD functions from the value of the global $\chi^2 = 1.06$ per data point. This result means that we have a very good description of data coming from two different processes in a wide kinematic range. We want to emphasize the fact that we introduce a normalization factor for the SIDIS data to recover a good agreement with data at N^{3}LL, as proposed in [9].

In fig. 1 we show the unpolarized TMD PDF for an up quark in the proton in different kinematic regions and we can see that the bands are very narrow except to the $x = 0.001$ region, where there are only few data points.

In fig. 2, we show the unpolarized TMD FF for an up quark fragmenting in a π^+ hadron in different kinematic regions. The error bands behave in a similar way to the TMD PDF. The TMD FF shows an increase at intermediate $|P_\perp|$, which calls for further

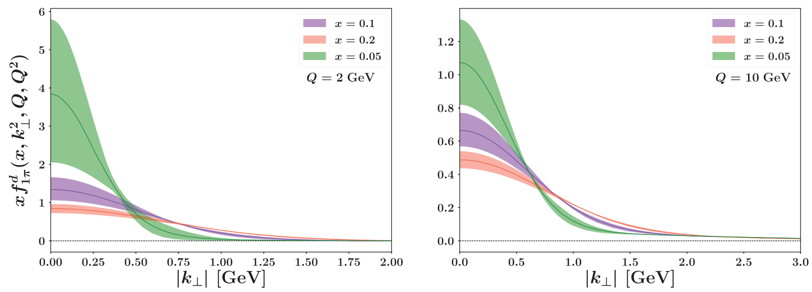

Fig. 3. – The TMD PDF of the down quark in a pion at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for different values of x [6]. The uncertainty bands represent 68% confidence level (CL).

investigations and would benefit from new data, especially for electron-positron annihilation in two back-to-back hadrons (Double-Inclusive Annihilation, DIA).

From the pion side, we obtain a fairly good description of data with a global $\chi^2 = 1.55$ per data point. We point out that 60% of that value comes from fully correlated normalization errors. This feature, which was found also in another work [5], means the necessity of new data to reduce these normalization errors. Note that in analysing pion-nucleon scattering data, we fix the proton TMD PDFs based on our previous, proton-only fit. See ref. [6] for more detailed information.

In fig. 3 we show the unpolarized TMD PDF for a down quark in a π^- in different kinematic regions. We can see that the error bands are much larger than for the proton. This feature means that the available set of data does not constrain the TMD PDFs so well and more data are needed.

The error analysis of both works is performed by using the bootstrap method, *i.e.*, by fitting an ensemble of 250 Monte Carlo replicas of the data.

The complete list of results obtained from the two fits presented in this work is available at the following public Git repository: `https://github.com/MapCollaboration/NangaParbat`.

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